

FORMULAS FOR LIFE

All formulas you'll ever need

Stefano Marton

A helpful tool for scientific studies

Contents

| | | | | | |
|----------|----------------------------------|----------|--|----------|--|
| 1 | Statistics | 2 | | | |
| 1.1 | Mean | 2 | | 4.3 | Internal Energy 5 |
| 1.1.1 | Arithmetic mean | 2 | | 4.4 | Enthalpy 5 |
| 1.1.2 | Geometric mean | 2 | | 4.4.1 | T dependence 5 |
| 1.1.3 | Harmonic mean | 2 | | 4.5 | Entropy 5 |
| 1.2 | Standard Deviation | 2 | | 4.5.1 | T dependence 5 |
| 1.2.1 | Absolute | 2 | | 4.6 | Isothermal Transformations 5 |
| 1.2.2 | Relative | 2 | | 4.6.1 | Free expansion 5 |
| 2 | Pure Physics | 3 | | 4.6.2 | Expansion vs. p_{ext} 6 |
| 3 | Pure Chemistry | 4 | | 4.6.3 | Reversible expansion 6 |
| 3.1 | Concentration | 4 | | 4.7 | Adiabatic Transformations 6 |
| 3.1.1 | Molarity | 4 | | 4.7.1 | Reversible process 6 |
| 3.1.2 | Mole | 4 | | 4.7.2 | Irreversible 6 |
| 4 | Thermodynamics | 5 | | 4.7.3 | Free expansion 6 |
| 4.1 | Perfect gases | 5 | | 4.7.4 | Expansion vs. p_{ext} 6 |
| 4.1.1 | State equation | 5 | | 4.7.5 | Reversible expansion 6 |
| 4.1.2 | Properties | 5 | | 4.8 | Isobaric transformations 6 |
| 4.2 | Real gases | 5 | | 4.8.1 | Reversible 6 |
| 4.2.1 | Virial equation | 5 | | 4.9 | Thermodynamic cycles 6 |
| 4.2.2 | van der Waals equation | 5 | | 4.9.1 | Carnot cycle 6 |
| | | | | 5 | Radiations |
| | | | | 5.1 | Bragg Equation 7 |

1 Statistics

1.1 Mean

1.1.1 Arithmetic mean

$$\bar{x} = \sum_{i=1}^n \frac{x_i}{N}$$

1.1.2 Geometric mean

$$\bar{x} = \left(\prod_{i=1}^n x_i \right)^{\frac{1}{n}}$$

1.1.3 Harmonic mean

$$\bar{x} = n \left(\sum_{i=1}^n \frac{1}{x_i} \right)^{-1}$$

1.2 Standard Deviation

1.2.1 Absolute

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{N - 1}}$$

1.2.2 Relative

$$RSD = \frac{s}{\bar{x}}$$

2 Pure Physics

3 Pure Chemistry

3.1 Concentration

3.1.1 Molarity

$$M = \frac{n}{V}$$

3.1.2 Mole

4 Thermodynamics

4.1 Perfect gases

4.1.1 State equation

$$pV = nRT$$
$$pV_m = RT$$

4.1.2 Properties

Molar mass

$$d_{gas} \cdot V_m = MW$$

Boyle's law

$$p_1 V_1 = p_2 V_2$$

Charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Gay-Lussac's Law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

4.2 Real gases

4.2.1 Virial equation

$$pV_m = RT \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \dots \right)$$

4.2.2 van der Waals equation

$$p = \frac{nRT}{V - nb} - a \left(\frac{n}{V} \right)^2$$
$$p = \frac{nRT}{V_m - b} - a \left(\frac{1}{V_m} \right)^2$$

4.3 Internal Energy

$$dU = \delta w + \delta q$$
$$dU = TdS - pdV$$

4.4 Entalphy

$$H = U + pV$$

4.4.1 T dependence

Pure substance

$$\Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} C_p dT.$$

Chemical reaction

$$\Delta(T_2) = \Delta H_r T_1 = \Delta C_p (T_2 - T_1)$$
$$\Delta C_p = [cC_{p,C} + dC_{p,D}] - [aC_{p,A} + bC_{p,B}]$$

4.5 Entropy

$$\Delta S_{TOT} \geq 0$$

$$\Delta S = \frac{q}{T} \implies \Delta S = \int_i^f \frac{q_{rev}}{T}$$

For an ideal gas it can be simplified:

$$\Delta S = nC_m \ln \left(\frac{T_f}{T_i} \right) + nR \ln \left(\frac{V_f}{V_i} \right)$$

4.5.1 T dependence

Constant pressure

$$\Delta S = S(T_f) - S(T_i) = C_p \int_i^f \frac{dT}{T} = C_p \ln \left(\frac{T_f}{T_i} \right)$$

Constant volume

$$\Delta S = S(T_f) - S(T_i) = C_v \int_i^f \frac{dT}{T} = C_v \ln \left(\frac{T_f}{T_i} \right)$$

