FORMULAS FOR LIFE

All formulas you'll ever need

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A helpful tool for scientific studies

Contents

1 Statistics

1.1 Mean

1.1.1 Arithmetic mean

$$\overline{x} = \sum_{i=1}^{n} \frac{x_i}{N}$$

1.1.2 Geometric mean

$$\overline{x} = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}}$$

1.1.3 Harmonic mean

$$\overline{x} = n\left(\sum_{i=1}^{n} \frac{1}{x_i}\right)^{-1}$$

1.2 Standard Deviation

1.2.1 Absolute

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{N-1}}$$

1.2.2 Relative

$$RSD = \frac{s}{\overline{x}}$$

2   Pure Physics

2.1   Uniform Rectilinear Motion

$$x(t) = x_0 + v_0 t$$
$$v = v_0$$

2.2   Uniform Accelerated Rectilinear Motion

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \implies x - x_0 = \frac{v^2 - v_0^2}{2 a_0}$$
$$v(t) = v_0 + a t$$
$$a(t) = a_0$$

2.3   Parabolic motion

On the x axis:

$$\begin{cases} a_x = 0 \\ v_x = v_{0x} \\ x(t) = x_0 + v_{0x} t \end{cases}$$

On the y axis:

$$\begin{cases} a_y = a_{0y} = g \\ v_y = v_{0y} + g t \\ y(t) = y_0 + v_{0y} t = \frac{1}{2} g t^2 \end{cases}$$

The range can be calculated as:

$$R = \frac{2 v_{0x} v_{oy}}{g} = \frac{v_0^2 \operatorname{sen} 2\alpha}{g}$$

The maximum height can be calculated as:

$$h_{max} = \frac{v_0^2 \operatorname{sen}^2 \alpha}{2g}$$

2.4   Uniform Circular Motion

2.4.1   Angular velocity

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta \theta}{\Delta t}$$

2.4.2   Angular acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

2.4.3   Relation with linear variables

Linear velocity

$$v = \omega r \quad \text{(radian measure)}$$

Linear acceleration

$$a_{tangential} = \alpha r \quad \text{(radian measure)}$$
$$a_{radial} = \frac{v^2}{r} = w^2 r$$

Period

If the point moves in uniform circular motion :

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

2.5   Rotation with constant acceleration

$$\omega = \omega_0 + \alpha t$$
$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

2.6   Kinetic of rotation

$$E_k = \frac{1}{2} I \omega^2$$

I is the rotational inertia of the body; for a system of discrete particles it is defines as:

$$I = \sum m_i r_i^2$$

It has different formulas for different shapes

2.6.1   Armonic Motion

$$x(t) = A \operatorname{sen}(\omega * t + \phi)$$

It can be also calculated as:

$$\omega = \sqrt{\frac{k}{m}}$$

2.6.2 Centripetal and Centrifugal acceleration

$$a_c = \omega^2 r = \frac{v^2}{r}$$

2.6.3 Teorema dell'impulso

$$I = \Delta p = m\Delta V = F\delta t$$

2.6.4 Newton Second Law's for rotation

$$\tau = I\alpha$$

2.7 Angular momentum

$$L = I\omega = r\,prodvet\,m\vec{v}$$

$$\Delta L = \tau \quad \Delta l = I\,vect\,r$$

2.8 Frictional Force

$$\vec{F}_a = -\mu N$$

2.9 Centripetal Force

$$F_c = m\frac{v^2}{r} = n\omega^2 r$$

2.10 Rope's tension

$$I\,\alpha = T\,R$$

2.11 Gravitational Law

$$F = G\frac{m_1 m_2}{r^2} \quad 6.6743 \cdot 10^{11} \text{m}^3\text{kg s}^2$$

Is also possible to calculate the Gravitational Acceleration

$$g = \frac{GM}{r^2}$$

2.12 Pendulum Equation

$$m\frac{d^2 x}{dt^2} + mg\,sen\theta = 0$$

$$\frac{d^2}{dt^2} + \frac{g}{L}sen\theta = 0 \quad \implies \quad \frac{g}{L} = \omega^2$$

