Minimum Warehouse and Transportation Cost

University of Colorado **Denver**

CU IN THE CITY

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Mathematical and Statistical Sciences

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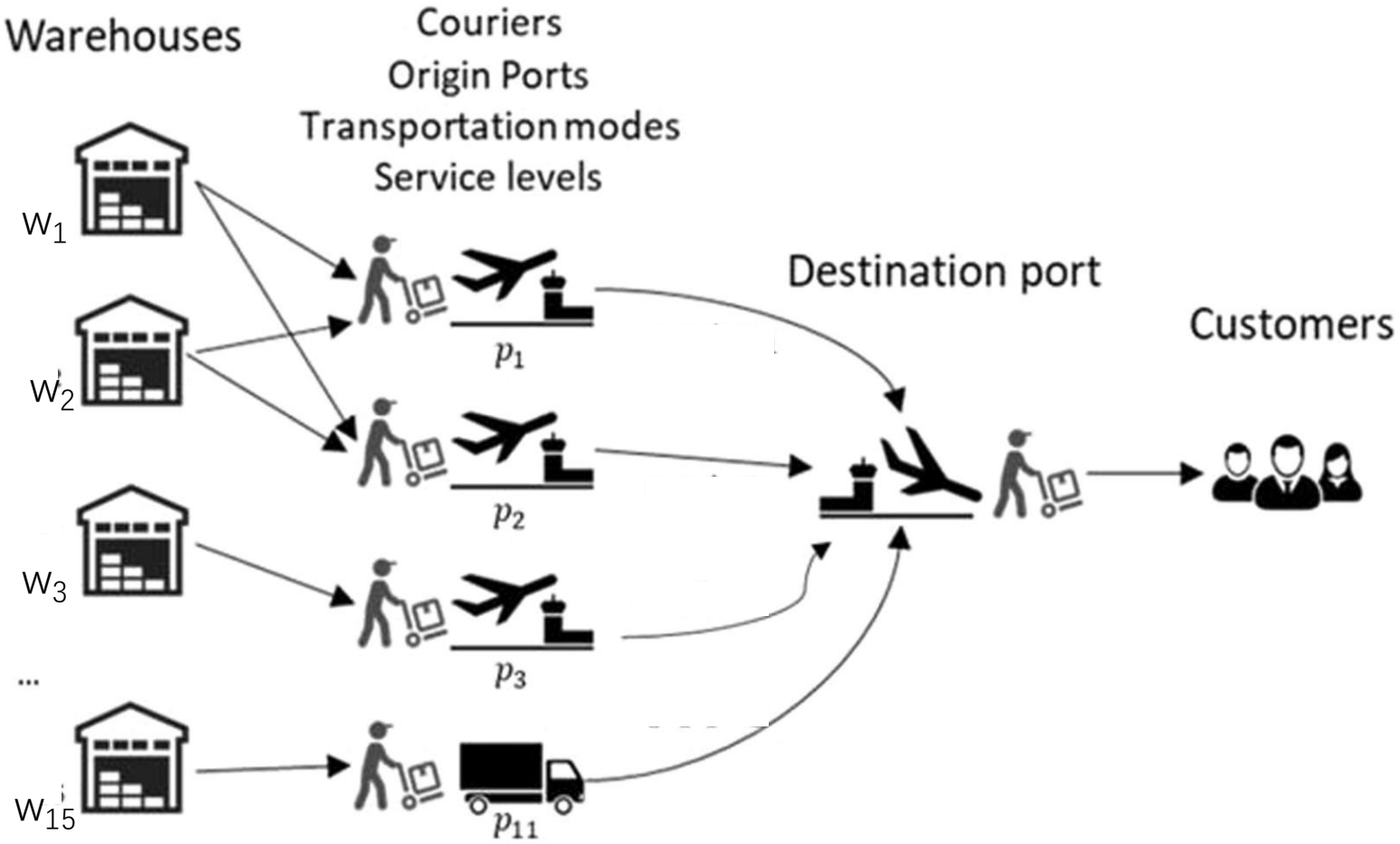
1. Introduction



Microchip producer

9216 Orders

15 Warehouses, 11 Origin Ports

One Destination Port

9 Couriers, offering different rates for different weight bands.

Mixed Integer Linear Programming (**MILP) Models**

https://www.sciencedirect.com/science/article/pii/S0360835220303442?via%3Dihub#f0010

https://brunel.figshare.com/articles/dataset/Supply_Chain_Logistics_Problem_Dataset/7558679?file=20162015

