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Interfaces with Other Disciplines

Decomposing the Luenberger–Hicks–Moorsteen Total Factor Productivity indicator: An application to U.S. agriculture[☆]

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ABSTRACT

This paper introduces a decomposition of the additively complete Luenberger–Hicks–Moorsteen Total Factor Productivity indicator into the usual components: technical change, technical inefficiency change and scale inefficiency change. Our approach is general in that it does not require differentiability or convexity of the production technology. Using a nonparametric framework, the empirical application focuses on the agricultural sector at the state-level in the U.S. over the period 1960–2004. The results show that Luenberger–Hicks–Moorsteen productivity increased substantially in the considered period. This productivity growth is due to output growth rather than input decline, although the extent depends on the convexity assumption of the technology. Technical change is the main driver, while the role of technical inefficiency change and scale inefficiency change also depends on the convexity assumption of the technology.

1. Introduction

Assessing the drivers of productivity growth is important for business and economic policy. Their identification allows monitoring of industries and can guide policymakers in their decisions. Hence, an abundant literature has sought to decompose various measures of productivity growth into components of technical change, efficiency change and scale efficiency change.[1] The literature has largely focused on ratio-based productivity "indexes". Yet, O'Donnell (2012a) recently shows that not all such decomposable indexes are "multiplicatively complete" (*i.e.* consisting of a *ratio* of an output aggregator to an input aggregator), while all multiplicatively complete indexes are decomposable in this way. He demonstrates that the class of multiplicatively complete productivity indexes includes Laspeyres, Paasche, Fischer, Törnqvist and Bjurek's (1996) Hicks–Moorsteen indexes, but does not include the popular Malmquist index of Caves, Christensen, and Diewert (1982).

Ratio-based productivity indexes are undefined when one or more of the variables are equal or close to zero (Balk, Färe, & Grosskopf, 2003). Difference-based productivity "indicators" do not suffer from this problem and are thus particularly useful in regulatory contexts.

Difference-based indicators were developed to measure Total Factor Productivity (TFP) growth based on Luenberger's (1992) shortage function. This directional distance function, introduced by Chambers, Färe, and Grosskopf (1996) in a production context, extends the Shephard input and output distance functions by allowing for simultaneous contraction of inputs and expansion of outputs. Chambers (2002) introduced a general difference-based Luenberger productivity indicator which can be decomposed in a technical change and efficiency change component (Chambers et al., 1996).[2] Since its introduction, it has frequently been applied in empirical applications (*e.g.* Nakano & Managi, 2008) and additional decompositions of its technical change component (*e.g.* Briec & Peypoch, 2007) and efficiency change component (*e.g.*

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[1] See Färe, Grosskopf, and Roos (1998) and Grosskopf (2003) for historical overviews.

[2] The "economic" approach to productivity measurement requires price information and if in addition (i) some assumptions can be made about firm behavior and (ii) the technology is approximated by a known flexible functional form up to the second order, then one can use a "superlative" index as advocated by Diewert (1976). Chambers (2002) showed that the Bennet–Bowley indicators are exact and superlative approximations of the Luenberger productivity indicator under (i) profit-maximizing behavior and (ii) a quadratic technology directional distance function. A corresponding superlative indicator for the LHM TFP indicator is currently not known.

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Epure, Kerstens, & Prior, 2011) have been proposed in the literature. However, the Luenberger productivity indicator is not "additively complete" (*i.e.* consisting of a *difference* between an output aggregator and an input aggregator) and thus cannot be disentangled into components of output growth and input growth. Briec and Kerstens (2004) introduced the Luenberger–Hicks–Moorsteen (LHM) TFP indicator, which is a difference-based, additively complete alternative to the ratio-based, multiplicatively complete Hicks–Moorsteen index.[3] Notwithstanding the attractive properties of the LHM TFP indicator, only few empirical studies can be found in the literature (*e.g.* Barros, Ibiwoye, and Managi, 2008; Managi, 2010). One possible reason for the limited number of applications is the fact that a full decomposition into components of technical change, technical inefficiency change and scale inefficiency change has hitherto not been developed. A first effort was made by Managi (2010) who decomposed the LHM TFP indicator into components of technical change and (in)efficiency change. However, this decomposition lacks a scale inefficiency change component and does not correctly capture technical change and technical inefficiency change (see Appendix A). No full decomposition of a difference-based TFP indicator being additively complete is thus presently known in the literature.

The current paper contributes to the existing literature by introducing a decomposition of the additively complete LHM TFP indicator into components of technical change, technical inefficiency change and scale inefficiency change. Our decomposition is general in that it does not require convexity or differentiability of the technology set. It is similar to Diewert and Fox's (2014,2017) decomposition of the ratio-based Hicks–Moorsteen TFP index.

Using a nonparametric framework, we illustrate the decomposition with an empirical application to state-level data of the U.S. agricultural sector over the period 1960–2004. Since our decomposition is suitable for non-convex as well as convex technologies, we demonstrate its flexibility by using the Free Disposal Hull as well as Data Envelopment Analysis. To the best of our knowledge, no other studies using the same dataset have investigated the issue of potential non-convexities. However, we believe that such an investigation is particularly relevant in the context of the agricultural sector. Inputs such as capital equipment are nondivisible, potentially leading to non-convexities.

This paper is structured as follows. The next section describes Luenberger's directional distance function and the LHM TFP indicator. We then introduce our complete decomposition and apply this to state-level data of the U.S. agricultural sector over the period 1960–2004. The final section concludes.

2. The Luenberger–Hicks–Moorsteen TFP indicator

Let $\boldsymbol{x}_t \in \mathbb{R}^n_+$ be the nonnegative inputs that are used to produce nonnegative outputs $\boldsymbol{y}_t \in \mathbb{R}^m_+$. We define the technology set in the usual way:

$$\mathcal{Y}_t = \left\{(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \mathbb{R}^{n+m}_+ | \boldsymbol{x}_t \text{ can produce } \boldsymbol{y}_t\right\}.$$

Furthermore, we make the following minimal assumptions on the technology set (Chambers, 2002):

Axiom 1 (Closedness)**.** $\mathcal{Y}_t$ is closed.

Axiom 2 (Free disposability of inputs and outputs)**.** if $(\boldsymbol{x}'_t, -\boldsymbol{y}'_t) \geq (\boldsymbol{x}_t, -\boldsymbol{y}_t)$ then $(\boldsymbol{x}_t, \boldsymbol{y}_t) \in \mathcal{Y}_t \Rightarrow (\boldsymbol{x}'_t, \boldsymbol{x}'_t) \in \mathcal{Y}_t$.

Axiom 3 (Inaction)**.** Inaction is possible: $(\mathbf{0}^n, \mathbf{0}^m) \in \mathcal{Y}_t$.

[3] See Briec, Kerstens, and Peypoch (2012) for exact relations between the Luenberger–Hicks–Moorsteen TFP indicator and the Hicks–Moorsteen TFP index.

Convexity of the technology set is thus not a necessary condition for our decomposition.[4] We illustrate this in our empirical application.

Luenberger's directional distance function is a measure of technical inefficiency as it simultaneously contracts inputs and expands outputs. The directional distance function proposed by Chambers et al. (1996) is:

$$D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t) = \sup\left\{\beta \in \mathbb{R} : (\boldsymbol{x}_t - \beta \boldsymbol{g}^i_t,\ \boldsymbol{y}_t + \beta \boldsymbol{g}^o_t) \in \mathcal{Y}_t\right\}, \tag{1}$$

if $(\boldsymbol{x}_t - \beta \boldsymbol{g}^i_t, \boldsymbol{y}_t + \beta \boldsymbol{g}^o_t) \in \mathcal{Y}_t$ for some β and $D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t) = -\infty$ otherwise. Here, $\boldsymbol{g}_t = (\boldsymbol{g}^i_t, \boldsymbol{g}^o_t)$ represents the direction vector. The directional distance function is a special case of Luenberger's (1992) shortage function.

We denote the time-related directional distance function for $(a, b) \in \{t, t+1\} \times \{t, t+1\}$:

$$D_b(\boldsymbol{x}_a, \boldsymbol{y}_a; \boldsymbol{g}_a) = \sup\left\{\beta \in \mathbb{R} : (\boldsymbol{x}_a - \beta \boldsymbol{g}^i_a,\ \boldsymbol{y}_a + \beta \boldsymbol{g}^o_a) \in \mathcal{Y}_b\right\}.$$

Next, we turn to the Luenberger–Hicks–Moorsteen (LHM) TFP indicator proposed by Briec and Kerstens (2004). This can be seen as the difference-based equivalent of the ratio-based Hicks–Moorsteen (HM) TFP index. They define the LHM TFP indicator with base period t as the difference between a Luenberger output quantity indicator and a Luenberger input quantity indicator:

$$\begin{aligned} & LHM_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t, \boldsymbol{g}_{t+1}) \\ &\quad = \left[D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right] \\ &\qquad - \left[D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\right] \\ &\quad \equiv LO_t(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{y}_{t+1}; \boldsymbol{g}^o_t, \boldsymbol{g}^o_{t+1}) - LI_t(\boldsymbol{x}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_t; \boldsymbol{g}^i_t, \boldsymbol{g}^i_{t+1}). \end{aligned} \tag{2}$$

Similarly, a base period $t+1$ LHM TFP indicator is defined as:

$$\begin{aligned} & LHM_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t, \boldsymbol{g}_{t+1}) \\ &\quad = \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right] \\ &\qquad - \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\right] \\ &\quad \equiv LO_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}, \boldsymbol{y}_t; \boldsymbol{g}^o_t, \boldsymbol{g}^o_{t+1}) - LI_{t+1}(\boldsymbol{x}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; \boldsymbol{g}^i_t, \boldsymbol{g}^i_{t+1}). \end{aligned} \tag{3}$$

O'Donnell (2012a) (p. 258, footnote 5) defines additive completeness as follows:

Definition 1 (Additive completeness)**.** Formally, let *TFPI*(x_t, q_t, x_s, q_s) denote an index number that compares TFP in period s with TFP in period t using period s as a base. *TFPI*(x_t, q_t, x_s, q_s) is additively complete if and only if it can be expressed in the form $TFPI(x_t, q_t, x_s, q_s) = Q(q_t) - Q(q_s) - X(x_t) + X(x_s)$ where $Q(\cdot)$ and $X(\cdot)$ are non-negative non-decreasing functions satisfying the translation property $Q(q + \lambda q) = Q(q) + \lambda$ and $X(x + \lambda x) = X(x) + \lambda$ for $\lambda > 0$.

$LHM_t(\cdot)$ and $LHM_{t+1}(\cdot)$ are "additively complete" in O'Donnell's sense. This can be verified from their definitions above where the directional distance function, along with its corresponding direction vector, serves as the output (using $(0, \boldsymbol{g}^o_s)$) and input (using $(\boldsymbol{g}^i_s, 0)$) aggregator functions.[5]

[4] In fact, the LHM TFP indicator and our decomposition are applicable to a wider range of non-convex models that satisfy the above axioms and for which the directional distance function can be defined. Examples of these non-convex models include the Constant-Elasticity-of-Substitution–Constant-Elasticity-of-Transformation model of Färe, Grosskopf, and Njinkeu (1988), relaxed convexity model of Petersen (1990) and Bogetoft (1996), selective convexity model of Podinovski (2005) and B-convexity model of Briec and Liang (2011).

