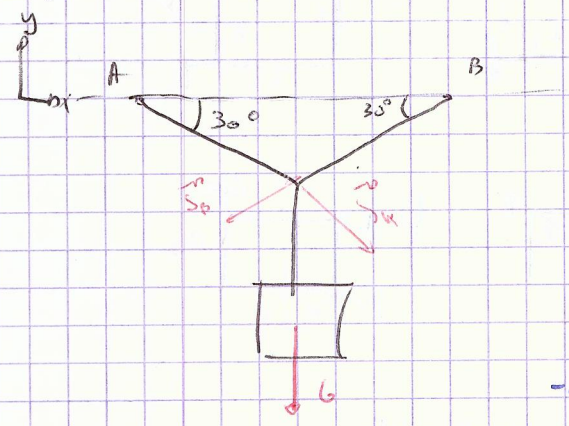
Toegepaste mechanica : oefeningen

1

oef 1.1



$$S_A + S_B + G = 0$$

$$\begin{Bmatrix} S_A \cdot \cos 30^\circ \\ -S_A \cdot \sin 30^\circ \end{Bmatrix} + \begin{Bmatrix} -S_B \cdot \cos 30^\circ \\ -S_B \cdot \sin 30^\circ \end{Bmatrix} + \begin{Bmatrix} 0 \\ -1000 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$S_A = S_B$$

$$-2S_A \cdot \frac{1}{2} = 1000 \text{ N}$$

$$\Downarrow$$

$$S_A = 1000 \text{ N}$$

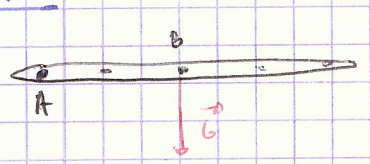
$$S_B = 1000 \text{ N}$$

$$\Rightarrow \vec{S_A} = 500\sqrt{3}\,\vec{e_x} - 500\,\vec{e_y}$$

$$\vec{S_B} = -500\sqrt{3}\,\vec{e_x} - 500\,\vec{e_y}$$

oef 1.2

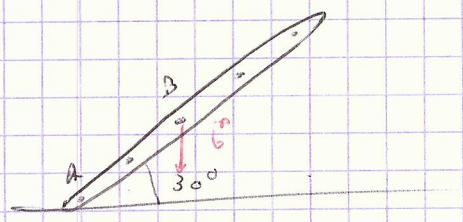
a)



$$\vec{M} = (\vec{r_B} - \vec{r_A}) \times \vec{G}$$

$$= 0{,}10 \cdot -1\text{N} = -0{,}10 \text{ N}$$

b)



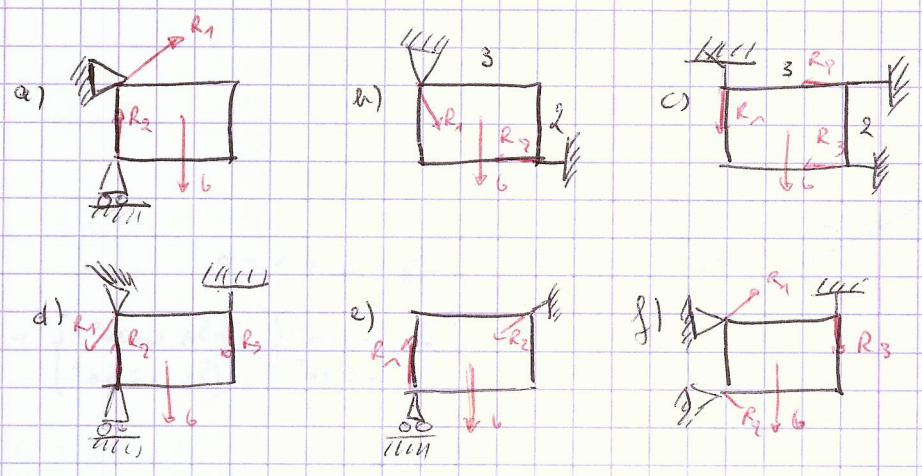
$$\vec{M} = (\vec{r_B} - \vec{r_A}) \times \vec{G}$$

$$= 0{,}1 \cdot \cos 30^\circ \cdot -1\text{N} = -0{,}1 \cdot \frac{\sqrt{3}}{2}$$

c)

$$\vec{M} = 0$$

oef 1.3



a) x : 1 onbekende
y : 3 onbekende → gaat niet

b) x : 2 onbekenden
y : 2 onbekenden → gaat wel

c) x : 2 onbekenden
y : 2 onbekenden → gaat wel

d) x : 1 onbekende
y : 4 onbekenden → gaat niet

e) x : 1 onbekende
y : 3 onbekenden → gaat niet

d) x : 2 onbekenden
y : 4 onbekenden → gaat niet

b) $R_1 + R_2 + G = 0$

$$\begin{Bmatrix} R_{1x} \\ R_{1y} \end{Bmatrix} + \begin{Bmatrix} R_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ G \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_{1x} = -R_2 \\ R_{1y} = -G \end{cases}$$

$R_1 = 1500\ N$
$R_2 = 2000\ N$

$$+ \quad G \cdot \frac{3}{2} - 2 \cdot R_2 = 0$$

$$-R_2 = -G \cdot \frac{3}{4}$$
$$R_2 = 1500\ N$$

c) $\vec{R_1} + \vec{R_2} + \vec{R_3} + \vec{G} = 0$

$$\begin{Bmatrix} 0 \\ -R_1 \end{Bmatrix} + \begin{Bmatrix} -R_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -R_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_2 = -R_3 \\ R_1 = -G \end{cases} \Rightarrow R_1 = 2000\ N$$

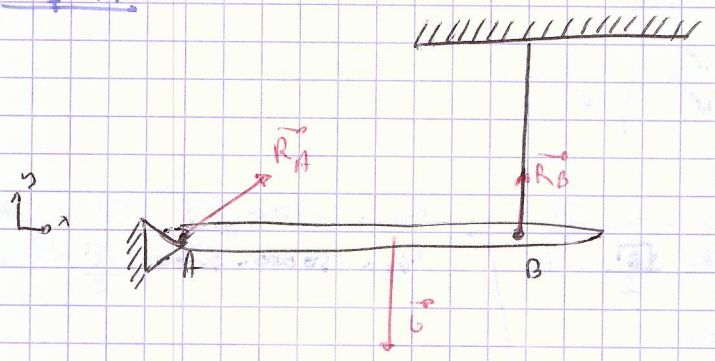
* $3 . R_1 + \frac{3}{2} G - 2 R_3 = 0$

$$R_3 = \frac{3}{2} R_1 + \frac{3}{4} G$$

$$R_3 = 3000\,N + 1500\,N$$
$$= 1500\,N$$

$$\Rightarrow R_4 = -1500\,N$$

<u>ex 1.4</u>



* $\vec{R_A} + \vec{G} + \vec{R_B} = 0$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

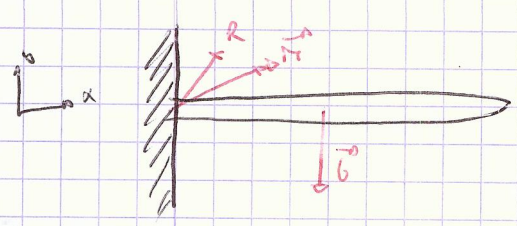
$$\begin{cases} R_{Ax} = 0 \\ R_{Ay} - G + R_B = 0 \end{cases}$$