If $\theta \approx 0$ then:

$$T = \frac{2\pi}{\omega} = \sqrt{\frac{L}{g}} \quad \theta(t) = \theta_0 + \cos(\omega t + \phi)$$

2.13 Energy

2.13.1 Work

$$\vec{L} = \vec{F} \cdot \Delta r = F\Delta r \cos\theta$$

2.13.2 Kinetic Energy

$$E_c = \frac{1}{2}mv^2$$

$$L = \Delta E_c = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

2.13.3 Elastic potential energy

$$U_{el} = \frac{1}{2}kx^2$$

2.13.4 Gravitational Potential Energy

$$U_g = mgh$$

2.13.5 Mechanical energy

$$E_m = E_c + U$$

2.13.6 Power

$$P = \frac{W}{\Delta t}$$

2.14 Momentum

$$\vec{p} = m\vec{v}$$

2.15 Electrostatic

2.15.1 Coulomb force

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

2.15.2 Electric Field

$$E = \frac{F}{q_0}$$

Point-like charge

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}r$$

Continuos distribution

$$E = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}dr$$

Plates electric field

$$E = \frac{\sigma}{2\epsilon_0}$$

Capacitor or conductor electric field

$$E = \frac{\sigma}{\epsilon_0}$$

Dipole electric field

$$E(P) = -\frac{1}{4\pi\varepsilon_0}\frac{p}{R^3}$$

Where p is called dipole momentum and is equal to:

$$p = 2qa\hat{e}_z$$

Cable electric field

$$E = \frac{\Delta V}{L}$$

2.15.3 Electric field flux

$$\Phi = \int E \cdot dS$$

2.15.4 Gauss Law

$$\Phi(E) = \frac{\sum_{k=1}^{N} q_{k\ int}}{\epsilon_0}$$

2.15.5 Potential electric energy

$$L_{AB} = q_0 \int_A^B E \cdot dl = U(B) - U(A)$$

$$L_{AB} = \sum_{k=1}^{N}[U_k(B) - U_k(A)] \quad U_k = k\frac{q_0 q_k}{r}$$

Displacement along a plates

$$L_{AB} = qE(r_B - r_A)$$

2.15.6 Electrical Potential

$$V_k = k\frac{q_k}{r} \quad U = q_0 V$$

2.15.7 Capacitor capacity

$$C = \frac{q}{V} \quad [C] = \frac{C}{V} = \text{Farad}$$

$$C = \varepsilon_0 \frac{A}{d}$$

Capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in parallel

$$C_{eq} = \sum_{i=1}^{N} C_i$$

Stored energy

$$\Delta U = \frac{1}{2}CV^2$$

$$\Delta U = \frac{1}{2}\varepsilon_0 AE^2 d$$

2.15.8 Dielectric Properties

The relative Dielectric constant is calculated as:

$$\varepsilon = \frac{V_0}{V}$$

Where V_0 is the Potential measured in empty space. It can be useful to calculate capacitor capacity, considering different material between the two plates.

$$C = C_0 \varepsilon_r$$

2.15.9 Electric Current

$$I = \frac{dq}{dt} \quad [I] = \frac{C}{s} = \text{Ampere}$$

It can be expressed in function to the number of charge carriage

$$I = nqAv_d$$

Electric Density

$$j = nqv_d$$

$$I = \int J \cdot \hat{n} dS$$

$$\Phi(j) = I = \int J \cos\theta dS$$

2.15.10 Ohm's Law

The general expression can be written as:

$$j = \sigma E$$

Where σ is the conductivity. For a wire it can be written as follows:

$$\begin{cases} \Delta V = RI \\ R = \rho \frac{l}{A} \end{cases}$$

Where ρ is equal to $\frac{1}{\sigma}$ and it is called resistivity

Resistor in series

$$R_{eq} = \sum_{i=1}^{N} R_i$$

Resistor in parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Circuit fem

$$RI = \Delta V = \varepsilon - rI$$

Where r is the circuit resistance

2.15.11 Power and Joule's effect

$$P = \frac{dU}{dT} = \frac{dq}{dt}\Delta V = I\Delta V$$

Using Ohm's law it can be rewritten as:

$$P = RI^2$$

n The energy dissipated by a resistor is equal to:

$$E_d = \frac{1}{2}C\varepsilon^2$$

2.16 Magnetism

2.16.1 Lorentz force

A particle of charge q moving with a velocity v in an electric field E and a magnetic field B experiences a force equal to:

$$\vec{F} = q\vec{v} \times \vec{B}$$

For a wire it can be also calculated as:

$$\vec{F} = \vec{B}IL$$

seconda legge di la place

2.16.2 La Place Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I}{r^2}d\vec{l} \times \hat{r}$$

$$\vec{B}(P) = \int d\vec{B}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{I}{r^2}d\vec{l}\operatorname{sen}\theta$$