4.6 Isothermal Transformations

4.6.1 Free expansion

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$

$$w = q = -p_{ext} \Delta V = 0$$

$$\Delta H = \Delta PV \quad (0 \text{ for perfect gas})$$

$$\Delta S = nR \ln \left(\frac{V_f}{V_i} \right) \quad \Delta S' = 0 \quad \Delta S_{tot} = \Delta S$$

4.6.2 Expansion vs. p_{ext}

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$

$$w = q = -p_{ext}\Delta V$$

$$\Delta H = \Delta PV \text{ (0 for perfect gas)}$$

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \quad \Delta S' = \frac{-q_{sistema}}{T} \quad \Delta S_{tot} = \Delta S + \Delta S'$$

4.6.3 Reversible expansion

$$\Delta T = 0 \implies \Delta U = 0 \implies q = w$$

$$q = w = -nRT \ln\left(\frac{V_f}{V_i}\right)$$

$$\Delta H = \Delta PV \text{ (0 for perfect gas)}$$

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \quad \Delta S' = -nR \ln\left(\frac{V_f}{V_i}\right) \implies \Delta S_{tot} = 0$$

4.7 Adiabatic Transformations

$$dU = dw$$

$$C_v dT = -pdV$$

$$w_{Adiabatic} = nC_{V,m}\Delta T$$

For an adiabatic process is also true that:

$$\gamma = \frac{C_{p,m}}{C_{V,m}} = \frac{C_p}{C_V} \quad P_1(V_1)^\gamma = P_2(V_2)^\gamma$$

4.7.1 Reversible process

$$C_V dT = \frac{-nRT}{V} dV$$

$$\int_{T_1}^{T_2} \frac{C_V dT}{T} = -nR \int_{T_1}^{T_2} \frac{dV}{V}$$

$$C_V \ln\left(\frac{T_1}{T_2}\right) = -R \ln\frac{V_2}{V_1}$$

$$\frac{T_2}{T_1} = \left[\frac{V_2}{V_1}\right]^{-\frac{R}{C_V}}$$

4.7.2 Irreversible

Take P as constant

$$\int_{T_1}^{T_2} C_V dT = -p \int_{V_1}^{V_2} dV$$

Assuming $C_V = \text{cost}$

$$C_V \Delta T = -p \Delta V$$

4.7.3 Free expansion

$$q = 0, \quad w = -p_{ext}\Delta V = 0 \implies \Delta U = 0$$

$$\Delta H =$$

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) \quad \Delta S' = 0 \quad \Delta S_{tot} = 0$$

4.7.4 Expansion vs. p_{ext}

$$q = 0 \implies \Delta U = w$$

$$\Delta U = nC_{V,m}\Delta T = w = -p_{ext}\Delta V$$

$$\Delta T = -\frac{p_{ext}\Delta V}{nC_{v,m}}$$

$$\Delta H = V\Delta p$$

$$\Delta S = nC_{V,m} \ln\left(\frac{T_f}{T_i}\right) + nR \ln\left(\frac{V_f}{V_i}\right)$$

$$\Delta S' = 0 \implies \Delta S_{TOT} = \Delta S$$

4.7.5 Reversible expansion

$$q = 0 \implies \Delta U = w$$

$$\Delta U = nC_{V,m}\Delta T$$

$$\Delta S = \Delta S' = \Delta_{tot} = 0$$

4.8 Isobaric transformations

4.8.1 Reversible

$$q = \Delta H = nC_{p,m}\Delta T$$

$$w = -pdV$$

$$\Delta S = nC_{p,m} \ln\left(\frac{T_f}{T_i}\right) \quad \Delta S' = -nC_{p,m} \ln\frac{V_f}{V_i}$$

$$\implies \Delta S_{TOT} = 0$$

4.9 Thermodynamic cycles

$$\Delta U = 0, \quad \Delta S = 0.$$

4.9.1 Carnot cycle

There is 4 stage (ABCD):

AB Reversible Isothermal expansion

$$\Delta U = 0 \implies q_{AB} = -w_{AB}$$

$$w_{AB} = -q_{AB} = -nRT_h \ln\left(\frac{V_B}{V_A}\right)$$

BC Reversible Adiabatic Expansion

$$q_{BC} = 0 \implies \Delta U_{BC} = w_{BC}$$

$$w_{BC} = nC_{V,m}(T_C - T_h)$$

CD Reversible Isothermal compression

$$\Delta U = 0 \implies q_{CD} = -w_{CD}$$

$$w_{CD} = -q_{CD} = -nRT_h \ln\left(\frac{V_D}{V_C}\right)$$

DA Reversible Adiabatic compression

$$q_{DA} = 0 \implies \Delta U_{DA} = w_{DA}$$

$$w_{DA} = nC_{V,m}(T_C - T_h)$$

5 Radiations

5.1 Bragg Equation

$$n\lambda = 2d_{hkl} \sin \theta$$