2. Data and Business Context

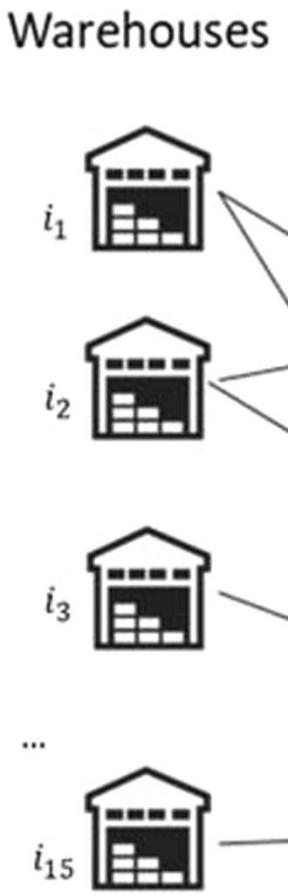
OrderList

Order ID
Product ID
Unit Quantity
Weight
Destination Port

We have 9216 orders, each Order has a uniq Order ID, and we have the weight, Unit Quantity and Destination Port of each order. The Product ID can be linked to the *ProductsPerPlant* table to determine the corresponding warehouse and origin port for that order.

A	B	C	D	E
Product ID	**Order ID**	**Unit quantity**	**Weight**	**Destination Port**
1700106	1447296447	808	14.3	PORT09
1700106	1447158015	3188	87.94	PORT09
1700106	1447138899	2331	61.2	PORT09
1700106	1447363528	847	16.16	PORT09
1700106	1447363981	2163	52.34	PORT09
1700106	1447351441	3332	92.8	PORT09
1700106	1447320236	1782	46.9	PORT09
1700106	1447158019	427	2.86	PORT09
1700106	1447219341	1291	26.6	PORT09
1700106	1447398416	2294	62.2	PORT09
1700106	1447381679	2766	75.5	PORT09
1700106	1447170785	798	14.3	PORT09
1697884	1447155056	739	73.9	PORT09
1697884	1447257265	280	7.8	PORT09
1697884	1447240989	574	59.5	PORT09
1697884	1447257231	556	52.7	PORT09
1697884	1447260653	544	58.5	PORT09
1697884	1447139375	1151	181.7	PORT09
1697884	1447308590	1404	227.2	PORT09
1697884	1447191271	662	74.1	PORT09
1699336	1447191284	836	80.43	PORT09
1702652	1447352426	2063	265.1	PORT09
1699337	1447211829	2114	267.1	PORT09
1699337	1447232833	2108	271.1	PORT09

ProductsPerPlant



	A	B
	Product ID	Plant Code
	1698815	PLANT15
	1664419	PLANT17
	1664426	PLANT17
	1672826	PLANT17
	1674916	PLANT17
	1674918	PLANT17
	1675507	PLANT17
	1676151	PLANT17
	1676152	PLANT17
	1677864	PLANT17
	1677865	PLANT17
	1679124	PLANT17
	1685369	PLANT17
	1685370	PLANT17
	1685378	PLANT17
	1685979	PLANT17
	1691969	PLANT17
	1694139	PLANT17
	1694217	PLANT17
	1696107	PLANT17
	1697052	PLANT17
	1660883	PLANT18
	1664500	PLANT18
	1666994	PLANT18