* $0,1 . -G + 0,15 . R_B = 0$

$$R_B = \frac{0,1 G}{0,15}$$
$$= 2\,N$$

$$\Rightarrow R_A = 1N$$

<u>ex 1.5</u>



* $\vec{R} + \vec{G} = \vec{0}$

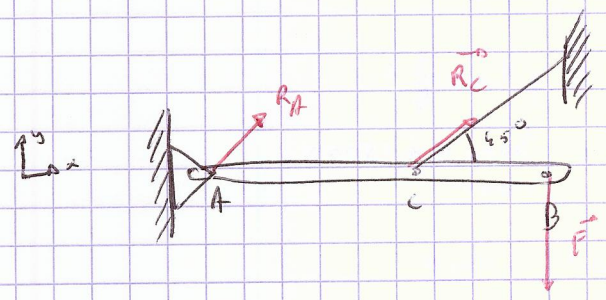
$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$R = G \qquad = 3N$$

* $M + 0,1 . G = 0$

$$M = 0,1 . G \quad = 0,1 . 3N = 0,3\,N$$

ex 1.6



- $\vec{R_A} + \vec{R_C} + \vec{F} = 0$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} R_C \cos 45° \\ R_C \sin 45° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F \end{Bmatrix} = 0$$

$$\begin{cases} R_{Ax} = -R_C \frac{\sqrt{2}}{2} \\ R_{Ay} + R_C \frac{\sqrt{2}}{2} = F \end{cases}$$

$R_{Ax} = -75000 \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2} = -75000$

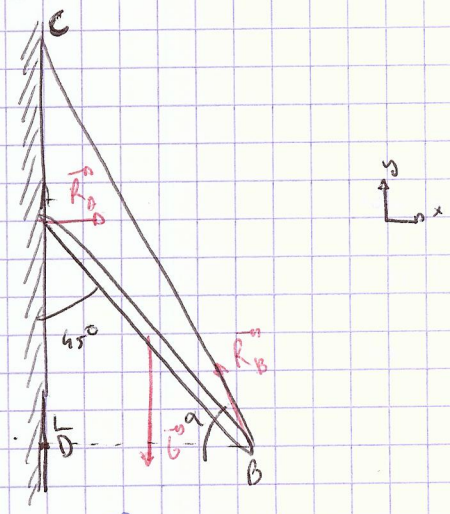
$R_{Ay} = 50000 - 75000 \frac{\sqrt{2}}{2} \cdot \sqrt{2} = -25000$

- $\ell \frac{\sqrt{2}}{1} R_C - 3F = 0$

$R_C = \frac{3F}{\sqrt{2}}$

$R_C = 75000\sqrt{2}$

ex 1.7



- $\vec{R_A} + \vec{G} + \vec{R_B} = 0$

$$\begin{Bmatrix} R_A \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} -R_{Bx} \\ R_{By} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_A = R_{Bx} = 25\,N \\ G = R_{By} = 50\,N \end{cases}$$

$\|R_B\| = \sqrt{3125} = 56\,N$

- $\frac{\sqrt{2}}{2} G \cdot 1 + \ell \cdot (-R_A) \frac{\sqrt{2}}{2} = 0$

$R_A = \frac{G}{2} = 25\,N$

- $\alpha = \cos^{-1}\left(\frac{25}{56}\right) = 63{,}43°$

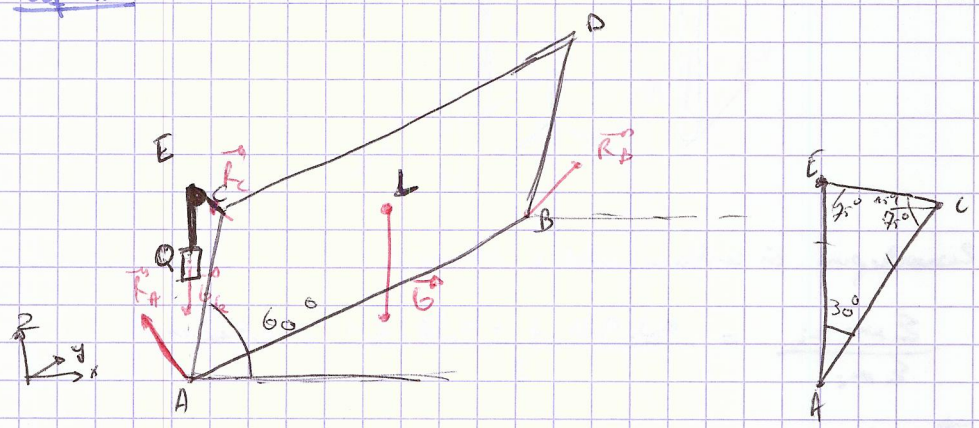
$\tan \alpha = \frac{|CA|}{|DB|}$

$|CD| = \frac{\sqrt{2}}{2} \cdot 2 \cdot \tan 9$

$|CD| = 2{,}82$

$* \; |AC| = |CD| - |AD|$
$= 2{,}82 - \frac{2\sqrt{2}}{2}$
$= 1{,}41$

Ex 1.8



$* \; \vec{R_A} + \vec{R_C} + \vec{G} + \vec{R_B} = 0$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{Bmatrix} + \begin{Bmatrix} R_{Bx} \\ R_{By} \\ R_{Bz} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} R_{Cx} \\ 0 \\ R_{Cz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_{Ax} + R_{Bx} + R_{Cx} = 0 \\ R_{Ay} + R_{By} = 0 \\ R_{Az} + R_{Bz} - G + R_{Cz} = 0 \end{cases} \Rightarrow \begin{cases} R_{Ax} + 0 + R_{Cx} = 0 \\ R_{Ay} = -R_{By} \\ R_{Az} + 200N - 400N + R_{Cz} = 0 \end{cases} \Rightarrow \begin{cases} R_{Ax} = 100\,N \\ R_{Ay} = -R_{By} \\ R_{Az} = 173\,N \end{cases}$$

$* \; |AB| \times \vec{R_B} + |AL| \times \vec{G} + |AC| \times \vec{R_C}$

$$\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & |AB| & 0 \\ R_{Bx} & R_{By} & R_{Bz} \end{vmatrix} = |AB| \cdot R_{Bz} \cdot \vec{e_x} - |AB| \, R_{Bx} \, \vec{e_z}$$

$$\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \frac{1}{2}|AC| \cos 60° & \frac{1}{2}|AB| & \frac{1}{2}|AC| \sin 60° \\ 0 & 0 & -G \end{vmatrix} = \frac{1}{2}|AB|(-G) \, \vec{e_x} + \frac{1}{2}|AC| \cos 60° \cdot G \, \vec{e_y}$$

$$\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ |AC| \cos 60° & 0 & |AC| \sin 60° \\ R_{Cx} & 0 & R_{Cz} \end{vmatrix} = (-|AC| \cos 60° \cdot R_{Cz} + |AC| \sin 60° \, R_{Cx}) \, \vec{e_y}$$

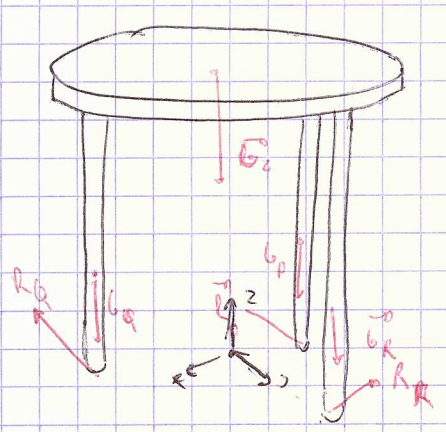
$$\begin{cases} \cancel{|AB|} \cdot R_{Bz} - \frac{1}{2} \cancel{|AB|} \cdot G = 0 \\ \frac{1}{2} \cancel{|AC|} \cos 60° \cdot G + \cancel{|AC|} \sin 60° \, R_{Cx} - \cancel{|AC|} \cos 60° \, R_{Cz} = 0 \\ -|AB| R_{Bx} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} R_{Bz} = \frac{1}{2} \cdot G = 200\,N \\ \frac{1}{4} G + \frac{\sqrt{3}}{2} R_{Cx} - R_{Cz} = 0 \\ R_{Bx} = 0 \end{cases}$$

en $R_{Cx} = R_C \cdot \cos 45°$ $\Rightarrow R_C = 103{,}5\,N$
$R_{Cz} = R_C \cdot \sin 45°$ $\quad Q = 10{,}35\,kg$