[5] Luenberger's (1992) *shortage function* differs from Chambers (2002)' Luenberger *productivity indicator*. The shortage function satisfies the translation property. It is an aggregator function that can be used to compute components of an additively complete indicator (such as the LHM TFP indicator), but is *not* additively complete.

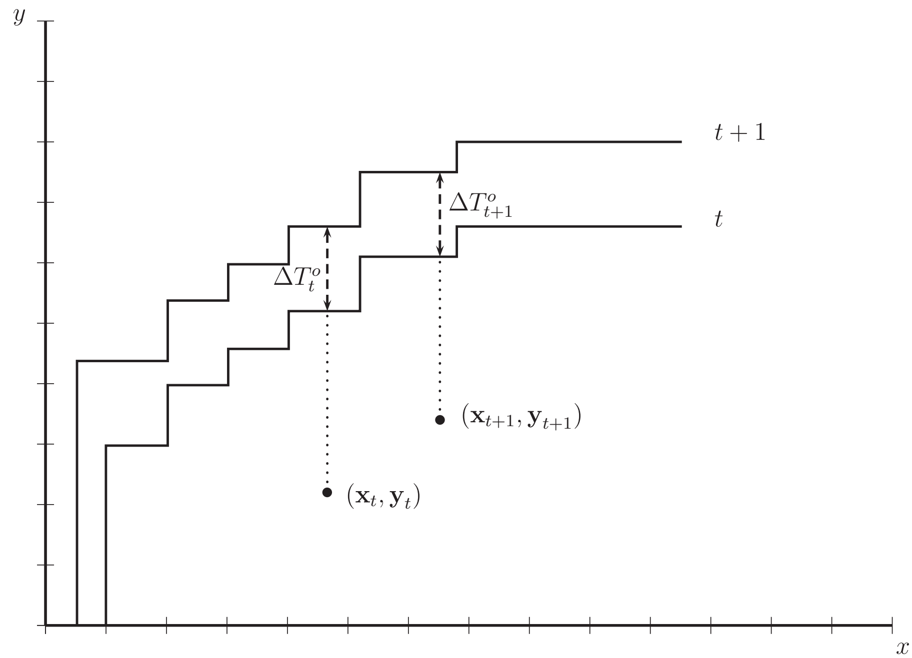


Fig. 1. Technical change.

Finally, one takes an arithmetic average of LHM_t and LHM_{t+1} to avoid an arbitrary choice of base periods:[6]

$$LHM_{t,t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; \boldsymbol{g}_t, \boldsymbol{g}_{t+1}) = \frac{1}{2}[LHM_t + LHM_{t+1}]. \tag{4}$$

The HM TFP index is defined as the ratio of an output index to an input index. Similarly, we can show that the LHM TFP indicator equals the difference between an output indicator and an input indicator, which are themselves arithmetic averages of two output and two input indicators:

$$\begin{aligned} LHM_{t,t+1} &= \frac{1}{2}[LO_t + LO_{t+1}] - \frac{1}{2}[LI_t + LI_{t+1}] \\ &\equiv LO_{t,t+1} - LI_{t,t+1}. \end{aligned} \tag{5}$$

3. Decomposition of the Luenberger–Hicks–Moorsteen indicator

This section introduces our LHM decomposition along with illustrative figures in the one input – one output dimension to provide the intuition. We show an example with a non-convex technology (*i.e.* Free Disposal Hull), as convexity is not a necessary assumption for our decomposition. Note, however, that one can also use our approach for a convex technology.

In line with the decomposition of the HM TFP index, the LHM TFP indicator can be decomposed using the output direction or input direction.[7] We focus on the decomposition using the output direction, but provide a similar decomposition using the input direction in Appendix B. Our LHM decomposition is a specific case (analogous to the multiplicatively complete case discussed in Section 3.7 of O'Donnell, 2012a) of an additively complete indicator that uses the directional distance function as the aggregator function for both inputs and outputs. Hence, in our case the mix efficiency change components are all 0 and our decomposition consists of three components:

$$LHM_{t,t+1} = \Delta T^o_{t,t+1} + \Delta TEI^o_{t,t+1} + \Delta SEC^o_{t,t+1}, \tag{6}$$

representing technical change, technical inefficiency change and scale inefficiency change respectively.[8] Given the close relation to the HM TFP index, it is no surprise that our decomposition is similar to Diewert and Fox's (2014,2017) decomposition of the HM TFP index.

The technical change component is

$$\begin{aligned} \Delta T^o_{t,t+1} &= \frac{1}{2}\{[D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t))] \\ &\quad + \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1})) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right]\right\} \\ &\equiv \frac{1}{2}\left\{\Delta T^o_t + \Delta T^o_{t+1}\right\}. \end{aligned} \tag{7}$$

Technical change $\Delta T^o_{t,t+1}$ is the arithmetic average of ΔT^o_t and ΔT^o_{t+1}. Fig. 1 depicts these technical change components. The arithmetic average is used to avoid an arbitrary choice of the observation under evaluation. Here, ΔT^o_t measures the difference in efficiency for observation $(\boldsymbol{x}_t, \boldsymbol{y}_t)$ evaluated against production frontier $t+1$ and t. An upward (downward) shift of the production frontier

Chambers (2002) defines the Luenberger productivity indicator as follows:

$$\begin{aligned} &L_{t,t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t, \boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; \boldsymbol{g}_t, \boldsymbol{g}_{t+1}) \\ &= \frac{1}{2}[(D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; \boldsymbol{g}_{t+1})) \\ &\quad + (D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; \boldsymbol{g}_t) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; \boldsymbol{g}_{t+1}))], \end{aligned}$$

All directional vectors are determined in the input direction as well as the output direction, *i.e.* $\boldsymbol{g}_a = (\boldsymbol{g}^i_a, \boldsymbol{g}^o_a) > 0$. This prevents us from disentangling the indicator into separate output and input aggregator functions.

[6] This average can be harder to interpret in regulatory and managerial contexts in which a clearer target is required. This can easily be accounted for by a different choice of weights for both periods: *i.e.* we can define $LHM_{t,t+1} = \zeta LHM_t + (1-\zeta)LHM_{t+1}$ with weights $\zeta \in [0, 1]$. One can then for example set $\zeta = 0$ or $\zeta = 1$. These weights trickle down in the technical change and scale inefficiency change components of our decomposition in a straightforward way.

[7] The technical change and technical inefficiency change components in particular are completely determined by this choice. The additive completeness property of the LHM TFP indicator can guide this decision by checking whether LHM TFP is mostly driven by $LO_{t,t+1}$ or $LI_{t,t+1}$. This contrasts with the Luenberger productivity indicator where both inputs and outputs contribute to its components.

[8] Managi's (2010) decomposition lacks a scale inefficiency change component. We refer to Appendix A for a discussion.

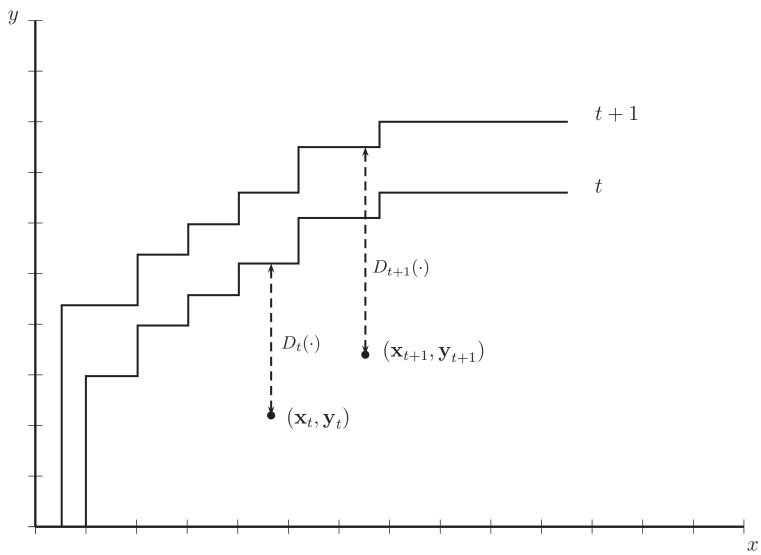


Fig. 2. Technical inefficiency change.

between t and $t+1$, indicating technical progress (regress), results in a positive (negative) difference. ΔT^o_{t+1} is similar to ΔT^o_t but evaluated for observation $(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1})$. Thus, technical change measures (local) shifts of the production frontier itself.

The technical inefficiency change component is

$$\Delta TEI^o_{t,t+1} = D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1})), \tag{8}$$

and measures the change between period t and period $t+1$ in the relative position to the production frontier. Positive (negative) values of $\Delta TEI^o_{t,t+1}$ indicate efficiency improvement (deterioration) over time: $(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1})$ is located closer (farther from) to the $t+1$ frontier than $(\boldsymbol{x}_t, \boldsymbol{y}_t)$ was to the t frontier. In Fig. 2 this means that $D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1})$ is smaller (larger) than $D_t(\boldsymbol{x}_t, \boldsymbol{y}_t)$. Note that ΔTEI^o only measures the evolution in technical efficiency of the observation under consideration without taking into account changes of the production frontier over time.

This technical inefficiency change component can be further decomposed in the same way as done by Epure et al. (2011) for the Luenberger indicator into "pure" inefficiency and, for example, congestion changes.

Finally, from the residual

$$\begin{aligned} &LHM_{t,t+1} - \Delta T^o_{t,t+1} - \Delta TEI^o_{t,t+1} \\ &= \frac{1}{2}\Big\{\big[D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1})) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\big] \\ &\quad + [D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t))]\Big\} \\ &\quad - \frac{1}{2}\Big\{\big[D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\big] \\ &\quad + \big[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\big]\Big\}, \end{aligned} \tag{9}$$

we can distill the scale inefficiency change component as follows. First, we define the projections of $\boldsymbol{y}_t$ and $\boldsymbol{y}_{t+1}$ on the production frontier at time t using notation of Diewert and Fox (2017):

$$\boldsymbol{y}^*_t = \boldsymbol{y}_t + D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t))\boldsymbol{g}^o_t \tag{10a}$$

$$\boldsymbol{y}^{**}_{t+1} = \boldsymbol{y}_{t+1} + D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\boldsymbol{g}^o_{t+1} \tag{10b}$$

Similarly, we define the projections of $\boldsymbol{y}_t$ and $\boldsymbol{y}_{t+1}$ on the production frontier at time $t+1$:

$$\boldsymbol{y}^{**}_t = \boldsymbol{y}_t + D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t))\boldsymbol{g}^o_t \tag{11a}$$

$$\boldsymbol{y}^*_{t+1} = \boldsymbol{y}_{t+1} + D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\boldsymbol{g}^o_{t+1} \tag{11b}$$

Then, respectively adding and subtracting $D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t))$ and $D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))$ to and from (9), and using the translation property of the directional distance function and the definitions of the projections above, we find the scale inefficiency change component:

$$\begin{aligned} \Delta SEC^o_{t,t+1} &= \frac{1}{2}\Big\{\big[D_t(\boldsymbol{x}_t, \boldsymbol{y}^*_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}^{**}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\big] \\ &\quad - \big[D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\big] \\ &\quad + \big[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}^{**}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}^*_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\big] \\ &\quad - \big[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\big]\Big\} \\ &\equiv \frac{1}{2}\Big\{SOC^o_t - SIC^o_t + SOC^o_{t+1} - SIC^o_{t+1}\Big\} \\ &\equiv \frac{1}{2}\Big\{\Delta SEC^o_t + \Delta SEC^o_{t+1}\Big\}, \end{aligned} \tag{12}$$

which has the interpretation of measuring changes in "global" returns to scale in line with Diewert and Fox (2017). As a result, our scale inefficiency change component does not require differentiability or convexity of the production technology. Fig. 3 illustrates the intuition behind (12). Again, the arithmetic average of ΔSEC^o_t and ΔSEC^o_{t+1} is used to avoid an arbitrary choice of base period for the technology. Both components have a similar interpretation as a finite difference approximation of the frontier's gradient. ΔSEC^o_t is a finite difference approximation of the frontier t's gradient and measures the change in inputs and outputs *along* the frontier when going from $(\boldsymbol{x}_t, \boldsymbol{y}_t)$ to $(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1})$. The change in inputs and outputs is measured separately: the SOC^o_t (SIC^o_t) subcomponent of ΔSEC^o_t keeps the inputs (outputs) constant while measuring the change in the level of outputs (inputs).

