# FORMULAS FOR LIFE

All formulas you'll ever need

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A helpful tool for scientific studies

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## 1 Statistics

## 1.1 Mean

## 1.1.1 Arithmetic mean

$$\overline{x} = \sum_{i=1}^n \frac{x_i}{N}$$

### 1.1.2 Geometric mean

$$\overline{x}=(\prod_{i=1}^n x_i)^{\frac{1}{n}}$$

## 1.1.3 Harmonic mean

$$\overline{x} = n(\sum_{i=1}^n \frac{1}{x_i}^{-1})$$

## 1.2 Standard Deviation

## 1.2.1 Absolute

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \overline{x})^2}{N-1}}$$

#### 1.2.2 Relative

$$RSD = \frac{s}{\overline{x}}$$

## 2 Pure Physics

## 2.1 Uniform Rectilinear Motion

$$\begin{aligned} \mathbf{x}(\mathbf{t}) &= \mathbf{x}_0 + \mathbf{v}_0 \mathbf{t} \\ \mathbf{v} &= \mathbf{v}_0 \end{aligned}$$

## 2.2 Uniform Accelerated Rectilinear Motion

$$\begin{split} \mathbf{x}(\mathbf{t}) &= \mathbf{x}_0 + \mathbf{v}_0 \mathbf{t} + \frac{1}{2} \mathbf{a} \mathbf{t}^2 \implies \mathbf{x} - \mathbf{x}_0 = \frac{\mathbf{v}^2 - \mathbf{v}_0^2}{2 \mathbf{a}_0} \\ \mathbf{v}(\mathbf{t}) &= \mathbf{v}_0 + \mathbf{a} \mathbf{t} \\ \mathbf{a}(\mathbf{t}) &= \mathbf{a}_0 \end{split}$$

#### 2.3 Parabolic motion

On the x axis:

$$\begin{cases} a_x = 0 \\ v_x = v_{0x} \\ x(t) = x_0 + v_{0x}t \end{cases}$$

On the y axis:

$$\begin{cases} a_y = a_{0y} = g \\ v_y = v_{0y} + gt \\ y(t) = y_0 + v_{0y}t = \frac{1}{2}gt^2 \end{cases}$$

The range can be calculated as:

$$R = \frac{2v_{0x}v_{oy}}{g} = \frac{v_0^2 sen 2\alpha}{g}$$

The maximum height can be calculated as:

$$h_{max} = \frac{v_0^2 sen^2 \alpha}{2g}$$

#### 2.4 Uniform Circular Motion

#### 2.4.1 Angular velocity

$$\omega_{\rm avg} = \frac{\theta_2 - \theta_1}{\mathbf{t}_2 - \mathbf{t}_1} = \frac{\Delta \theta}{\Delta \mathbf{t}}$$

#### 2.4.2 Angular acceleration

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{\mathbf{t}_2 - \mathbf{t}_1}$$

#### 2.4.3 Relation with linear variables

#### Linear velocity

$$v = \omega r$$
 (radian measure)

#### Linear acceleration

$$a_{\rm tangential} = \alpha r \quad ({\rm radian\ measure})$$
 
$$a_{\rm radial} = \frac{v^2}{r} = w^2 r$$

#### Period

If the point moves in uniform circular motion:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

# 2.5 Rotation with constant acceleration

$$\omega = \omega_0 + \alpha t$$
 
$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

#### 2.6 Kinetic of rotation

$$E_k = \frac{1}{2} I \omega^2$$

I is the rotational inertia of the body; for a system of discrete particles it is defines as:

$$I = \sum m_i r_i^2$$

It has different formulas for different shapes

#### 2.6.1 Armonic Motion

$$x(t) = Asen(\omega * t + \phi)$$

It can be also calculated as:

$$\omega = \sqrt{\frac{k}{m}}$$

# 2.6.2 Centripetal and Centrifugal acceleration

$$a_c = \omega^2 r = \frac{v^2}{r}$$

#### 2.6.3 Teorema dell'impulso

$$I = \Delta p = m\Delta V = F\delta t$$

## 2.6.4 Newton Second Law's for rotation

$$\tau = I\alpha$$

## 2.7 Angular momentum

$$L = I\omega = rprodvetm\vec{v}$$

$$\Delta L = \tau$$
  $\Delta l = Ivectr$ 

## 2.8 Frictional Force

$$\vec{F}_{a} = -\mu N$$

## 2.9 Centripetal Force

$$F_c = m \frac{v^2}{r} = n\omega^2 r$$

## 2.10 Rope's tension

I 
$$\alpha = T R$$

## 2.11 Gravitational Law

$$F = G \frac{m_1 m_2}{r^2} - 6.6743 \cdot 10^{11} \text{m}^3 \text{kg s}^2$$

Is also possible to calculate the Gravitational Acceleration

$$g = \frac{GM}{r^2}$$

## 2.12 Pendulum Equation

$$m\frac{d^2x}{dt^2} + mgsen\theta = 0$$
 
$$\frac{d^2}{dt^2} + \frac{g}{L}sen\theta = 0 \implies \frac{g}{L} = \omega^2$$

If  $\theta \approx 0$  then:

$$T = \frac{2\pi}{\omega} = \sqrt{\frac{L}{g}} \quad \theta(t) = \theta_0 + \cos(\omega t + \phi)$$

