1. Introduction

Warehouses

Couriers
Origin Ports
Transportation modes
Service levels

Destination port

Customers

w_1

w_2

w_3

...

w_{15}

p_1

p_2

p_3

p_{11}

Microchip producer

9216 Orders

15 Warehouses, 11 Origin Ports

One Destination Port

9 Couriers, offering different rates for different weight bands.

Mixed Integer Linear Programming (**MILP) Models**

2. Data and Business Context

OrderList

Order ID
Product ID
Unit Quantity
Weight
Destination Port

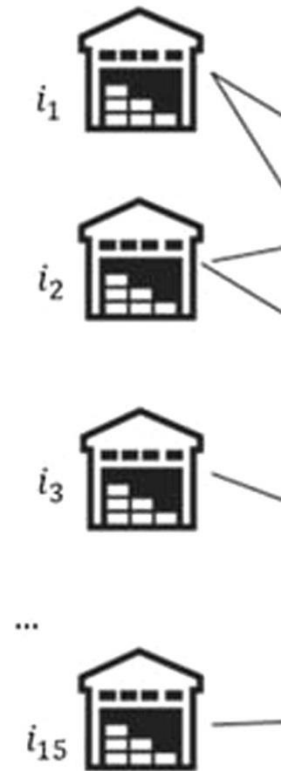We have 9216 orders, each Order has a uniq Order ID, and we have the weight, Unit Quantity and Destination Port of each order. The Product ID can be linked to the *ProductsPerPlant* table to determine the corresponding warehouse and origin port for that order.

Product ID	Order ID	Unit quantity	Weight	Destination Port
1700106	1447296447	808	14.3	PORT09
1700106	1447158015	3188	87.94	PORT09
1700106	1447138899	2331	61.2	PORT09
1700106	1447363528	847	16.16	PORT09
1700106	1447363981	2163	52.34	PORT09
1700106	1447351441	3332	92.8	PORT09
1700106	1447320236	1782	46.9	PORT09
1700106	1447158019	427	2.86	PORT09
1700106	1447219341	1291	26.6	PORT09
1700106	1447398416	2294	62.2	PORT09
1700106	1447381679	2766	75.5	PORT09
1700106	1447170785	798	14.3	PORT09
1697884	1447155056	739	73.9	PORT09
1697884	1447257265	280	7.8	PORT09
1697884	1447240989	574	59.5	PORT09
1697884	1447257231	556	52.7	PORT09
1697884	1447260653	544	58.5	PORT09
1697884	1447139375	1151	181.7	PORT09
1697884	1447308590	1404	227.2	PORT09
1697884	1447191271	662	74.1	PORT09
1699336	1447191284	836	80.43	PORT09
1702652	1447352426	2063	265.1	PORT09
1699337	1447211829	2114	267.1	PORT09
1699337	1447232833	2108	271.1	PORT09

ProductsPerPlant

PLANT Capacity and cost

Warehouses

Product ID	Plant Code
1698815	PLANT15
1664419	PLANT17
1664426	PLANT17
1672826	PLANT17
1674916	PLANT17
1674918	PLANT17
1675507	PLANT17
1676151	PLANT17
1676152	PLANT17
1677864	PLANT17
1677865	PLANT17
1679124	PLANT17
1685369	PLANT17
1685370	PLANT17
1685378	PLANT17
1685979	PLANT17
1691969	PLANT17
1694139	PLANT17
1694217	PLANT17
1696107	PLANT17
1697052	PLANT17
1660883	PLANT18
1664500	PLANT18
1666994	PLANT18

Plant ID	Daily Capacity	Cost/unit
PLANT15	66	1.42
PLANT17	48	0.43
PLANT18	666	2.04
PLANT05	2310	0.49
PLANT02	828	0.48
PLANT01	6420	0.57
PLANT06	294	0.55
PLANT10	708	0.49
PLANT07	1590	0.37
PLANT14	3294	0.63
PLANT16	2742	1.92
PLANT12	1254	0.77
PLANT11	1992	0.56
PLANT09	66	0.47
PLANT03	6078	0.52
PLANT13	2940	0.47
PLANT19	42	0.64
PLANT08	84	0.52
PLANT04	3324	0.43

Daily capacity refers to the number of orders the warehouse can process in a single day.

Each product has its designated warehouse.