This "residual" approach of Diewert and Fox (2017) differs from the traditional "Constant-Returns-to-Scale–Variable-Returns-to-Scale" (CRS–VRS) approach of Färe, Grosskopf, Norris, and Zhang (1994) for the Malmquist index and Epure et al. (2011) for the Luenberger indicator. The CRS–VRS approach compares the VRS frontier to a (hypothetical) benchmark CRS frontier to detect changes in returns to scale over time. In contrast, the residual approach directly considers changes in the frontiers gradient over time to assess scale inefficiency change. Thus, the main difference is that the Färe et al. (1994) approach relies on two frontiers (VRS and CRS) to measure scale inefficiency change, while the residual approach

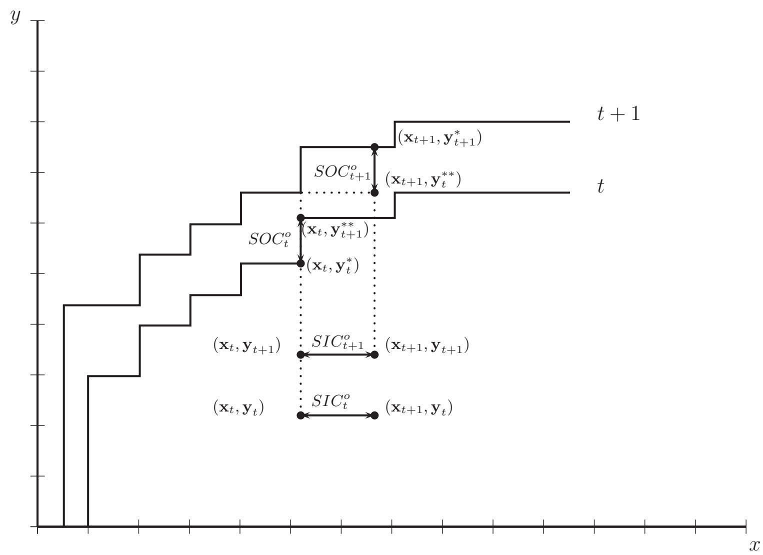


Fig. 3. Scale inefficiency change.

of Diewert and Fox (2017) only uses one frontier (VRS in our case).

From a theoretical point of view, CRS is often not a realistic assumption whereby this hypothetical CRS frontier to measure changes in returns-to-scale is not appropriate. In contrast, the main strength of the residual approach is that we do not need to introduce a CRS component into the LHM TFP indicator to detect changes in returns-to-scale. If the technology exhibits CRS then this will be automatically reflected in zero values for the $\Delta SEC^o_{t,t+1}$ component even if we use a VRS approximation. Of course, depending on the application at hand and results of a preliminary test on returns-to-scale, the LHM TFP indicator and our decomposition can also be computed under other returns-to-scale assumptions. From a practical point of view, an obvious drawback to the "CRS–VRS" approach is that it is sensitive to outliers, because the CRS frontier can be spanned by a few (extreme) observations. This drawback can be reduced by using appropriate techniques such as order-m (Cazals, Florens, & Simar, 2002) or order-α (Aragon, Daouia, & Thomas-Agnan, 2005).

The accuracy of the residual approach to approximate the gradient of the frontier depends on the "step-size", *i.e.* the gap SIC^o_t and SIC^o_{t+1} between the frontier projections of $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+1}$ for the decomposition using output directions. The larger the step-size, the cruder the approximation.[9] Thus, a big change in inputs for a DMU from period t to period $t+1$ can give a cruder approximation of the frontiers gradient.

As a final remark, observe that both $\Delta T^o_{t,t+1}$ and $\Delta SEC^o_{t,t+1}$ are the arithmetic average of a Laspeyres (using base period t) and a Paasche (using base period $t+1$) type indicator.

4. Empirical application: U.S. agriculture

We investigate LHM TFP growth of U.S. agriculture across 48 states[10]. We use our newly developed LHM decomposition to determine the main drivers of productivity growth. Specifically, we investigate the extent to which LHM TFP growth is driven by output growth and input growth, on the one hand, and technical change, technical inefficiency change and scale inefficiency change, on the other hand.

4.1. Data description

We use U.S. state-level agricultural panel data compiled by the U.S. Department of Agriculture (USDA). The data ranges from 1960 to 2004 and includes prices and quantities for 3 outputs (crops, livestock and other) and 4 inputs (land, intermediate, capital and labor). Table 1 contains mean values and the coefficient of variation per subperiod of 11 years. A full description of the data can be found in USDA (2016). The summary statistics suggest that aggregate production has substantially increased. Aggregate use of land, labor and to a lesser extent capital have decreased, while aggregate intermediate input use has increased. The low coefficient of variation of land use reveals that this production factor cannot be adjusted instantaneously.

The USDA identifies 10 regions of agricultural production in the U.S. An overview is provided in Table 2.

We compute LHM TFP growth and its output-oriented decomposition for every state over the selected time period. We compare across all 48 states when computing the necessary distance functions and thus assume that all states have access to a similar production technology. This is also the approach of Zofío and Lovell (2001) and Ball, Färe, Grosskopf, and Margaritis (2010). Alternatively, we could compare states within the same agricultural region (see Table 2). However, this would limit the set of observations to 2 or 3 for some regions, which may be insufficient.[11]

We first conduct the analysis for a non-convex technology (using Free Disposal Hull under a variable-returns-to-scale assumption) and then repeat the analysis for a convex technology (using Data Envelopment Analysis under a variable-returns-to-scale assumption). This shows the applicability of our decomposition for both technologies and highlights potential differences that can arise due to convexity assumptions of the production technology.

[9] This step-size is analogous to h in the commonly used definition of a derivative of a function f: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. The more h approaches zero, the better the approximation of the derivative at the evaluated point. Likewise, the smaller SIC^o_t and SIC^o_{t+1}, the better the approximation of the frontier's gradient.

[10] The dataset does not include data from Alaska and Hawaii.

[11] O'Donnell (2012b) applies window analysis to circumvent this problem, but uses rather large windows for some regions. This can dampen the estimated rates of technical change.

Table 1
Mean and coefficient of variation (CV) for quantities per subperiod in 1996 US dollars ($\times$ 10^3).