Oef 1.9

a)



b) Zwaartepunt in centrum, enkel z belangrijk:

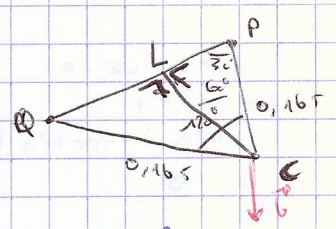
$$\frac{\sum m_i x_i}{\sum m_i} = \frac{0{,}3 \cdot 0{,}24 \cdot 3 + 0{,}50 \cdot 2{,}1}{3} = 0{,}42\,m$$

c) $\vec{M} = G_R \times 0{,}165$

$$= \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & 0{,}165 & 0 \\ 0 & 0 & -3 \end{vmatrix} = \vec{e_x}\,|-0{,}495|$$

d) 0

e)



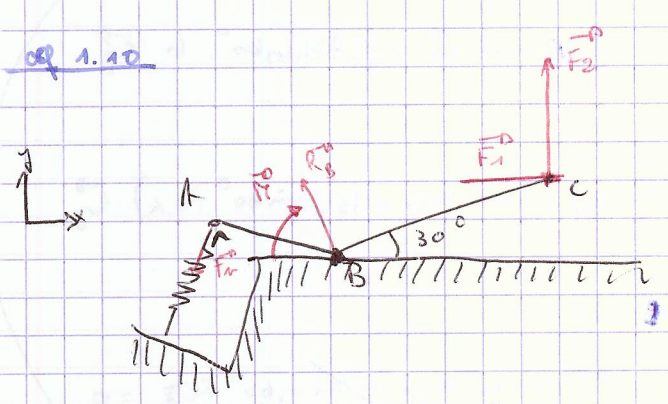
$$\cos 60° = \frac{|CL|}{0{,}165} \Rightarrow |CL| = \frac{0{,}165}{2}$$

$$\vec{M} = |CL| \times \vec{G}$$

$$= \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & -|CL| & 0 \\ 0 & 0 & -G \end{vmatrix} = G\,|CL|\,\vec{e_x} = 30\,N \cdot \frac{0{,}165}{2}\,m\,\vec{e_x}$$

$$= 2{,}475\,Nm$$

Oef 1.10



$$* \quad \vec{F_1} + \vec{F_2} + \vec{R_B} + \vec{F_W} = 0$$

$$\begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_2 \end{Bmatrix} + \begin{Bmatrix} R_{Bx} \\ R_{By} \end{Bmatrix} + \begin{Bmatrix} F_{Wx} \\ F_{Wy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} F_1 + R_{Bx} + F_{Wx} = 0 \\ F_2 + R_{By} + F_{Wy} = 0 \end{cases}$$

$$* \quad \vec{M} + |AB| \times \vec{F_v} + |BC| \times \vec{F_1} + |BC| \, \vec{F_2} = \vec{0}$$

$$\vec{M} + |AB| \times \vec{F_v} + |BC| \times (\vec{F_1} + \vec{F_2}) = \vec{0}$$

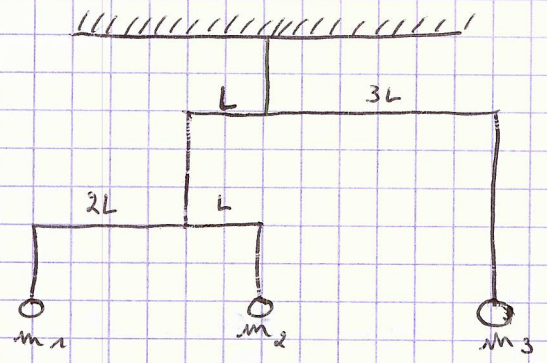
$$500 \, Nm + \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0{,}5 \cdot \cos 30^\circ & 0{,}5 \cdot \sin 30^\circ & 0 \\ 500 & 1000 & 0 \end{vmatrix} + 0{,}2 \, m \; F_v = \vec{0}$$

$$-500 \, Nm + 1000\sqrt{3} \cdot 125 \, Nm + 0{,}2 \, m \; F_v = 0$$

$$F_v = 5535 \, N$$

(2)

cf 2.1

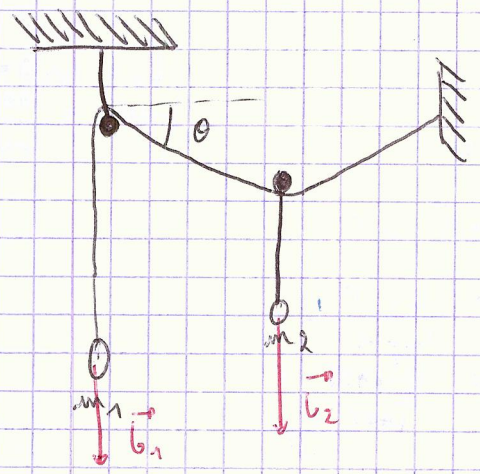


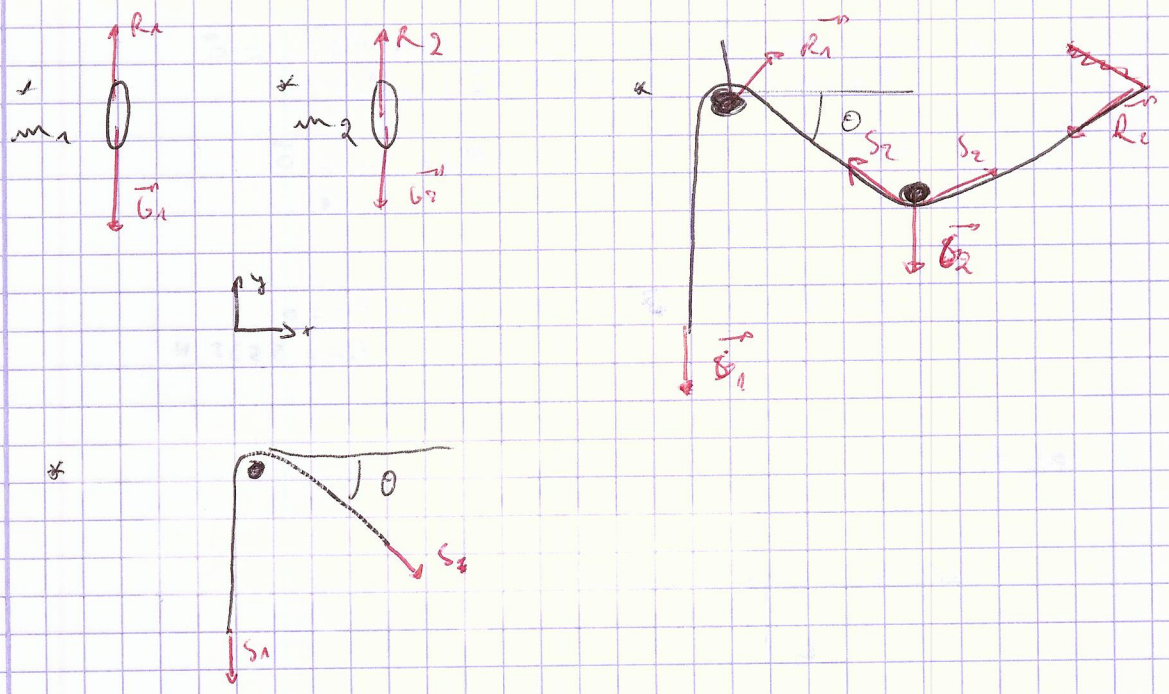
* $m_1 + m_2 > m_3$
* $m_2 > m_1$
* $m_1 + m_2 = 3m_3$
* $m_2 = 2m_1$

