

An isometric illustration of a warehouse and truck. In the background, a yellow truck is parked with its back open. In the foreground, several workers in green uniforms are handling cardboard boxes. Some are using pallet jacks to move boxes on pallets, while others are stacking boxes. The warehouse interior is filled with tall racks of boxes. The scene is brightly lit, suggesting a sunny day.

# Minimum Warehouse and Transportation Cost



University of Colorado **Denver**

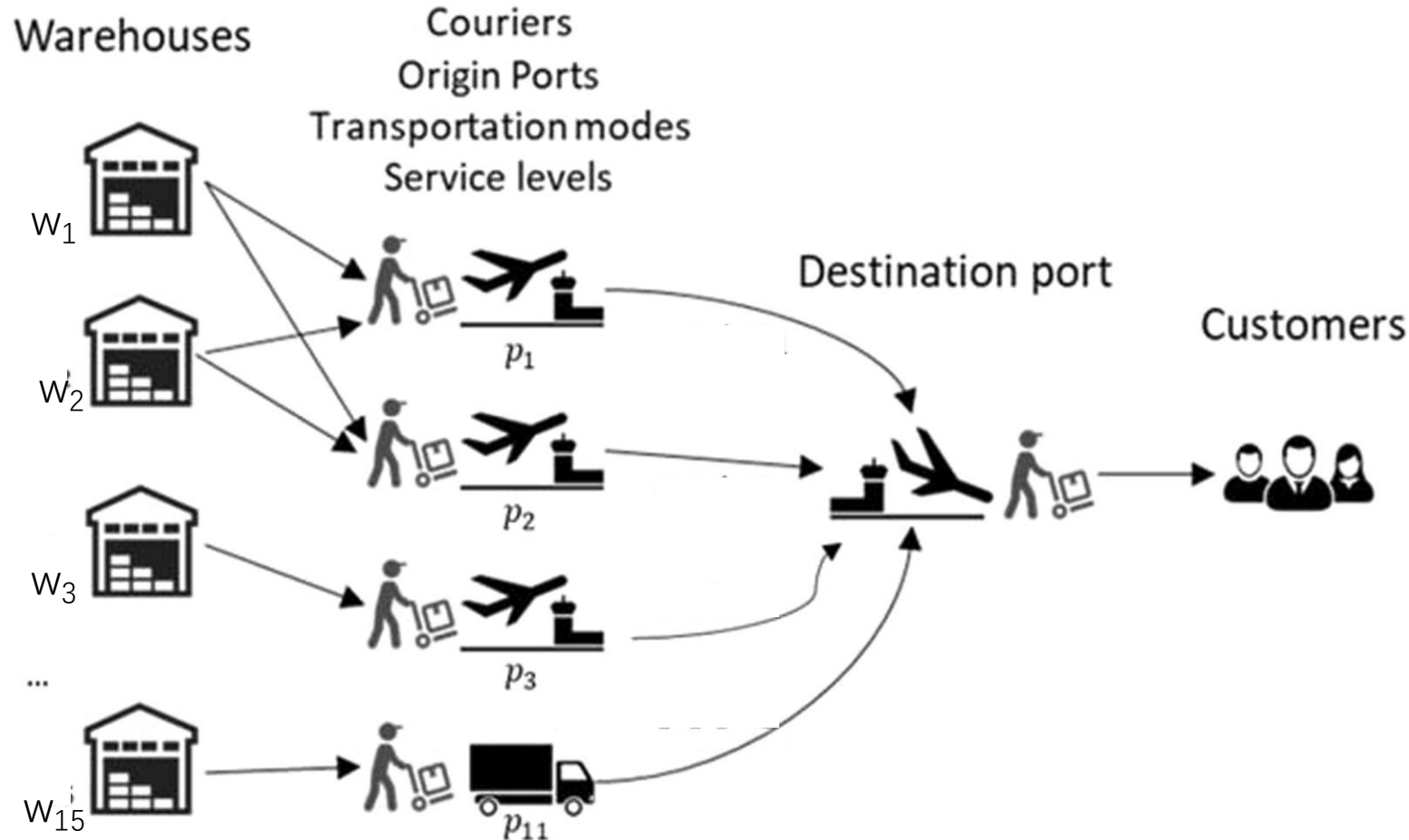
**CU IN THE CITY**

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Mathematical and Statistical Sciences

