

FORMULAS FOR LIFE

All formulas you'll ever need

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A helpful tool for scientific studies

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1 Statistics

1.1 Mean

1.1.1 Arithmetic mean

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{N}$$

1.1.2 Geometric mean

$$\bar{x} = \left(\prod_{i=1}^n x_i \right)^{\frac{1}{n}}$$

1.1.3 Harmonic mean

$$\bar{x} = n \left(\sum_{i=1}^n \frac{1}{x_i} \right)^{-1}$$

1.2 Standard Deviation

1.2.1 Absolute

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{N - 1}}$$

1.2.2 Relative

$$RSD = \frac{s}{\bar{x}}$$

2 PurePhysics

2.1 Uniform Rectilinear Motion

$$\begin{aligned}x(t) &= x_0 + v_0 t \\v &= v_0\end{aligned}$$

2.2 Uniform Accelerated Rectilinear Motion

$$\begin{aligned}x(t) &= x_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a_0} \\v(t) &= v_0 + at \\a(t) &= a_0\end{aligned}$$

2.3 Parabolic motion

On the x axis:

$$\begin{cases} a_x = 0 \\ v_x = v_{0x} \\ x(t) = x_0 + v_{0x}t \end{cases}$$

On the y axis:

$$\begin{cases} a_y = a_{0y} = g \\ v_y = v_{0y} + gt \\ y(t) = y_0 + v_{0y}t + \frac{1}{2}gt^2 \end{cases}$$

The range can be calculated as:

$$R = \frac{2v_{0x}v_{0y}}{g} = \frac{v_0^2 \sin 2\alpha}{g}$$

The maximum height can be calculated as:

$$h_{\max} = \frac{v_0^2 \sin^2 \alpha}{2g}$$

2.4 Uniform Circular Motion

2.4.1 Angular velocity

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

2.4.2 Angular acceleration

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

2.4.3 Relation with linear variables

Linear velocity

$$v = \omega r \quad (\text{radian measure})$$

Linear acceleration

$$\begin{aligned}a_{\text{tangential}} &= \alpha r \quad (\text{radian measure}) \\a_{\text{radial}} &= \frac{v^2}{r} = \omega^2 r\end{aligned}$$

Period

If the point moves in uniform circular motion :

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

2.5 Rotation with constant acceleration

$$\begin{aligned}\omega &= \omega_0 + \alpha t \\ \theta - \theta_0 &= \omega_0 t + \frac{1}{2}\alpha t^2\end{aligned}$$

2.6 Kinetic of rotation

$$E_k = \frac{1}{2}I\omega^2$$

I is the rotational inertia of the body; for a system of discrete particles it is defines as:

$$I = \sum m_i r_i^2$$

It has different formulas for different shapes

2.6.1 Armonic Motion

$$x(t) = A \sin(\omega * t + \phi)$$

It can be also calculated as:

$$\omega = \sqrt{\frac{k}{m}}$$

2.6.2 Centripetal and Centrifugal acceleration

$$a_c = \omega^2 r = \frac{v^2}{r}$$

2.6.3 Teorema dell'impulso

$$I = \Delta p = m\Delta v = F\delta t$$

2.6.4 Newton Second Law's for rotation

$$\tau = I\alpha$$

2.7 Angular momentum

$$L = I\omega = r \text{prodvet} m\vec{v}$$

$$\Delta L = \tau \quad \Delta l = I \text{vectr}$$

2.8 Frictional Force

$$\vec{F}_a = -\mu N$$

2.9 Centripetal Force

$$F_c = m \frac{v^2}{r} = n\omega^2 r$$

2.10 Rope's tension

$$I \propto T R$$

2.11 Gravitational Law

$$F = G \frac{m_1 m_2}{r^2} \quad 6.6743 \cdot 10^{11} \text{m}^3 \text{kg s}^2$$

Is also possible to calculate the Gravitational Acceleration

$$g = \frac{GM}{r^2}$$

2.12 Pendulum Equation

$$m \frac{d^2 x}{dt^2} + mg \sin \theta = 0$$

$$\frac{d^2}{dt^2} + \frac{g}{L} \sin \theta = 0 \quad \Rightarrow \quad \frac{g}{L} = \omega^2$$