2.16.3 Ampere teoreme

Solenoide

$$B = \mu_0 I \frac{N}{L}$$

Wire

$$B = \mu_0\frac{I}{2\pi r}$$

2.16.4 Magnetic Flux

$$\Phi_B = N\int \vec{B} \cdot d\vec{\sigma} = NBS\cos\alpha$$

For a rotational motion dependence

$$\Psi = BS\cos\omega t$$

2.16.5 Electromagnetic induction

Is also known as Faraday-Newman-Lentz Law

$$\varepsilon = -\frac{d}{dt}$$

The inductive effect can also be expressed as

$$\varepsilon_{\mathrm{L}} = -\mathrm{L} = \frac{di}{dt}$$

For a solenoid is true that:

$$\mathrm{L} = \mu_0 \left(\frac{\mathrm{N}}{\mathrm{L}}\right)^2 \mathrm{V}$$

Where V is the volume included in the solenoid. The relationship can be also writte as:

$$\frac{d}{dt} = \mathrm{L}\frac{d\mathrm{I}}{dt}$$
$$\mathrm{L} = \frac{d}{d\mathrm{I}}$$

For variable area

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\mathrm{BdS}}{dt} = \mathrm{B}\frac{\mathrm{ldx}}{dt} = \mathrm{Blv}$$

For a variable area dependent on circular motion

$$\varepsilon = \omega \mathrm{BS}\sin\omega t$$

2.16.6 Magnetic Energy

$$\mathrm{U_m} = \frac{1}{2}\mathrm{LI}^2$$

2.16.7 LC Circuit

$$q(t) = q_0 \cos(\omega t + \phi) \quad \frac{1}{\sqrt{\mathrm{LC}}}$$

2.16.8 LR Circuit

$$i(t) = -\frac{\varepsilon_0}{\mathrm{R}}[1 - e^{-\frac{\mathrm{R}}{\mathrm{L}}t}]$$

2.16.9

3 Pure Chemistry

3.1 Concentration

3.1.1 Mole

$$n = \frac{m}{MW}$$

3.1.2 Molarity

$$M = \frac{n}{V}$$

4 Thermodynamics

4.1 Perfect gases

4.1.1 State equation

$$pV = nRT$$
$$pV_m = RT$$

4.1.2 Properties

Molar mass

$$d_{gas} \cdot V_m = MW$$

Boyle's law

$$p_1 V_1 = p_2 V_2 \quad T = const$$

Charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = const$$

Gay-Lussac's Law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = const$$

4.1.3 Compression factor

$$Z = \frac{PV}{nRT}$$

$Z = 1$ for perfect gases

4.2 Real gases

4.2.1 Virial equation

$$pV_m = RT \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + ... \right)$$

4.2.2 van der Waals equation

$$p = \frac{nRT}{V - nb} - a \left(\frac{n}{V} \right)^2$$
$$p = \frac{nRT}{V_m - b} - \frac{a}{V_m^2}$$

4.2.3 Compression factor

$$Z = \frac{V_m}{V_m - b} - \frac{a}{RTV_m}$$
$$\text{if } y = \frac{1}{V_m} \implies \frac{1}{1 - by} - \frac{ay}{RT}$$

4.2.4 Boyle Temperature

$$\lim_{p \to 0} \frac{dZ}{dp} = 0 \quad p \to 0 \implies \frac{1}{V_m} \to 0$$
$$y = \frac{1}{V_m} \quad \lim_{y \to 0} \frac{1}{1 - by} - \frac{ay}{RT} =$$
$$T_b = \frac{a}{Rb}$$