## 2.13 Energy

#### 2.13.1 Work

$$\vec{L} = \vec{F} \cdot \Delta r = F \Delta r \cos \theta$$

#### 2.13.2 Kinetic Energy

$$E_{c}=\frac{1}{2}mv^{2}$$
 
$$L=\Delta E_{c}=\frac{1}{2}mv_{f}^{2}-\frac{1}{2}mv_{i}^{2}$$

#### 2.13.3 Elastic potential energy

$$U_{el} = \frac{1}{2}kx^2$$

#### 2.13.4 Gravitational Potential Energy

$$U_{\rm g}=mgh$$

#### 2.13.5 Mechanical energy

$$E_m = E_c + U$$

#### 2.13.6 Power

$$P = \frac{W}{\Delta t}$$

#### 2.14 Momentum

$$\vec{p} = m\vec{v}$$

#### 2.15 Electrostatic

### 2.15.1 Coulomb force

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

#### 2.15.2 Electric Field

$$E = \frac{F}{q_0}$$

Point-like charge

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} r$$

Continuos distribution

$$E = \frac{1}{4\pi\epsilon_0} \int \frac{\mathrm{dq}}{\mathrm{r}^2} \mathrm{dr}$$

Plates electric field

$$E = \frac{\sigma}{2\epsilon_0}$$

Capacitor or conductor electric field

$$E = \frac{\sigma}{\epsilon_0}$$

Dipole electric field

$$\mathrm{E}(\mathrm{P}) = -\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{p}}{\mathrm{R}^3}$$

Where p is called dipole momentum and is equal to:

$$p = 2qa\hat{e}_z$$

Cable electric field

$$E = \frac{\Delta V}{L}$$

#### 2.15.3 Electric field flux

$$\Phi = \int \mathbf{E} \cdot \mathbf{dS}$$

#### 2.15.4 Gauss Law

$$\Phi(E) = \frac{\sum_{k=1}^{N} q_{k \text{ int}}}{\epsilon_0}$$

#### 2.15.5 Potential electric energy

$$\begin{split} L_{AB} &= q_0 \int_A^B E \cdot dl = U(B) - U(A) \\ L_{AB} &= \sum_{k=1}^N [U_k(B) - U_k(A)] \quad U_k = k \frac{q_0 q_k}{r} \end{split}$$

#### Displacement along a plates

$$L_{AB} = qE(r_B - r_A)$$

#### 2.15.6 Electrical Potential

$$V_k = k \frac{q_k}{r} \quad U = q_0 V$$

### 2.15.7 Capacitor capacity

$$C = \frac{q}{V}$$
  $[C] = \frac{C}{V} = Farad$   $C = \varepsilon_0 \frac{A}{d}$ 

#### Capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

#### Capacitors in parallel

$$C_{eq} = \sum_{i=1}^{N} C_i$$

#### Stored energy

$$\Delta U = \frac{1}{2}CV^{2}$$
$$\Delta U = \frac{1}{2}\varepsilon_{0}AE^{2}d$$

#### 2.15.8 Dielectric Properties

The relative Dielectric constant is calculated as:

$$\varepsilon = \frac{V_0}{V}$$

Where  $V_0$  is the Potential measured in empty space. It can be useful to calculate capacitor capacity, considering different material between the two plates.

$$C=C_0\varepsilon_r$$

#### 2.15.9 Electric Current

$$I = \frac{dq}{dt}$$
  $[I] = \frac{C}{s} = Ampere$ 

It can be expressed in function to the number of charge carriage

$$I = nqAv_d$$

#### **Electric Density**

$$\begin{split} \mathbf{j} &= \mathbf{nq} \mathbf{v_d} \\ \mathbf{I} &= \int \mathbf{J} \cdot \hat{\mathbf{n}} d\mathbf{S} \\ \Phi(\mathbf{j}) &= \mathbf{I} = \int \mathbf{J} \cos \theta d\mathbf{S} \end{split}$$

#### 2.15.10 Ohm's Law

The general expression can be written as:

$$j = \sigma E$$

Where  $\sigma$  is the conductivity. For a wire it can be written as follows:

$$\begin{cases} \Delta V = RI \\ R = \rho \frac{1}{A} \end{cases}$$

Where  $\rho$  is equal to  $\frac{1}{\sigma}$  and it is called resistivity

#### Resistor in series

$$R_{eq} = \sum_{i=1}^{N} R_i$$

#### Resistor in parallel

$$\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i}$$

#### Circuit fem

$$RI = \Delta V = \varepsilon - rI$$

Where r is the circuit resistance

#### 2.15.11 Power and Joule's effect

$$P = \frac{dU}{dT} = \frac{dq}{dt}\Delta V = I\Delta V$$

Using Ohm's law it can be rewritten as:

$$P = RI^2$$

n The energy dissipated by a resistor is equal to:

$$E_d = \frac{1}{2} C \varepsilon^2$$

## 2.16 Magnetism

#### 2.16.1 Lorentz force

A particle of charge q moving with a velocity v in an electric field E and a magnetic field B experiences a force equal to:

$$\vec{F} = q\vec{v} \times \vec{B}$$

For a wire it can be also calculated as:

$$\vec{F} = \vec{B}IL$$

seconda legge di la place

#### 2.16.2 La Place Law

$$\begin{split} \mathrm{d}\vec{\mathrm{B}} &= \frac{\mu_0}{4\pi} \frac{\mathrm{I}}{\mathrm{r}^2} \mathrm{d}\vec{\mathrm{I}} \times \hat{\mathrm{r}} \\ \vec{\mathrm{B}}(\mathrm{P}) &= \int \mathrm{d}\vec{\mathrm{B}} \\ |\mathrm{d}\vec{\mathrm{B}}| &= \frac{\mu_0}{4\pi} \frac{\mathrm{I}}{\mathrm{r}^2} \; \mathrm{d}\vec{\mathrm{I}} \; \mathrm{sen}\theta \end{split}$$

#### 2.16.3 Ampere teoreme

#### Solenoide

$$B = \mu_0 I \frac{N}{L}$$

Wire

$$B = \mu_0 \frac{I}{2\pi r}$$

#### 2.16.4 Magnetic Flux

$$\Phi_{\rm B} = N \int \vec{B} \cdot d\vec{\sigma} = NBS \cos \alpha$$

For a rotational motion dependence

$$\Psi = BS \cos \omega t$$

#### 2.16.5 Electromagnetic induction

Is also known as Faraday-Newman-Lentz Law

$$\varepsilon = -\frac{\mathrm{d}}{\mathrm{dt}}$$

The inductive effect can also be expressed as

$$\varepsilon_{\rm L} = -{\rm L} = \frac{{\rm di}}{{\rm dt}}$$

For a solenoid is true that:

$$L = \mu_0 \left(\frac{N}{L}\right)^2 V$$

Where V is the volume included in the solenoid. The relationship can be also writte as:

$$\frac{d}{dt} = L \frac{dI}{dt}$$
 
$$L = \frac{d}{dI}$$

For variable area

$$\varepsilon = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\mathrm{BdS}}{\mathrm{d}t} = \mathrm{B}\frac{\mathrm{l}\mathrm{d}x}{\mathrm{d}t} = \mathrm{Blv}$$

For a variable area dependent on circular motion

$$\varepsilon = \omega BS \sin \omega t$$

2.16.6 Magnetic Energy

$$U_m = \frac{1}{2}LI^2$$

2.16.7 LC Circuit

$$q(t) = q_0 \cos(\omega t + \phi) \quad \frac{1}{\sqrt{LC}}$$

2.16.8 LR Circuit

$$i(t) = -\frac{\varepsilon_0}{R}[1 - e^{-\frac{R}{L}t}]$$

2.16.9

# 3 Pure Chemistry

3.1 Concentration

3.1.1 Mole

m

3.1.2 Molarity

$$M = \frac{n}{V}$$

## 4 Thermodynamics

## 4.1 Perfect gases

#### 4.1.1 State equation

$$pV = nRT$$
$$pV_m = RT$$

#### 4.1.2 Properties

Molar mass

$$d_{gas} \cdot V_m = MW$$

Boyle's law

$$p_1V_1 = p_2V_2 \quad T = const$$

Charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad p = const$$

Gay-Lussac's Law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad V = const$$

#### 4.1.3 Compression factor

$$Z = \frac{PV}{nRT}$$

Z = 1 for perfect gases

## 4.2 Real gases

#### 4.2.1 Virial equation

$$pV_{m} = RT \left( 1 + \frac{B}{V_{m}} + \frac{C}{V_{m}^{2}} + \dots \right)$$

#### 4.2.2 van der Waals equation

$$\begin{aligned} p &= \frac{nRT}{V - nb} - a \left(\frac{n}{V}\right)^2 \\ p &= \frac{nRT}{V_m - b} - \frac{a}{V_m^2} \end{aligned}$$

#### 4.2.3 Compression factor

$$\begin{split} Z &= \frac{V_m}{V_m - b} - \frac{a}{RTV_m} \\ &\text{if } y = \frac{1}{V_m} \implies \frac{1}{1 - by} - \frac{ay}{RT} \end{split}$$

#### 4.2.4 Boyle Temperature

$$\begin{split} \lim_{p\to 0} \frac{dZ}{dp} &= 0 \quad p\to 0 \implies \frac{1}{V_m} \to 0 \\ y &= \frac{1}{V_m} \quad \lim_{y\to 0} \frac{1}{1-by} - \frac{ay}{RT} = \\ T_b &= \frac{a}{Rb} \end{split}$$

#### 4.2.5 Critical and reduced variables

$$T_c = \frac{8a}{27Rb}$$
  $p_c = \frac{a}{27b^2}$   $V_c = 3b$   $Z_c = \frac{p_c V_c}{RT_c}$ 

The van der Waals equation can be rewritten:

$$p_{r} = \frac{8T_{r}}{3V_{r} - 1} - \frac{3}{V_{r}^{2}}$$

## 4.3 Internal Energy

$$\begin{split} \Delta U &= q + w \\ dU &= \delta w + \delta q \\ dU &= T dS - p dV \\ dU &= \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \end{split}$$

