

UNIVERSITY OF SALENTO  
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# Solow vs Lucas

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An Empirical Analysis of OECD and BRICS  
Countries

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*Is there some action a government of India could take  
that would lead the Indian economy to grow like  
Indonesia's or Egypt's? If so, what, exactly? If not, what is it about  
the "nature of India" that makes it so?*

*The consequences for human welfare involved in questions like these are simply staggering:  
Once one starts to think about them, it is hard to think about anything else.*

*Robert E. Lucas, Jr. (1988)*

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A special thank to Professor Mastromarco for her availability and capacity to spur students in loving econometrics. We are also grateful to our university for rooms and labs provided to students. A final thank to professor Grassi for his constant presence in our academic path.

# **Solow vs Lucas**

## **An Empirical Analysis of OECD and BRICS Countries**

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## **1. INTRODUCTION**

The purpose of this dissertation is to discuss the suitability of some growth models, using the Engel-Granger representation. Two models have been considered: Solow-Swan (1956) and Lucas-Uzawa (1988) model. The sample consists on some of OECD and BRICS countries.

The approach considered does not aim at verifying a simple co-integration relationship, but the objective is to test whether under specific conditions the long-run equilibrium is verified or not. In particular, the aforementioned conditions are settled by growth models.

In the second chapter the theoretical background has been analyzed. We focused on the historical evolution of growth models, their assumptions and theoretical predictions.

In the third chapter data sample is disclosed, and the main descriptive statistics are shown.

In the first part of fourth chapter the adopted procedure is explained. In particular the dissertation focused on Augmented Dickey-Fuller test, Co-integration and Engel-Granger representation.

In the second part of fourth chapter the model conditions are tested for each country, and the most suitable model is chosen.

Finally, in the fifth chapter economic and political hints are discussed on the light of previous results.

## **2. THEORETICAL BACKGROUND**

Our research has been conducted under the theoretical predictions of two main, and counterpoised, models: the Solow-Swan model and the Uzawa-Lucas model. The first could be framed into the neo-classical growth theory, as long as production function is supposed to meet some proprieties, and the second belongs to the new growth theory recently developed by Lucas and Romer. The debate about growth accounting has always been very complicate and many results have been highlighted. From an historical point of view, economists focused on the relevance of physical capital in output growth and this is confirmed by the aforementioned Solow (1956) and Swan (1956) work and the Cass and Koopmans's (1965) version of Ramsey growth model. The difference between the two could be highlighted: the relevance of saving rate, which is exogenous in Solow-Swan model and endogenous in the latest version of Ramsey growth model. However, the way through which endogenous saving rate in Ramsey model is defined is not plausible from an empirical point of view, hence economists took apart growth theory during the seventies. For about fifteen years macroeconomics research focused on short-term fluctuations and this is the moment of real business cycle models.

After mid-1980s research on economic growth experienced a boom with the works of Romer and Lucas. The aim of these models was to explain the long-run variables of growth endogenously. In the first wave of these models, growth may go with no limits because the returns to investment in a class of capital goods, human capital included, don't diminish as economies grow. In this context, knowledge spillovers and external benefits from human capital are relevant. The attempt to introduce R&D sector in endogenous models has been proposed by Romer (1987,1990) and others, such as Aghion and Howitt (1992), Grossman and Helpman (1991). These models call for monopoly power and the growth rate is endured by government plans about taxation, property rights and other issues. During the last decades, many economists focused on technology diffusion as cause of neoclassical convergence. The difference between old growth theory and new one is the care about empirical application.

## 2.1. Solow-Swan Model

The first model considered is Solow-Swan (1956). Our intention is to test whether it is possible to experience positive growth rate over years. The main assumptions and results will be shown in the following section.

### Assumptions

- *Economic background:* families own inputs (capital and labour) and assets; firms hire from households the necessary inputs in order to produce goods. It is possible to set up a market in which families sell inputs to firms and these sell good to the former. It is relevant to note that this model is a very simplified version, in which government is absent, labour market is on equilibrium, an aggregate production is assumed, and rates of saving, technological progress and population are constant and exogenously given.
- *Production function:* it is defined in three main inputs: labour, capital and knowledge. Hence it assumes the form:

$$Y(t) = F[K(t), L(t), T(t)]$$

Where  $K(t)$  is the capital,  $L(t)$  is the labour and  $T(t)$  is the knowledge. It has a neoclassical form and properties, in particular using homogeneity of degree one

$$Y = F(K, L, T) = L \cdot F(K/L, 1, T) = L \cdot f(k)$$

Per capita output is:

$$\frac{Y}{L} = y = f(k)$$

It is worth noting that the capital and labour are rivalry goods, while technology is a non-rivalry one. Assuming a one sector production technology, the output can be consumed ( $C(t)$ ) or invested ( $I(t)$ ) incrementing capital stock. The saved fraction of output depends on the level of saving rate  $s$ , which is exogenous as said before. Population is supposed to grow according this specification  $L(t) = e^{nt}$ .

To simplify the following dissertation and focus only on capital effects, the growth of technology input is supposed to be null. Relaxing this hypothesis leads to a different interpretation of the model.

## Theoretical predictions

The role of physical capital is relevant in this model as long as it accounts for economies' growth. The main theoretical prediction is about output growth rate. The model shows that, in the steady state, the growth rate of physical capital and output are constant over time. Once the steady state level of capital is reached, the economy doesn't develop anymore. The fundamental equation of Solow-Swan model is

$$\dot{K} = I(t) - \delta K(t) = s * F[K(t), L(t), T(t)] - \delta K(t)$$

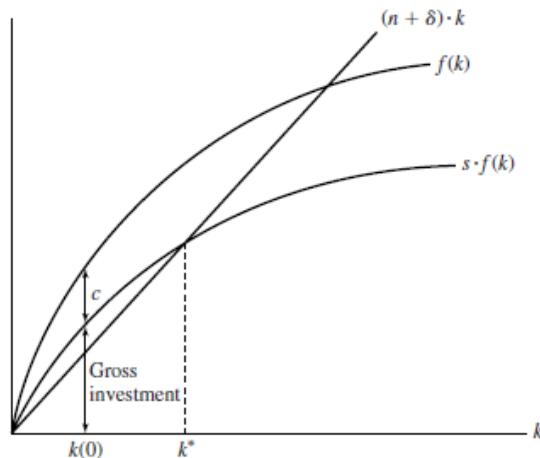
Where dot variables are time differentiated and  $\delta$  is the rate of depreciation of capital.

In per capita term the fundamental equation becomes:

$$\dot{k} = s * f(k) - (n + \delta) * k$$

a graphical interpretation is given in Figure 2.1

Figure 2.1: *The Steady State. Picture from Barro-Sala-i-Martin.*



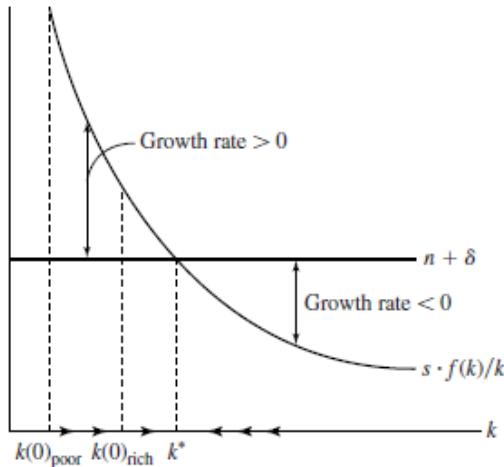
In this model, the steady state corresponds to  $\dot{k} = 0$ , and this is satisfied only if:

$$s * f(k^*) = (n + \delta) * k^*$$

Where  $k^*$  is the steady state level of capital, which is constant over time.

The second prediction of the model is about convergence. The model contemplates two kind of convergence concepts: relative and absolute. The latter states that all countries similar for their structure (same  $s$ ,  $n$ ,  $\delta$ ) will have the same value of capital and output in the steady state, but the speed of convergence is different according to the difference between actual level of capital and its steady state level.

Figure 2.2: *Convergence in Solow-Swan Model. Picture from Barro-Sala-i-Martin.*



The relative convergence states that a poor country (with a low  $s$ ) moves toward its steady state level of capital and output which is lower than the level of a rich one (with an high  $s$ ). Both countries move toward their own steady state but the speed movement and the nominal level are different. In Figure 2.2 the aforementioned situation is depicted.