$\Big\}$ $3m_1 = 3m_3 \Rightarrow m_1 = m_3$

$\Rightarrow \quad m_1 = m_3 < m_2$

cf 2.2



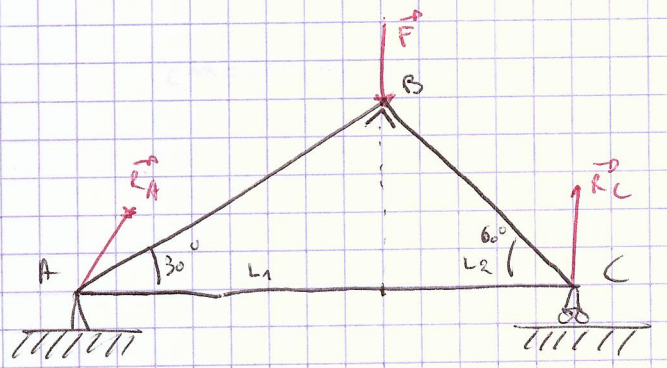


* $S_1 = S_2 = 380$ N

* $2 \cdot \sin\theta \cdot \vec{S_1} = \vec{G_2}$

$\theta = \arcsin\left(\frac{G_2}{S_1 \cdot 2}\right) = \arcsin\left(\frac{340}{380 \cdot 2}\right) = 10°$

ex 2.3



* $\vec{R_A} + \vec{R_C} + \vec{F} = \vec{0}$

$\begin{cases} R_{Ax} = 0 \\ R_{Ay} + R_C - F = 0 \end{cases}$

$\left(\begin{array}{l} L_1 = \cos 30° \cdot |AB| \\ L_1 + L_2 = \frac{|AB|}{\cos 30°} \end{array} \right)$

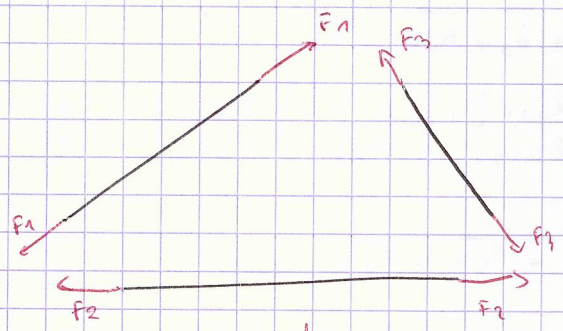
* $-L_1 \times \vec{F} + (L_1 + L_2) \times \vec{R_C} = 0$

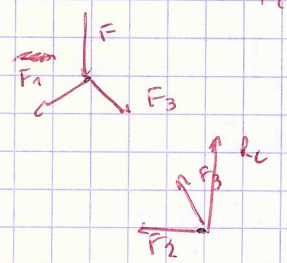
$R_C = \frac{L_1 F}{L_1 + L_2}$

$= \frac{\frac{\sqrt{3}}{2}|AB| \; F}{|AB| \cdot \frac{2}{\sqrt{3}}}$

$= \frac{3}{4} F = 6000$ N

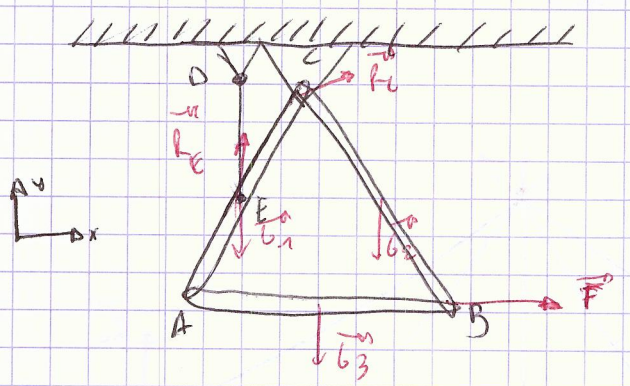
$\Rightarrow R_A = \frac{1}{4} F = 2000$ N

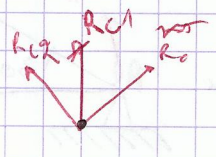




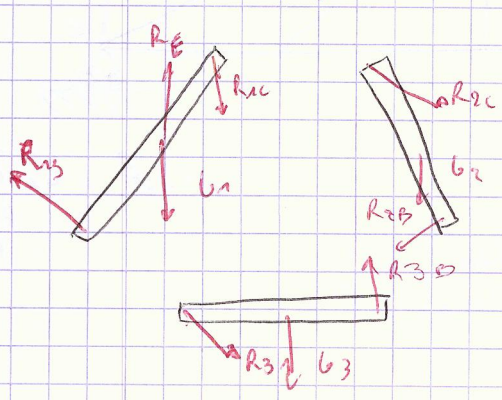
$$\begin{cases} F + F_1 + F_3 = 0 \\ R_A + F_1 + F_2 = 0 \\ R_C + F_3 + F_2 = 0 \end{cases} \Rightarrow \begin{cases} F_1 = -F - F_3 \\ F_2 = -R_A - F_1 \\ F_3 = -F_2 - R_C \end{cases} \Rightarrow \begin{cases} F_1 = -2000\ N \\ F_2 = 0 \\ F_3 = -6000\ N \end{cases}$$

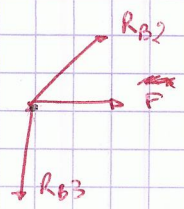
oef 2.4





1)





2) Staaf 1: $\vec{R_{13}} + \vec{G_1} + \vec{R_{1C}} + \vec{R_E} = \vec{0}$

$$\begin{Bmatrix} R_{13x} \\ R_{13y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_1 \end{Bmatrix} + \begin{Bmatrix} R_{1Cx} \\ R_{1Cy} \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_E \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Staaf 2: $\vec{R_{2C}} + \vec{G_2} + \vec{R_{2B}} = \vec{0}$

$$\begin{Bmatrix} R_{2Cx} \\ R_{2Cy} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_2 \end{Bmatrix} + \begin{Bmatrix} R_{2Bx} \\ R_{2By} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Staaf 3: $\vec{R_{31}} + \vec{G_3} + \vec{R_{3B}} = \vec{0}$

$$\begin{Bmatrix} R_{31x} \\ R_{31y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_3 \end{Bmatrix} + \begin{Bmatrix} R_{3Bx} \\ R_{3By} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Knooppunt C: $\vec{R_C} + \vec{R_{C1}} + \vec{R_{C2}} = \vec{0}$

$$\begin{Bmatrix} R_{Cx} \\ R_{Cy} \end{Bmatrix} + \begin{Bmatrix} R_{C1x} \\ R_{C1y} \end{Bmatrix} + \begin{Bmatrix} R_{C2x} \\ R_{C2y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Knooppunt B: $\vec{F} + \vec{R_{B2}} + \vec{R_{B3}} = \vec{0}$

$$\begin{Bmatrix} F \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{B2x} \\ R_{B2y} \end{Bmatrix} + \begin{Bmatrix} R_{B3x} \\ R_{B3y} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