4.2.5 Critical and reduced variables

$$T_c = \frac{8a}{27Rb} \quad p_c = \frac{a}{27b^2} \quad V_c = 3b \quad Z_c = \frac{p_c V_c}{RT_c}$$

The van der Waals equation can be rewritten :

$$p_r = \frac{8T_r}{3V_r - 1} - \frac{3}{V_r^2}$$

4.3 Internal Energy

$$\Delta U = q + w$$
$$dU = \delta w + \delta q$$
$$dU = TdS - pdV$$
$$dU = \left(\frac{\partial U}{\partial S} \right)_V dS + \left(\frac{\partial U}{\partial V} \right)_S dV$$

Is always true that:

$$dU = \int_{T_1}^{T_2} C_V dT$$
$$\Delta U = nC_{v,m} \Delta T$$

4.4 Enthalpy

$$H = U + pV$$
$$\Delta H = \Delta U + \Delta pV$$
$$dH = -SdT + Vdp$$

For the perfect gas is also true that:

$$\Delta H = \Delta U + \Delta(nRT)$$

4.4.1 Temperature dependence

Pure substance

$$\Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} C_p dT.$$

Chemical reaction or phase transition

$$\Delta H(T_2) = \Delta H_r T_1 + \Delta C_p (T_2 - T_1)$$
$$\Delta C_p = [cC_{p,C} + dC_{p,D}] - [aC_{p,A} + bC_{p,B}]$$

4.5 Entropy

$$\Delta S_{TOT} \geq 0$$
$$\Delta S = \frac{q}{T} \implies \Delta S = \int_i^f \frac{q_{rev}}{T}$$

For an ideal gas it is always true that:

$$\Delta S = nC_m \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$

Only mixing two gases results in:

$$\Delta S_{mix} = -R \sum_{i=1}^{N} n_i \ln \chi_i$$

4.5.1 Trouton's Rules

$$\Delta S_{trs} = \frac{\Delta H_{trs}}{T_{trs}}$$

4.5.2 Temperature dependence

Constant pressure

$$\Delta S = S(T_f) - S(T_i) = C_p \int_i^f \frac{dT}{T} = C_p \ln\left(\frac{T_f}{T_i}\right)$$

Constant volume

$$\Delta S = S(T_f) - S(T_i) = C_v \int_i^f \frac{dT}{T} = C_v \ln\left(\frac{T_f}{T_i}\right)$$

4.5.3 Chemical reaction

$$aA + bB \rightarrow cC + dD$$
$$\Delta_r S^0 = (cS_C^0 + dS_D^0) - (aS_A^0 + bS_B^0)$$
$$\Delta_r S^0 = \sum_{i=1}^{N} \nu_i S_{reagents}^0 - \sum_{i=1}^{N} \nu_i S_{products}^0$$

If calculating $T \neq 298K$:

$$\Delta_r S(T_2) = \Delta_r S(T_1) + \Delta C_p \ln \frac{T_2}{T_1}$$
$$\Delta C_p = (cC_{c,p} + dC_{d,p}) - (aC_{a,p} + bC_{b,p})$$

4.6 Isothermal Transformations

4.6.1 Free expansion

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$
$$w = q = -p_{ext}\Delta V = 0$$
$$\Delta H = \Delta PV \text{ (0 for perfect gas)}$$
$$\Delta S = nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = 0 \quad \Delta S_{tot} = \Delta S$$

4.6.2 Expansion vs. p_{ext}

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$
$$w = q = -p_{ext}\Delta V$$
$$\Delta H = \Delta PV \text{ (0 for perfect gas)}$$
$$\Delta S = nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = \frac{-q_{sistema}}{T}$$
$$\Delta S_{tot} = \Delta S + \Delta S'$$

4.6.3 Reversible expansion

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$
$$q = w = -nRT \ln(\frac{V_f}{V_i})$$
$$\Delta H = \Delta PV \text{ (0 for perfect gas)}$$
$$\Delta S = nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = -nR \ln(\frac{V_f}{V_i})$$
$$\Delta S_{tot} = 0$$