Is always true that:

$$\begin{split} dU &= \int_{T_1}^{T_2} C_V dT \\ \Delta U &= n C_{v,m} \Delta T \end{split}$$

## 4.4 Enthalpy

$$\begin{split} \mathbf{H} &= \mathbf{U} + \mathbf{p} \mathbf{V} \\ \Delta \mathbf{H} &= \Delta \mathbf{U} + \Delta \mathbf{p} \mathbf{V} \\ \mathbf{d} \mathbf{H} &= -\mathbf{S} \mathbf{d} \mathbf{T} + \mathbf{V} \mathbf{d} \mathbf{p} \end{split}$$

For the perfect gas is also true that:

$$\Delta H = \Delta U + \Delta (nRT)$$

## 4.4.1 Temperature dependence

Pure substance

$$\Delta H(T_2) = \Delta H(T_1) + \int_{T_1}^{T_2} C_p dT.$$

Chemical reaction or phase transition

$$\begin{split} \Delta H(T_2) &= \Delta H_r T_1 + \Delta C_p (T_2 - T_1) \\ \Delta C_p &= [cC_{p,C} + dC_{p,D}] - [aC_{p,A} + bC_{p,B}] \end{split}$$

## 4.5 Entropy

$$\begin{split} \Delta S_{TOT} &\geq 0 \\ \Delta S &= \frac{q}{T} \implies \Delta S = \int_{i}^{f} \frac{q_{rev}}{T} \end{split}$$

For an ideal gas it is always true that:

$$\Delta S = n C_m \ln \left(\frac{T_f}{T_i}\right) + n R \ln \left(\frac{V_f}{V_i}\right)$$

Only mixing two gases results in:

$$\Delta S_{mix} = -R \sum_{i=1}^N n_i \ln \chi_i$$

#### 4.5.1 Trouton's Rules

$$\Delta S_{trs} = \frac{\Delta H_{trs}}{T_{trs}}$$

#### 4.5.2 Temperature dependence

Constant pressure

$$\Delta S = S(T_f) - S(T_i) = C_p \int_i^f \frac{dT}{T} = C_p \ln \left( \frac{T_f}{T_i} \right)$$

#### Constant volume

$$\Delta S = S(T_f) - S(T_i) = C_v \int_i^f \frac{dT}{T} = C_v \ln \left( \frac{T_f}{T_i} \right)$$

#### 4.5.3 Chemical reaction

$$\begin{split} aA + bB &\rightarrow cC + dD \\ \Delta_r S^0 &= (cS_C^0 + dS_D^0) - (aS_A^0 + bS_B^0) \\ \Delta_r S^0 &= \sum_{i=1}^N \nu_i S_{reagents}^0 - \sum_{i=1}^N \nu_i S_{products}^0 \end{split}$$

If calculating  $T \neq 298K$ :

$$\begin{split} \Delta_{r}S(T_{2}) &= \Delta_{r}S(T_{1}) + \Delta C_{p} \ln \frac{T_{2}}{T_{1}} \\ \Delta C_{p} &= (cC_{c,p} + dC_{d,p}) - (aC_{a,p} + bC_{b,p}) \end{split}$$

#### 4.6 Isothermal Transformations

#### 4.6.1 Free expansion

$$\begin{split} \Delta T &= 0 \implies \Delta U = 0 \implies q = w \\ w &= q = -p_{ext} \Delta V = 0 \\ \Delta H &= \Delta PV \; (0 \text{ for perfect gas}) \\ \Delta S &= nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = 0 \quad \Delta S_{tot} = \Delta S \end{split}$$

## 4.6.2 Expansion vs. $p_{ext}$

$$\begin{split} \Delta T &= 0 \implies \Delta U = 0 \implies q = w \\ w &= q = -p_{ext} \Delta V \\ \Delta H &= \Delta PV \; (0 \; \text{for perfect gas}) \\ \Delta S &= nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = \frac{-q_{sistema}}{T} \\ \Delta S_{tot} &= \Delta S + \Delta S' \end{split}$$

#### 4.6.3 Reversible expansion

$$\begin{split} \Delta T &= 0 \implies \Delta U = 0 \implies q = w \\ q &= w = -nRT \ln(\frac{V_f}{V_i}) \\ \Delta H &= \Delta PV \; (0 \; \text{for perfect gas}) \\ \Delta S &= nR \ln(\frac{V_f}{V_i}) \quad \Delta S' = -nR \ln(\frac{V_f}{V_i}) \\ \Delta S_{tot} &= 0 \end{split}$$

#### 4.7 Adiabatic Transformations

$$\begin{aligned} \mathrm{d} \mathbf{U} &= \mathrm{d} \mathbf{w} \\ \mathbf{C}_{\mathbf{v}} \mathrm{d} \mathbf{T} &= -\mathrm{p} \mathrm{d} \mathbf{V} \\ \mathbf{w}_{\mathrm{Adiabatic}} &= \mathrm{n} \mathbf{C}_{\mathrm{V,m}} \Delta \mathbf{T} \end{aligned}$$

For an adiabatic process is also true that:

$$\gamma = \frac{C_{p,m}}{C_{V,m}} = \frac{C_p}{C_V} \quad P_1(V_1)^\gamma = P_2(V_2)^\gamma$$