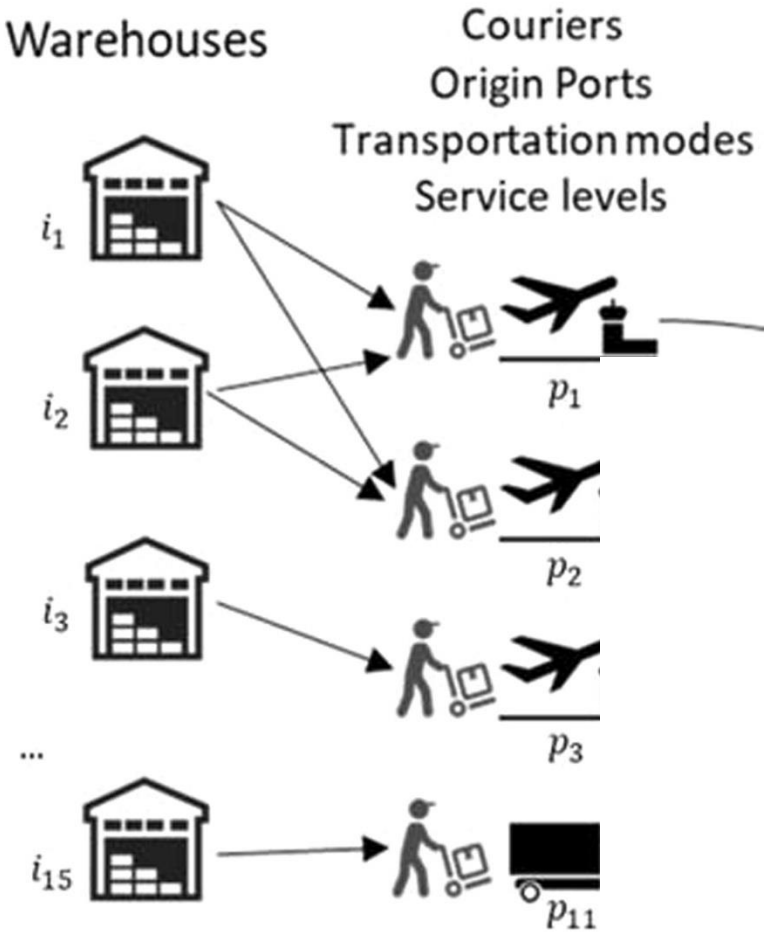
Each product has its designated warehouse.

PLANT Capacity and cost

Plant ID	Daily Capacity	Cost/unit
PLANT15	11	1.42
PLANT17	8	0.43
PLANT18	111	2.04
PLANT05	385	0.49
PLANT02	138	0.48
PLANT01	1070	0.57
PLANT06	49	0.55
PLANT10	118	0.49
PLANT07	265	0.37
PLANT14	549	0.63
PLANT16	457	1.92
PLANT12	209	0.77
PLANT11	332	0.56
PLANT09	11	0.47
PLANT03	1013	0.52
PLANT13	490	0.47
PLANT19	7	0.64
PLANT08	14	0.52
PLANT04	554	0.43

Daily capacity refers to the number of orders the warehouse can process in a single day.