4.7 Adiabatic Transformations

$$dU = dw$$
$$C_v dT = -p dV$$
$$w_{Adiabatic} = nC_{V,m} \Delta T$$

For an adiabatic process is also true that:

$$\gamma = \frac{C_{p,m}}{C_{V,m}} = \frac{C_p}{C_V} \quad P_1(V_1)^\gamma = P_2(V_2)^\gamma$$

4.7.1 Reversible process

$$C_V dT = \frac{-nRT}{V} dV$$
$$\int_{T_1}^{T_2} \frac{C_V dT}{T} = -nR \int_{T_1}^{T_2} \frac{dV}{V}$$
$$C_V \ln\left(\frac{T_1}{T_2}\right) = -R \ln \frac{V_2}{V_1}$$
$$\frac{T_2}{T_1} = \left[\frac{V_2}{V_1}\right]^{-\frac{R}{C_V}}$$

4.7.2 Irreversible

Take P as constant

$$\int_{T_1}^{T_2} Cv dT = -p \int_{V_1}^{V_2} dV$$
$$\text{Assuming } C_V = \text{cost}$$
$$C_V \Delta T = -p \Delta V$$

4.7.3 Free expansion

$$q = 0, \ w = -p_{ext}\Delta V = 0 \implies \Delta U = 0$$
$$\Delta H =$$
$$\Delta S = nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = 0 \quad \Delta S_{tot} = 0$$

4.7.4 Expansion vs. p_{ext}

$$q = 0 \implies \Delta U = w$$
$$\Delta U = nC_{V,m}\Delta T = w = -p_{ext}\Delta V$$
$$\Delta T = -\frac{p_{ext}\Delta V}{nC_{v,m}}$$
$$\Delta H = V\Delta p$$
$$\Delta S = nC_{V,m} \ln(\frac{T_f}{T_i}) + nR \ln(\frac{V_f}{V_i})$$
$$\Delta S' = 0 \implies \Delta S_{TOT} = \Delta S$$

4.7.5 Reversible expansion

$$q = 0 \implies \Delta U = w$$
$$\Delta U = nC_{V,m}\Delta T$$
$$\Delta S = \Delta S' = \Delta_{tot} = 0$$

4.8 Isobaric transformations

$$dp = 0$$
$$q = \Delta H = nC_{p,m}\Delta T$$
$$w = -p dV$$
$$\Delta S = nC_{p,m} \ln(\frac{T_f}{T_i}) \quad \Delta S' = -nC_{p,m} \ln \frac{V_f}{V_i}$$
$$\implies \Delta S_{TOT} = 0$$

4.9 Thermodynamic cycles

$$\Delta U = 0, \ \Delta S = 0, \ \Delta H = 0.$$

4.9.1 Carnot cycle

There is 4 stage (ABCD):

AB Reversible Isothermal expansion

$$\Delta U = 0 \implies q_{AB} = -w_{AB}$$
$$w_{AB} = -q_{AB} = -nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

BC Reversible Adiabatic Expansion

$$q_{BC} = 0 \implies \Delta U_{BC} = w_{BC}$$
$$w_{BC} = nC_{V,m}(T_C - T_h)$$

CD Reversible Isothermal compression

$$\Delta U = 0 \implies q_{CD} = -w_{CD}$$
$$w_{CD} = -q_{CD} = -nRT_h \ln\left(\frac{V_D}{V_C}\right)$$

DA Reversible Adiabatic compression

$$q_{DA} = 0 \implies \Delta U_{DA} = w_{DA}$$
$$w_{DA} = nC_{V,m}(T_C - T_h)$$

4.10 Helmholtz's Energy

$$A = U - ST$$
$$\Delta A_V = \Delta U_V - T\Delta S_V$$

ΔA is the maximum (more negative) work that can be done by the system if dT=0.

$$0 \geq w \geq \Delta A$$

It can also be expressed with natural variables:

$$dA = -SdT - pdV$$
$$dA = \left(\frac{\partial A}{\partial T}\right)_V dT + \left(\frac{\partial A}{\partial V}\right)_T dV$$

4.11 Gibbs's Energy

$$G = H - ST$$
$$\Delta G_p = \Delta H_p - T\Delta S_p$$

ΔG is the maximum (more negative) work that can be done by the system if dT=0.

$$0 \geq w_{ne} \geq \Delta G$$

It can also be expressed with natural variables:

$$dG = -SdT + Vdp$$
$$dG = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp$$