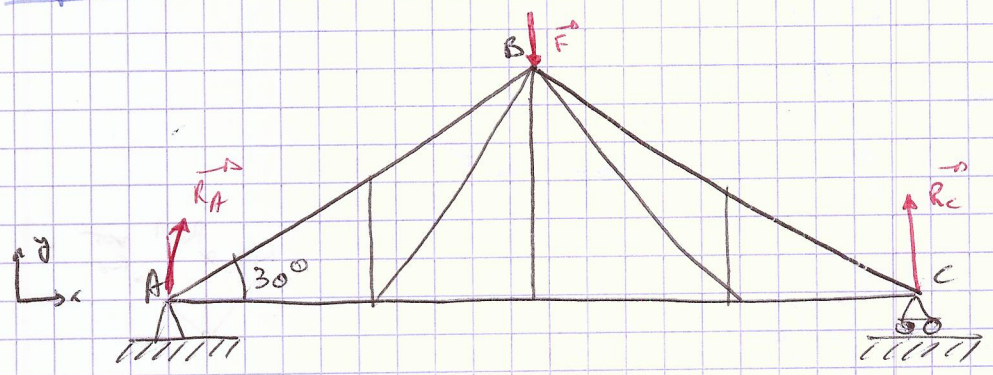
Staaf 1: rond A: $\frac{1}{2} R_E - \frac{1}{2} G + 1 . R_{1C} = 0$

Staaf 2: rond B: $\frac{1}{2} G + 1 . R_{2C} = 0$

Staaf 3: rond B: $R_{31} . 1 + \frac{1}{2} G_3 = 0$

$\Rightarrow$ 13 vgln, 13 onbekenden

oef 2.5



$+ \vec{F} + \vec{R_C} + \vec{R_A} = 0$

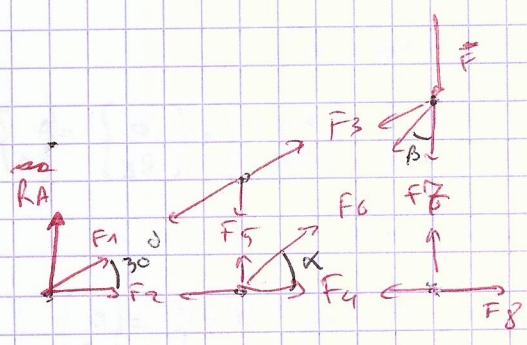
$$\begin{Bmatrix} 0 \\ -F \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_C \end{Bmatrix} + \begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_{Ax} = 0 \\ R_C + R_{Ay} - F = 0 \end{cases}$$

$\times \; -2 \times F + 4 \times \vec{R_C} = 0$

$R_C = 3\,kN$

$R_A = 5\,kN$



$$* \begin{cases} R_A + F_1 . \sin 30^\circ = 0 \\ F_1 . \cos 30^\circ + F_2 = 0 \end{cases}$$

$$\begin{cases} F_3 . \sin 30^\circ - F_5 - F_1 . \sin 30^\circ = 0 \\ F_3 \cos 30^\circ - F_1 \cos 30^\circ = 0 \end{cases}$$

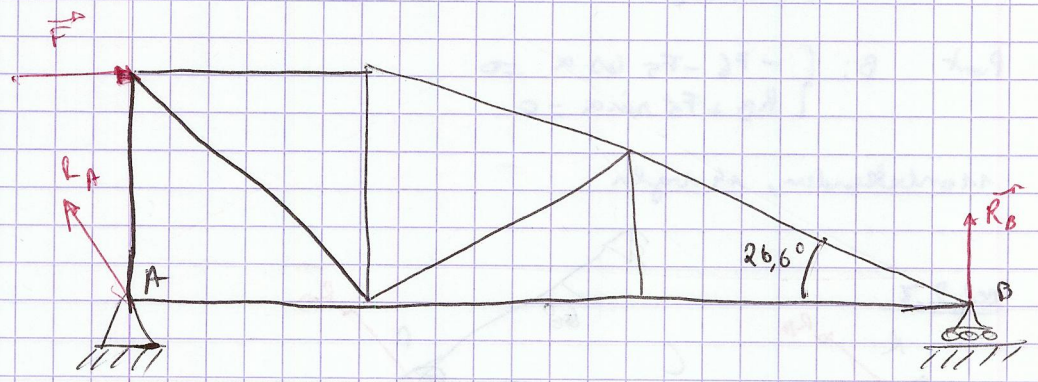
$$\begin{cases} F_5 + F_6 \sin \alpha = 0 \\ F_6 \cos \alpha + F_4 - F_2 = 0 \end{cases}$$

$$\begin{cases} -F - F_7 - F_3 \sin 30^\circ - F_6 . \sin \beta = 0 \\ -F_3 . \cos 30^\circ - F_6 . \sin \beta = 0 \end{cases}$$

$$\begin{cases} F_7 = 0 \\ F_8 - F_4 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} F_1 = -10 \text{ kN} \\ F_2 = 5\sqrt{3} \text{ kN} \\ F_3 = -10 \text{ kN} \\ F_4 = 5\sqrt{3} \text{ kN} \\ F_5 = 0 \\ F_6 = 0 \\ F_7 = 0 \end{cases}$$

exf 2.6



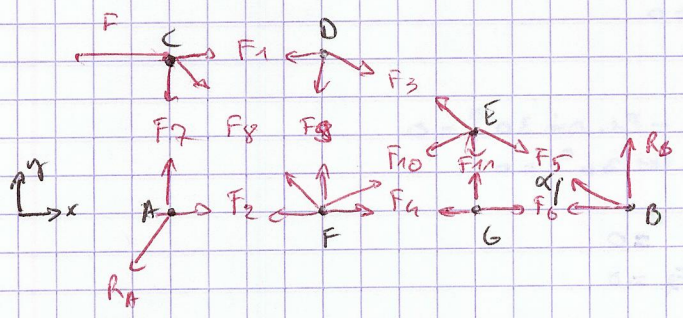
$$* \quad \vec{F} + \vec{R_B} + \vec{R_A} = \vec{0}$$

$$\begin{Bmatrix} F \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_B \end{Bmatrix} = \begin{Bmatrix} -R_{Ax} \\ -R_{Ay} \end{Bmatrix}$$

$$\begin{cases} F = -R_{Ax} \\ R_B = -R_{Ay} \end{cases} \Rightarrow \begin{cases} R_{Ax} = -1000 \text{ N} \\ R_{Ay} = \frac{-1000}{3} \text{ N} \\ R_B = \frac{1000}{3} \text{ N} \end{cases}$$

$$* \quad -1 \times \vec{F} + 3m \times \vec{R_B} = 0$$

$$R_B = \frac{F}{3} = \frac{1000}{3} \text{ N}$$



Punt C: $\begin{cases} F + F_1 + F_8 . \cos 45^\circ = 0 \\ -F_7 - F_8 . \sin 45^\circ = 0 \end{cases}$

Punt A: $\begin{cases} F_2 - R_{Ax} = 0 \\ F_7 - R_{Ay} = 0 \end{cases}$

Punt D: $\begin{cases} -F_1 + F_3 . \cos \alpha = 0 \\ -F_9 - F_3 . \sin \alpha = 0 \end{cases}$

Punt F: $\begin{cases} F_4 - F_2 + F_{10} . \cos \alpha - F_8 . \cos 45^\circ = 0 \\ F_9 + F_{10} . \sin \alpha + F_8 . \sin \alpha = 0 \end{cases}$