#### 4.7.1 Reversible process

$$\begin{split} C_V \mathrm{d}T &= \frac{-\mathrm{n}RT}{V} \mathrm{d}V \\ \int_{T_1}^{T_2} \frac{C_V \mathrm{d}T}{T} &= -\mathrm{n}R \int_{T_1}^{T_2} \frac{\mathrm{d}V}{V} \\ C_V \ln \left(\frac{T_1}{T_2}\right) &= -R \ln \frac{V_2}{V_1} \\ \frac{T_2}{T_1} &= \left[\frac{V_2}{V_1}\right]^{-\frac{R}{C_V}} \end{split}$$

#### 4.7.2 Irreversible

Take P as constant

$$\begin{split} \int_{T_1}^{T_2} Cv dT &= -p \int_{V_1}^{V_2} dV \\ Assuming \ C_V &= cost \\ C_V \Delta T &= -p \Delta V \end{split}$$

#### 4.7.3 Free expansion

$$\begin{split} \mathbf{q} &= \mathbf{0}, \ \mathbf{w} = -\mathbf{p}_{\mathrm{ext}} \Delta \mathbf{V} = \mathbf{0} \implies \Delta \mathbf{U} = \mathbf{0} \\ \Delta \mathbf{H} &= \\ \Delta \mathbf{S} &= \mathbf{n} \mathbf{R} \ln(\frac{\mathbf{V}_{\mathrm{f}}}{\mathbf{V}_{\mathrm{i}}}) \quad \Delta \mathbf{S}' = \mathbf{0} \quad \Delta \mathbf{S}_{\mathrm{tot}} = \mathbf{0} \end{split}$$

#### 4.7.4 Expansion vs. $p_{ext}$

$$\begin{split} q &= 0 \implies \Delta U = w \\ \Delta U &= nC_{V,m} \Delta T = w = -p_{ext} \Delta V \\ \Delta T &= -\frac{p_{ext} \Delta V}{nC_{v,m}} \\ \Delta H &= V \Delta p \\ \Delta S &= nC_{V,m} \ln(\frac{T_f}{T_i}) + nR \ln(\frac{V_f}{V_i}) \\ \Delta S' &= 0 \implies \Delta S_{TOT} = \Delta S \end{split}$$

#### 4.7.5 Reversible expansion

$$\begin{split} \mathbf{q} &= 0 \implies \Delta \mathbf{U} = \mathbf{w} \\ \Delta \mathbf{U} &= \mathbf{n} \mathbf{C}_{\mathrm{V,m}} \Delta \mathbf{T} \\ \Delta \mathbf{S} &= \Delta \mathbf{S}' = \Delta_{\mathrm{tot}} = 0 \end{split}$$

#### 4.8 Isobaric transformations

$$\begin{split} dp &= 0 \\ q &= \Delta H = nC_{p,m} \Delta T \\ w &= -pdV \\ \Delta S &= nC_{p,m} \ln(\frac{T_f}{T_i}) \quad \Delta S' = -nC_{p,m} \ln \frac{V_f}{V_i} \\ \Longrightarrow \Delta S_{TOT} &= 0 \end{split}$$

## 4.9 Thermodynamic cycles

$$\Delta U = 0$$
,  $\Delta S = 0$ ,  $\Delta H = 0$ .

#### 4.9.1 Carnot cycle

There is 4 stage (ABCD):

#### AB Reversible Isothermal expansion

$$\begin{split} \Delta U &= 0 \implies q_{AB} = -w_{AB} \\ w_{AB} &= -q_{AB} = -nRT_h \ln \left( \frac{V_B}{V_A} \right) \end{split}$$

#### BC Reversible Adiabatic Expansion

$$\begin{aligned} q_{BC} &= 0 \implies \Delta U_{BC} = w_{BC} \\ w_{BC} &= nC_{V,m}(T_C - T_h) \end{aligned}$$

#### CD Reversible Isothermal compression

$$\begin{split} \Delta U &= 0 \implies q_{CD} = -w_{CD} \\ w_{CD} &= -q_{CD} = -nRT_h \ln \left(\frac{V_D}{V_C}\right) \end{split}$$

#### DA Reversible Adiabatic compression

$$\begin{split} q_{DA} &= 0 \implies \Delta U_{DA} = w_{DA} \\ w_{DA} &= n C_{V,m} (T_C - T_h) \end{split}$$

## 4.10 Helmholtz's Energy

$$\begin{aligned} \mathbf{A} &= \mathbf{U} - \mathbf{S} \mathbf{T} \\ \Delta \mathbf{A}_{\mathbf{V}} &= \Delta \mathbf{U}_{\mathbf{V}} - \mathbf{T} \Delta \mathbf{S}_{\mathbf{V}} \end{aligned}$$

 $\Delta A$  is the maximum (more negative) work that can be done by the system if dT=0.