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# 1. Introduction



Microchip producer

9216 Orders

15 Warehouses, 11 Origin Ports

One Destination Port

9 Couriers, offering different rates for different weight bands.

Mixed Integer Linear Programming (**MILP**) Models

<https://www.sciencedirect.com/science/article/pii/S0360835220303442?via%3Dihub#f0010>

[https://brunel.figshare.com/articles/dataset/Supply\\_Chain\\_Logistics\\_Problem\\_Dataset/7558679?file=20162015](https://brunel.figshare.com/articles/dataset/Supply_Chain_Logistics_Problem_Dataset/7558679?file=20162015)

## 2. Data and Business Context

### *OrderList*

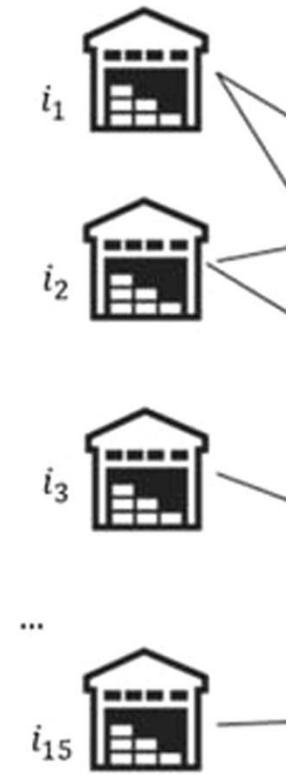
Order ID  
Product ID  
Unit Quantity  
Weight  
Destination Port

We have 9216 orders, each Order has a unique Order ID, and we have the weight, Unit Quantity and Destination Port of each order. The Product ID can be linked to the *ProductsPerPlant* table to determine the corresponding warehouse and origin port for that order.

A	B	C	D	E
Product ID	Order ID	Unit quantity	Weight	Destination Port
1700106	1447296447	808	14.3	PORT09
1700106	1447158015	3188	87.94	PORT09
1700106	1447138899	2331	61.2	PORT09
1700106	1447363528	847	16.16	PORT09
1700106	1447363981	2163	52.34	PORT09
1700106	1447351441	3332	92.8	PORT09
1700106	1447320236	1782	46.9	PORT09
1700106	1447158019	427	2.86	PORT09
1700106	1447219341	1291	26.6	PORT09
1700106	1447398416	2294	62.2	PORT09
1700106	1447381679	2766	75.5	PORT09
1700106	1447170785	798	14.3	PORT09
1697884	1447155056	739	73.9	PORT09
1697884	1447257265	280	7.8	PORT09
1697884	1447240989	574	59.5	PORT09
1697884	1447257231	556	52.7	PORT09
1697884	1447260653	544	58.5	PORT09
1697884	1447139375	1151	181.7	PORT09
1697884	1447308590	1404	227.2	PORT09
1697884	1447191271	662	74.1	PORT09
1699336	1447191284	836	80.43	PORT09
1702652	1447352426	2063	265.1	PORT09
1699337	1447211829	2114	267.1	PORT09
1699337	1447232833	2108	271.1	PORT09

*ProductsPerPlant*

Warehouses



A		B	
Product ID		Plant Code	
1698815		PLANT15	
1664419		PLANT17	
1664426		PLANT17	
1672826		PLANT17	
1674916		PLANT17	
1674918		PLANT17	
1675507		PLANT17	
1676151		PLANT17	
1676152		PLANT17	
1677864		PLANT17	
1677865		PLANT17	
1679124		PLANT17	
1685369		PLANT17	
1685370		PLANT17	
1685378		PLANT17	
1685979		PLANT17	
1691969		PLANT17	
1694139		PLANT17	
1694217		PLANT17	
1696107		PLANT17	
1697052		PLANT17	
1660883		PLANT18	
1664500		PLANT18	
1666994		PLANT18	
1666994		PLANT18	