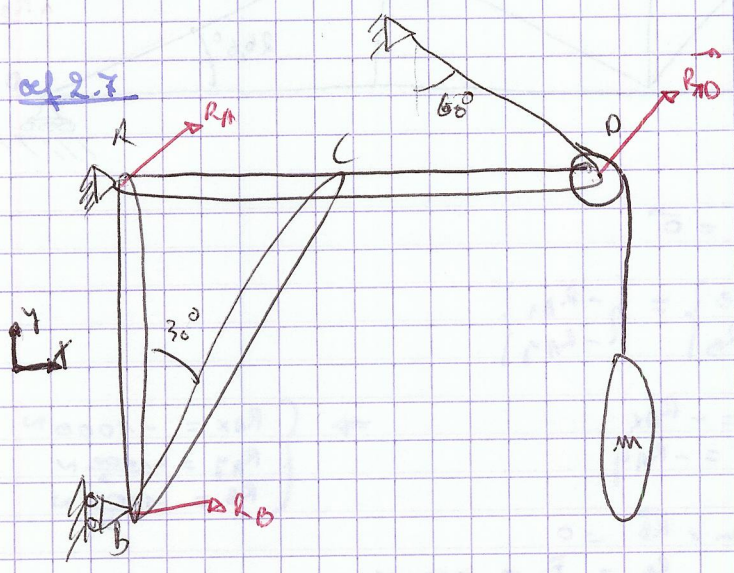
Punt E: $\begin{cases} F_5 . \cos \alpha - F_3 \cos \alpha - F_{10} \cos \alpha = 0 \\ -F_{11} + F_3 \sin \alpha - F_{10} \sin \alpha = 0 \end{cases}$

Punt G: $\begin{cases} F_6 - F_4 = 0 \\ F_{11} = 0 \end{cases}$

Punt B: $\begin{cases} -F_6 - F_5 \cos \alpha = 0 \\ R_B + F_5 \sin \alpha = 0 \end{cases}$

11 onbekenden, 14 vgln

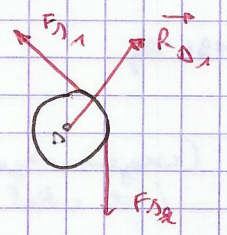
<u>oef 2.7</u>



* $\vec{R_A} + \vec{R_B} + \vec{R_D} = \vec{0}$

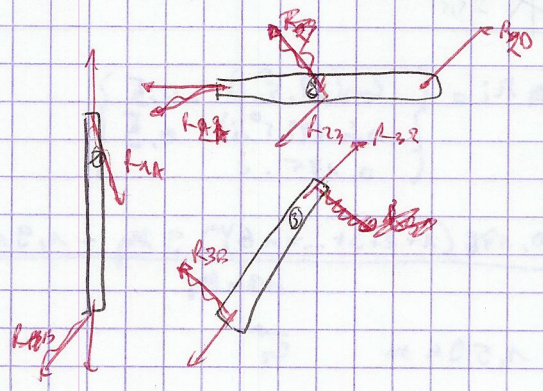
$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} R_B \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{Dx} \\ R_{Dy} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

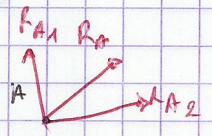
* $|AB| \times \vec{R_B} + |AD| \times \vec{R_D} = 0$

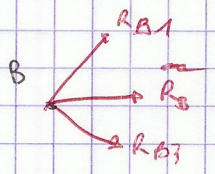


$$* \quad \vec{R_{D1}} + \vec{F_{D1}} + \vec{F_{D2}} = 0$$

$$\begin{Bmatrix} R_{D1x} \\ R_{D1y} \end{Bmatrix} + \begin{Bmatrix} F_D \cdot \cos 30^\circ \\ F_D \cdot \sin 30^\circ \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F_D \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (F_D = G_d)$$





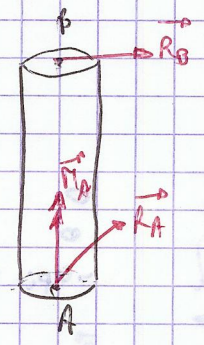


(krachten volgens staaf gelegen in staaf 1 en 3)

7 onbekenden, 7 vgln

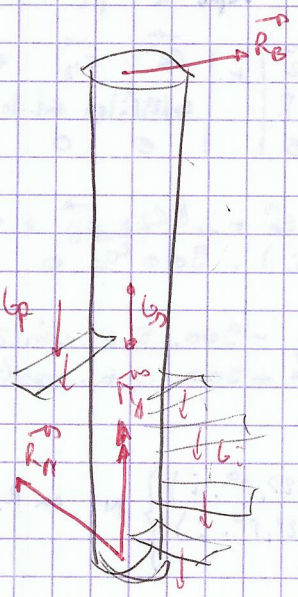
oef 2.8

1)



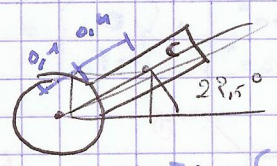
$$R_A \begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{Bmatrix} \qquad R_B \begin{Bmatrix} R_{Bx} \\ R_{By} \\ 0 \end{Bmatrix} \qquad M_A \begin{Bmatrix} 0 \\ 0 \\ M_{Az} \end{Bmatrix}$$

2)



3) 16 treden, elk 9 kg
1 spil, 50 kg $\Big\} \Rightarrow m = 194\ kg$

zwaartepunt : $\dfrac{\sum m_i r_i}{\sum m_i}$ (wegens symmetrie in spil gelegen)
($\Rightarrow$ enkel z-component belangrijk)

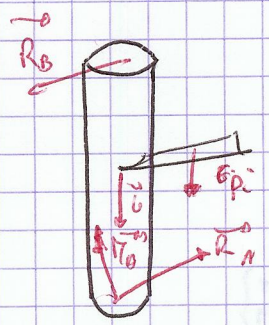


$$\Rightarrow r_i = \begin{Bmatrix} \cos(22,5^\circ . i) \cdot 0,5 \\ \sin(22,5^\circ . i) \cdot 0,5 \\ 0,175 \cdot i \end{Bmatrix}$$

$$\Rightarrow \text{zwaartepunt} : \frac{0,175\,(1+2+3+\ldots+16)\,m \cdot 9\,kg + 1,9\,m \cdot 50\,kg}{194\,kg}$$

$$= 1,594\ m \quad \vec{e_z}$$

4)



$$\vec{R_B} + \vec{R_A} + \vec{G} + \vec{G_{pi}} = \vec{0}$$

$$\begin{Bmatrix} R_{Bx} \\ R_{By} \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -G_{pi} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\vec{M_A} + |AB| \times \vec{R_B} + \vec{r_{pi}} \times \vec{G_{pi}} = \vec{0}$$

$$\vec{M_A} + \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0 & 0 & 3,8 \\ R_{Bx} & R_{By} & 0 \end{vmatrix} + \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ \cos(22,5^\circ . i) \cdot 0,6 & \sin(22,5^\circ . i) \cdot 0,6 & 0,175 . i \\ 0 & 0 & -900 \end{vmatrix} = \vec{0}$$

$$M_A\,\vec{e_z} + -3,8\,R_{By}\,\vec{e_x} + 3,8\,R_{Bx}\,\vec{e_y} + 0,6\sin(22,5^\circ . i)(-900)\,\vec{e_x} + 0,6\cos(22,5^\circ . i) \cdot 900\,\vec{e_y} = 0$$

$$\Rightarrow \begin{cases} -3,8\,R_{By} - 900 \cdot 0,6 \cdot \sin(22,5^\circ . i) = 0 \\ 3,8\,R_{Bx} + 900 \cdot 0,6 \cdot \cos(22,5^\circ . i) = 0 \\ M_A = 0 \end{cases}$$

$$\Rightarrow R_A = \begin{Bmatrix} 142\cos(22,5^\circ . i) \\ 142\sin(22,5^\circ . i) \\ 2840 \end{Bmatrix} N \quad \text{en } R_B = \begin{Bmatrix} -142\cos(22,5^\circ . i) \\ -142\sin(22,5^\circ . i) \\ 0 \end{Bmatrix} N$$