With no technological progress, this model foresees that capital is the engine of the economy: the more capital a country accumulates, the higher is the propensity to save, the higher will be the output in steady state. It is possible to note that a change in  $s$  leads to a change of steady state level of  $k$  and  $y$  but in the steady state the dynamic of capital is null, and no more investments are done.

Although this model has been very popular in his simplicity, it does not provide a good picture. One could say to increase saving rate infinitely, but it is a ratio, hence bounded. The initial question about income per capita growth has not been answered, and this leads to a conclusion: investing and accumulating capital do not lead to the perpetual growth of income per capita.

At this point it is worth assessing whether a growth in technology causes a permanent growth in income per capita.

## 2.2. The Lucas-Uzawa Model

In neoclassical models, no attention is given to human capital. Growth of physical capital had an effect on the growth of GDP. The rise of human capital theory (Schultz 1961; Becker 1964) and the awareness that neo-classical models was not able to explain positive and persistence growth rates led to the introduction of the new growth theories. One of the first models presented in literature is the one of Romer (1986), however the most used models are of Lucas (1988) and Romer (1990). In this section the former will be taken into consideration. The basic difference from neo-classical models is about the number of sectors involved in the economy. In the first, human capital and physical capital are used to produce output, according to the following form:

$$Y = AK^\alpha(uhL)^{1-\alpha}h_a^\gamma$$

where:

- $A$  is the level of technology;
- $K$  is physical capital;
- $u$  is the time devoted to productive activities;
- $h$  is the per capita human capital;
- $L$  is the labour force;
- $h_a^\gamma$  is the average positive external effect of human capital (it must not be confused with the human capital);

Note that in this sector human capital presents diminishing returns.

The second sector includes the share of human capital not utilized in the productive sector, but used for producing extra human capital. Only if this exhibits non-diminishing returns an endogenous growth is possible. This is written as:

$$\dot{h}_t = h_t B(1 - u_t) - \delta h_t$$

where:

- $\delta$  is the depreciation of human capital;
- $B(1-u)$  indicates the increase in the amount of human capital;

In other words,  $B$  is a technical parameter indicating at what rate investments in the second sector are converted to a growth of human capital, and  $(1-u)$  is the share of human capital that is devoted to its formation.

By manipulating the previous equations, the output dynamic, in term of rate, is given:

$$\frac{\dot{y}}{y} = \frac{1-\alpha+\gamma}{1-\alpha} B(1-u) = \frac{1-\alpha-\gamma}{1-\alpha} \frac{\dot{h}}{h}$$

Assuming

- $u$  constant over time;
- $\delta = 0$ ;
- a balanced growth path  $\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y}$

according to Diebolt and Monteils (2000).

### Theoretical predictions

Growth can thus be caused by the effectiveness of human capital accumulation,  $B$ , the positive externalities of human capital,  $\gamma$ , and the share of human capital devoted to human capital accumulation,  $1-u$ . All growth in output is thus derived from human capital growth. This means that endogenous growth can only exist if there is a constant growth of human capital, which in turn can only be the case if there are constant or increasing marginal returns to human capital accumulation. As can be seen in the  $\frac{\dot{y}}{y}$  equation, this causes growth in production even without the presence of positive external effects. Positive external effects can accelerate growth, but in itself cannot cause endogenous growth.

### 3. DATA DESCRIPTION

#### 3.1. The sample

We used PENN WORLD table as data source. The dataset is composed by three main variables.

##### Variables

- The *GDP* at constant (2005) national prices in the Penn world tables means GDP converted to dollars using purchasing power parity (PPP) rates. We computed the pro capita variable dividing by the population (available on Penn world table)
- The *capital stock* in the aforementioned tables reports capital stock levels in terms of the constant (2005) prices. The same treatment has been done for collecting per capita variable.
- The *index of human capital* per capita provides an index of human capital per person, which is related the average years of schooling and the return to education.

The time period 1950 to 2011 has been considered, collecting 62 observations for each series. No missing data are observed, with exception for China, as long as the first two years are missing values.

As said before, many economists have been focusing on the definition of human capital. Although capital stock is easy to be measured, for human capital is a hard matter. Penn World tables use Barro-Lee (2012) dataset, about the average years of schooling, and the one of Psacharopoulos about rates of education return, in order to build up an index. In this intention, the developer did not aim at comparing differences in the number of workers in a country, but rather differences in labour service. Workers with different amounts of human capital will have different marginal products and in a broad sense, this will depend on a worker's innate talent and the amount and quality of formal schooling, on-the-job training and experience. In practice, the amount of human capital can be approximated by a limited number of observable characteristics, primarily the amount of formal schooling. The quality of education, as reflected in internationally comparable test scores, is also increasingly flagged as an important dimension of human capital. We think that many other kinds of measures are available in literature, but for this dissertation we do not provide any comparison of them.

### 3.2. Descriptive Statistics

#### GDP per capita

In this section the main statistics will be shown in order to depict a better figure of the sample.

Tables 3.1 - 3.2 show main descriptive statistics, while Figure 3.1 - 3.2 depict the real GDP per capita over the considered period.

Table 3.1: *GDP of BRICS Countries from 1950 to 2011.*

Country	Mean	Std. Dev.	Min	Max
BRAZIL	5398.37	2259.11	1700.03	9390.63
CHINA	2141.83	2586.55	315.12	10538.37
INDIA	1388.36	790.59	639.22	3779.88

Table 3.2: *GDP of OECD Countries from 1950 to 2011.*

Country	Mean	Std. Dev.	Min	Max
AUSTRALIA	23262.64	8236.37	11283.40	38513.48
CANADA	23321.38	8285.67	10881.71	36926.90
GERMANY	21021.27	8842.88	5429.05	35361.09
FRANCE	18998.09	7642.29	6499.32	30248.95
GBR	19644.75	7358.51	9673.37	33723.98
JAPAN	18747.70	10356.52	2832.43	32879.67
NORWAY	35735.52	17098.34	12568.30	63954.03
SWEDEN	20694.25	7561.31	9042.09	35034.85
USA	26747.71	9863.09	12725.07	43483.53