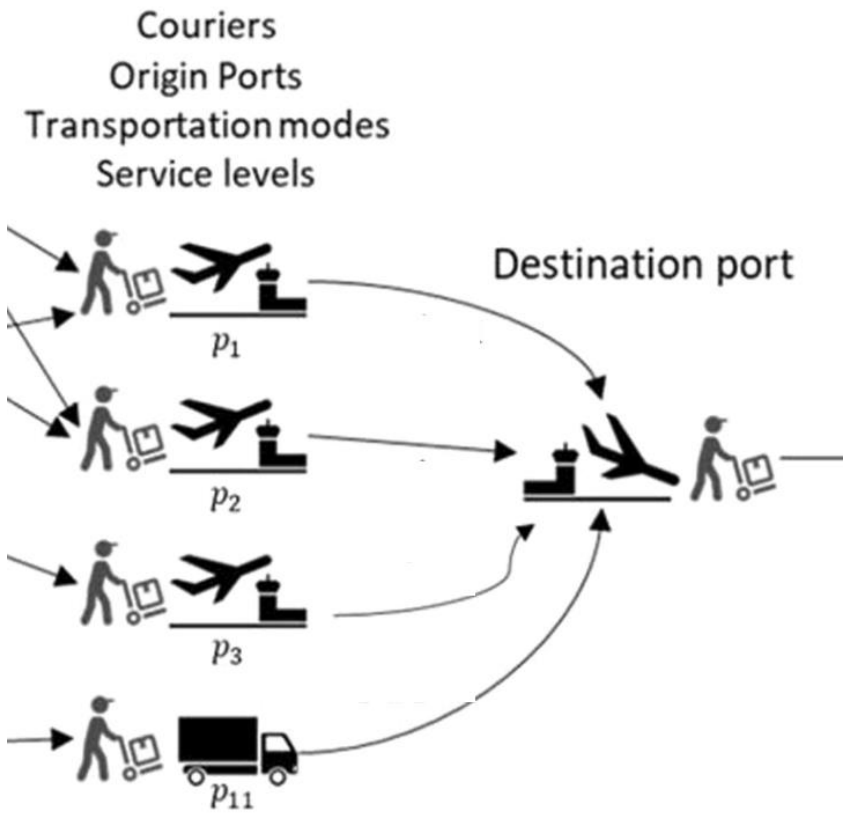
PlantPorts



Plant Code	Port
PLANT01	PORT01
PLANT01	PORT02
PLANT02	PORT03
PLANT03	PORT04
PLANT04	PORT05
PLANT05	PORT06
PLANT06	PORT06
PLANT07	PORT01
PLANT07	PORT02
PLANT08	PORT04
PLANT09	PORT04
PLANT10	PORT01
PLANT10	PORT02
PLANT11	PORT04
PLANT12	PORT04
PLANT13	PORT04
PLANT14	PORT07
PLANT15	PORT08
PLANT16	PORT09
PLANT17	PORT10
PLANT18	PORT11
PLANT19	PORT04

Each warehouse has its specific Origin ports.

FreightRates



Carrier	orig_port_cd	dest_port_cd	minm_wgh_qty	max_wgh_qty	minimum cost	rate
V444_6	PORT08	PORT09	250	499.99	$ 43.23	$ 0.71
V444_6	PORT08	PORT09	65	69.99	$ 43.23	$ 0.75
V444_6	PORT08	PORT09	60	64.99	$ 43.23	$ 0.79
V444_6	PORT08	PORT09	50	54.99	$ 43.23	$ 0.83
V444_6	PORT08	PORT09	35	39.99	$ 43.23	$ 1.06
V444_6	PORT08	PORT09	100	249.99	$ 43.23	$ 0.71
V444_6	PORT08	PORT09	500	1999.99	$ 43.23	$ 0.68
V444_6	PORT08	PORT09	55	59.99	$ 43.23	$ 0.83
V444_6	PORT08	PORT09	25	29.99	$ 43.23	$ 1.25
V444_6	PORT08	PORT09	30	34.99	$ 43.23	$ 1.13
V444_6	PORT08	PORT09	70	99.99	$ 43.23	$ 0.75
V444_6	PORT08	PORT09	10	14.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	0	4.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	5	9.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	2000	99999.99	$ 43.23	$ 0.64
V444_6	PORT08	PORT09	45	49.99	$ 43.23	$ 0.94
V444_6	PORT08	PORT09	20	24.99	$ 43.23	$ 1.32
V444_6	PORT08	PORT09	40	44.99	$ 43.23	$ 0.98
V444_6	PORT08	PORT09	15	19.99	$ 43.23	$ 1.59
V444_6	PORT10	PORT09	65	69.99	$ 43.23	$ 0.75
V444_6	PORT10	PORT09	45	49.99	$ 43.23	$ 0.94
V444_6	PORT10	PORT09	50	54.99	$ 43.23	$ 0.83
V444_6	PORT10	PORT09	250	499.99	$ 43.23	$ 0.71
V444_6	PORT10	PORT09	500	1999.99	$ 43.23	$ 0.68

9 carriers offering different rates for different weight bands.

LP Model Formulation

Decision Variables (Binary decision variables)

- $x_{i,w,p,c} = 1$ if order i is assigned to warehouse w, shipped through port p, and transported by carrier c.
- $y_{w,p,c} = 1$ if the warehouse–port–carrier transportation channel is activated.

Objective Function

$$\min \sum_{i,w,p,c} \left(warehouse\_cost_{i,w} + transport\_cost_{i,w,p,c} \right) x_{i,w,p,c}$$

That is, we minimize the total logistics cost for all orders.

We define binary variables x to choose how each order is shipped through which warehouse w, port p, and carrier c.
and binary variable y to indicate whether a transportation channel is activated.

Our objective function is to minimize the total logistics cost.
For each feasible route, warehouse, port, and carrier, we add together the warehouse cost and the transportation cost, and multiply that by the decision variable $x_{i,w,p,c}$.
The model then chooses the combination of routes that has the lowest cost.

Constraints

1. **Every order must be assigned exactly once:**

$$\sum_{w,p,c} x_{i,w,p,c} = 1, \quad \forall i$$

2. **Warehouse capacity limits:**

$$\sum_{i,p,c} x_{i,w,p,c} \leq Capacity_w, \quad \forall w$$

3. **Carrier weight-band feasibility:**

$$minW_c \cdot y_{w,p,c} \leq \sum_i Weight_i \, x_{i,w,p,c} \leq maxW_c \cdot y_{w,p,c}$$

4. **Channel activation:** If a channel is not activated, no orders may flow through it:

$$x_{i,w,p,c} \leq y_{w,p,c}, \quad \forall i, w, p, c$$

Four constraints:

First, each order must be assigned exactly once, meaning it must choose one valid combination of warehouse, port, and carrier.