5) Wegens symmetrie enkel z-component van belang

$$* \quad \vec{R_A} + \cancel{\vec{R_B}} + \vec{G} + \vec{F_2} + \vec{F_6} + \vec{F_8} + \vec{F_{14}} = 0$$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{Bmatrix} + \begin{Bmatrix} R_{Bx} \\ R_{By} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -1940\,N \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -3600 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$\Rightarrow R_{Az} = 5540\,N$

$\Rightarrow R_B = 0$

$* \quad$ Moment $\rightarrow \vec{M_A} = 0$

6) $$* \quad \vec{R_A} + \vec{R_B} + \vec{G} + \vec{F_8} + \vec{F_{10}} + \vec{F_{12}} + \vec{F_{14}} = \vec{0}$$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \end{Bmatrix} + \begin{Bmatrix} R_{Bx} \\ R_{By} \\ R_{Bz} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -1940\,N \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -2000\,N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

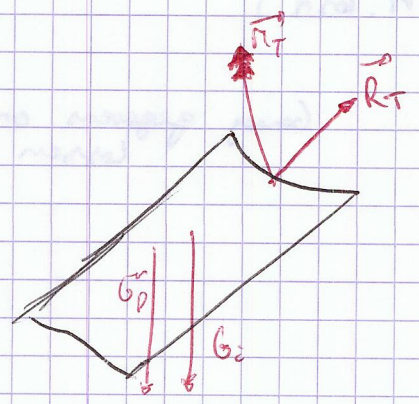
$$* \quad \vec{M_A} + (\vec{M_B} - \vec{M_A}) \times \vec{R_B} + \dots + (\vec{M_{14}} - \vec{M_A}) \times \vec{F_{14}} = \vec{0}$$

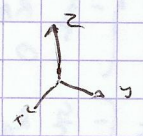
$$\Rightarrow \vec{R_A} = \begin{Bmatrix} 79 \\ -191 \\ 3940 \end{Bmatrix} N$$

$$\Rightarrow \vec{R_B} = \begin{Bmatrix} 79 \\ 191 \\ 0 \end{Bmatrix} N$$

$$\Rightarrow \vec{M_A} = 0$$

7)





$$* \quad \vec{R_T} + \vec{G_i} + \vec{G_p} = \vec{0}$$

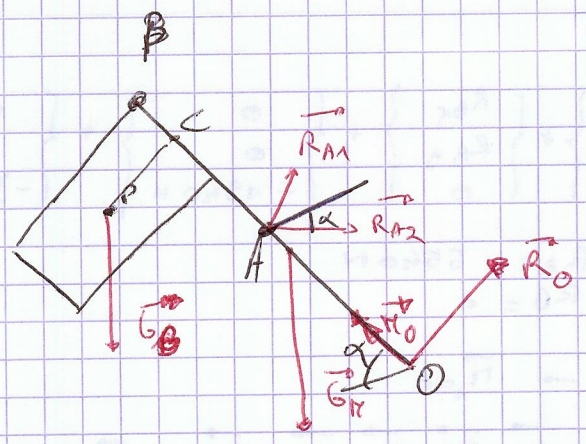
$$\vec{R_T} = \begin{Bmatrix} 0 \\ 0 \\ 990 \end{Bmatrix} N$$

$$* \quad \vec{M_T} + 0{,}4 \times \vec{G_i} + 0{,}5\,\vec{G_p} = \vec{0}$$

$$\vec{M_T} = \begin{Bmatrix} 0 \\ -486 \\ 0 \end{Bmatrix}$$

Oef 2.9

1)



2) * $\vec{G_B} + \vec{G_M} + \vec{R_{A1}} + \vec{R_{A2}} + \vec{R_O} = 0$

$$\begin{Bmatrix} 0 \\ 0 \\ -G_B \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -G_M \end{Bmatrix} + \begin{Bmatrix} R_{A1x} \\ R_{A1y} \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{A2x} \\ R_{A2y} \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{Ox} \\ R_{Oy} \\ R_{Oz} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{cases} R_{A1x} + R_{A2x} + R_{Ox} = 0 \\ R_{A1y} + R_{A2y} + R_{Oy} = 0 \\ R_{Oz} - G_B - G_M = 0 \end{cases}$$

* $\vec{M_O} + (\vec{R_{A1}} + \vec{R_{A2}}) \times |AO| + \frac{|BO|}{2} \times \vec{G_M} + |OP| . \vec{G_B}$

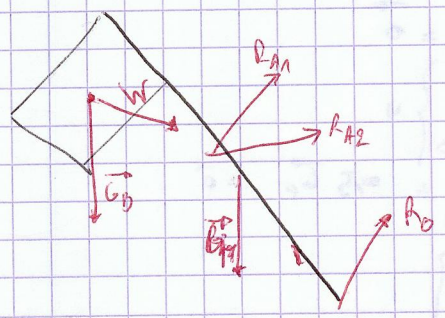
Niet gegeven: $|OP| = \begin{Bmatrix} |OC| . \cos\alpha + |CP| . \sin\alpha \\ 0 \\ |OC| . \sin\alpha - |CP| . \cos\alpha \end{Bmatrix}$

$\vec{R_{A1}} = \begin{Bmatrix} -\frac{4}{5} R_{A1} \\ -\frac{3}{5} R_{A1} \\ 0 \end{Bmatrix}$

Genoeg gegevens om op te lossen

$\vec{R_{A2}} = \begin{Bmatrix} -\frac{4}{5} R_{A2} \\ \frac{3}{5} R_{A2} \\ 0 \end{Bmatrix}$

3)



Extra kracht + extra moment

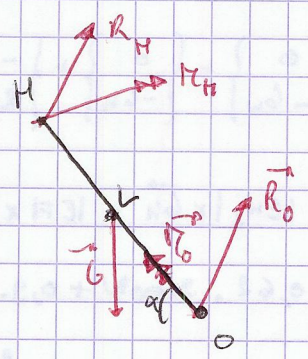
$$+ \quad \vec{G_B} + \vec{G_M} + \vec{R_{A1}} + \vec{R_{A2}} + \vec{R_O} + \vec{W} = 0$$

$$\vec{W} = \begin{Bmatrix} 0 \\ W \\ 0 \end{Bmatrix}$$

$$+ \quad |OP| \times \vec{W} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 7{,}4\,m & 0 & 4{,}3\,m \\ 0 & W & 0 \end{vmatrix} = \begin{Bmatrix} -4{,}3\,m.W \\ 0 \\ 7{,}4\,m.W \end{Bmatrix}$$

Genoeg gegevens om op te lossen

4)



$$+ \quad \vec{R_H} + \vec{R_O} + \vec{G} = \vec{0}$$

$$\begin{Bmatrix} R_{Hx} \\ R_{Hy} \\ R_{Hz} \end{Bmatrix} + \begin{Bmatrix} R_{Ox} \\ R_{Oy} \\ R_{Oz} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