If $\theta \approx 0$ then:

$$T = \frac{2\pi}{\omega} = \sqrt{\frac{L}{g}} \quad \theta(t) = \theta_0 + \cos(\omega t + \phi)$$

2.13 Energy

2.13.1 Work

$$\vec{L} = \vec{F} \cdot \Delta \mathbf{r} = F \Delta r \cos \theta$$

2.13.2 Kinetic Energy

$$E_c = \frac{1}{2} m v^2$$

$$L = \Delta E_c = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

2.13.3 Elastic potential energy

$$U_{el} = \frac{1}{2} k x^2$$

2.13.4 Gravitational Potential Energy

$$U_g = mgh$$

2.13.5 Mechanical energy

$$E_m = E_c + U$$

2.13.6 Power

$$P = \frac{W}{\Delta t}$$

2.14 Momentum

$$\vec{p} = m\vec{v}$$

2.15 Electrostatic

2.15.1 Coulomb force

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

2.15.2 Electric Field

$$E = \frac{F}{q_0}$$

Carica puntiforme

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \mathbf{r}$$

Distribuzione continua

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} d\mathbf{r}$$

Campo elettrico lamina

$$E = \frac{\sigma}{2\epsilon_0}$$

Campo elettrico condensatore o conduttore

$$E = \frac{\sigma}{\epsilon_0}$$

Dipole electric field

$$E(P) = -\frac{1}{4\pi\epsilon_0} \frac{\mathbf{p}}{R^3}$$

Where \mathbf{p} is called dipole momentum and is equal to:

$$\mathbf{p} = 2qa\hat{\mathbf{e}}_z$$

Cable electric field

$$E = \frac{\Delta V}{L}$$

2.15.3 Flusso del campo elettrico

$$\Phi = \int \mathbf{E} \cdot d\mathbf{S}$$

2.15.4 Gauss Law

$$\Phi(E) = \frac{\sum_{k=1}^N q_{k \text{ int}}}{\epsilon_0}$$

2.15.5 Potential electric energy

$$L_{AB} = q_0 \int_A^B \mathbf{E} \cdot d\mathbf{l} = U(B) - U(A)$$

$$L_{AB} = \sum_{k=1}^N [U_k(B) - U_k(A)] \quad U_k = k \frac{q_0 q_k}{r}$$

Spostamento lungo una lamina

$$L_{AB} = qE(r_B - r_A)$$

2.15.6 Potenziale

$$V_k = k \frac{q_k}{r} \quad U = q_0 V$$

2.15.7 Capacità di un condensatore

$$C = \frac{q}{V} \quad [C] = \frac{C}{V} = \text{Farad}$$

$$C = \epsilon_0 \frac{A}{d}$$

Condensatori in serie

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Condensatori in parallelo

$$C_{eq} = \sum_{i=1}^N C_i$$

Stored energy

$$\Delta U = \frac{1}{2} CV^2$$

$$\Delta U = \frac{1}{2} \epsilon_0 A E^2 d$$

2.15.8 Dielectric Properties

The relative Dielectric constant is calculated as:

$$\epsilon = \frac{V_0}{V}$$

Where V_0 is the Potential measured in empty space. It can be useful to calculate capacitor capacity, considering different material between the two PIASTRE.

$$C = C_0 \epsilon_r$$

2.15.9 Electric Current

$$I = \frac{dq}{dt} \quad [I] = \frac{C}{s} = \text{Ampere}$$

It can be expressed in function to the number of charge carriage

$$I = nqAv_d$$

Electric Density

$$j = nqv_d$$

$$I = \int J \cdot \hat{n} dS$$

$$\Phi(j) = I = \int J \cos \theta dS$$

2.15.10 Ohm's Law

The general expression can be written as:

$$j = \sigma E$$

Where σ is the conductivity. For a wire it can be written as follows:

$$\begin{cases} \Delta V = RI \\ R = \rho \frac{1}{A} \end{cases}$$

Where ρ is equal to $\frac{1}{\sigma}$ and it is called resistivity

Resistor in series

$$R_{eq} = \sum_{i=1}^N R_i$$

Resistor in parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i}$$

Circuit fem

$$RI = \Delta V = \varepsilon - rI$$

Where r is the circuit resistance

2.15.11 Power and Joule's effect

$$P = \frac{dU}{dT} = \frac{dq}{dt} \Delta V = I \Delta V$$

Using Ohm's law it can be rewritten as:

$$P = RI^2$$

n The energy dissipated by a resistor is equal to:

$$E_d = \frac{1}{2} C \varepsilon^2$$

2.16 Magnetism

2.16.1 Lorentz force

A particle of charge q moving with a velocity v in an electric field E and a magnetic field B experiences a force equal to:

$$\vec{F} = q\vec{v} \times \vec{B}$$

For a wire it can be also calculated as:

$$\vec{F} = \vec{B}IL$$

seconda legge di Laplace

2.16.2 Laplace Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I}{r^2} d\vec{l} \times \hat{r}$$

$$\vec{B}(P) = \int d\vec{B}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I}{r^2} d\vec{l} \sin \theta$$

2.16.3 Ampere teoreme

Solenoid

$$B = \mu_0 I \frac{N}{L}$$

Wire

$$B = \mu_0 \frac{I}{2\pi r}$$

2.16.4 Magnetic Flux

$$\Phi_B = N \int \vec{B} \cdot d\vec{\sigma} = NBS \cos \alpha$$

For a rotational motion dependence

$$\Psi = BS \cos \omega t$$

2.16.5 Electromagnetic induction

Is also known as Faraday-Newman-Lenz Law

$$\varepsilon = -\frac{d}{dt}$$

The inductive effect can also be expressed as

$$\varepsilon_L = -L = \frac{di}{dt}$$

For a solenoid is true that:

$$L = \mu_0 \left(\frac{N}{L} \right)^2 V$$

Where V is the volume included in the solenoid.

The relationship can be also written as:

$$\begin{aligned} \frac{d}{dt} &= L \frac{dI}{dt} \\ L &= \frac{d}{dI} \end{aligned}$$

For variable area

$$\varepsilon = \frac{d\Phi}{dt} = \frac{BdS}{dt} = B \frac{dx}{dt} = Blv$$

For a variable area dependent on circular motion

$$\varepsilon = \omega BS \sin \omega t$$

2.16.6 Magnetic Energy

$$U_m = \frac{1}{2} LI^2$$

2.16.7 LC Circuit

$$q(t) = q_0 \cos(\omega t + \phi) \quad \frac{1}{\sqrt{LC}}$$

2.16.8 LR Circuit

$$i(t) = -\frac{\varepsilon_0}{R} [1 - e^{-\frac{R}{L}t}]$$

2.16.9

3 PureChemistry

3.1 Concentration

3.1.1 Mole

$$n = \frac{m}{MW}$$

3.1.2 Molarity

$$M = \frac{n}{V}$$

4 Thermodynamics

4.1 Perfect gases

4.1.1 State equation

$$pV = nRT$$

$$pV_m = RT$$

4.1.2 Properties

Molar mass

$$d_{\text{gas}} \cdot V_m = MW$$

Boyle's law

$$p_1 V_1 = p_2 V_2 \quad T = \text{const}$$

Charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = \text{const}$$

Gay-Lussac's Law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = \text{const}$$

4.1.3 Compression factor

$$Z = \frac{PV}{nRT}$$

$Z = 1$ for perfect gases

4.2 Real gases

4.2.1 Virial equation

$$pV_m = RT \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \dots \right)$$

4.2.2 van der Waals equation

$$p = \frac{nRT}{V - nb} - a \left(\frac{n}{V} \right)^2$$
$$p = \frac{nRT}{V_m - b} - \frac{a}{V_m^2}$$

4.2.3 Compression factor

$$Z = \frac{V_m}{V_m - b} - \frac{a}{RTV_m}$$
$$\text{if } y = \frac{1}{V_m} \Rightarrow \frac{1}{1 - by} - \frac{ay}{RT}$$