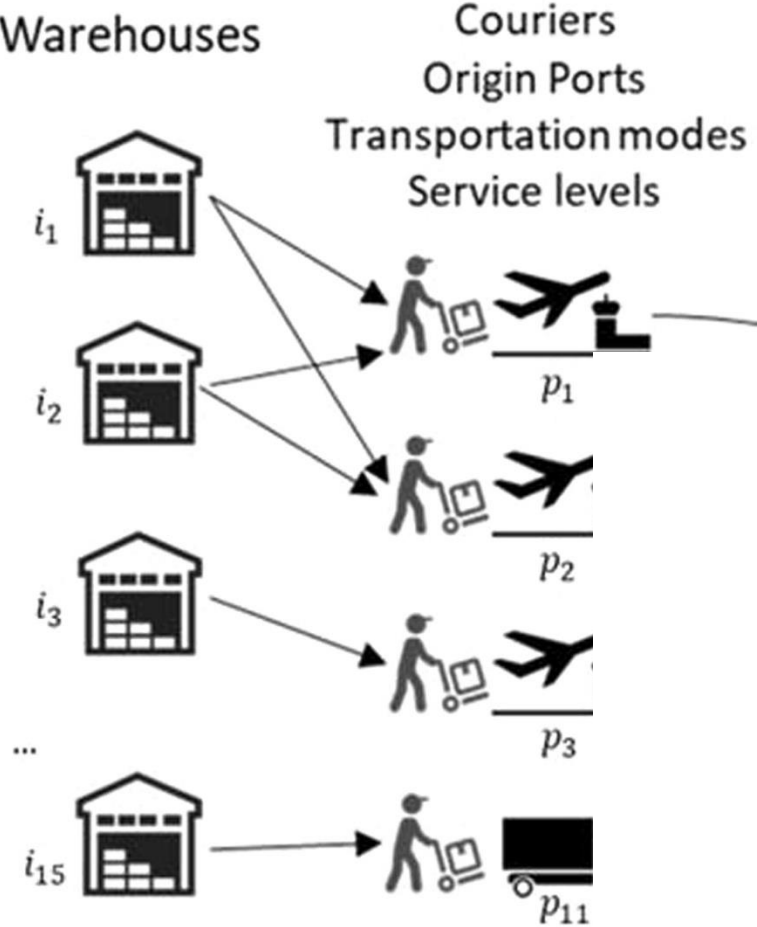
*PLANT Capacity and cost*

Plant ID	Daily Capacity	Cost/unit
PLANT15	66	1.42
PLANT17	48	0.43
PLANT18	666	2.04
PLANT05	2310	0.49
PLANT02	828	0.48
PLANT01	6420	0.57
PLANT06	294	0.55
PLANT10	708	0.49
PLANT07	1590	0.37
PLANT14	3294	0.63
PLANT16	2742	1.92
PLANT12	1254	0.77
PLANT11	1992	0.56
PLANT09	66	0.47
PLANT03	6078	0.52
PLANT13	2940	0.47
PLANT19	42	0.64
PLANT08	84	0.52
PLANT04	3324	0.43

Daily capacity refers to the number of orders the warehouse can process in a single day.

Each product has its designated warehouse.