4.11.1 Gibbs-Helmholtz's equation

$$\frac{\partial}{\partial T}\left(\frac{\Delta G}{T}\right)_p = -\frac{\Delta H}{T^2}$$
$$\frac{\Delta G(T_2)}{T_2} - \frac{\Delta G(T_1)}{T_1} = \Delta H\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

4.11.2 Pressure dependence

$$\Delta G = G(p_2) - \Delta G(p_1) = nRT\ln\left(\frac{p_2}{p_1}\right)$$

It can be also used to find $G_m(T_2)$ for a reaction:

$$nA \rightarrow mB$$
$$nG(A, p_2) = nG^0 + nRT\ln p_2$$
$$nG(B, p_2) = mG^0 + mRT\ln p_2$$
$$\Delta G(p_2) = mG(B, p_2) - nG(A, p_2) =$$
$$\Delta G^0 + (m - n)RT\ln p_2$$

4.12 Chemical potential

G is an extensive variable, otherwise a p, T = cost it would be always true that dG = 0. G depends on the composition of the system: $G = G(p, T, n_1, n_2, ...)$ The partial derivative of G is the chemical potential:

$$\mu = \left(\frac{\partial G}{\partial n_i}\right)_{p,T,n_j} \implies dG = \sum_{i=1}^N \mu_i dn_i$$

4.12.1 Gibbs-Duhem equation

$$\sum_{i=1}^N n_i d\mu_i = -SdT + Vdp$$

If temperature and pressure are constant:

$$\sum_{i=1}^N n_i d\mu_i = 0$$

4.12.2 Natural variables

$$\left(\frac{d}{dp}\right)_T = V_m \quad \left(\frac{d}{dT}\right)_p = -S_m$$

4.12.3 Perfect gas chemical potential

$$\left(\frac{d}{dp}\right)_T$$
$$\int_{p^0}^p = \int_{P_0}^p \frac{RT}{p} = dp$$
$$\mu(p, T) = \mu_0(p_0, T) + RT\ln\left(\frac{p}{p_0}\right)$$

4.13 Chemical equilibrium

$\Delta_r G$ is related to G variation at constant temperature and pressure

$$\Delta_r G = \left(\frac{\partial G}{\partial \xi}\right)_{p,T} = \sum_{i=1}^n \nu_i \mu_i$$

Is possible to relate $\Delta_r G^0$ and K :

$$K = e^{-\frac{\Delta_r G^0}{RT}}$$

Different K calculation are related easily:

$$K_c = K_n \left(\frac{1}{V}\right)^{\Delta\nu} \quad K_\chi = K_n \left(\frac{1}{n_{tot}}\right)^{\Delta\nu}$$

$$K_p = K_n \left(\frac{p_{tot}}{n_{tot}}\right)^{\Delta\nu}$$

4.13.1 Pressure dependence

$$\frac{\partial \ln K}{\partial p} = -\frac{1}{RT}\left(\frac{\partial \Delta_r G}{\partial p}\right)$$

$$\frac{\partial \ln K}{\partial p} = -\frac{\Delta_r V}{RT}$$

$$\Delta_r V \approx 0 \implies \left(\frac{\partial K}{\partial p}\right)_T = 0$$

Still, the equilibrium's composition is not independent of the pressure.

4.13.2 Temperature dependence

$$\frac{\partial d \ln K}{\partial \frac{1}{T}} = -\frac{\Delta_r H^0}{R}$$

$$\ln\left(\frac{K_2}{K_1}\right) = -\frac{\Delta_r H^0}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$K_2 = e^{\left(-\frac{\Delta_r H^0}{R}\left(\frac{1}{T_2} - \frac{1}{T_2}\right)\right)} \cdot K_1$$

4.14 Clausius-Clapeyron equation

$$\begin{array}{ll} \text{Clausius' equation} & \dfrac{dp}{dT} = \dfrac{\Delta S_m}{\Delta V_m} \\[2ex] \text{Clausius-Clapeyron} & \dfrac{dp}{dT} = \dfrac{\Delta H_m}{T\Delta V_m} \end{array}$$