Figure 3.1: *GDP of BRICS Countries.*

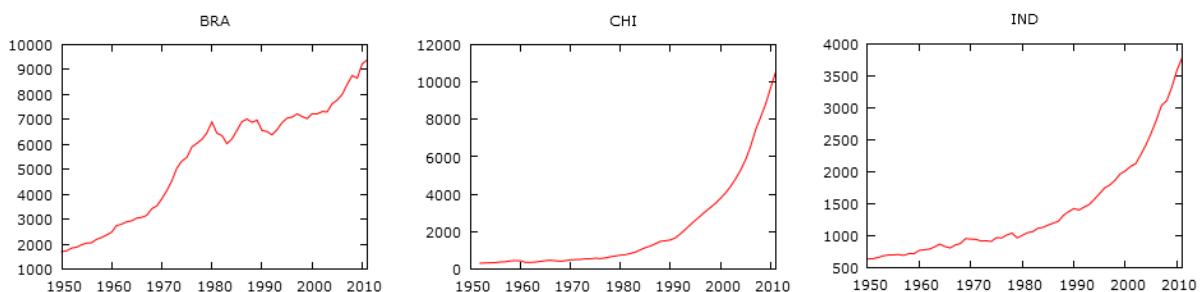


Figure 3.2: *GDP of OECD Countries.*

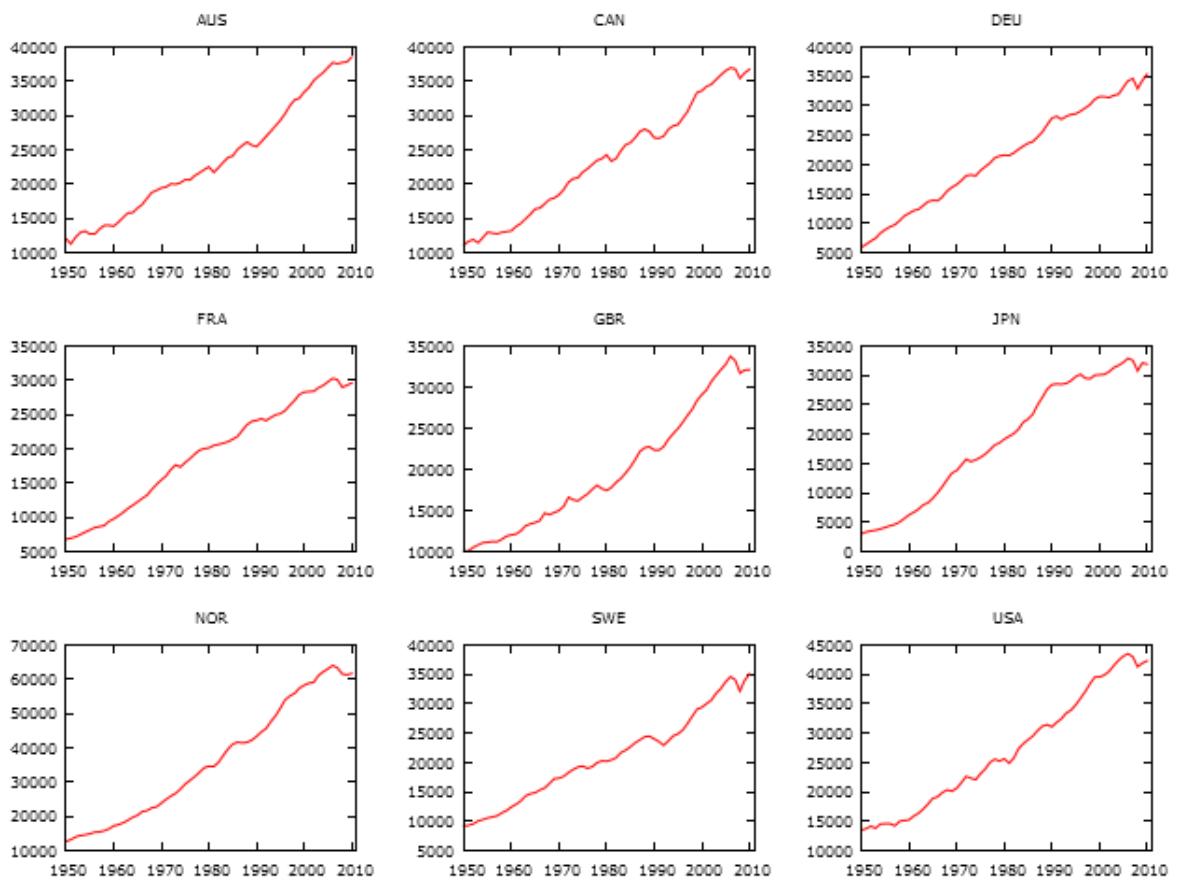
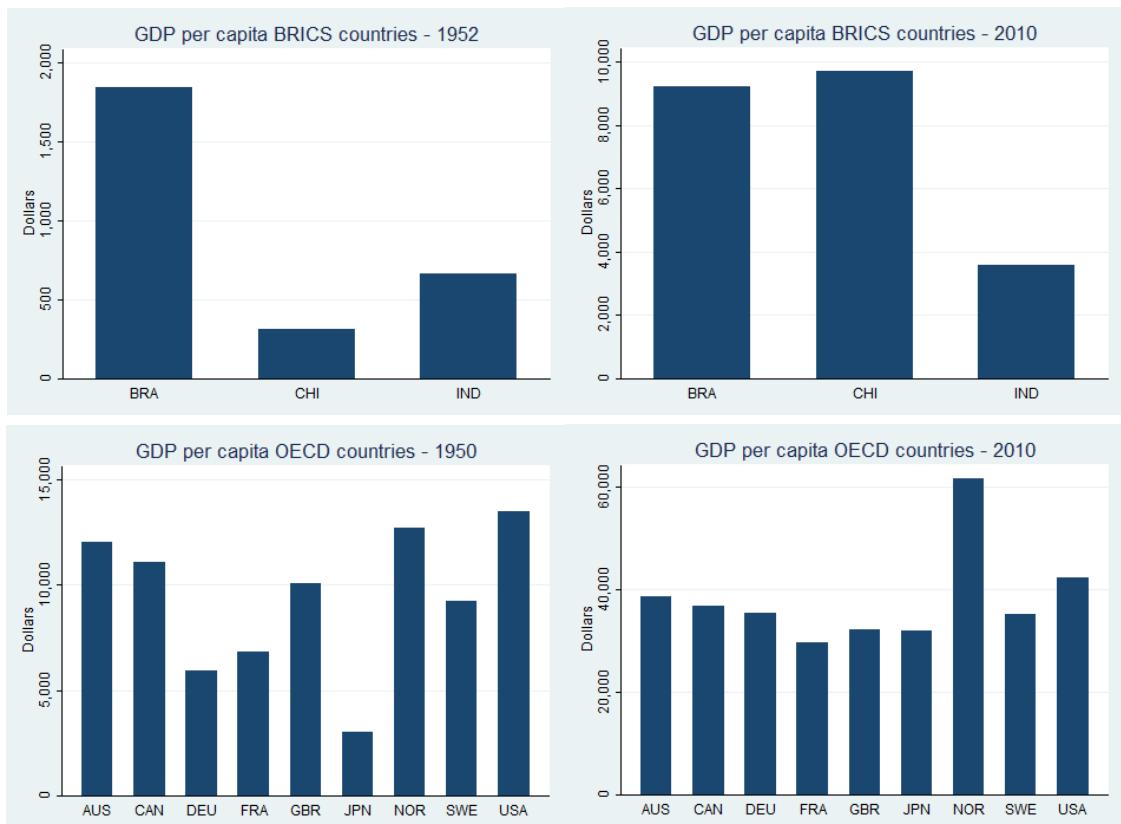


Figure 3.3: *Comparison between Initial and Final GDP per capita.*



As it is possible to see from the above tables, the average income per capita during the period considered is so much higher in OECD countries than BRICS countries. In the former, the highest income belongs to USA and Brazil for the latter. Brazil has a different level in income per capita yet from the beginning of the series. This is a clear signal that this country has experienced a growth before the 1950s. As highlighted in Figure 3.1 – 3.3, the time series of Brazil shows a higher level of GDP per capita yet from 1950 than the other countries.

According to this observation, the Wilcoxon test (Table 3.3) has been performed in order to verify a possible significant difference in income per capita between Brazil and other BRICS countries.

Table 3.3: *p-values of Wilcoxon test.*

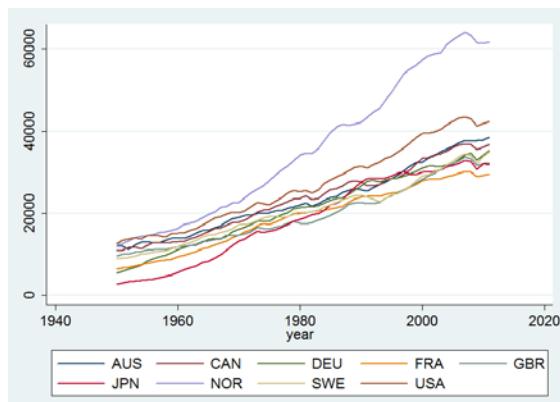
p-values	BRAZIL	CHINA	INDIA
BRAZIL	-	-	-
CHINA	0	-	-
INDIA	0	0.14	-

From Table 3.3 it is possible to highlight the significant difference in BRICS countries with respect to Brazil. The history of Brazil supports this evidence as long as, during the half of XX century, its economy boosted sharply. As depicted in Figure 3.1 – 3.3, China and India have grown later but at a higher rate.

OECD countries seem to have experienced the same trend during the past years, as represented in Figure 3.2 – 3.3.

In the following graph, the concept of absolute convergence is shown for OECD countries, according to Solow-Swan model.

Figure 3.4: *Convergence of OECD Countries.*



## Capital Stock per capita

As it is possible to see from Figure 3.5 – 3.7, the level of capital stock per capita during the period considered is so much higher in OECD countries than BRICS countries. Also in this case, Brazil shows a higher initial level of capital stock than other BRICS countries.