# PlantPorts

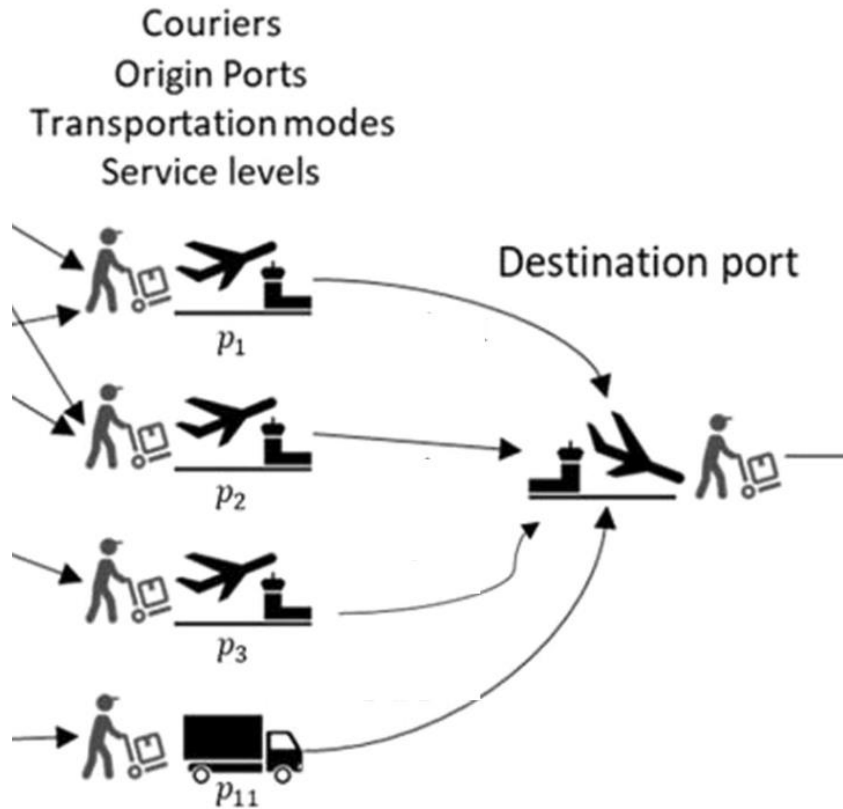


Each warehouse has its specific Origin ports.

Plant Code	Port	
PLANT01	PORT01	
PLANT01	PORT02	
PLANT02	PORT03	
PLANT03	PORT04	
PLANT04	PORT05	
PLANT05	PORT06	
PLANT06	PORT06	
PLANT07	PORT01	
PLANT07	PORT02	
PLANT08	PORT04	
PLANT09	PORT04	
PLANT10	PORT01	
PLANT10	PORT02	
PLANT11	PORT04	
PLANT12	PORT04	
PLANT13	PORT04	
PLANT14	PORT07	
PLANT15	PORT08	
PLANT16	PORT09	
PLANT17	PORT10	
PLANT18	PORT11	
PLANT19	PORT04	



# FreightRates



Carrier	orig_port_cd	dest_port_cd	minm_wgh_qty	max_wgh_qty	minimum cost	rate
V444_6	PORT08	PORT09	250	499.99	\$ 43.23	\$ 0.71
V444_6	PORT08	PORT09	65	69.99	\$ 43.23	\$ 0.75
V444_6	PORT08	PORT09	60	64.99	\$ 43.23	\$ 0.79
V444_6	PORT08	PORT09	50	54.99	\$ 43.23	\$ 0.83
V444_6	PORT08	PORT09	35	39.99	\$ 43.23	\$ 1.06
V444_6	PORT08	PORT09	100	249.99	\$ 43.23	\$ 0.71
V444_6	PORT08	PORT09	500	1999.99	\$ 43.23	\$ 0.68
V444_6	PORT08	PORT09	55	59.99	\$ 43.23	\$ 0.83
V444_6	PORT08	PORT09	25	29.99	\$ 43.23	\$ 1.25
V444_6	PORT08	PORT09	30	34.99	\$ 43.23	\$ 1.13
V444_6	PORT08	PORT09	70	99.99	\$ 43.23	\$ 0.75
V444_6	PORT08	PORT09	10	14.99	\$ 43.23	\$ 1.83
V444_6	PORT08	PORT09	0	4.99	\$ 43.23	\$ 1.83
V444_6	PORT08	PORT09	5	9.99	\$ 43.23	\$ 1.83
V444_6	PORT08	PORT09	2000	99999.99	\$ 43.23	\$ 0.64
V444_6	PORT08	PORT09	45	49.99	\$ 43.23	\$ 0.94
V444_6	PORT08	PORT09	20	24.99	\$ 43.23	\$ 1.32
V444_6	PORT08	PORT09	40	44.99	\$ 43.23	\$ 0.98
V444_6	PORT08	PORT09	15	19.99	\$ 43.23	\$ 1.59
V444_6	PORT10	PORT09	65	69.99	\$ 43.23	\$ 0.75
V444_6	PORT10	PORT09	45	49.99	\$ 43.23	\$ 0.94
V444_6	PORT10	PORT09	50	54.99	\$ 43.23	\$ 0.83
V444_6	PORT10	PORT09	250	499.99	\$ 43.23	\$ 0.71
V444_6	PORT10	PORT09	500	1999.99	\$ 43.23	\$ 0.68