Second, each warehouse has a daily processing capacity, so the total number of assigned orders cannot exceed its limit.

Third, each carrier has a feasible weight band. The total shipment weight assigned to a carrier must fall between its lower bound and upper bound.

Fourth, we use channel activation constraints. If a warehouse–port–carrier channel is not activated, no order is allowed to flow through it. This ensures the model uses only valid and consistent transportation routes.

Implementation Code

(Use Python Gurobi solver to solve this problem)

Definition of decision variables:
Model build to minimize the plant cost and transportation cost for each grouped product

```
#Model build to minimize the plant cost and transportation cost for each grouped product
model = Model()
x = model.addVars(C.keys(), obj=C, vtype=GRB.BINARY, name='x')
y = model.addVars(N.keys(), vtype=GRB.BINARY, name='y')
```

Introduce two binary variables:

x, indicating whether order i is assigned, and

y, indicating whether a transportation channel is activated.

Constraint 1: all orders have to be assigned.

```
#Constraint 1: all orders have to be placed and delivered.
for i in Orders_PID.keys():
    if Orders_PID[i][11] == 0:
        model.addConstr(x.sum(i, '*', '*', '*') == 1)
```

Constraint 2: the capacity limitation of the plant.

```
#Constraint 2: the capacity limitation of the plant.
for i in N.keys():
    model.addConstr(x.prod(T, '*', 'i[0]', '*', '*') <= N[i]*y[i])
```

Constraint 3: the capacity limitation of m-th Carrier and transporter

```
#Constraint 3: the capacity limitation of m-th Carrier and transporter
for i in N.keys():
    model.addConstr(x.prod(W,'*', '*', '*', i[2]) <= i[2][3]*y[i])
    model.addConstr(x.prod(W,'*', '*', '*', i[2]) >= i[2][2]*y[i])
```

Run the model and results

```
#model.setParam(GRB.Param.LogToConsole, 0)
model.setParam(GRB.Param.TimeLimit, 10)
model.optimize()
```

```
CPU model: Apple M4
Thread count: 10 physical cores, 10 logical
processors, using up to 10 threads

Non-default parameters:
TimeLimit  10

Optimize a model with 1634 rows, 8736 columns
and 9695 nonzeros (Min)
Model fingerprint: 0x2958504f
Model has 8244 linear objective coefficients
Variable types: 0 continuous, 8736 integer (873
binary)
Coefficient statistics:
  Matrix range     [1e-02, 1e+05]
  Objective range  [1e+02, 2e+05]
  Bounds range     [1e+00, 1e+00]
  RHS range        [1e+00, 1e+00]
Found heuristic solution: objective 1421348.443
Presolve removed 1634 rows and 8736 columns
Presolve time: 0.01s
Presolve: All rows and columns removed

Explored 0 nodes (0 simplex iterations) in 0.01
seconds (0.00 work units)
Thread count was 1 (of 10 available processors)

Solution count 2: 1.31306e+06 1.42135e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.313057154265e+06, best bound
1.313057154265e+06, gap 0.0000%
```

Optimal Solutions

Product ID	Order ID	Unit quantity	weight	Destination p	Plant code	Origin Port	Carrier type	minm_wgh_q	max_wgh_qty	Service Level	Plant cost for	Transportation cost for group of Product		
1700106	1447296447	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447158015	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447138899	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447363528	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447363981	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447351441	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447320236	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447158019	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447219341	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447398416	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447381679	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1700106	1447170785	798	14.3	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469076.167	1731.98527		
1697884	1447155056	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447257265	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447240989	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447257231	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447260653	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447139375	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447308590	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1697884	1447191271	662	74.1	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469337.261	1724.83519		
1699336	1447191284	801	78.34	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469070.408	1722.49971		
1702652	1447352426	605	46.35	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469446.69	1722.49971		
1699337	1447211829	897	89.7	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	468886.106	1722.49971		
1699337	1447232833	897	89.7	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	468886.106	1722.49971		
1699336	1447205081	801	78.34	PORT09	PLANT16	PORT03	V444_0	0	5000	DTD	469070.408	1722.49971		


THANK YOU!