4.14.1 Solid-liquid equilibrium

$$\frac{dp}{dT} = \frac{\Delta_{fus} H_m}{\Delta V_{fus}}\frac{dT}{T}$$

$$p_2 = p_1 + \frac{\Delta_{fus} H_m}{\Delta_{fus} V_m}\ln\left(\frac{T_2}{T_1}\right)$$

4.14.2 Liquid-vapor equilibrium

Temperature dependence

$$\frac{dp}{dT} = \frac{\Delta_{vap} H_m}{T\Delta_{vap} V_m}$$

$$d\ln p = \frac{\Delta_{vap} H_m}{RT^2}dT$$

$$p_2 = p_1 \cdot e^{\frac{\Delta_{vap} H_m}{RT^2}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$$

Pressure dependence

$$p = p^* \cdot e^{\left(\frac{V_{m,liq}}{RT}\Delta p\right)}$$

4.14.3 Solid-vapor equilibrium

$$\frac{dp}{dT} = \frac{\Delta_{sub} H_m}{T\Delta_{sub} V_m}$$

$$d\ln p = \frac{\Delta_{sub} H_m}{RT^2}dT$$

$$p_2 = p_1 \cdot e^{\frac{\Delta_{sub} H_m}{RT^2}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$$

5 Chemical Kinetics

5.1 Reaction rate

Is always true that:

$$r = \frac{dx}{dt} = \frac{1}{\nu_B}\frac{dCb}{dt}$$

5.2 Advance coefficient

$$x = \frac{C_A^0 - C_A}{\nu_A}$$

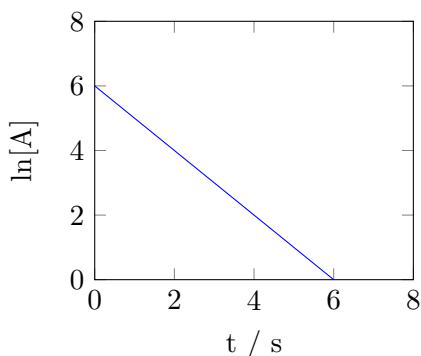### 5.2.1 Adimensional advance coefficient

$$g = \frac{C_A^0 - C_A}{C_A^0}$$

$$g = \frac{x}{C_A^0} \quad 0 \le g \le 1$$

$$x = gC_A^0 \implies C_A = C_A^0 - x$$

$$C_A = C_A^0(1 - g)$$

5.3 First order reactions

5.3.1 Rate expression

$$A \to P$$

$$r = -\frac{dC_A}{dt} = k[A]$$

$$-\ln C_A + \ln C_{0A} = kt$$

$$\ln(\frac{C_A^0}{C_A}) = e^{-kt}$$

$$\frac{C_A}{C_A^0} = e^{-kt}$$

Alternative rate expression

One can use g to express the rate:

$$CheckTickTick$$

5.3.2 Half-life time

$$t_{\frac{1}{2}} = \frac{\ln 2}{k} \quad [s^-]$$

In theory one can calculate the reactant concentration

$$\ln\left(\frac{C_A^0}{C_A}\right) = knt_{1/2}$$

$$kt_{1/2} = \ln 2 \implies C_A = \frac{C_A^0}{2^n}$$

5.3.3 Life time

It is define as the time needed to reduce the reactant concentrationby a factor of $\frac{1}{e}$

$$C_A = \frac{C_A^0}{e}$$

$$\frac{C_A}{C_A^0} = e^{-1}$$

$$kt = 1 \implies \tau = t = \frac{1}{k}$$

$$C_A = C_A^0 e^{-t/\tau}$$

5.4 Second order reactions

5.4.1 Unimolecolar reaction

$$r = \frac{dC_A}{dt} = k[A]^2$$

$$\frac{1}{C_A} - \frac{1}{C_A^0} = kt$$

Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_A^0 k}$$

5.4.2 Bimolecolar reaction

$$r = -\frac{dC_A}{dt} = -\frac{dC_B}{dt} = \frac{dx}{dt} = k[A][B]$$

$$\frac{1}{C_A^0 - C_B^0} \ln\left[\frac{(C_A^0 - x)C_B^0}{(C_B^0 - x)C_A^0}\right] = kt$$

$$\frac{C_B}{C_A} = \frac{C_B^0}{C_A^0}e^{(C_A^0 - C_B^0)kt}$$

Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_A^0 - C_B^0 k} \ln\left(\frac{2C_A^0 - C_B^0}{C_A^0}\right)$$