4.2.4 Boyle Temperature

$$\lim_{p \rightarrow 0} \frac{dZ}{dp} = 0 \quad p \rightarrow 0 \Rightarrow \frac{1}{V_m} \rightarrow 0$$
$$y = \frac{1}{V_m} \quad \lim_{y \rightarrow 0} \frac{1}{1 - by} - \frac{ay}{RT} =$$
$$T_b = \frac{a}{Rb}$$

4.2.5 Critical and reduced variables

$$T_c = \frac{8a}{27Rb} \quad p_c = \frac{a}{27b^2} \quad V_c = 3b \quad Z_c = \frac{p_c V_c}{RT_c}$$

The van der Waals equation can be rewritten :

$$p_r = \frac{8T_r}{3V_r - 1} - \frac{3}{V_r^2}$$

4.3 Internal Energy

$$\Delta U = q + w$$

$$dU = \delta w + \delta q$$

$$dU = TdS - pdV$$

$$dU = \left(\frac{\partial U}{\partial S} \right)_V dS + \left(\frac{\partial U}{\partial V} \right)_S dV$$

Is always true that:

$$dU = \int_{T_1}^{T_2} C_V dT$$

$$\Delta U = nC_{V,m} \Delta T$$

4.4 Enthalpy

$$\begin{aligned} H &= U + pV \\ \Delta H &= \Delta U + \Delta pV \\ dH &= -SdT + Vdp \end{aligned}$$

For the perfect gas is also true that:

$$\Delta H = \Delta U + \Delta(nRT)$$

4.4.1 Temperature dependence

Pure substance

$$\Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} C_p dT.$$

Chemical reaction or phase transition

$$\begin{aligned} \Delta H(T_2) &= \Delta H_r T_1 + \Delta C_p (T_2 - T_1) \\ \Delta C_p &= [cC_{p,C} + dC_{p,D}] - [aC_{p,A} + bC_{p,B}] \end{aligned}$$

4.5 Entropy

$$\begin{aligned} \Delta S_{TOT} &\geq 0 \\ \Delta S &= \frac{q}{T} \Rightarrow \Delta S = \int_i^f \frac{q_{rev}}{T} \end{aligned}$$

For an ideal gas it is always true that:

$$\Delta S = nC_m \ln \left(\frac{T_f}{T_i} \right) + nR \ln \left(\frac{V_f}{V_i} \right)$$

Only mixing two gases results in:

$$\Delta S_{mix} = -R \sum_{i=1}^N n_i \ln \chi_i$$

4.5.1 Trouton's Rules

$$\Delta S_{trs} = \frac{\Delta H_{trs}}{T_{trs}}$$

4.5.2 Temperature dependence

Constant pressure

$$\Delta S = S(T_f) - S(T_i) = C_p \int_i^f \frac{dT}{T} = C_p \ln \left(\frac{T_f}{T_i} \right)$$

Constant volume

$$\Delta S = S(T_f) - S(T_i) = C_v \int_i^f \frac{dT}{T} = C_v \ln \left(\frac{T_f}{T_i} \right)$$

4.5.3 Chemical reaction

$$\begin{aligned} aA + bB &\rightarrow cC + dD \\ \Delta_r S^0 &= (cS_C^0 + dS_D^0) - (aS_A^0 + bS_B^0) \\ \Delta_r S^0 &= \sum_{i=1}^N \nu_i S_{reagents}^0 - \sum_{i=1}^N \nu_i S_{products}^0 \end{aligned}$$

If calculating $T \neq 298K$:

$$\begin{aligned} \Delta_r S(T_2) &= \Delta_r S(T_1) + \Delta C_p \ln \frac{T_2}{T_1} \\ \Delta C_p &= (cC_{c,p} + dC_{d,p}) - (aC_{a,p} + bC_{b,p}) \end{aligned}$$

4.6 Isothermal Transformations

4.6.1 Free expansion

$$\begin{aligned} \Delta T = 0 &\Rightarrow \Delta U = 0 \Rightarrow q = w \\ w = q &= -p_{ext} \Delta V = 0 \\ \Delta H &= \Delta PV \text{ (0 for perfect gas)} \\ \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) \quad \Delta S' = 0 \quad \Delta S_{tot} = \Delta S \end{aligned}$$