Figure 3.5: *Capital Stock of OECD Countries.*

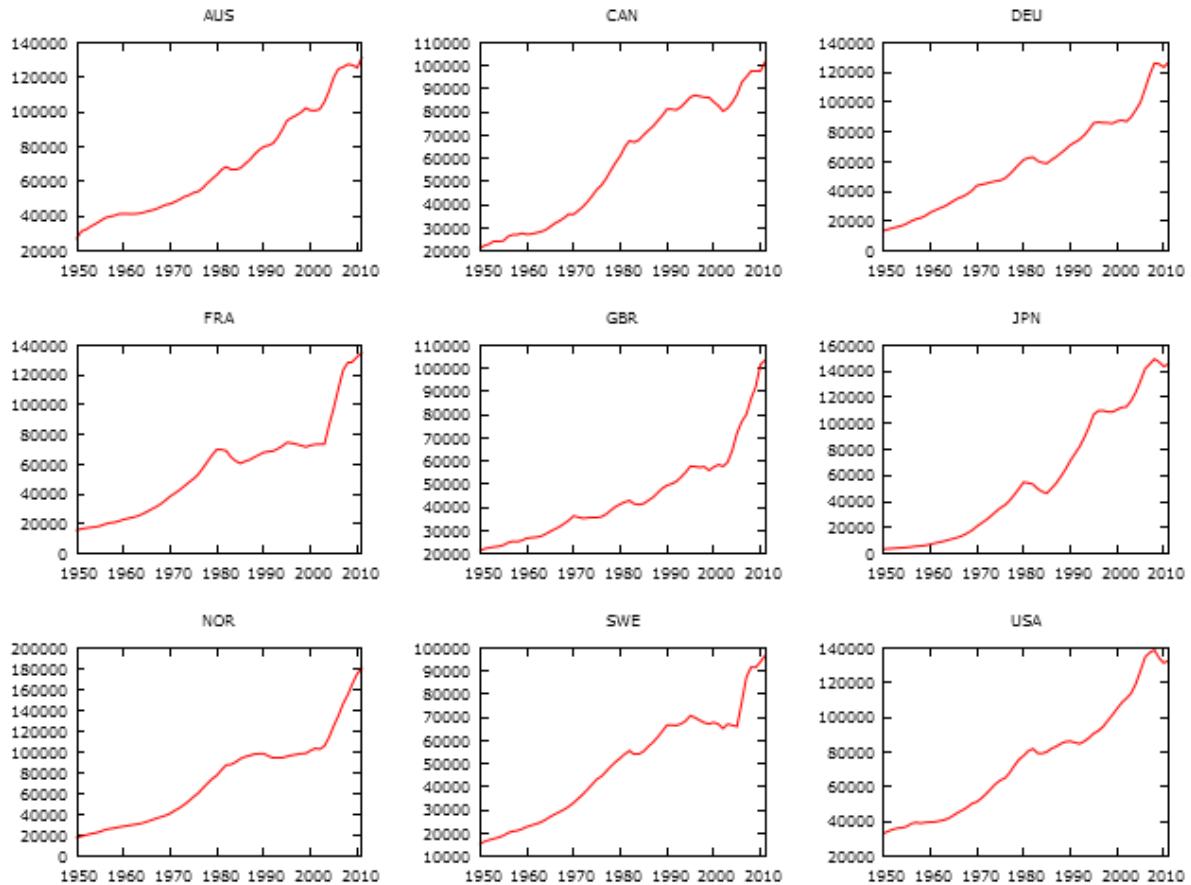


Figure 3.6: *Capital Stock of BRICS Countries.*

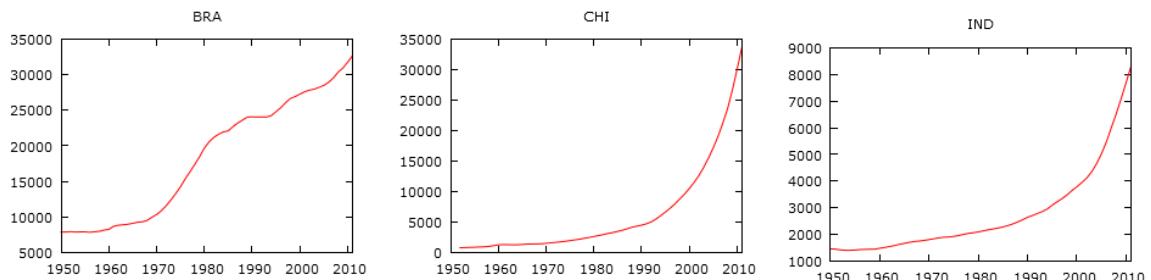
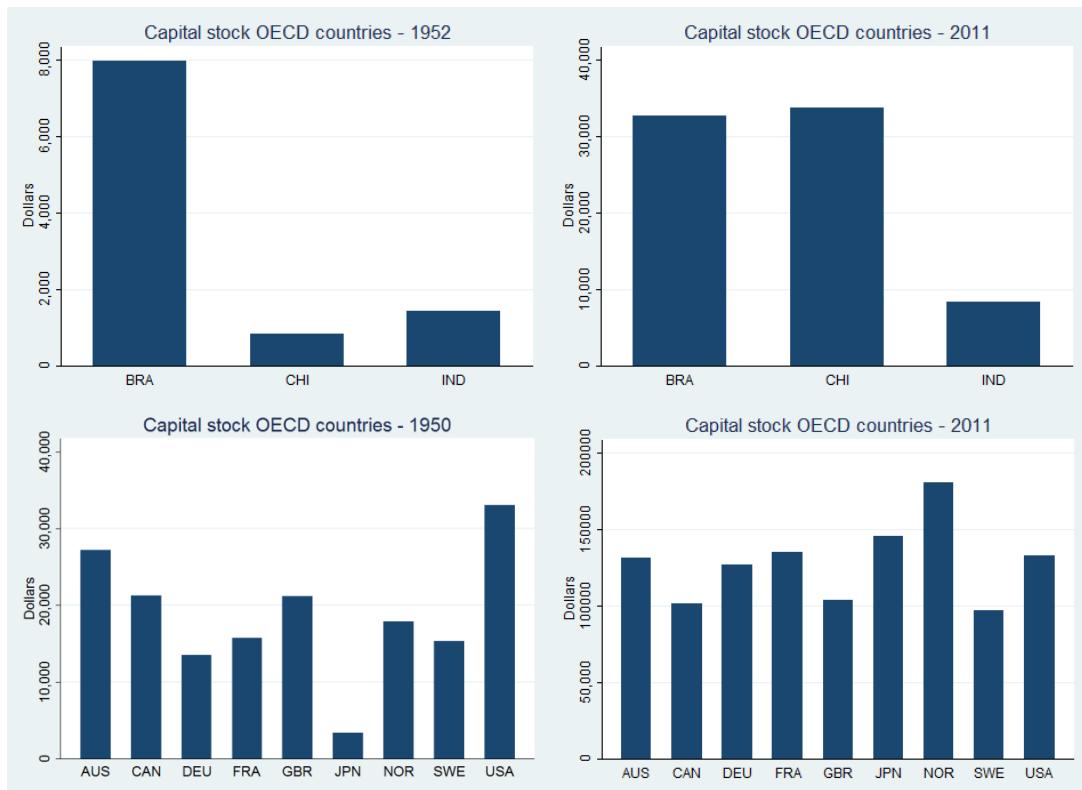


Figure 3.7: Comparison between Initial and Final Capital Stock per capita.



## Human Capital (Index)

From Figure 3.9 – 3.11, it is possible to note that BRICS countries started from similar level of human capital index. Among OECD countries, the initial level of human capital index is quite different, but the gap has been reduced in past few years. It is relevant to note that many authors have found a strong correlation between R&D sector and human capital, as long as the latter could be the input of the former. In Figure 3.8 is shown which countries have the highest level of expenditure in R&D.