	Period		Pacific	Mountain	Northern Plains	Southern Plains	Corn Belt	Southeast	Northeast	Lake States	Appalachian	Delta States
Land	1960/71	Mean	3300931.169	7255411.046	5840331.457	6292994.186	5004879.143	2215366.354	1768427.738	2444184.002	2356925.337	1520081.600
		CV	0.041	0.039	0.007	0.010	0.007	0.070	0.107	0.043	0.056	0.029
	1971/82	Mean	2893980.439	6720070.751	5687708.224	5781558.555	4767466.397	1773007.009	1394621.216	2199276.592	1956513.091	1343990.332
		CV	0.015	0.007	0.011	0.017	0.009	0.027	0.014	0.014	0.021	0.016
	1982/93	Mean	2710142.283	6508197.735	5544756.703	5535859.150	4632340.008	1484494.228	1267039.795	2112296.486	1818028.473	1206858.098
		CV	0.029	0.011	0.004	0.004	0.011	0.067	0.055	0.024	0.030	0.039
	1993/04	Mean	2590402.638	5967026.474	5581123.690	5718432.955	4582192.192	1428024.529	1190460.233	2085066.363	1802684.458	1209153.491
		CV	0.016	0.035	0.005	0.011	0.007	0.020	0.021	0.009	0.013	0.018
Intermediate	1960/71	Mean	6388062.631	5374177.714	9020391.093	5551339.202	16939475.421	4322275.998	5303636.777	8170759.798	4824174.845	3346989.032
		CV	0.047	0.125	0.092	0.126	0.048	0.119	0.034	0.027	0.064	0.125
	1971/82	Mean	7546312.374	7223653.517	11783582.906	7805959.230	18720865.969	5445467.048	5643208.149	9328635.249	5770043.712	4124055.132
		CV	0.111	0.091	0.128	0.094	0.074	0.112	0.090	0.112	0.093	0.086
	1982/93	Mean	8462752.863	6899795.667	12869397.188	7703305.897	16985319.256	5663158.403	5712198.784	9970093.780	5939957.239	4740621.090
		CV	0.058	0.029	0.032	0.070	0.056	0.037	0.026	0.057	0.022	0.130
	1993/04	Mean	11556159.900	8073898.489	14492935.479	8891404.135	17845564.638	6924523.321	6077658.726	11068114.621	7896907.296	6121353.879
		CV	0.082	0.070	0.078	0.064	0.047	0.066	0.054	0.059	0.119	0.031
Capital	1960/71	Mean	2277278.160	1965743.390	3839212.101	2420531.367	7485812.020	1409850.653	2761049.666	4235861.165	2550447.895	1202072.627
		CV	0.020	0.058	0.054	0.056	0.084	0.070	0.021	0.030	0.067	0.115
	1971/82	Mean	2583856.991	2443132.645	4662093.957	3052573.807	9993671.366	1854854.361	3040945.252	4891432.936	3233950.577	1718949.272
		CV	0.079	0.084	0.072	0.085	0.095	0.103	0.068	0.075	0.086	0.106
	1982/93	Mean	2340101.255	2292679.329	4213514.172	2901421.560	8602936.055	1665912.719	2711313.433	4626726.086	2817472.143	1619430.276
		CV	0.128	0.114	0.108	0.108	0.159	0.147	0.124	0.131	0.141	0.144
	1993/04	Mean	1983069.814	1936337.760	3376963.177	2316924.548	6070538.603	1370479.997	1983837.703	3562676.158	2361032.305	1264252.022
		CV	0.029	0.016	0.026	0.032	0.056	0.023	0.052	0.040	0.016	0.020
Labor	1960/71	Mean	11826308.223	7251927.972	11451651.884	10457257.796	26640405.728	8806948.892	12416331.593	17517663.881	16278275.980	8020645.246
		CV	0.124	0.089	0.134	0.141	0.164	0.117	0.182	0.141	0.171	0.192
	1971/82	Mean	10262405.279	6501263.073	10321361.426	7673646.814	20171125.527	6484264.056	9595326.458	13841201.345	9745925.952	4745991.646
		CV	0.059	0.033	0.062	0.096	0.066	0.101	0.040	0.030	0.137	0.149
	1982/93	Mean	9271807.049	6202223.028	9061470.223	6519170.104	16138668.519	4774474.559	8008139.689	12006149.317	7133646.807	3299937.462
		CV	0.064	0.088	0.117	0.051	0.095	0.074	0.136	0.126	0.163	0.090
	1993/04	Mean	10210352.950	5202355.190	7081823.093	7023374.727	11939104.652	4363279.813	6389996.042	7731857.592	6268745.715	2921548.411
		CV	0.083	0.043	0.056	0.047	0.097	0.042	0.052	0.136	0.050	0.059
Crops	1960/71	Mean	9125685.221	4287806.931	7041572.037	4374994.511	14332254.967	4522798.287	4261514.605	5899717.197	5730387.079	3013945.475
		CV	0.080	0.081	0.111	0.061	0.087	0.054	0.036	0.063	0.050	0.093
	1971/82	Mean	13188542.391	5597955.159	10329731.388	5405400.050	20625602.933	6242424.518	4699512.097	8385250.909	6594263.573	3872567.263
		CV	0.157	0.115	0.143	0.169	0.142	0.114	0.084	0.179	0.085	0.134
	1982/93	Mean	17643771.954	6701908.463	12996711.723	5879907.506	22994725.677	7229992.378	5461209.103	10186991.736	6992866.349	4664010.212
		CV	0.084	0.061	0.138	0.089	0.170	0.056	0.048	0.134	0.120	0.131
	1993/04	Mean	23286483.981	7886660.357	16120799.908	6506193.595	27046336.520	8590499.524	5693994.670	12044270.033	7794004.214	5409819.779
		CV	0.068	0.049	0.120	0.090	0.091	0.046	0.041	0.108	0.054	0.110
Livestock	1960/71	Mean	5327584.310	4919007.684	7454614.625	5173132.409	17052320.948	4112320.193	6257852.014	9324537.634	4515637.364	2939895.358
		CV	0.057	0.129	0.098	0.101	0.026	0.158	0.014	0.030	0.063	0.165
	1971/82	Mean	6152491.657	6380074.865	9212170.296	7281860.962	15516508.437	5433833.885	6445471.439	9292008.630	5271046.811	3749067.268
		CV	0.044	0.034	0.043	0.031	0.047	0.054	0.073	0.053	0.069	0.028
	1982/93	Mean	7493949.676	6568411.982	10365093.348	7989950.726	14086364.477	6324043.452	7531591.704	10371877.289	6873033.353	4410247.573
		CV	0.082	0.043	0.050	0.063	0.025	0.071	0.025	0.021	0.077	0.122
	1993/04	Mean	9685007.165	8609192.715	11405247.563	9848376.630	14512438.991	8009618.671	8382637.716	10506374.382	9332273.382	6095279.863
		CV	0.076	0.098	0.036	0.041	0.034	0.060	0.028	0.033	0.052	0.048
Other	1960/71	Mean	1505214.023	754115.275	763723.724	1373528.695	973630.348	1109452.294	542393.806	574042.980	631465.964	510886.261
		CV	0.072	0.074	0.111	0.261	0.085	0.066	0.202	0.186	0.154	0.123
	1971/82	Mean	1287921.873	565772.514	619433.118	715874.669	779303.889	807393.456	366832.739	430656.260	433338.235	391387.399
		CV	0.062	0.101	0.163	0.123	0.104	0.086	0.089	0.095	0.059	0.065
	1982/93	Mean	1666910.125	968193.737	1430545.782	1362035.393	843864.319	869125.637	491187.178	646790.108	588487.411	619379.070
		CV	0.121	0.141	0.136	0.175	0.171	0.193	0.091	0.126	0.295	0.408
	1993/04	Mean	2550514.545	1351626.269	1934947.718	1728703.994	1218992.400	1394982.468	667759.874	904455.293	1049200.724	992821.059
		CV	0.178	0.142	0.134	0.135	0.106	0.230	0.209	0.181	0.216	0.128

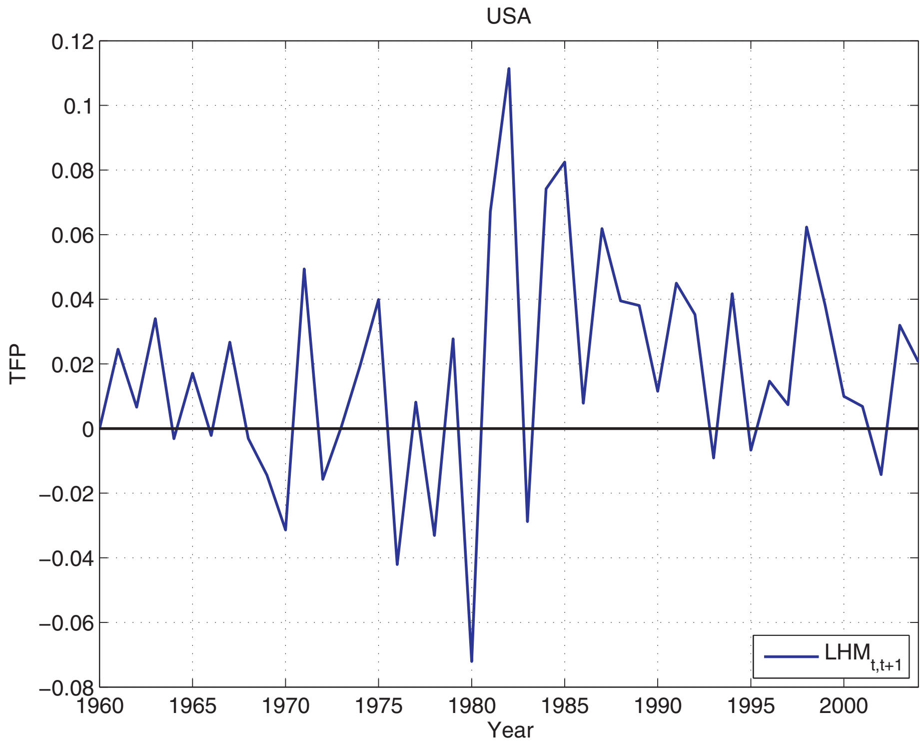


Fig. 4. Mean TFP change in the U.S. using a non-convex technology.

Table 2
Regions of agricultural production.

Region	States
Pacific	CA, OR, WA
Mountain	AZ, CO, ID, MT, NM, NV, UT, WY
Northern Plains	KS, ND, NE, SD
Southern Plains	OK, TX
Corn Belt	IA, IL, IN, MO, OH
Southeast	AL, FL, GA, SC
Northeast	CT, DE, MA, MD, ME, NH, NJ, NY, PA, RI, VT
Lake States	MI, MN, WI
Appalachian	KY, NC, TN, VA, WV
Delta States	AR, LA, MS

4.2. Non-convex technology

In practice, $\mathcal{Y}_t$ is unknown and needs to be estimated from the K observations in the dataset. The smallest enveloping non-convex approximation under variable-returns-to-scale (VRS) is given by:

$$\hat{\mathcal{Y}}_t = \left\{ (\boldsymbol{x}_{0t}, \boldsymbol{y}_{0t}) | \sum_{k=1}^{K} \lambda_k \boldsymbol{x}_{kt} \le \boldsymbol{x}_{0t}, \quad \sum_{k=1}^{K} \lambda_k \boldsymbol{y}_{kt} \ge \boldsymbol{y}_{0t}, \right.$$
$$\left. \sum_{k=1}^{K} \lambda_k = 1, \quad \lambda_k \in \{0, 1\} \right\}, \tag{13}$$

and can be plugged in (1) to compute the directional distance function in practice. The resulting program is a mixed-integer program and can be computationally harder to solve than the usual linear program. As first pointed out by Tulkens (1993), there exists an equivalent formulation based on enumeration which is considerably easier to solve. The enumeration formulation for directional distance functions with $\boldsymbol{g}_t > 0$ proposed by Cherchye, Kuosmanen, and Post (2001) is:

$$D_b(\boldsymbol{x}_{0a}, \boldsymbol{y}_{0a}; \boldsymbol{g}_a) = \max_{k \in \{1,\ldots,K\}} \left\{ \min_{\substack{j \in \{1,\ldots,m\}, \\ v \in \{1,\ldots,n\}}} \left\{ \frac{Y^j_{kb} - Y^j_{0a}}{g^{oj}_a}, \frac{X^v_{0a} - X^v_{kb}}{g^{iv}_a} \right\} \right\}, \tag{14}$$

with $(a, b) \in \{t, t+1\} \times \{t, t+1\}$. This allows us to compute all distance functions needed for the LHM TFP indicator and its decomposition. In line with the literature, we choose $\boldsymbol{g}^i_a = \boldsymbol{x}_{0a}$ and $\boldsymbol{g}^o_a = \boldsymbol{y}_{0a}$ such that β can be interpreted as the maximum proportional expansion (contraction) in the output (input) direction.[12] Since we work with aggregate data, all of our chosen directional vectors are nonzero. Moreover, the data set only contains nonnegative outputs $\boldsymbol{y}_t \in \mathbb{R}^m_+$. As a result, we can use the simplified formula (14).[13]

4.2.1. Main findings for the U.S

We first present the results for the U.S. as a whole before presenting individual results for the agricultural regions. We first consider the average LHM TFP change in Fig. 4. This is computed in a given year by taking the average LHM TFP of all states. This figure shows several considerable LHM TFP changes over time. Until 1979–1980, bad years offset good years resulting in only marginal cumulative LHM TFP growth over this period. After this period, positive growth rates dominate negative growth rates resulting in a positive cumulative LHM TFP growth of 78.61% in 2004. This boils down to an average LHM TFP growth of 1.79% per year.

[12] This choice of the direction vector takes into account state heterogeneity and projects each observation in a different direction onto the frontier. Recently, more advanced data-driven approaches were developed that determine the direction vectors using the analyzed firm's configuration (see Daraio and Simar, 2016 for technical details and Epure, 2016 for a management-oriented discussion). Finally, a homogeneous direction vector is more desirable, for example, for regulators in sectors where heterogeneity in input–output configurations is low.

[13] We use Bogetoft and Otto's (2015) Benchmarking package in R to compute the distance functions.