$$0 \ge w \ge \Delta A$$

It can also be expressed with natural variables:

$$\begin{split} \mathrm{d}\mathbf{A} &= -\mathrm{S}\mathrm{d}\mathbf{T} - \mathrm{p}\mathrm{d}\mathbf{V} \\ \mathrm{d}\mathbf{A} &= \left(\frac{\partial \mathbf{A}}{\partial \mathbf{T}}\right)_{\mathbf{V}}\mathrm{d}\mathbf{T} + \left(\frac{\partial \mathbf{A}}{\partial \mathbf{V}}\right)_{\mathbf{T}}\mathrm{d}\mathbf{V} \end{split}$$

## 4.11 Gibbs's Energy

$$\begin{aligned} \mathbf{G} &= \mathbf{H} - \mathbf{S} \mathbf{T} \\ \Delta \mathbf{G}_{\mathbf{p}} &= \Delta \mathbf{H}_{\mathbf{p}} - \mathbf{T} \Delta \mathbf{S}_{\mathbf{p}} \end{aligned}$$

 $\Delta G$  is the maximum (more negative) work that can be done by the system if dT=0.

$$0 \ge w_{ne} \ge \Delta G$$

It can also be expressed with natural variables:

$$\begin{split} dG &= -SdT + Vdp \\ dG &= \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp \end{split}$$

#### 4.11.1 Gibbs-Helmholtz's equation

$$\begin{split} \frac{\partial}{\partial T} \left(\frac{\Delta G}{T}\right)_p &= -\frac{\Delta H}{T^2} \\ \frac{\Delta G(T_2)}{T_2} &- \frac{\Delta G(T_1)}{T_1} = \Delta H \left(\frac{1}{T_2} - \frac{1}{T_1}\right) \end{split}$$

#### 4.11.2 Pressure dependence

$$\Delta G = G(p_2) - \Delta G(p_1) = nRT \ln \left(\frac{p_2}{p_1}\right)$$

It can be also used to find  $G_m(T_2)$  for a reaction:

$$\begin{split} \mathrm{nA} &\to \mathrm{mB} \\ \mathrm{nG}(\mathrm{A},\mathrm{p}_2) &= \mathrm{nG}^0 + \mathrm{nRT} \ln \mathrm{p}_2 \\ \mathrm{nG}(\mathrm{B},\mathrm{p}_2) &= \mathrm{mG}^0 + \mathrm{mRT} \ln \mathrm{p}_2 \\ \Delta \mathrm{G}(\mathrm{p}_2) &= \mathrm{mG}(\mathrm{B},\mathrm{p}_2) - \mathrm{nG}(\mathrm{A},\mathrm{p}_2) = \\ \Delta \mathrm{G}^0 + (\mathrm{m} - \mathrm{n}) \mathrm{RT} \ln \mathrm{p}_2 \end{split}$$

#### 4.12 Chemical potential

G is an extensive variable, otherwise a p, T = cost it would be always true that dG = 0. G depends on the composition of the system:  $G = G(p, T, n_1, n_2, ...)$  The partial derivative of G is the chemical potential:

$$\mu = \left(\frac{\partial G}{\partial n_i}\right)_{p,T,n_i} \implies dG = \sum_{i=1}^N \mu_i dn_i$$

#### 4.12.1 Gibbs-Duhem equation

$$\sum_{i=1}^{N} n_i d\mu_i = -SdT + Vdp$$

If temperature and pressure are constant:

$$\sum_{i=1}^N n_i d\mu_i = 0$$

#### 4.12.2 Natural variables

$$\left(\frac{\mathrm{d}}{\mathrm{dp}}\right)_{\mathrm{T}} = V_{\mathrm{m}} \quad \left(\frac{\mathrm{d}}{\mathrm{dT}}\right)_{\mathrm{p}} = -S_{\mathrm{m}}$$

## 4.12.3 Perfect gas chemical potential

$$\begin{split} \left(\frac{d}{dp}\right)_T \\ \int_{p^0}^p &= \int_{p_0}^p \frac{RT}{p} = dp \\ \mu(p,T) &= \mu_0(p_0,T) + RT \ln \left(\frac{p}{p_0}\right) \end{split}$$

## 4.13 Chemical equilibrium

 $\Delta_{\rm r} G$  is related to G variation at constant temperature and pressure

$$\Delta_r G = \left(\frac{\partial G}{\partial \xi}\right)_{p,T} = \sum_{i=1}^n \nu_i \mu_i$$

Is possible to relate  $\Delta_r G^0$  and K :

$$K = e^{-\frac{\Delta_r G^0}{RT}}$$

Different K calculation are related easily:

$$\begin{split} K_{c} &= K_{n} \left(\frac{1}{V}\right)^{\Delta \nu} \ K_{\chi} = K_{n} \left(\frac{1}{n_{tot}}\right)^{\Delta \nu} \\ K_{p} &= K_{n} \left(\frac{p_{tot}}{n_{tot}}\right)^{\Delta \nu} \end{split}$$

#### 4.13.1 Pressure dependence

$$\begin{split} \frac{\partial \ln K}{\partial p} &= -\frac{1}{RT} \left( \frac{\partial \Delta_r G}{\partial p} \right) \\ &\frac{\partial \ln K}{\partial p} = -\frac{\Delta_r V}{RT} \\ \Delta_r V &\approx 0 \implies \left( \frac{\partial K}{\partial p} \right)_T = 0 \end{split}$$

Still, the equilibrium's composition is not independent of the pressure.