5.5 Zero order reactions

$$r = k$$
$$C = C^0 - kt$$

5.5.1 Half-life time

$$t_{\frac{1}{2}} = \frac{1}{2}t_f$$

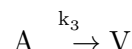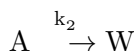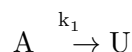## 5.6 nth order reactions

$$r = k[A]^n$$

$$\frac{1}{C^{n-1}} - \frac{1}{C^{0(n-1)}} = (n-1)kt$$

5.6.1 Half-life time

$$t_{\frac{1}{2}} = \frac{2^{n-1} - 1}{(C^0)^{n-1}k(n-1)}$$

5.7 Parallel reactions

In this condition A react with different velocity constant towards different products W,V and U.

$$A \xrightarrow{k_1} U$$
$$A \xrightarrow{k_2} W$$
$$A \xrightarrow{k_3} V$$

$$r = -\frac{dC_A}{dt} = k_1 C_A + k_2 C_A + k_3 C_A = kC_A$$

Equation for the products can be written as:

$$C_U - C_U^0 = \frac{k_1 C_A^0}{k}(1 - e^{-kt})$$

$$C_V - C_V^0 = \frac{k_2 C_A^0}{k}(1 - e^{-kt})$$

$$C_W - C_W^0 = \frac{k_3 C_A^0}{k}(1 - e^{-kt})$$

k can be determined sperimentally from the relation between C_A and time, in this case it would be a first order relation. Concentration of the products can be determined with sperimental methods. At this point the following system can be used:

$$\begin{cases} k_1 + k_2 + k_3 = k \\ \frac{C_U}{C_W} = \frac{k_1}{k_2} \\ \frac{C_U}{C_V} = \frac{k_2}{k_3} \end{cases}$$

5.8 Consecutive reactions

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
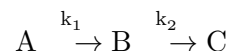
This is the simpliest case, with all first order reactions

$$r = -\frac{dC_A}{dt} = k_1 C_A$$

$$r = \frac{dC_B}{dt} = k_1 C_A - k_2 C_B$$

$$r = \frac{dC_C}{dt} = k_2 C_B$$

The C_C value in relation to time, in this case would be:

$$C_C = C_A^0\left[1 - \frac{k_2 e^{-k_1 t} - k_1 e^{-k_2 t}}{k_2 - k_1}\right] + C_B^0(1 - e^{-k_2 t}) + C_C^0$$

In case that $C_B^0 = 0, C_C^0 = 0$ then:

$$C_C = C_A^0\left[1 - \frac{k_2 e^{-k_1 t} - k_1 e^{-k_2 t}}{k_2 - k_1}\right]$$

5.9 Opposite reactions

5.10 Arrhenius's equation

$$k = A \exp\left(-\frac{Ea}{RT}\right)$$

$$\ln k = \ln A - \frac{E_a}{RT}$$

6 Electrochemistry

7 Wape Phenomena

7.1 Bragg Equation

$$n\lambda = 2d_{hkl}\sin\theta$$

7.2 Waves equation

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} = 0$$

The solution must be in the form:

$$\Psi(x,t) = g(x \pm vt)$$

7.3 Waves electric and magnetic field

$$B_0 = \frac{E_0}{c}$$

7.4 Waves variables

7.4.1 Wave lenght

$$\lambda = \frac{2\pi}{K}$$

7.4.2 Frequency

$$\nu = \frac{1}{T}$$

7.4.3 Pulsazione

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

7.4.4 Period

$$T = \frac{2\pi}{\omega}$$

7.5 Electromagnetic waves

$$E_x = B_x =$$
$$E_z^2 + E_y^2 = c^2(B_z^2 + B_y^2)$$
$$E = cB$$

Also you can calculate:

$$E_0 = Bv = B\frac{\omega}{k}$$

7.5.1 Power

$$S = \frac{1}{\mu_0}EvecB$$

7.5.2 Intensity

Planar wave

Sferic wave