PlantPorts

Warehouses

Couriers
Origin Ports
Transportation modes
Service levels

i_1

i_2

i_3

...

i_{15}

p_1

p_2

p_3

p_{11}
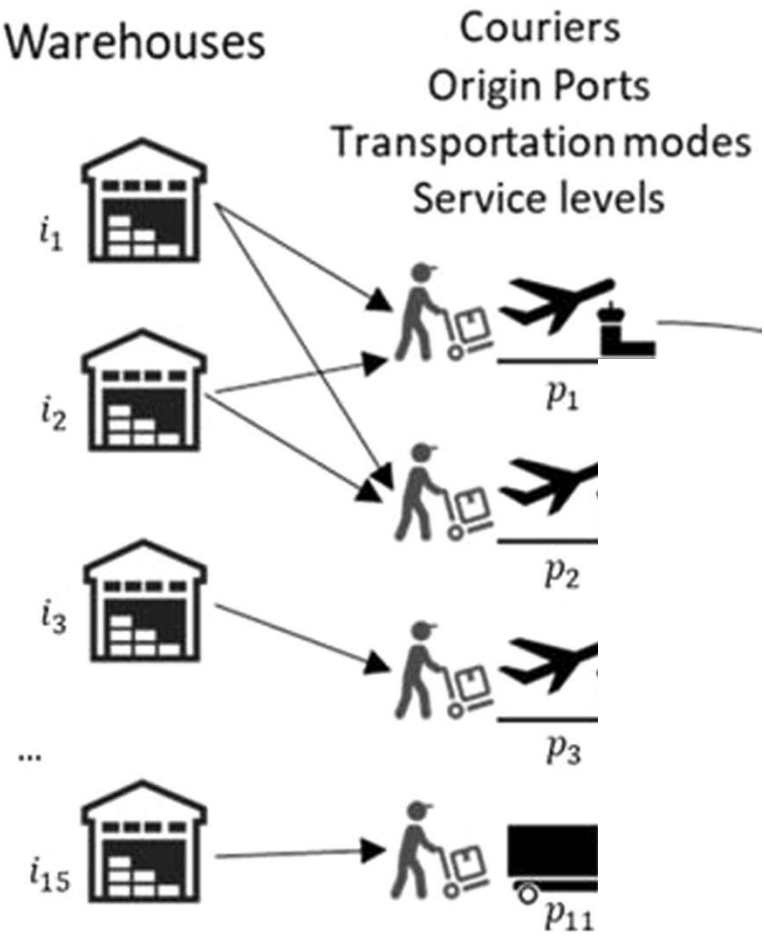
Each warehouse has its specific Origin ports.

Plant Code	Port
PLANT01	PORT01
PLANT01	PORT02
PLANT02	PORT03
PLANT03	PORT04
PLANT04	PORT05
PLANT05	PORT06
PLANT06	PORT06
PLANT07	PORT01
PLANT07	PORT02
PLANT08	PORT04
PLANT09	PORT04
PLANT10	PORT01
PLANT10	PORT02
PLANT11	PORT04
PLANT12	PORT04
PLANT13	PORT04
PLANT14	PORT07
PLANT15	PORT08
PLANT16	PORT09
PLANT17	PORT10
PLANT18	PORT11
PLANT19	PORT04