#### 4.13.2 Temperature dependence

$$\begin{split} \frac{\partial \mathrm{d} \ln \mathrm{K}}{\partial \frac{1}{\mathrm{T}}} &= -\frac{\Delta_{\mathrm{r}} \mathrm{H}^0}{\mathrm{R}} \\ \ln \left( \frac{\mathrm{K}_2}{\mathrm{K}_1} \right) &= -\frac{\Delta_{\mathrm{r}} \mathrm{H}^0}{\mathrm{R}} \left( \frac{1}{\mathrm{T}_2} - \frac{1}{\mathrm{T}_1} \right) \\ \mathrm{K}_2 &= \mathrm{e}^{\left( -\frac{\Delta_{\mathrm{r}} \mathrm{H}^0}{\mathrm{R}} \left( \frac{1}{\mathrm{T}_2} - \frac{1}{\mathrm{T}_2} \right) \right)} \cdot \mathrm{K}_1 \end{split}$$

## 4.14 Clausius-Clapeyron equation

$$\begin{split} & \text{Clausius' equation} & \quad \frac{dp}{dT} = \frac{\Delta S_m}{\Delta V_m} \\ & \text{Clausius-Clapeyron} & \quad \frac{dp}{dT} = \frac{\Delta H_m}{T\Delta V_m} \end{split}$$

#### 4.14.1 Solid-liquid equilibrium

$$\begin{split} \frac{dp}{dT} &= \frac{\Delta_{fus} H_m}{\Delta V_{fus}} \frac{dT}{T} \\ p_2 &= p_1 + \frac{\Delta_{fus} H_m}{\Delta_{fus} V_m} \ln \left( \frac{T_2}{T_1} \right) \end{split}$$

#### 4.14.2 Liquid-vapor equilibrium

Temperature dependence

$$\begin{split} \frac{dp}{dT} &= \frac{\Delta_{vap} H_m}{T \Delta_{vap} V_m} \\ d\ln p &= \frac{\Delta_{vap} H_m}{RT^2} dT \\ p_2 &= p_1 \cdot e^{\frac{\Delta_{vap} H_m}{RT^2} \left(\frac{1}{T_2} - \frac{1}{T_1}\right)} \end{split}$$

Pressure dependence

$$p = p^* \cdot e^{\left(\frac{V_{m,liq}}{RT}\Delta p\right)}$$

#### 4.14.3 Solid-vapor equilibrium

$$\begin{split} \frac{dp}{dT} &= \frac{\Delta_{sub} H_m}{T \Delta_{sub} V_m} \\ d\ln p &= \frac{\Delta_{sub} H_m}{RT^2} dT \\ p_2 &= p_1 \cdot e^{\frac{\Delta_{sub} H_m}{RT^2} \left(\frac{1}{T_2} - \frac{1}{T_1}\right)} \end{split}$$

## 5 Chemical Kinetics

#### 5.1 Reaction rate

Is always true that:

$$r = \frac{dx}{dt} = \frac{1}{\nu_B} \frac{dCb}{dt}$$

#### 5.2 Advance coefficient

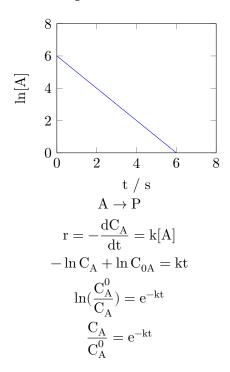
$$x = \frac{C_A^0 - C_A}{\nu_A}$$

#### 5.2.1 Adimensional advance coefficient

$$g = \frac{C_A^0 - C_A}{C_A^0}$$
 
$$g = \frac{x}{C_A^0} \quad 0 \le g \le 1$$
 
$$x = gC_A^0 \implies C_A = C_A^0 - x$$
 
$$C_A = C_A^0 (1 - g)$$

#### 5.3 First order reactions

#### 5.3.1 Rate expression



#### Alternative rate expression

One can use g to express the rate:

CheckTickTick

#### 5.3.2 Half-life time

$$t_{\frac{1}{2}} = \frac{\ln 2}{k} \quad [s^-]$$

In theory one can calculate the reactant concentration

$$\ln \left(\frac{C_A^0}{C_A}\right) = knt_{1/2}$$

$$kt_{1/2} = \ln 2 \implies C_A = \frac{C_A^0}{2^n}$$

#### 5.3.3 Life time

It is define as the time needed to reduce the reactant concentration by a factor of  $\frac{1}{e}$ 

$$\begin{split} C_A &= \frac{C_A^0}{e} \\ \frac{C_A}{C_A^0} &= e^{-1} \\ kt &= 1 \implies \tau = t = \frac{1}{k} \\ C_A &= C_A^0 e^{-t/\tau} \end{split}$$

#### 5.4 Second order reactions

#### 5.4.1 Unimolecolar reaction

$$\begin{split} r &= \frac{dC_A}{dt} = k[A]^2 \\ &\frac{1}{C_A} - \frac{1}{C_A^0} = kt \end{split}$$

Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_{\Delta}^0 k}$$

#### 5.4.2 Bimolecolar reaction

$$\begin{split} r &= -\frac{dC_A}{dt} = -\frac{dC_B}{dt} = \frac{dx}{dt} = k[A][B] \\ &\frac{1}{C_A^0 - C_B^0} \; \ln \left[ \frac{(C_A^0 - x)C_B^0}{(C_B^0 - x)C_A^0} \right] = kt \\ &\frac{C_B}{C_A} = \frac{C_B^0}{C_A^0} e^{(C_A^0 - C_B^0)kt} \end{split}$$

#### Half-life time

$$t_{\frac{1}{2}} = \frac{1}{C_A^0 - C_B^0 k} \ln \left( \frac{2C_A^0 - C_B^0}{C_A^0} \right)$$

### 5.5 Zero order reactions

$$r = k$$
$$C = C^0 - kt$$

#### 5.5.1 Half-life time

$$t_{\frac{1}{2}}=\frac{1}{2}t_f$$

#### 5.6 nth order reactions

$$r = k[A]^{n}$$

$$\frac{1}{C^{n-1}} - \frac{1}{C^{0(n-1)}} = (n-1)kt$$

#### 5.6.1 Half-life time

$$t_{\frac{1}{2}} = \frac{2^{n-1}-1}{(C^0)^{n-1}k(n-1)}$$

#### 5.7 Parallel reactions

In this condition A react with different velocity constant towards different products W,V and U.

$$\begin{split} A & \stackrel{k_1}{\rightarrow} U \\ A & \stackrel{k_2}{\rightarrow} W \\ A & \stackrel{k_3}{\rightarrow} V \\ r = -\frac{dC_A}{dt} = k_1C_A + k_2C_A + k_3C_A = kC_A \end{split}$$

Equation for the products can be written as:

$$\begin{split} C_{U} - C_{U}^{0} &= \frac{k_{1}C_{A}^{0}}{k}(1 - e^{-kt}) \\ C_{V} - C_{V}^{0} &= \frac{k_{2}C_{A}^{0}}{k}(1 - e^{-kt}) \\ C_{W} - C_{W}^{0} &= \frac{k_{3}C_{A}^{0}}{k}(1 - e^{-kt}) \end{split}$$

k can be determined sperimentally from the relation between  $C_A$  and time, in this case it would be a first order relation. Concentration of the products can be determined with sperimental methods. At this point the following system can be used:

$$\begin{cases} \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = \mathbf{k} \\ \frac{\mathbf{C}_{\mathbf{U}}}{\mathbf{C}_{\mathbf{U}}} = \frac{\mathbf{k}_1}{\mathbf{k}_2} \\ \frac{\mathbf{C}_{\mathbf{U}}}{\mathbf{C}_{\mathbf{V}}} = \frac{\mathbf{k}_2}{\mathbf{k}_3} \end{cases}$$

#### 5.8 Consecutive reactions

$$\mathbf{A} \overset{\mathbf{k}_1}{\to} \mathbf{B} \overset{\mathbf{k}_2}{\to} \mathbf{C}$$

This is the simpliest case, with all first order reactions

$$\begin{split} r &= -\frac{dC_A}{dt} = k_1C_A \\ r &= \frac{dC_B}{dt} = k_1C_A - k_2C_B \\ r &= \frac{dC_C}{dt} = k_2C_B \end{split}$$

The  $C_C$  value in relation to time, in this case would be:

$$C_{C} = C_{A}^{0} \left[ 1 - \frac{k_{2}e^{-k_{1}t} - k_{1}e^{-k_{2}t}}{k_{2} - k_{1}} \right] + C_{B}^{0} (1 - e^{-k_{2}t}) + C_{C}^{0}$$

In case that  $C_B^0 = 0, C_C^0 = 0$  then:

$$C_{C} = C_{A}^{0} \left[ 1 - \frac{k_{2}e^{-k_{1}t} - k_{1}e^{-k_{2}t}}{k_{2} - k_{1}} \right]$$

## 5.9 Opposite reactions

## 5.10 Arrhenius's equation

$$k = A \exp\left(-\frac{Ea}{RT}\right)$$
 
$$\ln k = \ln A - \frac{E_a}{RT}$$

# 6 Electrochemistry

## 7 Wape Phenomena

## 7.1 Bragg Equation

$$n\lambda = 2d_{hkl}\sin\theta$$

## 7.2 Waves equation

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

The solution must be in the form:

$$\Psi(x,t) = g(x \pm vt)$$

# 7.3 Waves electric and magnetic field

$$B_0 = \frac{E_0}{c}$$

## 7.4 Waves variables

## 7.4.1 Wave lenght

$$\lambda = \frac{2\pi}{K}$$

## 7.4.2 Frequency

$$\nu = \frac{1}{T}$$

#### 7.4.3 Pulsazione

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

#### **7.4.4** Period

$$T = \frac{2\pi}{\omega}$$

## 7.5 Electromagnetic waves

$$\begin{split} E_x &= B_x = \\ E_z^2 + E_y^2 &= c^2(B_z^2 + B_y^2) \\ E &= cB \end{split}$$

Also you can calculate:

$$E_0 = Bv = B\frac{\omega}{k}$$

#### 7.5.1 Power

$$S = \frac{1}{\mu_0} EvecB$$

### 7.5.2 Intensity

Planar wave

Sferic wave