4.6.2 Expansion vs. p_{ext}

$$\begin{aligned} \Delta T = 0 &\Rightarrow \Delta U = 0 \Rightarrow q = w \\ w = q &= -p_{ext} \Delta V \\ \Delta H &= \Delta PV \text{ (0 for perfect gas)} \\ \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) \quad \Delta S' = \frac{-q_{sistema}}{T} \\ \Delta S_{tot} &= \Delta S + \Delta S' \end{aligned}$$

4.6.3 Reversible expansion

$$\begin{aligned} \Delta T = 0 &\Rightarrow \Delta U = 0 \Rightarrow q = w \\ q = w &= -nRT \ln \left(\frac{V_f}{V_i} \right) \\ \Delta H &= \Delta PV \text{ (0 for perfect gas)} \\ \Delta S &= nR \ln \left(\frac{V_f}{V_i} \right) \quad \Delta S' = -nR \ln \left(\frac{V_f}{V_i} \right) \\ \Delta S_{tot} &= 0 \end{aligned}$$

4.7 Adiabatic Transformations

$$dU = dw$$

$$C_V dT = -pdV$$

$$w_{\text{Adiabatic}} = nC_{V,m}\Delta T$$

For an adiabatic process is also true that:

$$\gamma = \frac{C_{p,m}}{C_{V,m}} = \frac{C_p}{C_V} \quad P_1(V_1)^\gamma = P_2(V_2)^\gamma$$

4.7.1 Reversible process

$$C_V dT = \frac{-nRT}{V} dV$$

$$\int_{T_1}^{T_2} \frac{C_V dT}{T} = -nR \int_{V_1}^{V_2} \frac{dV}{V}$$

$$C_V \ln\left(\frac{T_2}{T_1}\right) = -nR \ln\frac{V_2}{V_1}$$

$$\frac{T_2}{T_1} = \left[\frac{V_2}{V_1}\right]^{-\frac{R}{C_V}}$$

4.7.2 Irreversible

Take P as constant

$$\int_{T_1}^{T_2} C_V dT = -p \int_{V_1}^{V_2} dV$$

Assuming $C_V = \text{const}$

$$C_V \Delta T = -p \Delta V$$

4.7.3 Free expansion

$$q = 0, \quad w = -p_{\text{ext}} \Delta V = 0 \implies \Delta U = 0$$

$$\Delta H =$$

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \quad \Delta S' = 0 \quad \Delta S_{\text{tot}} = 0$$

4.7.4 Expansion vs. p_{ext}

$$q = 0 \implies \Delta U = w$$

$$\Delta U = nC_{V,m}\Delta T = w = -p_{\text{ext}}\Delta V$$

$$\Delta T = -\frac{p_{\text{ext}}\Delta V}{nC_{V,m}}$$

$$\Delta H = V\Delta p$$

$$\Delta S = nC_{V,m} \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$

$$\Delta S' = 0 \implies \Delta S_{\text{TOT}} = \Delta S$$

4.7.5 Reversible expansion

$$q = 0 \implies \Delta U = w$$

$$\Delta U = nC_{V,m}\Delta T$$

$$\Delta S = \Delta S' = \Delta_{\text{tot}} = 0$$

4.8 Isobaric transformations

$$dp = 0$$

$$q = \Delta H = nC_{p,m}\Delta T$$

$$w = -pdV$$

$$\Delta S = nC_{p,m} \ln\left(\frac{T_f}{T_i}\right) \quad \Delta S' = -nC_{p,m} \ln\frac{V_f}{V_i}$$

$$\implies \Delta S_{\text{TOT}} = 0$$

4.9 Thermodynamic cycles

$$\Delta U = 0, \quad \Delta S = 0, \quad \Delta H = 0.$$

4.9.1 Carnot cycle

There is 4 stage (ABCD):