Figure 3.8: *Research and Development Expenditure Map.*

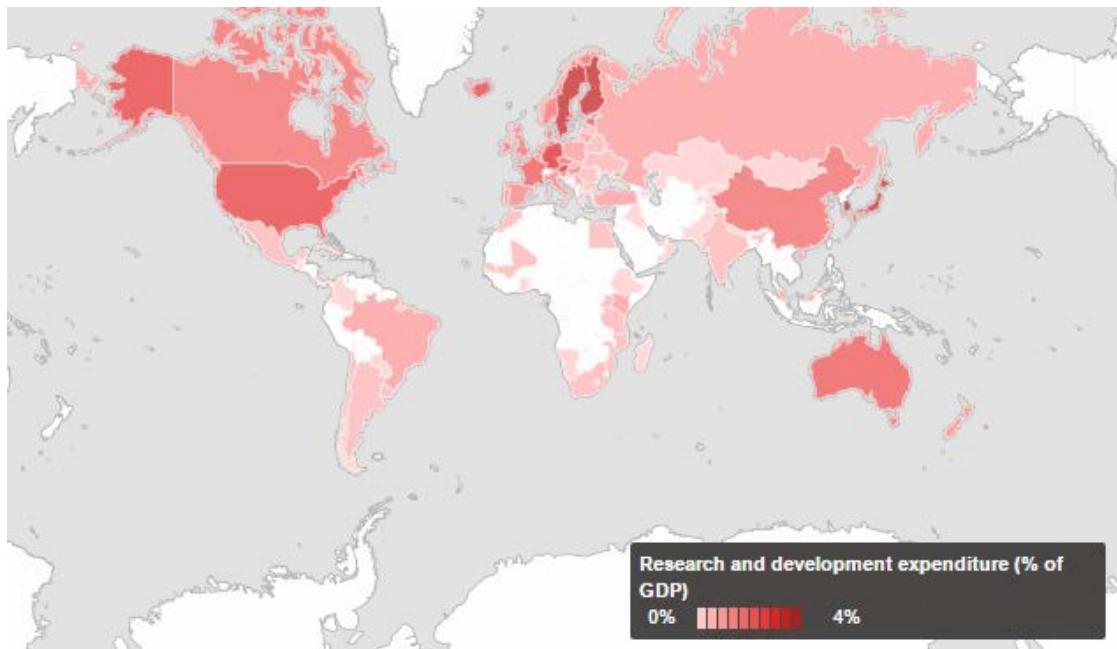


Figure 3.9: *Comparison between Initial and Final Human Capital Index.*

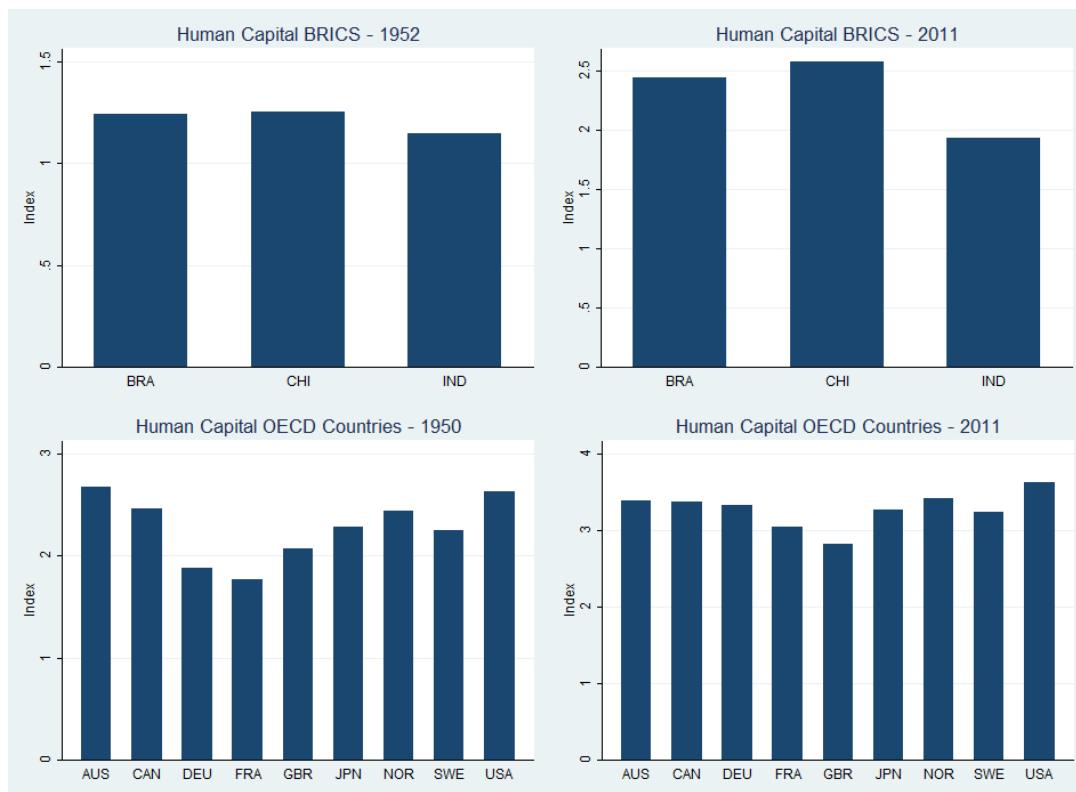


Figure 3.10: *Human Capital Index of OECD Countries.*

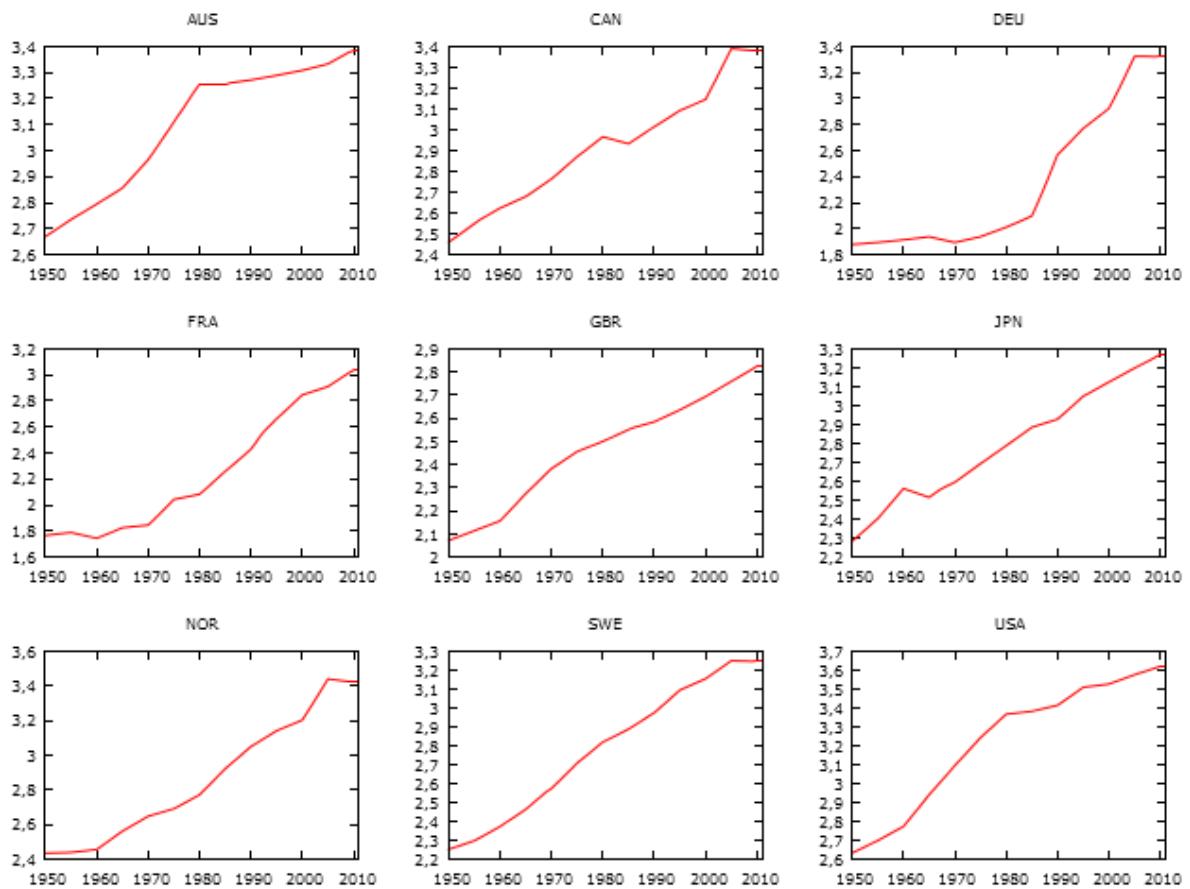
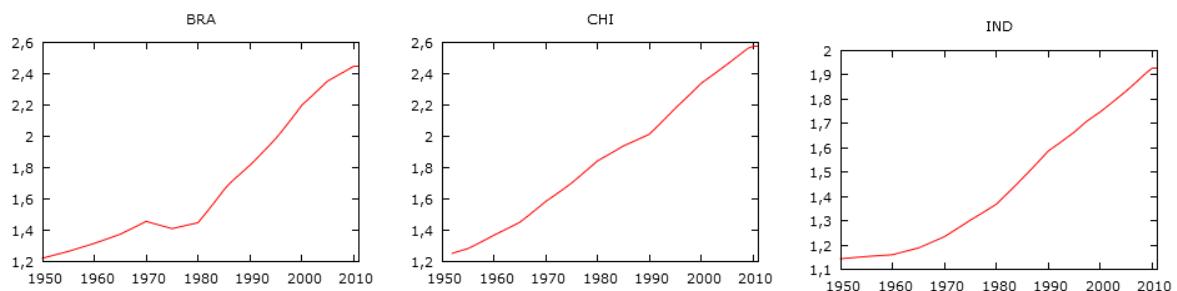


Figure 3.11: *Human Capital Index of BRICS Countries.*



## 4. THE CO-INTEGRATION ANALYSIS

In the following section the procedure adopted will be explained. It is worth noting that all the variables are expressed in logarithm, in order to handle with growth rates using differences (for instance, the variable labelled GDP stands for its logarithm).