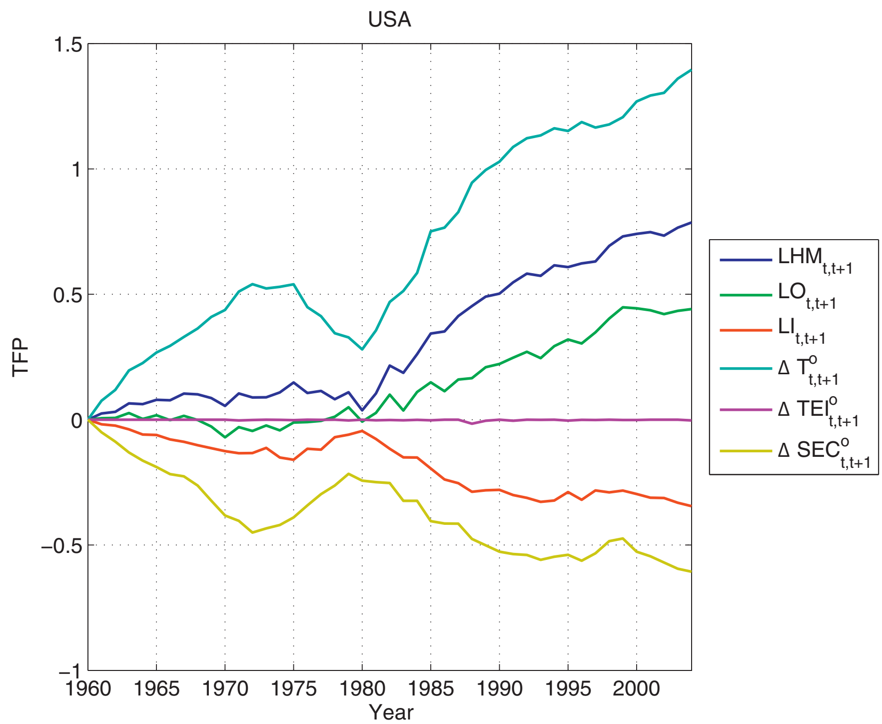


Fig. 5. Mean cumulative TFP growth in the U.S. and it components using a non-convex technology.

Table 3
LHM TFP growth and its components in the U.S. over 1960–2004 (in %) using a non-convex technology.

		$LHM_{t,t+1}$	$LO_{t,t+1}$	$LI_{t,t+1}$	$\Delta T^o_{t,t+1}$	$\Delta TEI^o_{t,t+1}$	$\Delta SEC^o_{t,t+1}$
$\sum_{t=1960}^{2004}$ mean (states)		78.61	44.10	−34.51	139.57	−0.32	−60.63
Avg growth rate		1.79	1.00	−0.78	3.17	-7.30×10^{-3}	−1.38
Min	1960/71	−5.72 (OK)	−4.70 (OK)	−6.61 (RI)	−8.77 (FL)	−1.45 (OK)	−30.20 (RI)
	1971/82	−1.99 (WY)	−1.18 (IN)	−2.46 (SC)	−13.81 (AZ)	−0.57 (PA)	−7.95 (DE)
	1982/93	0.30 (FL)	−2.02 (SD)	−4.76 (NH)	−1.22 (AR)	−0.28 (MO)	−14.44 (NH)
	1993/04	−1.03 (VT)	−0.60 (WY)	−3.11 (MA)	−3.38 (AL)	−1.40 (OK)	−10.13 (DE)
Max	1960/71	3.29 (RI)	2.99 (NV)	2.02 (CO)	33.49 (RI)	0.00 (all but OK)	9.68 (FL)
	1971/82	3.90 (OK)	4.12 (NE)	2.98 (ID)	7.64 (NH)	1.45 (OK)	14.60 (AZ)
	1982/93	7.45 (UT)	4.13 (AR)	1.42 (OK)	17.79 (NH)	0.57 (PA)	6.81 (UT)
	1993/04	5.34 (MA)	5.42 (SC)	1.99 (TN)	13.76 (DE)	0.28 (MO)	7.61 (AL)

Fig. 5 also shows the underlying drivers of these trends. Up to 1979–1980, cumulative LHM TFP growth is driven by $LI_{t,t+1}$. Subsequently, both input decline and output growth contribute to substantial LHM TFP growth. Cumulative output growth is 44.10%, while cumulative input decline is 34.51%. This means that U.S. agricultural production simultaneously increases output production at an average rate of 1% per year while decreasing input use at an average rate of 0.78% per year.

We now turn to our LHM TFP decomposition. Technical progress is the main driver of LHM TFP growth which is partly offset by scale inefficiency growth. Over the entire period, technical progress increased with 139.57% on average while cumulative scale inefficiency change reached −60.63%. Technical inefficiency change plays virtually no role. Table 3 summarizes these results and also lists the minimal and maximal values of the LHM TFP indicator and its components per subperiod of 11 years. It also lists the corresponding states.

4.2.2. Main findings per region

Fig. 6 depicts the average cumulative LHM TFP and its components for every region over time. The mean is computed with respect to all states in that particular agricultural production region. The highest cumulative TFP growth is achieved by the Northeast, Southeast, Corn Belt and Delta States with 84.47–95.62%. They are followed by the Pacific, Northern Plains, Appalachian, Lake States and Mountain regions with 63.56–74.36%. Finally, the Southern Plains region is severely behind the other regions with a cumulative TFP growth of 35.95%.

Although almost all regions experience technical progress, there are diverging trends among the different regions. Positive (negative) cumulative technical change over the whole time period indicates progress (regress) in terms of production technology. The Northeast experienced the largest cumulative technical progress (349%). The Pacific region is second with 162.2% and the Lake States are third with 104.9%. The Mountain, Corn Belt, Appalachian, Northern Plains, Delta States and Southeast experience milder technical progress between 49.23–83.65%. The Southern Plains is the only region with a cumulative technical regress of 11.42%, mainly due to a severe dip in the period 1975–1980 from which it only slowly recovers.

Technical inefficiency change generally plays a minor role. Positive (negative) cumulative technical inefficiency change indicates that the distance to the frontier decreases (increases) over the whole time period. Negative changes in cumulative technical in-

(A) Average cumulative TFP growth per region

(B) Average cumulative technical change per region

(C) Average cumulative technical inefficiency change per region

(D) Average cumulative scale inefficiency change per region

Fig. 6. Cumulative TFP growth and its decomposition per U.S. region using a non-convex technology.

efficiency change are quickly followed by positive changes. These spikes are visible in the Southern Plains, Northern Plains, Corn Belt, Delta States and Lake States. There is only a negative cumulative technical inefficiency change in the Southern Plains, due to a drop in technical inefficiency by 7.71% in 2004.

The trend in the scale inefficiency change is the mirror image of the trend in technical change: regions with positive (negative) technical change experience negative (positive) scale inefficiency. Positive (negative) cumulative scale inefficiency change indicates that the region operates at a more (less) optimal scale over the whole time period. The Southern Plains, Southeast, Delta States, Northern Plains, Corn Belt experience the highest positive cumulative scale inefficiency change between 4.48–55.07%. Cumulative scale inefficiency change is negative in the Appalachian, Mountain and Lake States (between −1.00% and −36.05%). Cumulative scale inefficiency change is most negative in the Pacific (−87.82%) and Northeast (−253.4%) regions.

4.3. Convex technology

Since we only have 48 observations per year, a non-convex technology might provide limited discriminating power resulting in many efficient observations. Therefore, we repeat the analysis for a convex VRS representation of the production technology using Data Envelopment Analysis (DEA). The smallest enveloping approximation is given by:

$$\hat{\mathcal{Y}}_t = \left\{ (\boldsymbol{x}_{0t}, \boldsymbol{y}_{0t}) \mid \sum_{k=1}^{K} \lambda_k \boldsymbol{x}_{kt} \leq \boldsymbol{x}_{0t}, \quad \sum_{k=1}^{K} \lambda_k \boldsymbol{y}_{kt} \geq \boldsymbol{y}_{0t}, \quad \sum_{k=1}^{K} \lambda_k = 1, \quad \lambda_k \geq 0 \right\}, \tag{15}$$

and can be plugged in (1) to compute the directional distance function in practice. The resulting linear program with $(a, b) \in \{t, t+1\} \times \{t, t+1\}$ is:

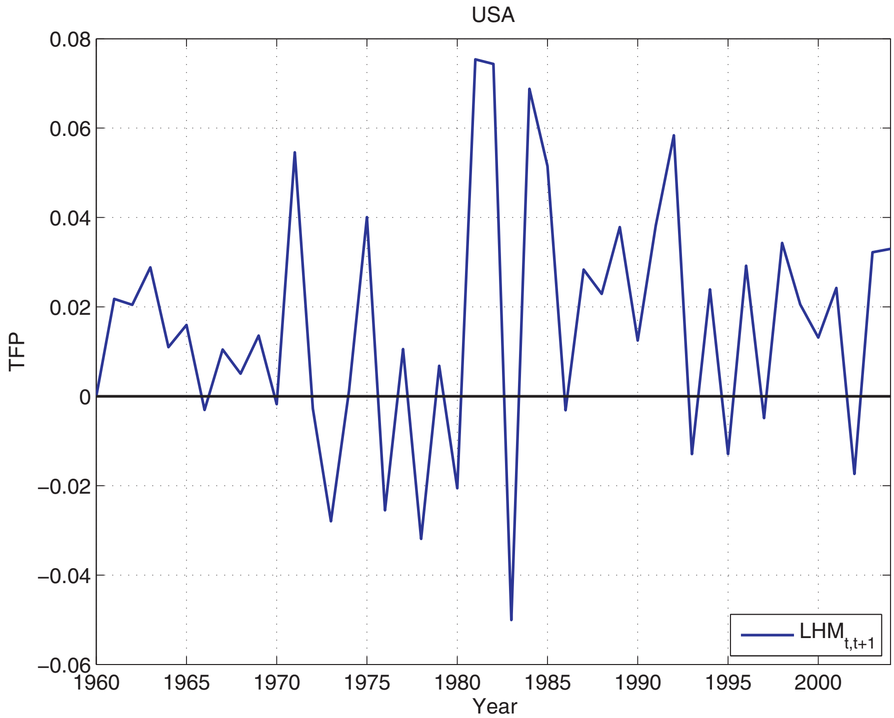


Fig. 7. Mean TFP change in the U.S. using a convex technology.

$$D_b(\boldsymbol{x}_{0a}, \boldsymbol{y}_{0a}; \boldsymbol{g}_a) = \max_{\beta, \lambda_k \geq 0} \beta \text{ s.t. } \sum_{k=1}^{K} \lambda_k \boldsymbol{x}_{kb} \leq \boldsymbol{x}_{0a} - \beta \boldsymbol{g}_a^i, \quad (16)$$
$$\sum_{k=1}^{K} \lambda_k \boldsymbol{y}_{kb} \geq \boldsymbol{y}_{0a} + \beta \boldsymbol{g}_a^o,$$
$$\sum_{k=1}^{K} \lambda_k = 1.$$

This allows us to compute all necessary distance functions needed for all the components of the LHM TFP indicator. As for the FDH analysis, we choose $\boldsymbol{g}_a^i = \boldsymbol{x}_{0a}$ and $\boldsymbol{g}_a^o = \boldsymbol{y}_{0a}$.

4.3.1. *Main findings for the U.S*

We present the results for the U.S. as a whole before presenting individual results for the agricultural regions[14]. We first consider the average annual LHM TFP change in Fig. 7. This is computed in a given year by taking the average LHM TFP of all states. This figure shows considerable fluctuations in annual LHM TFP changes over time. Overall, years with LHM TFP growth dominate years with LHM TFP decline.

Fig. 8 shows the cumulative LHM TFP growth and the underlying drivers. Our main finding for the U.S. as a whole is that LHM TFP clearly increases over time. The LHM TFP indicator increases by 70.46% between 1960 and 2004. This boils down to an average LHM TFP growth of 1.60% per year. LHM TFP growth is driven by output growth (+62.98%) rather than input decline (−7.47%). In the period 1977–1982, $LI_{t,t+1}$ contributes to a temporary slowdown in LHM TFP growth. $LI_{t,t+1}$ only plays a minor role in the remaining periods.