AB Reversible Isothermal expansion

$$\Delta U = 0 \implies q_{AB} = -w_{AB}$$

$$w_{AB} = -q_{AB} = -nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

BC Reversible Adiabatic Expansion

$$q_{BC} = 0 \implies \Delta U_{BC} = w_{BC}$$

$$w_{BC} = nC_{V,m}(T_C - T_h)$$

CD Reversible Isothermal compression

$$\Delta U = 0 \implies q_{CD} = -w_{CD}$$

$$w_{CD} = -q_{CD} = -nRT_h \ln\left(\frac{V_D}{V_C}\right)$$

DA Reversible Adiabatic compression

$$q_{DA} = 0 \implies \Delta U_{DA} = w_{DA}$$

$$w_{DA} = nC_{V,m}(T_C - T_h)$$

4.10 Helmholtz's Energy

$$A = U - ST$$

$$\Delta A_V = \Delta U_V - T\Delta S_V$$

ΔA is the maximum (more negative) work that can be done by the system if $dT=0$.

$$0 \geq w \geq \Delta A$$

It can also be expressed with natural variables:

$$dA = -SdT - pdV$$

$$dA = \left(\frac{\partial A}{\partial T}\right)_V dT + \left(\frac{\partial A}{\partial V}\right)_T dV$$

4.11 Gibbs's Energy

$$G = H - ST$$

$$\Delta G_p = \Delta H_p - T\Delta S_p$$

ΔG is the maximum (more negative) work that can be done by the system if $dT=0$.

$$0 \geq w_{ne} \geq \Delta G$$

It can also be expressed with natural variables:

$$dG = -SdT + Vdp$$

$$dG = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp$$

4.11.1 Gibbs-Helmholtz's equation

$$\frac{\partial}{\partial T} \left(\frac{\Delta G}{T} \right)_p = -\frac{\Delta H}{T^2}$$

$$\frac{\Delta G(T_2)}{T_2} - \frac{\Delta G(T_1)}{T_1} = \Delta H \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

4.11.2 Pressure dependence

$$\Delta G = G(p_2) - G(p_1) = nRT \ln \left(\frac{p_2}{p_1} \right)$$

It can be also used to find $G_m(T_2)$ for a reaction:

$$nA \rightarrow mB$$

$$nG(A, p_2) = nG^0 + nRT \ln p_2$$

$$nG(B, p_2) = mG^0 + mRT \ln p_2$$

$$\Delta G(p_2) = mG(B, p_2) - nG(A, p_2) =$$

$$\Delta G^0 + (m - n)RT \ln p_2$$

4.12 Chemical potential

G is an extensive variable, otherwise a p , T = cost it would be always true that $dG = 0$. G depends on the composition of the system: $G = G(p, T, n_1, n_2, \dots)$ The partial derivative of G is the chemical potential:

$$\mu = \left(\frac{\partial G}{\partial n_i} \right)_{p, T, n_j} \implies dG = \sum_{i=1}^N \mu_i dn_i$$

4.12.1 Gibbs-Duhem equation

$$\sum_{i=1}^N n_i d\mu_i = -SdT + Vdp$$

If temperature and pressure are constant:

$$\sum_{i=1}^N n_i d\mu_i = 0$$

4.12.2 Natural variables

$$\left(\frac{d}{dp} \right)_T = V_m \quad \left(\frac{d}{dT} \right)_p = -S_m$$

4.12.3 Perfect gas chemical potential

$$\left(\frac{d}{dp} \right)_T = \frac{V}{p}$$

$$\int_{p_0}^p \frac{V}{p} dp = \int_{p_0}^p \frac{RT}{p} dp = RT \ln \left(\frac{p}{p_0} \right)$$

$$\mu(p, T) = \mu_0(p_0, T) + RT \ln \left(\frac{p}{p_0} \right)$$

4.13 Chemical equilibrium

$\Delta_r G$ is related to G variation at constant temperature and pressure

$$\Delta_r G = \left(\frac{\partial G}{\partial \xi} \right)_{p, T} = \sum_{i=1}^n \nu_i \mu_i$$

Is possible to relate $\Delta_r G^0$ and K :

$$K = e^{-\frac{\Delta_r G^0}{RT}}$$

Different K calculation are related easily:

$$K_c = K_n \left(\frac{1}{V} \right)^{\Delta \nu} \quad K_\chi = K_n \left(\frac{1}{n_{\text{tot}}} \right)^{\Delta \nu}$$

$$K_p = K_n \left(\frac{p_{\text{tot}}}{n_{\text{tot}}} \right)^{\Delta \nu}$$

4.13.1 Pressure dependence

$$\frac{\partial \ln K}{\partial p} = -\frac{1}{RT} \left(\frac{\partial \Delta_r G}{\partial p} \right)$$

$$\frac{\partial \ln K}{\partial p} = -\frac{\Delta_r V}{RT}$$

$$\Delta_r V \approx 0 \Rightarrow \left(\frac{\partial K}{\partial p} \right)_T = 0$$

Still, the equilibrium's composition is not independent of the pressure.