Furthermore, the aforementioned growth models will be empirically tested using some econometric tools. In particular the procedure adopted involves Augmented – Dickey Fuller test (henceforth, ADF test) and Co-integration analysis following the Engle-Granger representation (1987). The procedure requires the following steps:

1. Defining the integration order of the series using DF/ADF tests. In order to carry on the co-integration analysis, the integration order of the series must be the same and at least equal to one (usually it is indicated with  $I(1)$ );
2. Estimating the long-run relationship between variables through a OLS regression model and collecting residuals. The super-consistent coefficients set up the co-integration vector;
3. Using the DF/ADF test to estimate the residuals integration order (using a pure ADF without constant).
4. If the residuals are stationary, than the error correction model (henceforth, ECM) can be estimated. The lags selection procedure contemplates the use of t and F tests for discarding insignificant regressors.

The formal specification of the ECM is:

$$\Delta y_t = \beta_0 + \gamma \hat{\varepsilon}_{t-1} + \beta_1 \Delta x_t + \sum_{j=1}^q \delta_{1+j} \Delta y_{t-j} + \sum_{i=1}^p \beta_{1+i} \Delta x_{t-i} + \zeta_t$$

where

- $\Delta y_t$  and  $\Delta x_t$  are the first differences of considered series.
- $\hat{\varepsilon}_{t-1} = y_t - (\beta_0 + \beta_1 x_t)$  is the error correction term (ECT) coming from the OLS model (point 2)
- $\zeta_t$  is the erratic component
- $q$  and  $p$  are the number of lagged variables respectively for  $\Delta y$  and  $\Delta x$

The coefficient of the residuals  $\gamma$  is supposed to be negative in order to correct the gap from long-run equilibrium. It is commonly known that the relationship between co-integration analysis and ECM is biunivocal.

## 4.1.ADF Test Procedure

In this section the ADF test results will be shown. According to the iter shown in the previous section, the test procedure is the following:

considering the GDP series, the hypothesis testing will be executed:

$$\Delta GDP_t = \beta GDP_{t-1} + \varepsilon_t$$

under null ( $H_0$ ) and alternative ( $H_1$ ) hypothesis:

$$\begin{aligned} H_0 &: \beta = 0 \\ H_1 &: \beta < 0 \end{aligned}$$

If  $H_0$  is rejected, the series is stationary, else it could be at least  $I(1)$ . In the last case, the test must be computed on the following regression:

$$\Delta^2 GDP_t = \beta \Delta GDP_{t-1} + \varepsilon_t$$

Table 4.1-4.6 show the ADF test results for the variables GDP per capita, index of human capital per capita and capital stock per capita for both BRICS and OECD countries.

The ADF test can be performed in different ways (with or without constant, with constant and trend or with constant and quadratic trend). This feature is specified in the column “Deterministic component”.

Table 4.1: *ADF tests on GDP of BRICS Countries.*

BRICS Country (logarithm value)	Integration Order GDP	Deterministic Component	p-value (ADF)	Lag selection
Brazil	2	No	***	0
China	2	No	***	0
India <sup>1</sup>	2	No	***	1

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

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<sup>1</sup> Lag selection method used consider MAIC with a maximum of 10 lags. Graphically the series of second differences are stationary.

Table 4.2: *ADF tests on Human Capital of BRICS Countries.*

BRICS Country (logarithm value)	Integration Order HC	Deterministic Component	p-value (ADF)	Lag selection
Brazil	2	No	***	0
China	2	No	***	0
India <sup>2</sup>	2	No	***	0

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

Table 4.3: *ADF tests on Capital Stock of BRICS Countries.*

BRICS Country (logarithm value)	Integration Order CS	Deterministic Component	p-value (ADF)	Lag selection
Brazil	2	No	***	0
China <sup>3</sup>	2	No	***	1
India <sup>4</sup>	2	No	**	0

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

Table 4.4: *ADF tests on GDP of OECD Countries.*

OECD Country (logarithm value)	Integration Order GDP	Deterministic Component	p-value (ADF)	Lag selection
Norway	1	No	*	2
United States	2	No	***	1
Australia	1	Constant ***	***	1
Canada	1	No	*	2
Germany	1	Trend *** Constant ***	***	1
Sweden	1	Constant **	**	1
United Kingdom	1	Constant **	*	0
Japan	2	No	***	1
France	1	Trend *** Constant ***	***	0

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

<sup>2</sup> We have deleted the last value of the series. It is a neglecting outlier.

<sup>3</sup> Lag selection method used involves MAIC with a maximum of 19 lags. Graphically the series of the second differences is stationary.

<sup>4</sup> Lag selection method used involves MAIC with a maximum of 7 lags. Graphically the series of the second differences is stationary.

Table 4.5: *ADF tests on Human Capital of OECD Countries.*

OECD Country (logarithm value)	Integration Order HC	Deterministic Component	p-value (ADF)	Lag selection
Norway	1	Constant **	*	0
United States	2	No	***	0
Australia	2	No	***	0
Canada	1	Constant **	**	0
Germany	2	No	***	0
Sweden	2	No	***	0
United Kingdom	2	No	***	0
Japan	1	Constant***	***	4
France	2	No	***	0

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

Table 4.6: *ADF tests on Capital Stock of OECD Countries.*

OECD Country (logarithm value)	Integration Order CS	Deterministic Component	p-value (ADF)	Lag selection
Norway	2	No	***	0
United States	2	No	***	1
Australia	1	Constant ***	***	0
Canada	1	No	**	0
Germany	1	Constant***	***	1
Sweden	1	No	*	2
United Kingdom	1	Constant ***	***	0
Japan	1	Trend * Constant ***	**	1
France	1	Constant **	**	0

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

## 4.2. Testing the Models

Once the integration orders have been computed, it is possible to test which countries are eligible for being described by theoretical models.

To do this, it is supposed that the countries must meet some requirements, according to the considered model.

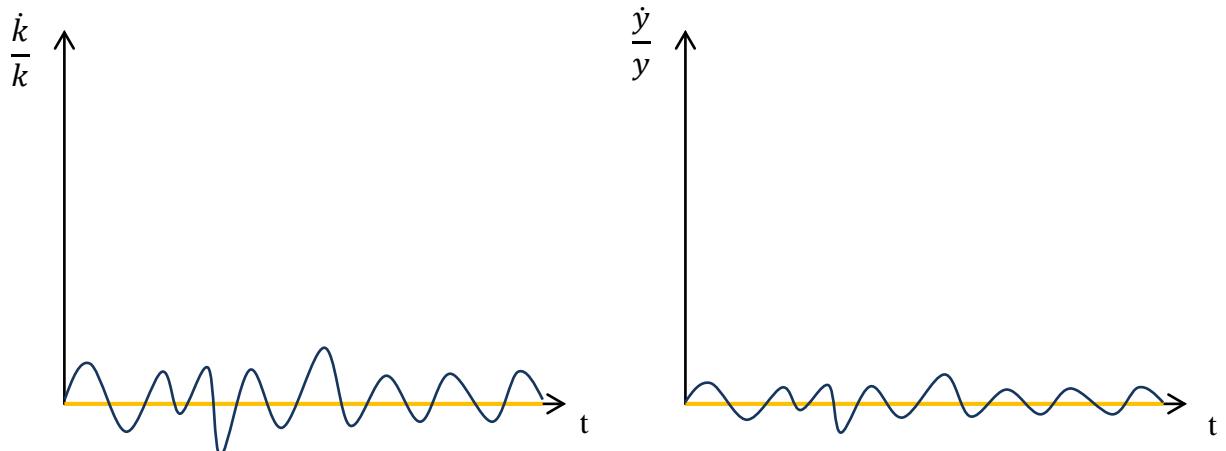
### 4.2.1 Solow-Swan Model Conditions

The Solow-Swan Model states that the growth rate of per capita variables (GDP and capital stock) must be zero in the steady state:

$$\frac{\dot{k}}{k} = 0 \quad \frac{\dot{y}}{y} = 0$$

Considering the Figure 4.1, the growth rates should move around the t axis.