We now turn to our LHM decomposition. Our decomposition shows that technical change (+70.55%) is the main driver, while technical inefficiency change (−1.99%) and scale inefficiency change (+0.42%) only play a minor role. Table 4 summarizes these results and lists the minimal and maximal values of the LHM TFP indicator and its components per subperiod of 11 years. It also lists the corresponding states.

4.3.2. *Main findings per region*

Fig. 9 depicts the mean cumulative LHM TFP and its components for every region over time. The mean is computed with respect to all states in that particular agricultural production region. The Northern Plains experienced the highest cumulative LHM TFP growth (119.3%) while the Southern Plains experienced the lowest cumulative LHM TFP growth (8.96%) over the entire period. Between them, Delta States experience the second highest cumulative LHM TFP growth of 107%. Pacific, Corn Belt, Southeast, Northeast and Mountain regions have similar levels of cumulative LHM TFP growth of 65.58–86.18%. The cumulative LHM TFP growth of Lake States and Appalachian regions varies in the range 35.36–46.83%.

Being the main driver of LHM TFP growth, similar trends occur for technical change. The Northern Plains region has the highest rate of cumulative technical change (117.9%) and the Southern Plains the lowest (6.25%). Again, Delta States experience the second highest rate of technical change of 95%. The other regions can roughly be classified in two clusters. The first cluster consists of the Corn Belt, Mountain, Southeast, Pacific and Northeast regions (63.89–81.67%). The second cluster consists of Lake States and Appalachian (43.6–52.09%).

In terms of cumulative technical inefficiency change, there are diverging trends among the different regions. Pacific, Northern Plains, Delta States and Northeast experience a positive cumulative technical inefficiency change between 4.86% and 10.82%. The six remaining regions experience a negative cumulative technical inefficiency change. Cumulative technical inefficiency change is mildly negative (between −7.01% and −1.58%) in the Southeast, Corn Belt, Lake States and Mountain regions. This is worse in the Southern Plains and Appalachian, where the cumulative technical inefficiency change is −12.96% and −23.61%, respectively.

[14] Infeasibilities may arise for the components where the year of the observation differs from the year of the reference technology. As there is no easy solution to solve this problem, Briec and Kerstens (2009) recommend to report the infeasibilities. There were only infeasibilities for Rhode Island.

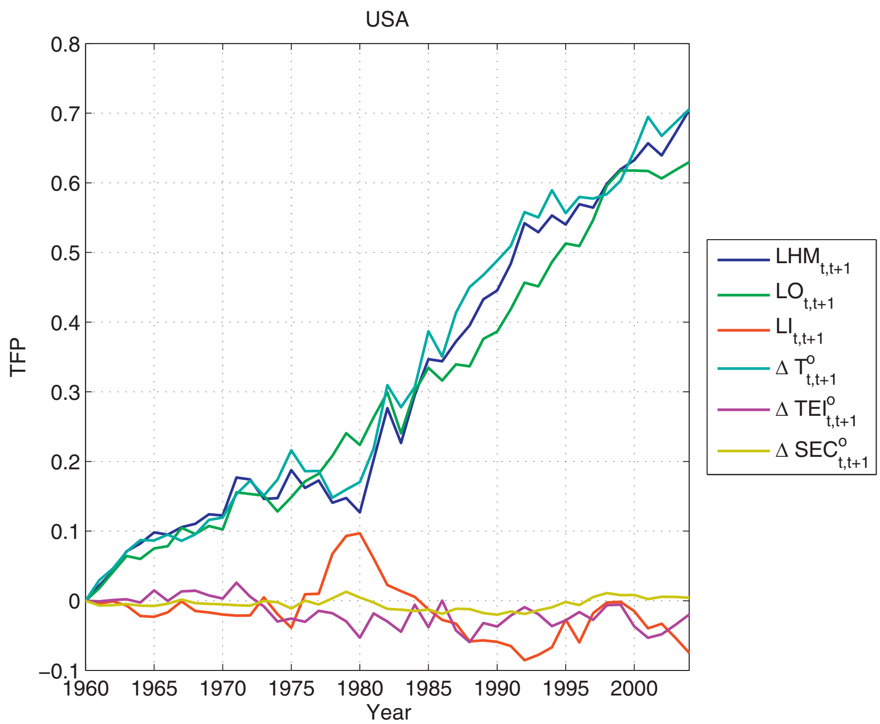


Fig. 8. Mean cumulative TFP growth and its components in the U.S. using a convex technology.

Table 4
TFP growth and its components in the U.S. covering the years 1960–2004 (in %) using a convex technology.

		$LHM_{t,t+1}$	$LO_{t,t+1}$	$LI_{t,t+1}$	$\Delta T^o_{t,t+1}$	$\Delta TEI^o_{t,t+1}$	$\Delta SEC^o_{t,t+1}$
$\sum_{t=1960}^{2004}$ mean (states)		70.46	62.98	−7.47	70.55	−1.99	0.42
Avg growth rate		1.60	1.43	−0.17	1.60	−0.05	9.50 $\times 10^{-3}$
Min	1960/71	−4.37 (OK)	−3.84 (NJ)	−6.61 (RI)	−3.83 (OK)	−1.60 (OK)	−3.32 (NV)
	1971/82	−1.28 (WV)	−1.55 (MO)	−1.84 (RI)	−0.67 (FL)	−2.92 (WY)	−1.65 (DE)
	1982/93	−0.36 (TN)	−1.53 (NH)	−3.52 (KS)	0.64 (FL)	−2.64 (MO)	−1.63 (SD)
	1993/04	−2.41 (WY)	−2.34 (WY)	−2.70 (RI)	−1.34 (KY)	−3.65 (WY)	−0.93 (LA)
Max	1960/71	7.16 (ND)	7.21 (AR)	3.18 (AR)	5.42 (NV)	3.72 (ND)	1.76 (LA)
	1971/82	3.77 (IL)	3.51 (WA)	2.63 (DE)	4.05 (ND)	1.59 (OK)	1.33 (OR)
	1982/93	5.64 (DE)	4.83 (WV)	1.99 (OK)	5.67 (DE)	3.62 (MT)	2.03 (IA)
	1993/04	4.24 (AL)	4.11 (SD)	2.53 (KY)	4.38 (MS)	2.89 (MO)	2.54 (TN)

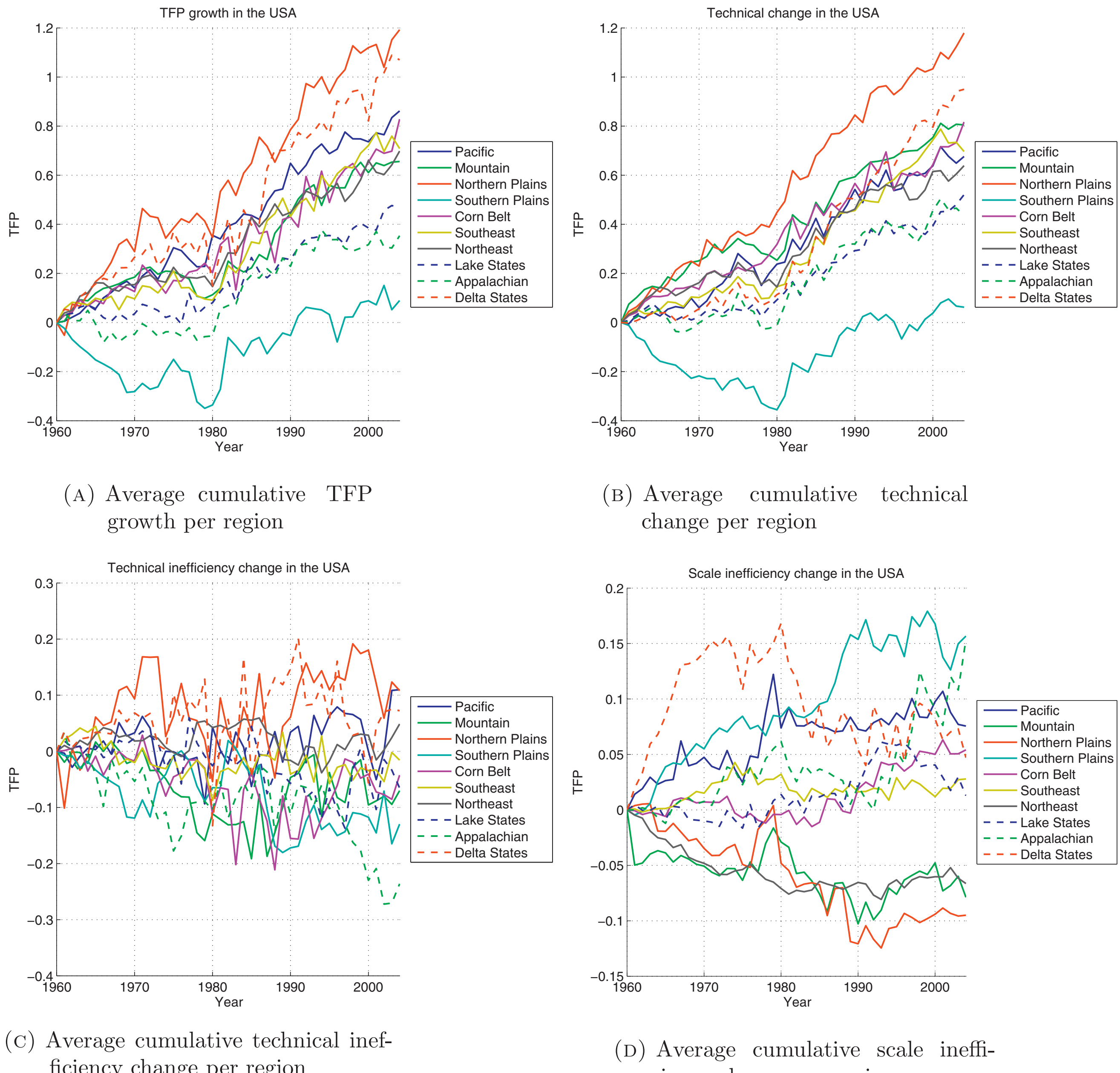
Again, there are diverging trends for cumulative scale inefficiency change. The Southern Plains experience the highest increase in cumulative scale inefficiency change (15.67%) followed closely by the Appalachian region (15.38%). The Northern Plains experience a negative cumulative scale inefficiency change (−9.5%). Between these extremes, the Pacific, Corn Belt, Delta States, Southeast and Lake States have a positive cumulative scale inefficiency change in the range of 1.29–7.56%. In contrast, the cumulative scale inefficiency change of the Northeast and Mountain regions is negative (−6.64% and −7.88%, respectively).

Although all U.S. regions experienced LHM TFP growth in the period 1960–2004, this analysis shows that the contribution of the underlying factors varies considerably per region. Technical change is the main driver of LHM TFP growth for all U.S. regions. In addition, several U.S. regions partly increased TFP by becoming more efficient over time and/or operating at a more optimal scale. Other regions mainly relied on technical change to increase LHM TFP.