9 carriers offering different rates for different weight bands.

# LP Model Formulation

## Decision Variables (Binary decision variables)

- $x_{i,w,p,c} = 1$  if order  $i$  is assigned to warehouse  $w$ , shipped through port  $p$ , and transported by carrier  $c$ .
- $y_{w,p,c} = 1$  if the warehouse–port–carrier transportation channel is activated.

## Objective Function

$$\min \sum_{i,w,p,c} (\text{warehouse\_cost}_{i,w} + \text{transport\_cost}_{i,w,p,c}) x_{i,w,p,c}$$

That is, we minimize the total logistics cost for all orders.

We define binary variables  $x$  to choose how each order is shipped through which warehouse **w**, port **p**, and carrier **c**.

and binary variable  $y$  to indicate whether a transportation channel is activated.

Our objective function is to minimize the total logistics cost.

For each feasible route, warehouse, port, and carrier, we add together the warehouse cost and the transportation cost, and multiply that by the decision variable  $x_{i,w,p,c}$ .

The model then chooses the combination of routes that has the lowest cost.

## Constraints

1. **Every order must be assigned exactly once:**

$$\sum_{w,p,c} x_{i,w,p,c} = 1, \quad \forall i$$

2. **Warehouse capacity limits:**

$$\sum_{i,p,c} x_{i,w,p,c} \leq Capacity_w, \quad \forall w$$

3. **Carrier weight-band feasibility:**

$$minW_c \cdot y_{w,p,c} \leq \sum_i Weight_i x_{i,w,p,c} \leq maxW_c \cdot y_{w,p,c}$$

4. **Channel activation:** If a channel is not activated, no orders may flow through it:

$$x_{i,w,p,c} \leq y_{w,p,c}, \quad \forall i, w, p, c$$

### Four constraints:

First, each order must be assigned exactly once, meaning it must choose one valid combination of warehouse, port, and carrier.

Second, each warehouse has a daily processing capacity, so the total number of assigned orders cannot exceed its limit.

Third, each carrier has a feasible weight band. The total shipment weight assigned to a carrier must fall between its lower bound and upper bound.

Fourth, we use channel activation constraints. If a warehouse–port–carrier channel is not activated, no order is allowed to flow through it. This ensures the model uses only valid and consistent transportation routes.



# Implementation Code

(Use Python Gurobi solver to solve this problem)

Definition of decision variables:

Model build to minimize the plant cost and transportation cost for each grouped product

```
m = Model("OutboundLogistics_withCapacity")
m.Params.OutputFlag = 1 if verbose else 0

# Decision variables
x = m.addVars(candidates, vtype=GRB.BINARY, name="x")
```

## Objective Function

```
# Objective: fixed_cost + freight_factor * var_cost
m.setObjective(
    quicksum((fixed_cost[idx] + freight_factor * var_cost[idx]) * x[idx]
              for idx in candidates),
    GRB.MINIMIZE
)
```