Figure 4.1: *The Steady State Path in Solow Model.*



For our purposes, the only time series that will be taken into account are:

- GDP and capital stock per capita  $I(1)$ , stationary without constant.

According to our results, almost none of the selected countries (neither BRICS nor OECD) seems to meet the aforementioned conditions, therefore the co-integration analysis cannot be carried on. The only country that seems to meet the conditions is Canada, but from our analysis, the OLS model is spurious and ECM is not valid<sup>5</sup>.

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<sup>5</sup> For layout reasons the analysis is not reported.

### 4.2.2 Lucas-Uzawa Model (1988) Conditions

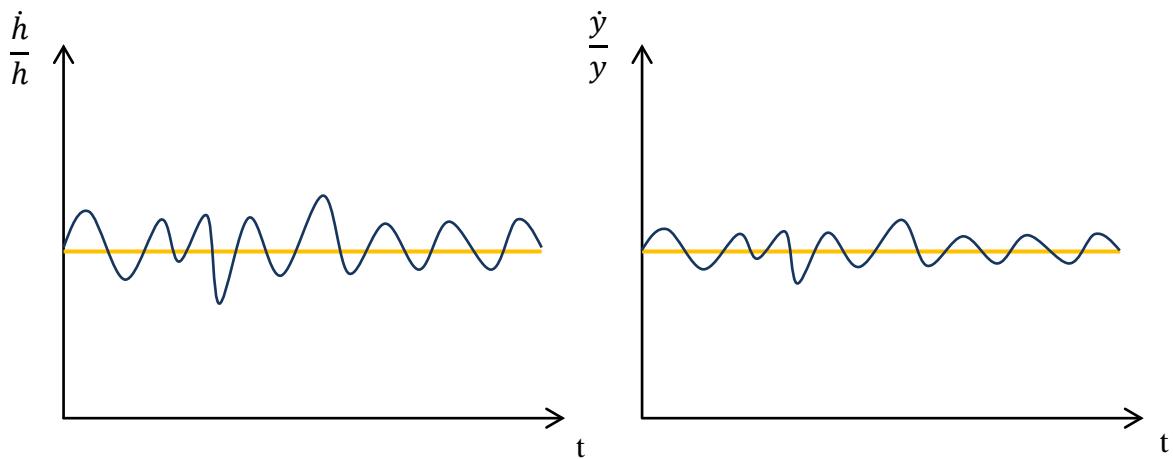
The Lucas-Uzawa Model states that the growth rate of per capita variables (GDP, capital stock and human capital) must be positive in the steady state:

$$\frac{\dot{y}}{y} > 0 \quad \frac{\dot{h}}{h} > 0 \quad \frac{\dot{k}}{k} > 0$$

In particular, according to section xxx, a *balanced growth path* is assumed.

Considering the Figure 4.2, the growth rates should move around a positive constant.

Figure 4.2: *The Steady State Path in Lucas Model.*

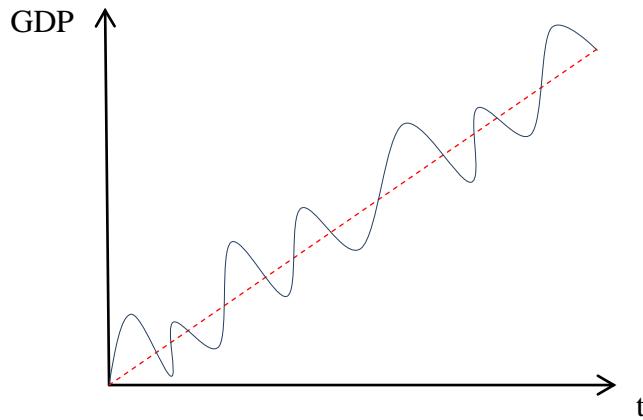


The time series of GDP, capital stock and human capital per capita that will be taken into account must be:

- $I(1)$ , in particular stationary with constant  $\Rightarrow$  first differences (growth rates) are stationary around a constant;
- or
- $I(2)$ , stationary without constant  $\Rightarrow$  second differences are stationary around t axis.

It is relevant to note that there is another eligible case in which a series could be  $I(1)$  and stationary around a trend without constant, but it is unfeasible, from an economic point of view, as long as the series should start from the origin of axis (Figure 4.3).

Figure 4.3: Time Series Starting from Origin and Stationary around a Trend.



Only BRICS countries satisfy the previous two assumptions and the hypothesis of balanced growth path is satisfied. Among OECD countries, only the USA seems to meet the conditions, but according to our analysis the OLS regression is not valid and co-integration analysis is unfeasible.

In the following, the results of BRICS countries are shown. All ECM tables are the results of lags selection procedure.

### Brazil

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The long-run relationship is going to be tested:

$$\Delta GDP_t = \beta_0 + \beta_1 \Delta HC_t + \varepsilon_t$$

The relative results are shown in Table 4.7.

Table 4.7: OLS Regression between  $\Delta GDP$  and  $\Delta HC$  of Brazil

	Coefficient	Standard error	t statistic	p-value	
const	0,0552	0,0061	8,9982	<0,0001	***
$\Delta HC_t$	-2,3970	0,5923	-4,0470	0,0002	***

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

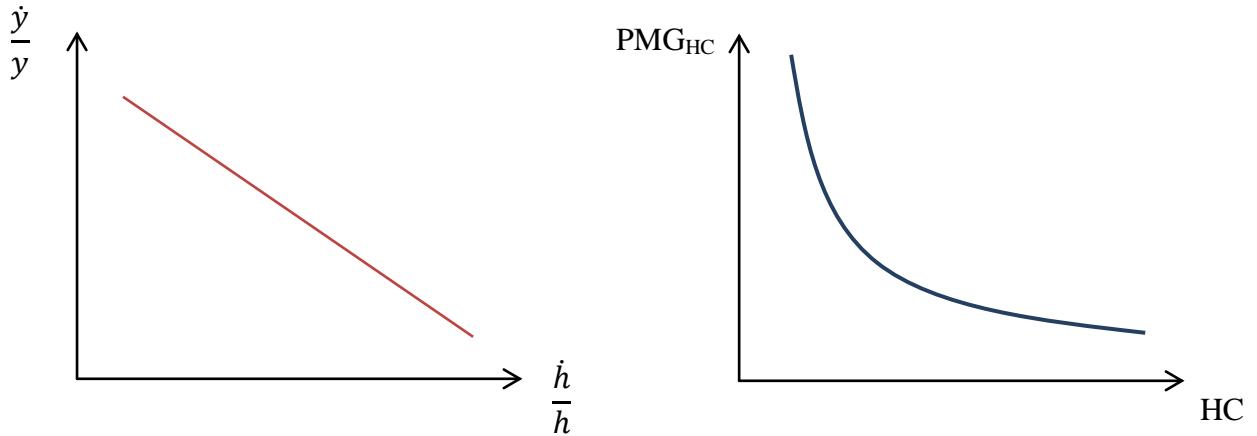
R-squared 0,323809

Durbin-Watson statistic = 1,934596, p-value = 0,354386

It is important to note that the sign of  $\Delta HC$  coefficient is negative. From an economic point of view, this sheds light on *neoclassical assumption* about productivity of factors.

Being that  $\Delta HC = \frac{\dot{h}}{h}$ , because of logarithm transformation, as  $\frac{\dot{h}}{h}$  increases, human capital increases and according to assumption on decreasing marginal productivity, the marginal effect on output decreases. Therefore, a slowdown of output growth rate is expected.

Figure 4.4: *The Marginal Effect*.



The next step is about testing residuals' stationarity. The asymptotic p-value of ADF test is lower than 0.0001, therefore the residuals are stationary.

In order to depict the short-run relationship according to the Engle-Granger representation, the ECM has been computed and the results are shown in Table 4.8. Note that the involved variables represent the variation of growth rate over time.

$$\Delta^2 GDP_t = \beta_0 + \gamma \hat{\varepsilon}_{t-1} + \beta_1 \Delta^2 HC_t + \sum_{j=1}^q \delta_{1+j} \Delta^2 GDP_{t-j} + \sum_{i=1}^p \beta_{1+i} \Delta^2 HC_{t-i} + \zeta_t$$

Table 4.8: *Error Correction Model of Brazil*.