4.4. Discussion

The results depend on the convexity assumption of the technology. We test the hypothesis whether the distributions of the LHM TFP indicator and its components for FDH and DEA are not significantly different using a Kolmogorov–Smirnov test. This non-parametrically tests the hypothesis H_0 whether two samples are drawn from the same underlying distribution. We conduct the test for every year separately, resulting in 44 different test hypotheses for every component. The results at the 10% significance level are presented in Table 5. For the majority of years, the distributions of the $LHM_{t,t+1}$ and its components $LO_{t,t+1}$ and $LI_{t,t+1}$ are not statistically different using FDH and DEA. In contrast, the distributions of $\Delta T^o_{t,t+1}$ are statistically significant for a majority of years and the distributions of $\Delta TEI^o_{t,t+1}$ and $\Delta SEC^o_{t,t+1}$ under both technologies are significantly different for all years. These results in conjunction with Tables 3 and 4 lead us to the following qualitative conclusions.

Both results suggest there is substantial LHM TFP growth over the entire period which is mainly driven by technical progress. Both the DEA and FDH results indicate that output growth dominates input decline, although this finding is much more pronounced for the DEA results. A possible explanation for the smaller contribution of input decline is that some quasi-fixed inputs (*e.g.* land) are not constantly adjusted over time or that input reduction is not an objective for some inputs such as land and labor.



(A) Average cumulative TFP growth per region

(B) Average cumulative technical change per region

(C) Average cumulative technical inefficiency change per region

(D) Average cumulative scale inefficiency change per region

Fig. 9. Cumulative TFP growth and its decomposition per U.S. region using a convex technology.

Table 5
Results of Kolmogorov–Smirnov test for distributions under non-convex and convex technologies.

	$LHM_{t,t+1}$	$LO_{t,t+1}$	$LI_{t,t+1}$	$\Delta T^o_{t,t+1}$	$\Delta TEI^o_{t,t+1}$	$\Delta SEC^o_{t,t+1}$
Reject H_0 per year at 10%	9/44	12/44	8/44	25/44	44/44	44/44
Reject H_0 at 10%	No	Yes	Yes	Yes	Yes	Yes

We analyze LHM TFP growth and technical change across time, farm types and agricultural intensity rates. Table 6 shows the results of the Kolmogorov–Smirnov test testing equality of distributions for LHM TFP growth rates and technical changes for consecutive subperiods of eleven years in line with Table 3. Regarding FDH, all distributions of consecutive LHM TFP growth rates and technical changes are significantly different at the 10% level. Regarding DEA, the distributions of the LHM TFP growth rates and technical changes between 1982/93 and 1993/04 are not significantly different at the 10% level, while these are significantly different comparing the preceding time periods. This suggests that distributional differences in productivity growth driven by shifts in technology may decrease in importance throughout time.

In line with Ang and Kerstens (2016), we assess whether there are distributional differences in LHM TFP growth rates and technical changes between farm types in Table 7. We rank the farm regions by the ratio of crop production to total production con-

Table 6
Results of Kolmogorov–Smirnov test for distributions of LHM TFP growth rates and technical changes for consecutive time periods.

	1960/71–1971/82	1971/82–1982/93	1982/93–1993/04
FDH: Reject H_0 at 10% $LHM_{t,t+1}$	Yes	Yes	Yes
FDH: Reject H_0 at 10% $\Delta T_{t,t+1}$	Yes	Yes	Yes
DEA: Reject H_0 at 10% $LHM_{t,t+1}$	Yes	Yes	No
DEA: Reject H_0 at 10% $\Delta T_{t,t+1}$	Yes	Yes	No

Table 7
Results of Kolmogorov–Smirnov test for distributions of LHM TFP growth rates and technical changes covering the whole time period among farm types.

	Crops – Mixed	Mixed – Livestock	Crops – Livestock
FDH: Reject H_0 at 10% $LHM_{t,t+1}$	No	Yes	No
FDH: Reject H_0 at 10% $\Delta T_{t,t+1}$	Yes	Yes	Yes
DEA: Reject H_0 at 10% $LHM_{t,t+1}$	No	No	Yes
DEA: Reject H_0 at 10% $\Delta T_{t,t+1}$	No	No	Yes

Table 8
Results of Kolmogorov–Smirnov test for distributions of LHM TFP growth rates and technical changes covering the whole time period among agricultural intensity rates.

	Low – Medium	Medium – High	Low – High
FDH: Reject H_0 at 10% $LHM_{t,t+1}$	No	No	No
FDH: Reject H_0 at 10% $\Delta T_{t,t+1}$	Yes	No	Yes
DEA: Reject H_0 at 10% $LHM_{t,t+1}$	No	Yes	No
DEA: Reject H_0 at 10% $\Delta T_{t,t+1}$	No	Yes	No

sidering the whole time period. This leads to a classification of 3 crop regions (Corn Belt, Northern Plains and Pacific), 4 mixed regions (Delta States, Southeast, Appalachian, and Northeast) and 3 livestock regions (Lake States, Mountain area and Southern Plains). Regarding the FDH results, the distributions of the LHM TFP growth rates of mixed regions and livestock regions are significantly different at the 10% level, while these are not significantly different at the 10% level comparing crop regions to mixed regions and livestock regions. Interestingly, regarding the FDH results, the distributions of the technical changes are significantly different at the 10% level comparing all types of regions. Regarding the DEA results, the distributions of the LHM TFP growth rates and technical changes of crop regions and mixed regions, and mixed regions and livestock regions, are not significant at the 10% level, while these are significantly different at the 10% level comparing crop regions to livestock regions. In summary, there seems to be ambiguity in how regional differences in specialization may drive differences in the distribution of LHM TFP growth and technical changes.

We also assess whether there are distributional differences in LHM TFP growth rates and technical changes between agricultural intensity rates in Table 8. We rank the farm regions by the Industry Specialization Index (ISI) for agriculture considering 1963–2004[15]. The U.S. Bureau of Economic Analysis (BEA) computes the ISI as the agricultural industry's share of the state-level Gross Domestic Product divided by the agricultural industry's share of the U.S. total for the same statistic. The complete dataset can be found in BEA (2016). We rank the regions by ISI, which leads to a classification of 3 low ISI regions (Northeast, Lake States and Southeast), 4 medium ISI regions (Appalachian, Southern Plains, Pacific and Corn Belt) and 3 high ISI regions (Mountain area, Delta States and Northern Plains). With respect to the FDH results, comparing distributions of the LHM TFP growth rates for all groups do not yield any significant difference at the 10% level. The distributions of the technical changes are significantly different at the 10% level comparing low ISI regions to medium and high ISI regions. Regarding the DEA results, the distributions of the LHM TFP growth rates and technical changes are significantly different at the 10% level comparing medium ISI regions to high ISI regions. Similar to the preceding section, there thus seems to be ambiguity in how regional differences in agricultural intensity may drive differences in the distribution of LHM TFP growth and technical changes.

The contribution of technical inefficiency change to LHM TFP growth is less clear-cut. Using FDH, technical inefficiency change is virtually nonexistent. Further inspection reveals that most (contemporaneous) technical inefficiency scores are zero using FDH. This drives the extremely low technical inefficiency change. Therefore, these remarkable results may be due to lower discriminatory power of FDH in this case since there are relatively few observations per year compared to the number of inputs and outputs. Using DEA, there is a small cumulative increase in technical inefficiency change.

The results differ more for the scale inefficiency change component. There is a substantial increase in cumulative scale inefficiency change using FDH, whereas there is almost no cumulative scale inefficiency change using DEA. Again, this may be due to the higher discriminatory power of DEA.

Our DEA results are in line with other empirical studies that analyze the TFP growth in the U.S. agricultural sector using the same data source. Zofío and Lovell (2001), Ball et al. (2010), O'Donnell (2012b) and Ball, Wang, and Nehring (2016) also find substantial TFP growth[16]. It is driven by technical progress rather than efficiency change in line with Zofío and Lovell (2001) and Ball et al. (2016). Following Ball et al. (2016), TFP growth is also due to output growth rather than changes in the input level.

5. Conclusions

This paper decomposes the additively complete LHM TFP indicator into components of technical change, technical inefficiency

[15] Data for 1960–1962 are unavailable.

[16] Zofío and Lovell (2001) only analyze TFP growth over the period 1960–1990.

change and scale inefficiency change. Our approach is general in that it does not require differentiability or convexity of the production technology. Using a nonparametric framework, the empirical application focuses on state-level data of the U.S. agricultural sector over the period 1960–2004. We compute the scores using FDH and DEA to show the flexibility of our decomposition and to investigate the potential issue of non-convexities in the agricultural sector. Furthermore, we analyze LHM TFP growth and technical change across time, farm types and agricultural intensity rates.

The FDH results show that LHM TFP has increased by 78.61% in the considered period. This is due to output growth (+44.10%) as well as input decline (−34.51%). Technical change (+130.57%) and scale inefficiency change (−60.63%) are the main drivers, while technical inefficiency change (−0.32%) only plays a minor role.

Following the DEA results, LHM TFP has increased by 70.46% for the considered period. This productivity growth is due to output growth (+62.98%) rather than changes in the input level (−7.47%). Technical change is the main driver (+70.55%), while technical inefficiency change (−1.99%) and scale inefficiency change (+0.42%) only play a minor role.

The results thus depend on whether we use FDH or DEA. Although this may partly be driven by the underlying true production technology, we note that FDH may result in too low discriminatory power to compute the distance functions given the relatively low number of observations for the number of variables in this application.

Following the Kolmogorov–Smirnov tests, there seem to be differences in the distributions of LHM TFP growth and technical change across time, farm types and agricultural intensity rates. We suspect that policy instruments and factor endowments (*e.g.* soil and weather conditions) may drive differences across time, farm types and agricultural intensity rates, potentially resulting in differing distributions in LHM TFP growth and technical change. For instance, agricultural support payments with restrictions on land use (Just & Kropp, 2013) and ethanol subsidies (Motamed, McPhail, & Williams, 2016) likely have an impact on geographical specialization. This information would be relevant for policy makers. Such an empirical investigation is left for future research.

Appendix A. Managi's (2010) decomposition

Managi (2010) decomposes the Luenberger–Hicks–Moorsteen indicator into technical change (TC):

$$TC = \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\right] - \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1})) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right],$$

and the residual being efficiency change (EC):

$$\begin{aligned}EC = \frac{1}{2}\Big\{&D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\\ &- D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) + D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\\ &+ D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) + D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\\ &- D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) + D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\Big\}\\ &- D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1})) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\\ &+ D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\end{aligned}$$

However, this decomposition is incomplete. First, it lacks a scale (in)efficiency change component. Second, there is no reason for TC to be defined as a difference between an output-oriented technical change component and an input-oriented technical change component. Furthermore, TC is only defined with respect to observations in period $t+1$, although there is no clear reason to favor those to observations in period t. Finally, the EC component does not capture technical (in)efficiency change.

Appendix B. Decomposition using the input direction

The decomposition using the input direction is:

$$LHM_{t,t+1} = \Delta T^i + \Delta TEI^i + \Delta SEC^i, \tag{B.1}$$

representing technical change, technical inefficiency change and scale inefficiency change, respectively.