FreightRates
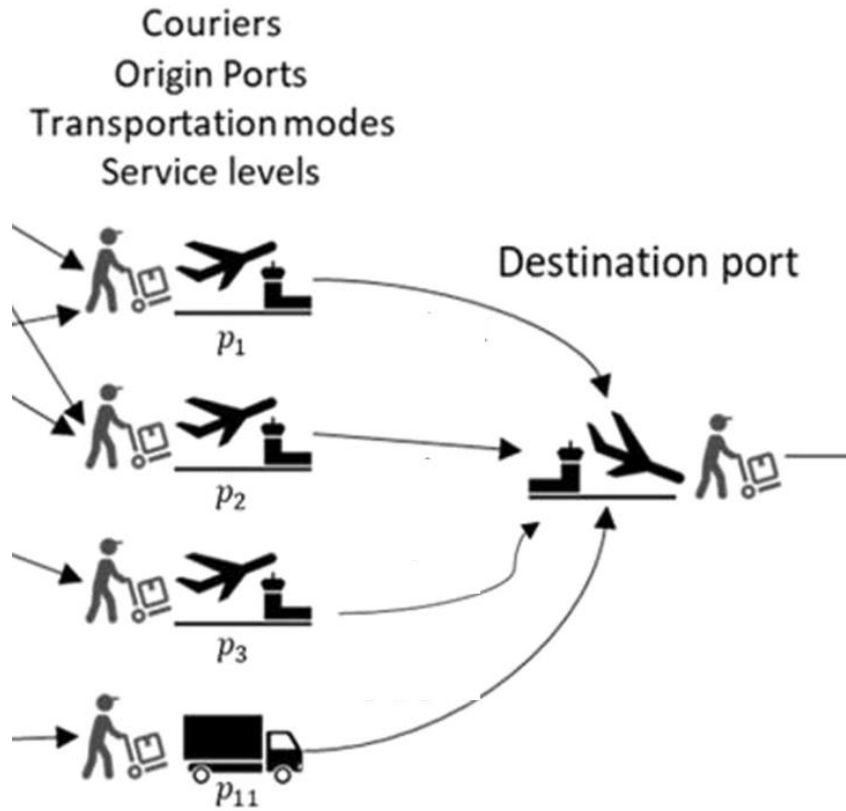
Carrier	orig_port_cd	dest_port_cd	minm_wgh_qty	max_wgh_qty	minimum cost	rate
V444_6	PORT08	PORT09	250	499.99	$ 43.23	$ 0.71
V444_6	PORT08	PORT09	65	69.99	$ 43.23	$ 0.75
V444_6	PORT08	PORT09	60	64.99	$ 43.23	$ 0.79
V444_6	PORT08	PORT09	50	54.99	$ 43.23	$ 0.83
V444_6	PORT08	PORT09	35	39.99	$ 43.23	$ 1.06
V444_6	PORT08	PORT09	100	249.99	$ 43.23	$ 0.71
V444_6	PORT08	PORT09	500	1999.99	$ 43.23	$ 0.68
V444_6	PORT08	PORT09	55	59.99	$ 43.23	$ 0.83
V444_6	PORT08	PORT09	25	29.99	$ 43.23	$ 1.25
V444_6	PORT08	PORT09	30	34.99	$ 43.23	$ 1.13
V444_6	PORT08	PORT09	70	99.99	$ 43.23	$ 0.75
V444_6	PORT08	PORT09	10	14.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	0	4.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	5	9.99	$ 43.23	$ 1.83
V444_6	PORT08	PORT09	2000	99999.99	$ 43.23	$ 0.64
V444_6	PORT08	PORT09	45	49.99	$ 43.23	$ 0.94
V444_6	PORT08	PORT09	20	24.99	$ 43.23	$ 1.32
V444_6	PORT08	PORT09	40	44.99	$ 43.23	$ 0.98
V444_6	PORT08	PORT09	15	19.99	$ 43.23	$ 1.59
V444_6	PORT10	PORT09	65	69.99	$ 43.23	$ 0.75
V444_6	PORT10	PORT09	45	49.99	$ 43.23	$ 0.94
V444_6	PORT10	PORT09	50	54.99	$ 43.23	$ 0.83
V444_6	PORT10	PORT09	250	499.99	$ 43.23	$ 0.71
V444_6	PORT10	PORT09	500	1999.99	$ 43.23	$ 0.68

9 carriers offering different rates for different weight bands.

LP Model Formulation

Decision Variables   (Binary decision variables)

- $x_{i,w,p,c} = 1$ if order i is assigned to warehouse w, shipped through port p, and transported by carrier c.

- $y_{w,p,c} = 1$ if the warehouse–port–carrier transportation channel is activated.

Objective Function

$$\min \sum_{i,w,p,c} \left(warehouse\_cost_{i,w} + transport\_cost_{i,w,p,c} \right) x_{i,w,p,c}$$

That is, we minimize the total logistics cost for all orders.

We define binary variables x to choose how each order is shipped through which warehouse w, port p, and carrier c.
and binary variable y to indicate whether a transportation channel is activated.

Our objective function is to minimize the total logistics cost.
For each feasible route, warehouse, port, and carrier, we add together the warehouse cost and the transportation cost, and multiply that by the decision variable $x_{i,w,p,c}$.
The model then chooses the combination of routes that has the lowest cost.

Constraints

1. **Every order must be assigned exactly once:**

$$\sum_{w,p,c} x_{i,w,p,c} = 1, \quad \forall i$$

2. **Warehouse capacity limits:**

$$\sum_{i,p,c} x_{i,w,p,c} \leq Capacity_w, \quad \forall w$$

3. **Carrier weight-band feasibility:**

$$minW_c \cdot y_{w,p,c} \leq \sum_i Weight_i \, x_{i,w,p,c} \leq maxW_c \cdot y_{w,p,c}$$

Four constraints:
First, each order must be assigned exactly once, meaning it must choose one valid combination of warehouse, port, and carrier.