	Coefficient	Standard error	t statistic	p-value
const	0,0006	0,0032	0,1692	0,8663
Residuals <sub>t-1</sub>	-1,4447	0,1795	-8,0459	<0,0001 ***
$\Delta^2 HC_t$	-3,0170	1,0591	-2,8485	0,0065 ***
$\Delta^2 HC_{t-3}$	1,7962	0,5873	3,0581	0,0036 ***
$\Delta^2 HC_{t-4}$	3,1723	0,4750	6,6772	<0,0000 ***
$\Delta^2 GDP_{t-1}$	0,3029	0,1359	2,2280	0,0307 **
$\Delta^2 GDP_{t-2}$	0,4799	0,1090	4,4009	0,0000 ***
$\Delta^2 GDP_{t-3}$	0,3796	0,1132	3,3533	0,0015 ***
$\Delta^2 GDP_{t-4}$	0,2038	0,1054	1,9328	0,0593 *

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

R-squared (corrected) = 0,601219.

Durbin-Watson statistic = 1,948240, p-value = 0,422098.

The ECT coefficient,  $\gamma$  is negative and significant, as expected, in order to correct the gap from long-run equilibrium.

### China

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The same empirical findings, both economic and econometric, of Brazil are valid for China (Table 4.9 - 4.10).

Table 4.9: *OLS Regression between GDP and HC of China*<sup>6</sup>.

	<i>Coefficient</i>	<i>Standard error</i>	<i>t statistic</i>	<i>p-value</i>	
const	0,0842	0,0093	9,0330	<0,0001	***
$\Delta HC_t$	-2,2111	0,8790	-2,5150	0,0147	**

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

R-squared = 0,009

Durbin-Watson statistic = 1,098

Table 4.10: *Error Correction Model of China*.

	<i>Coefficient</i>	<i>Standard error</i>	<i>t statistic</i>	<i>p-value</i>	
const	0,0030	0,0062	0,4913	0,6252	
Residuals <sub>t-1</sub>	-0,4478	0,1129	-3,9641	0,0002	***
$\Delta^2 GDP_{t-3}$	-0,2648	0,1408	-1,8810	0,0656	*

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

R-squared (corrected) = 0,304712.

Durbin-Watson statistic = 1,8521, p-value = 0,4016.

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<sup>6</sup> We performed a regression using HSK-WSL model, obtaining the same residuals of OLS.

## India

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In this case, empirical findings are different from other BRICS countries. It seems that there is no co-integration relationship, because the asymptotic p-value of ADF test does not reject the null hypothesis (p-value = 0,4132) and the coefficient of the error correction term is not significant. (Table 4.11 – 4.12). This undermines the existence of a co-integration relationship.

Table 4.11: *OLS Regression between GDP and HC of India.*

	<i>Coefficient</i>	<i>Standard error</i>	<i>t statistic</i>	<i>p-value</i>	
Const	0,0195	0,0065	3,0149	0,0038	***
$\Delta HC_t$	1,1173	0,6372	1,7534	0,0847	*

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

R-squared = 0,061828

Durbin-Watson statistic = 1,7158, p-value = 0,1067 <sup>7</sup>

Table 4.12: *Error Correction Model of India.*

	<i>Coefficient</i>	<i>Standard error</i>	<i>t statistic</i>	<i>p-value</i>	
const	0,0027	0,0038	0,7032	0,4853	
Residuals <sub>t-1</sub>	-0,3256	0,2098	-1,5520	0,1272	
$\Delta^2 GDP_{t-1}$	-0,6283	0,2006	-3,1319	0,0029	***
$\Delta^2 GDP_{t-2}$	-0,4589	0,1486	-3,0878	0,0033	***
$\Delta^2 GDP_{t-3}$	-0,4342	0,1317	-3,2952	0,0018	***
$\Delta^2 GDP_{t-4}$	-0,5302	0,1592	-3,3298	0,0016	***
$\Delta^2 GDP_{t-5}$	-0,3486	0,1102	-3,1622	0,0027	***

p-value: \* significant at 10%, \*\* significant at 5%, \*\*\* significant at 1%.

R-squared (corrected) = 0,4709.

Durbin-Watson statistic = 2,036452, p-value = 0,5344

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<sup>7</sup> The regression seems to be spurious

## 5. CONCLUSION

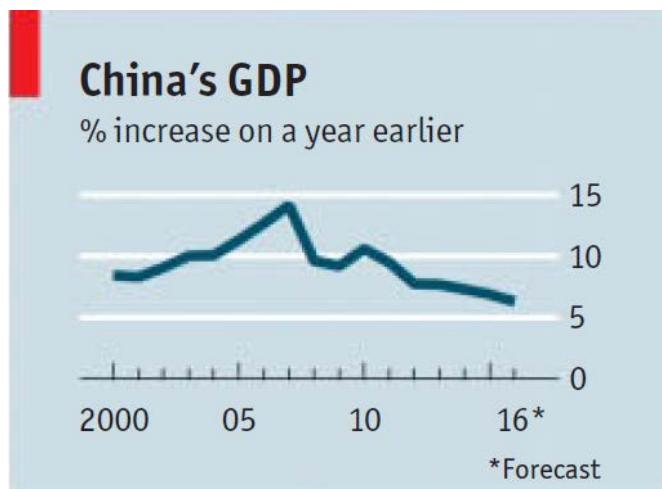
After having tested the suitability of Solow-Swan and Lucas-Uzawa models, it is possible to summarize the main empirical findings and to argue about their usefulness.

The Solow-Swan growth model advises policy maker to spur investments and accumulate capital which is the “engine” of the economy’s growth. Obviously, this point could be achieved by increasing the propensity to save using stimulating policies. By looking at the previous graphs in section 3, both OECD and BRICS countries have accumulated capital in the past years. Among BRICS, China has increased its capital stock more than other countries, reaching Brazil’s level that was the leader in 1950. This could lead to think about China’s capital-accumulating policies and the possibility that its great growth could be explained by some neo-classical model, such as Solow-Swan. The same results can be extended to Japan, which has experienced an epic growth in the last three decades. On the light of this observations, one could say that Solow-Swan model fits well on this empirical cases, but our findings highlight that no one of the sampled countries (both OECD and BRICS) is eligible to be described by this model. Besides this, it is important to note that a country cannot increase its propensity to save to infinity, hence once the economy reaches the steady state, growth rates stop. Nevertheless, in December an adviser to China’s government predicted that GDP growth would soon rebound (Figure 5.1). China’s prime minister announced a GDP growth target of 6.5%-7% for 2016, using budget deficit and credit growth. According to his opinion, China will avoid a sharp economic slowdown. But Mr. Li did not only open the macroeconomic issue, but he hinted that a fiscal boost will be designed to help rebalance the economy, reducing private firms’ fiscal burden. In this way, government seems to lay on its usual recipe of debt and investment fuelled growth.<sup>8</sup> According to the previous analysis, the role of capital seems not to be relevant in Chinese economy, and many experts confirm this opinion.

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<sup>8</sup> Information reported from The Economist (March 12<sup>th</sup>-18<sup>th</sup> 2016)

Figure 5.1: *China's GDP Forecast* (picture from *The Economist*).



The attention must be paid on the USA and India cases. These two countries are eligible to be described by Lucas-Uzawa model, but empirical results seem to deny this conclusion. It is worth noting that a test of Solow-Swan model with Technological progress could be conducted, in order to test whether the positive growth rates are not due to human capital impact but to the exogenous increase of knowledge. This requires the time series must be I(1) with constant or I(2) around time axis.

The Lucas-Uzawa model seems to explain well the BRICS economies (China and Brazil). According to this result, policy maker should increase the care of human capital, by improving education, government expenditure and returns from schooling. Barro (1999) in accounting growth factors considered jointly human and physical capital, in order to assess the effect on growth rate. These variables all depend on policy variables, national characteristics and on initial values of state variables. He focused on the ratio of human to physical capital. The higher is the ratio, the higher is the economic growth, for two main reasons: first more human capital facilitates the absorption of superior technologies from leading countries; this channel is likely to be especially important for schooling at secondary and higher levels. Second, human capital tends to be more difficult to adjust than physical capital. So a country that starts with a high ratio of human capital to physical capital tends to grow rapidly by adjusting upward the quantity of physical capital.

OECD countries have developed human capital yet before 1950s and it is possible to suppose that countries are saturated, hence the marginal effect is negligible. The situation is opposite for BRICS countries.

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