The technical change component is defined as:

$$\begin{aligned}\Delta T^i = \frac{1}{2}\Big\{&\left[D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\right]\\ &+ \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\right]\Big\},\end{aligned} \tag{B.2}$$

and has the same interpretation as before. Technical inefficiency change is:

$$\Delta TEI^i = D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)). \tag{B.3}$$

From the residual

$$\begin{aligned}&LHM_{t,t+1} - \Delta T^i - \Delta TEI^i\\ &= \frac{1}{2}\Big\{\left[D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right]\\ &+ \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right]\Big\}\\ &- \frac{1}{2}\Big\{\left[D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\right]\\ &+ \left[D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0)) - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\right]\Big\},\end{aligned} \tag{B.4}$$

we recover the scale inefficiency change component in a similar way as before. Define the projections of $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+1}$ on the production frontier at time t:

$$\boldsymbol{x}^*_t = \boldsymbol{x}_t - D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\boldsymbol{g}^i_t \tag{B.5a}$$

$$\boldsymbol{x}^{**}_{t+1} = \boldsymbol{x}_{t+1} - D_t(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\boldsymbol{g}^i_{t+1}, \tag{B.5b}$$

and the projections of $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+1}$ on the production frontier at time $t+1$:

$$\boldsymbol{x}^{**}_t = \boldsymbol{x}_t - D_{t+1}(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\boldsymbol{g}^i_t \tag{B.6a}$$

$$\boldsymbol{x}^*_{t+1} = \boldsymbol{x}_{t+1} - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))\boldsymbol{g}^i_{t+1}. \tag{B.6b}$$

Respectively adding and subtracting $D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))$ and $D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0))$ to and from (B.4), and using the translation property of the directional distance function and the definitions of the projections above, we find the scale inefficiency change component:

$$\begin{aligned}\Delta SEC^i = \frac{1}{2}\Big\{&\left[D_t(\boldsymbol{x}_t, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_t(\boldsymbol{x}_t, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right]\\ &- \left[D_t(\boldsymbol{x}^{**}_{t+1}, \boldsymbol{y}_t; (\boldsymbol{g}^i_{t+1}, 0)) - D_t(\boldsymbol{x}^*_t, \boldsymbol{y}_t; (\boldsymbol{g}^i_t, 0))\right]\\ &+ \left[D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_t; (0, \boldsymbol{g}^o_t)) - D_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{y}_{t+1}; (0, \boldsymbol{g}^o_{t+1}))\right]\\ &- \left[D_{t+1}(\boldsymbol{x}^*_{t+1}, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_{t+1}, 0)) - D_{t+1}(\boldsymbol{x}^{**}_t, \boldsymbol{y}_{t+1}; (\boldsymbol{g}^i_t, 0))\right]\Big\}\\ \equiv \frac{1}{2}\Big\{&\Delta SEC^i_t + \Delta SEC^i_{t+1}\Big\}.\end{aligned} \tag{B.7}$$

Appendix C. State-level TFP figures

This appendix includes the LHM TFP indicator and its components per agricultural region. Each figure is constructed by averaging over all states in that particular agricultural region in every year.

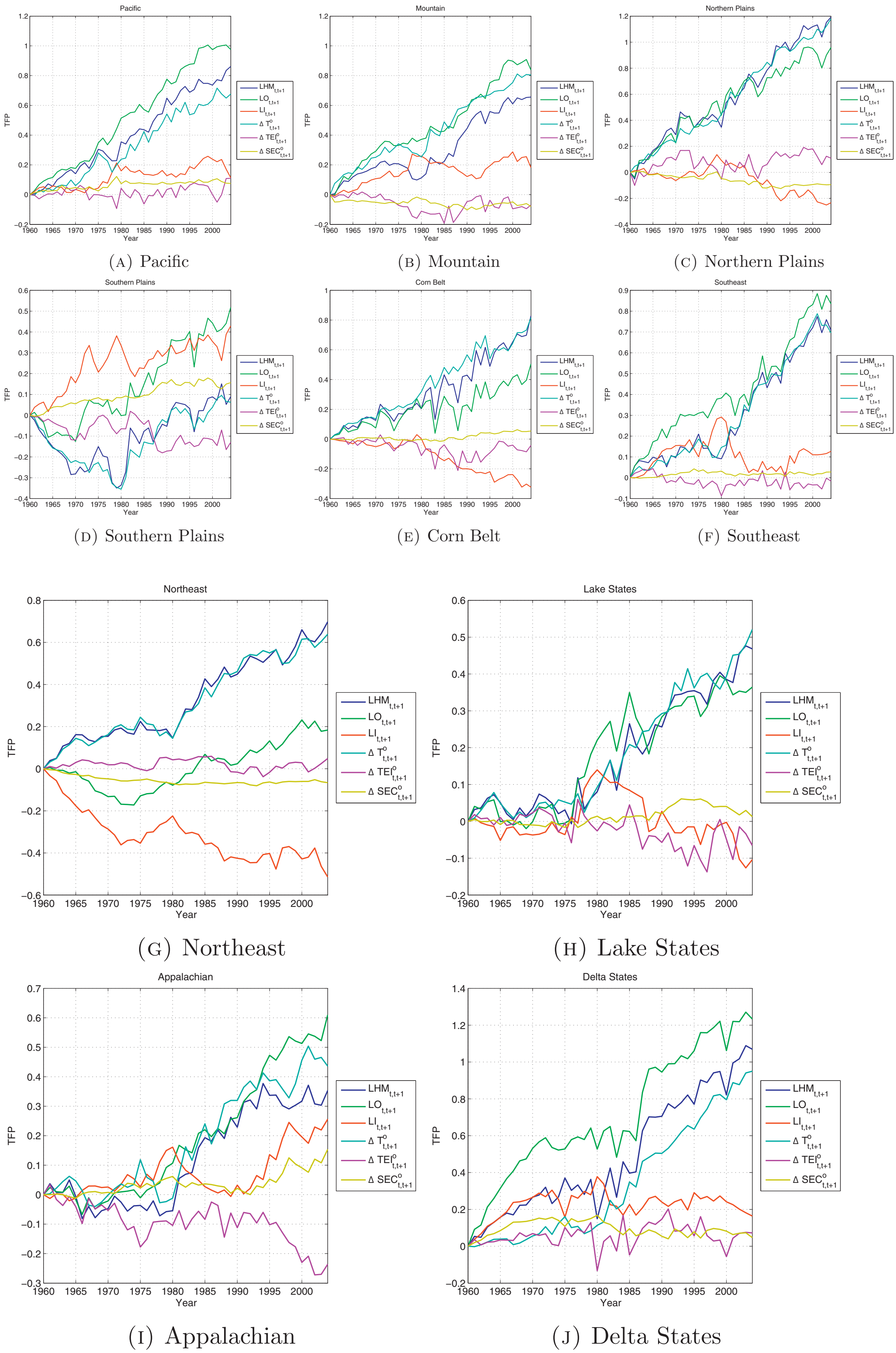
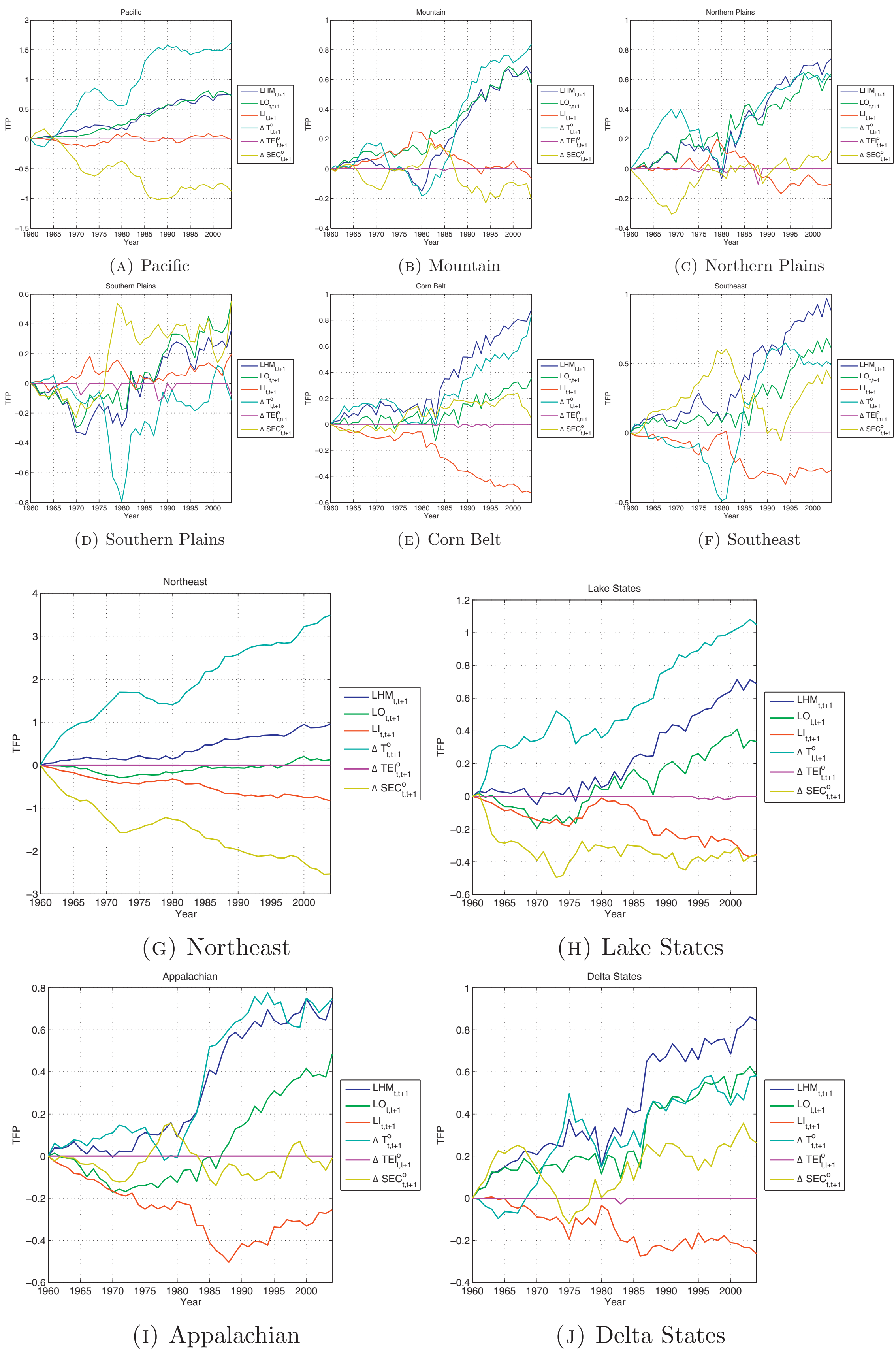


Fig. C.1. Cumulative LHM TFP indicator and its components per agricultural region under a convex technology.



(A) Pacific (B) Mountain (C) Northern Plains
(D) Southern Plains (E) Corn Belt (F) Southeast
(G) Northeast (H) Lake States
(I) Appalachian (J) Delta States

Fig. C.2. Cumulative LHM TFP indicator and its components per agricultural region under a non-convex technology.

C1. Convex technology

C2. Non-convex technology

Supplementary material

Supplementary material associated with this paper can be found, in the online version, at 10.1016/j.ejor.2016.12.015.

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