Second, each warehouse has a daily processing capacity, so the total number of assigned orders cannot exceed its limit.

Third, each carrier has a feasible weight band. The total shipment weight assigned to a carrier must fall between its lower bound and upper bound.

Implementation Code

(Use Python Gurobi solver to solve this problem)

Definition of decision variables:
Model build to minimize the plant cost and transportation cost for each grouped product

```python
m = Model("OutboundLogistics_withCapacity")
m.Params.OutputFlag = 1 if verbose else 0

# Decision variables
x = m.addVars(candidates, vtype=GRB.BINARY, name="x")
```

Objective Function

```python
# Objective: fixed_cost + freight_factor * var_cost
m.setObjective(
    quicksum((fixed_cost[idx] + freight_factor * var_cost[idx]) * x[idx]
            for idx in candidates),
    GRB.MINIMIZE
)
```

Constraint 1: all orders have to be assigned.

```python
# Assignment: each product k chooses exactly one route
for k in products:
    idx_k = [idx for idx in candidates if idx[0] == k]
    m.addConstr(quicksum(x[idx] for idx in idx_k) == 1,
                name=f"assign_{k}")
```

Constraint 2: the capacity limitation of the plant.

```python
# Capacity constraints in NUMBER OF ORDERS
for w in plants:
    idx_w = [idx for idx in candidates if idx[1] == w]
    if idx_w and plant_capacity[w] > 0:
        cap_eff = cap_factor * plant_capacity[w]
        m.addConstr(
            quicksum(prod_order_count[idx[0]] * x[idx] for idx in idx_w)
            <= cap_eff,
            name=f"capacity_{w}"
        )
```
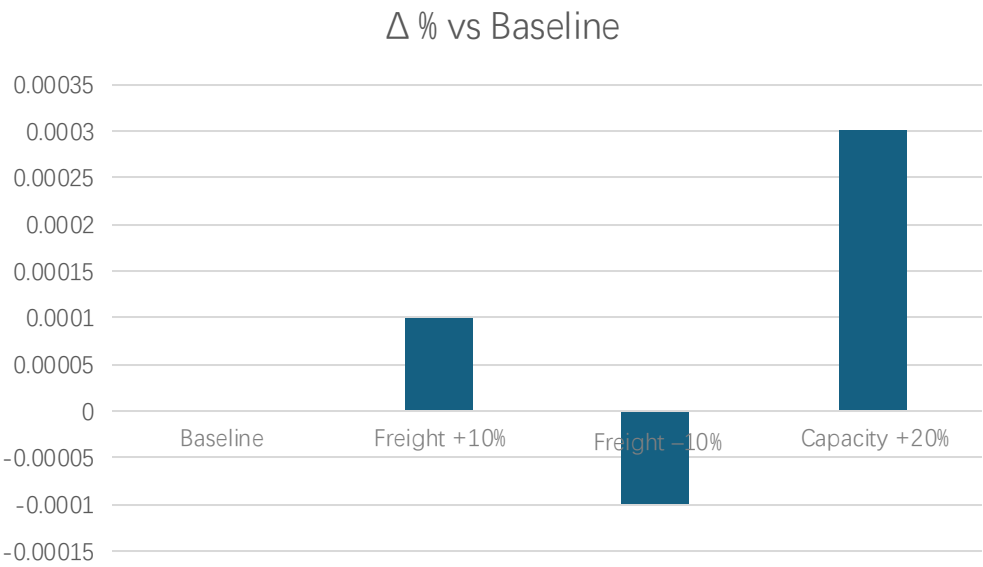
Constraint 3: Carrier weight-band feasibility

```python
# 3) product total weight must lie inside [minW, maxW]
if not (minW <= weight_k <= maxW):
    continue
```

Constraint 4: Implicit integer constraints

```python
vtype=GRB.BINARY,
```

Sensitivity Analysis: Capacity & Freight Cost

Δ % vs Baseline

Scenario	Capacity Factor	Freight Factor	Total Cost (USD)	Δ Cost vs Baseline	Δ % vs Baseline
Baseline	1	1	15480159.64	0	0
Freight +10%	1	1.1	15481123.75	964.11	0.01%
Freight –10%	1	0.9	15479195.29	-964.35	-0.01%
Capacity +20%	1.2	1	15484077.58	3917.94	0.03%
Capacity –20%	0.8	1	Model infeasible	Model infeasible	Model infeasible

THANK YOU!