4.13.2 Temperature dependence

$$\frac{\partial \ln K}{\partial \frac{1}{T}} = -\frac{\Delta_r H^0}{R}$$

$$\ln \left(\frac{K_2}{K_1} \right) = -\frac{\Delta_r H^0}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$K_2 = e^{\left(-\frac{\Delta_r H^0}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right) \right)} \cdot K_1$$

4.14 Clausius-Clapeyron equation

Clausius' equation $\frac{dp}{dT} = \frac{\Delta S_m}{\Delta V_m}$

Clausius-Clapeyron $\frac{dp}{dT} = \frac{\Delta H_m}{T \Delta V_m}$

4.14.1 Solid-liquid equilibrium

$$\frac{dp}{dT} = \frac{\Delta_{\text{fus}} H_m}{\Delta V_{\text{fus}} T}$$

$$p_2 = p_1 + \frac{\Delta_{\text{fus}} H_m}{\Delta_{\text{fus}} V_m} \ln \left(\frac{T_2}{T_1} \right)$$

4.14.2 Liquid-vapor equilibrium

Temperature dependence

$$\frac{dp}{dT} = \frac{\Delta_{\text{vap}} H_m}{T \Delta_{\text{vap}} V_m}$$

$$d \ln p = \frac{\Delta_{\text{vap}} H_m}{RT^2} dT$$

$$p_2 = p_1 \cdot e^{\frac{\Delta_{\text{vap}} H_m}{RT^2} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)}$$

Pressure dependence

$$p = p^* \cdot e^{\left(\frac{V_{m,\text{liq}}}{RT} \Delta p \right)}$$

4.14.3 Solid-vapor equilibrium

$$\frac{dp}{dT} = \frac{\Delta_{\text{sub}} H_m}{T \Delta_{\text{sub}} V_m}$$

$$d \ln p = \frac{\Delta_{\text{sub}} H_m}{RT^2} dT$$

$$p_2 = p_1 \cdot e^{\frac{\Delta_{\text{sub}} H_m}{RT^2} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)}$$

5 Chemical Kinetics

5.1 Reaction rate

Is always true that:

$$r = \frac{dx}{dt} = \frac{1}{\nu_B} \frac{dC_B}{dt}$$

5.2 First order reactions

$$\begin{aligned} A &\rightarrow P \\ r &= -\frac{dC_A}{dt} = k[A] \\ -\ln C_A + \ln C_{0A} &= kt \\ \frac{C_A}{C_A^0} &= e^{-kt} \end{aligned}$$

5.2.1 Half-life time

$$t_{\frac{1}{2}} = \frac{\ln 2}{k} \quad [s^{-1}]$$

5.2.2 Life time

5.3 Second order reactions

5.3.1 Unimolecular reaction

$$\begin{aligned} r &= \frac{dC_A}{dt} = k[A]^2 \\ \frac{1}{C_A} - \frac{1}{C_A^0} &= kt \end{aligned}$$

Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_A^0 k}$$

5.3.2 Bimolecular reaction

$$\begin{aligned} r &= -\frac{dC_A}{dt} = -\frac{dC_B}{dt} = \frac{dx}{dt} = k[A][B] \\ \frac{1}{C_A^0 - C_B^0} \ln \left[\frac{(C_A^0 - x)C_B^0}{(C_B^0 - x)C_A^0} \right] &= kt \\ \frac{C_B}{C_A} &= \frac{C_B^0}{C_A^0} e^{(C_A^0 - C_B^0)kt} \end{aligned}$$

Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_A^0 - C_B^0 k} \ln \left(\frac{2C_A^0 - C_B^0}{C_A^0} \right)$$