Constraint 1: all orders have to be assigned.

```
# Assignment: each product k chooses exactly one route
for k in products:
    idx_k = [idx for idx in candidates if idx[0] == k]
    m.addConstr(quicksum(x[idx] for idx in idx_k) == 1,
                name=f"assign_{k}")
```

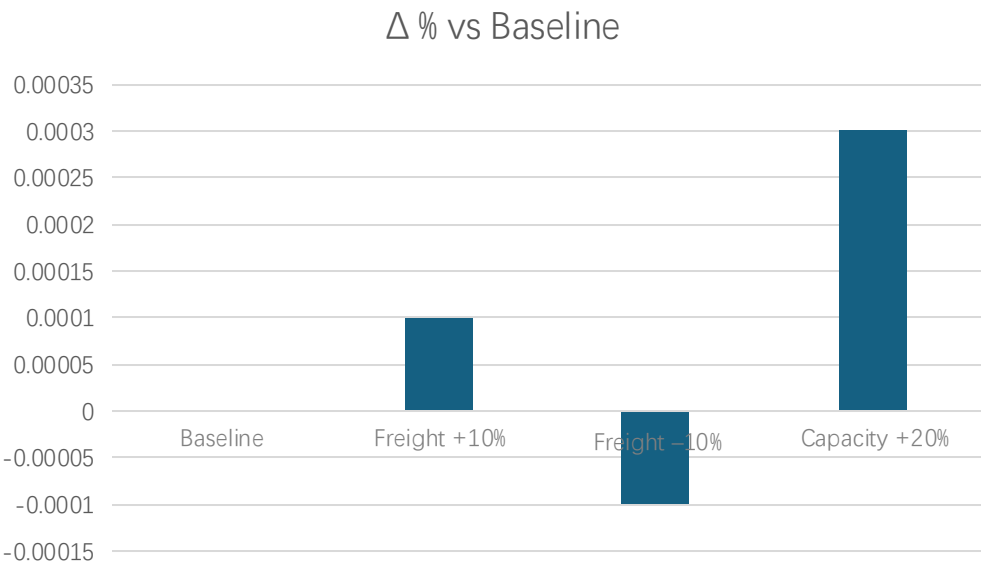
Constraint 2: the capacity limitation of the plant.

```
# Capacity constraints in NUMBER OF ORDERS
for w in plants:
    idx_w = [idx for idx in candidates if idx[1] == w]
    if idx_w and plant_capacity[w] > 0:
        cap_eff = cap_factor * plant_capacity[w]
        m.addConstr(
            quicksum(prod_order_count[idx[0]] * x[idx] for idx in idx_w)
            <= cap_eff,
            name=f"capacity_{w}"
        )
```

Constraint 3: Implicit integer constraints

```
vtype=GRB.BINARY,
```

# Sensitivity Analysis: Capacity & Freight Cost



Scenario	Capacity Factor	Freight Factor	Total Cost (USD)	Δ Cost vs Baseline	Δ % vs Baseline
Baseline	1	1	15480159.64	0	0
Freight +10%	1	1.1	15481123.75	964.11	0.01%
Freight -10%	1	0.9	15479195.29	-964.35	-0.01%
Capacity +20%	1.2	1	15484077.58	3917.94	0.03%
Capacity -20%	0.8	1	Model infeasible	Model infeasible	Model infeasible

An isometric illustration of a busy warehouse and truck loading area. In the background, a yellow truck with a white trailer is parked with its rear door open. A worker is using a forklift to load a pallet of boxes into the truck. To the left, several workers in blue overalls and hard hats are organizing stacks of cardboard boxes on pallets. A supervisor in a white shirt and tie is pointing towards the workers. In the foreground, more stacks of boxes are visible, along with a worker pushing a pallet jack loaded with boxes. A large, semi-transparent grey banner with the text "THANK YOU!" in white capital letters is centered over the middle of the scene. The overall color palette is dominated by browns, yellows, and blues, with a teal accent color for the truck's side and the banner's background.

**THANK YOU!**