5.4 Zero order reactions

$$\begin{aligned} r &= k \\ C &= C^0 - kt \end{aligned}$$

5.4.1 Half-life time

$$t_{\frac{1}{2}} = \frac{1}{2} t_f$$

5.5 nth order reactions

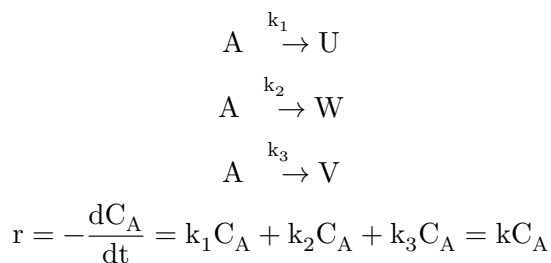
$$\begin{aligned} r &= k[A]^n \\ \frac{1}{C^{n-1}} - \frac{1}{C^{0(n-1)}} &= (n-1)kt \end{aligned}$$

5.5.1 Half-life time

$$t_{\frac{1}{2}} = \frac{2^{n-1} - 1}{(C^0)^{n-1} k(n-1)}$$

5.6 Parallel reactions

In this condition A react with different velocity constant towards different products W,V and U.



Equation for the products can be written as:

$$C_U - C_U^0 = \frac{k_1 C_A^0}{k} (1 - e^{-kt})$$

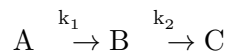
$$C_V - C_V^0 = \frac{k_2 C_A^0}{k} (1 - e^{-kt})$$

$$C_W - C_W^0 = \frac{k_3 C_A^0}{k} (1 - e^{-kt})$$

k can be determined sperimentally from the relation between C_A and time, in this case it would be a first order relation. Concentration of the products can be determined with sperimental methods. At this point the following system can be used:

$$\begin{cases} k_1 + k_2 + k_3 = k \\ \frac{C_U}{C_W} = \frac{k_1}{k_2} \\ \frac{C_U}{C_V} = \frac{k_2}{k_3} \end{cases}$$

5.7 Consecutive reactions



This is the simplest case, with all first order reac-

tions

$$r = -\frac{dC_A}{dt} = k_1 C_A$$

$$r = \frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$r = \frac{dC_C}{dt} = k_2 C_B$$

The C_C value in relation to time, in this case would be:

$$C_C = C_A^0 \left[1 - \frac{k_2 e^{-k_1 t} - k_1 e^{-k_2 t}}{k_2 - k_1} \right] + C_B^0 (1 - e^{-k_2 t}) + C_C^0$$

In case that $C_B^0 = 0, C_C^0 = 0$ then:

$$C_C = C_A^0 \left[1 - \frac{k_2 e^{-k_1 t} - k_1 e^{-k_2 t}}{k_2 - k_1} \right]$$

5.8 Opposite reactions

5.9 Arrhenius's equation

$$k = A \exp \left(-\frac{E_a}{RT} \right)$$

$$\ln k = \ln A - \frac{E_a}{RT}$$

6 Electrochemistry

7 Wave Phenomena

7.1 Bragg Equation

$$n\lambda = 2d_{hkl} \sin \theta$$

7.2 Waves equation

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

The solution must be in the form:

$$\Psi(x, t) = g(x \pm vt)$$

7.3 Waves electric and magnetic field

$$B_0 = \frac{E_0}{c}$$

7.4 Waves variables

7.4.1 Wave lenght

$$\lambda = \frac{2\pi}{K}$$

7.4.2 Frequency

$$\nu = \frac{1}{T}$$

7.4.3 Pulsazione

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

7.4.4 Period

$$T = \frac{2\pi}{\omega}$$

7.5 Electromagnetic waves

$$\begin{aligned} E_x &= B_y = \\ E_z^2 + E_y^2 &= c^2(B_z^2 + B_y^2) \\ E &= cB \end{aligned}$$

Also you can calculate:

$$E_0 = Bv = B \frac{\omega}{k}$$

7.5.1 Power

$$S = \frac{1}{\mu_0} E \text{vec} B$$

7.5.2 Intensity

Planar wave

Sferic wave