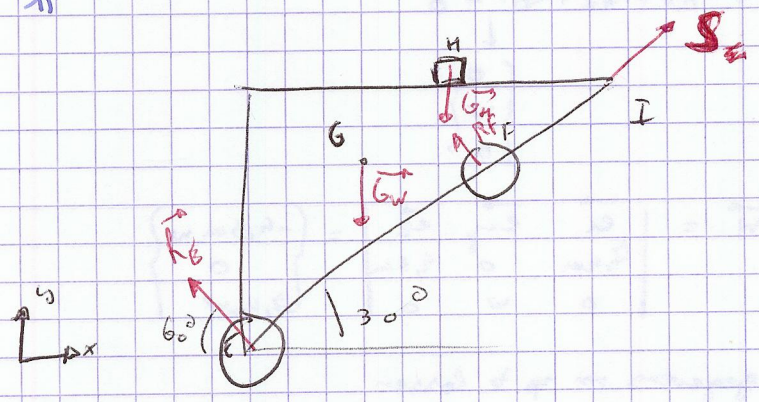
$$G = \frac{5}{18} \cdot 200\,N = 56\,N$$

$$+ \quad |HL| \times \vec{G} + |OH| \times \vec{R_O} + \vec{M_O} + \vec{M_H} = 0$$

Genoeg gegevens om op te lossen

ex 2.10

1)



* $\vec{R_E} + \vec{G_W} + \vec{G_H} + \vec{R_F} + \vec{S} = \vec{0}$

$$\begin{Bmatrix} -R_E \cdot \cos 60° \\ R_E \cdot \sin 60° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_W \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_H \end{Bmatrix} + \begin{Bmatrix} -R_F \cos 60° \\ R_F \sin 60° \end{Bmatrix} + \begin{Bmatrix} S \cdot \cos 30° \\ S \sin 30° \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

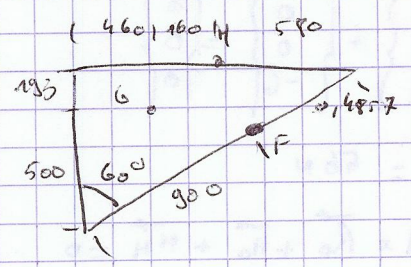
* rend E : $|EG_\perp| \times \vec{G_W} - |EH_\perp| \times \vec{G_H} + |EF| \times \vec{R_F} = \vec{0}$

$$-0{,}46 \cdot 450\,N - 0{,}62 \cdot 2000\,N + 0{,}9 \cdot R_F = 0$$

$$\Downarrow$$

$$R_F = 1608\,N$$

* rend F : $-|EF| \times \vec{R_E} + |GF_\perp| \times \vec{G_W} + |HF_\perp| \times \vec{G_H} = \vec{0}$



$\longrightarrow$ $|GF_\perp| = 0{,}32\,m$

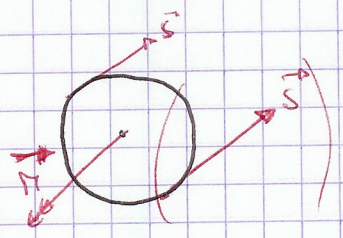
$|HF_\perp| = 0{,}16\,m$

$$-0{,}9\,m \cdot \vec{R_E} + 0{,}32\,m \cdot 450\,N + 0{,}16\,m \cdot 2000\,N = 0$$

$$R_E = 516\,N$$

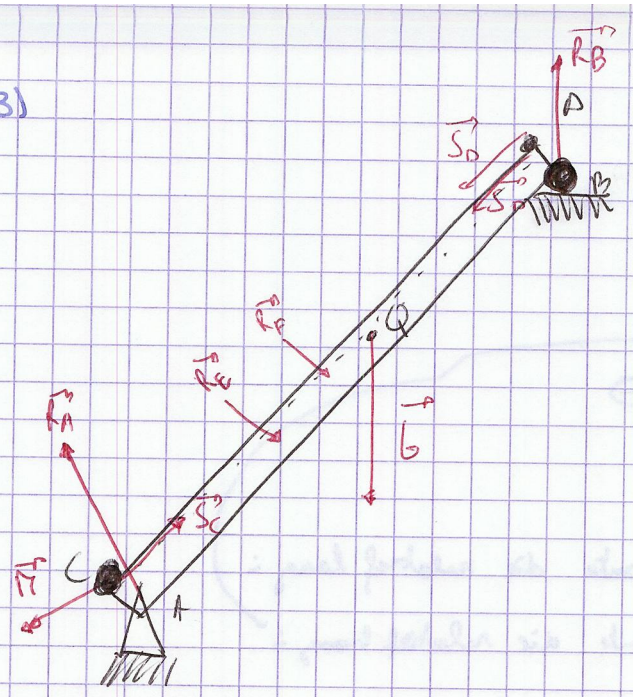
2) * $\begin{cases} -516\,N \cdot \cos 60° - 1608\,N \cdot \cos 60° + S \cdot \cos 30° = 0 \\ 516\,N \cdot \sin 60° - 450\,N - 2000\,N + 1608\,N \cdot \sin 60° + S \sin 30° = 0 \end{cases}$

$$\Rightarrow S = 1225\,N$$



$$\vec{M} = -0{,}1 \cdot \vec{S} = -122{,}5\,N$$

3)



* $\vec{R_A} + \vec{S_C} + \vec{R_E} + \vec{R_F} + \vec{G} + 2.\vec{S_D} + \vec{R_D}$

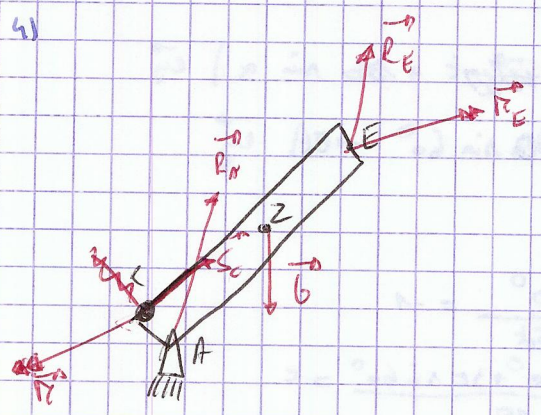
$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} S.\cos 30° \\ S\sin 30° \end{Bmatrix} + \begin{Bmatrix} R_E.\cos 60° \\ -R_E.\sin 60° \end{Bmatrix} + \begin{Bmatrix} R_F.\cos 60° \\ -R_F.\sin 60° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + 2.\begin{Bmatrix} -S.\cos 30° \\ S.\sin 30° \end{Bmatrix}$$

$$+ \begin{Bmatrix} 0 \\ R_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

* rond A: $\vec{M} - |AE| \times \vec{R_E} - |AF| \times \vec{R_F} - |AQ| \times \vec{G} + |AB| \times \vec{R_B} = \vec{0}$

benodigd gegevens om op te lossen

4)



* $\vec{R_A} + \vec{S_C} + \vec{G} + \vec{R_E} = \vec{0}$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} S.\cos 30° \\ S\sin 30° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} R_{Ex} \\ R_{Ey} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

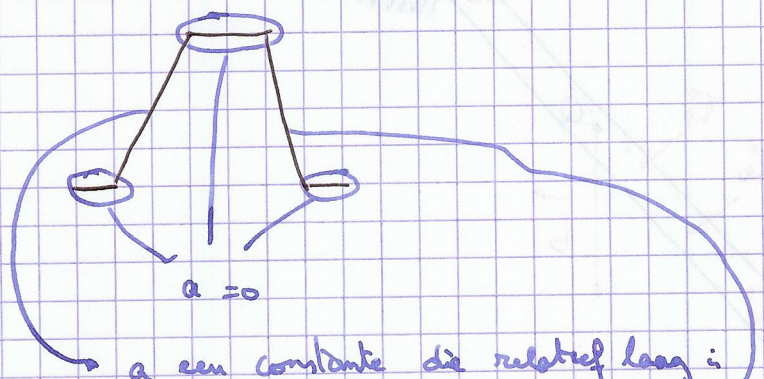
1/3 totaal

* $\vec{M} + \vec{M_E} + |AE| \times \vec{R_A} + |ZE| \times \vec{G} = \vec{0}$

benodigd gegevens om op te lossen

③

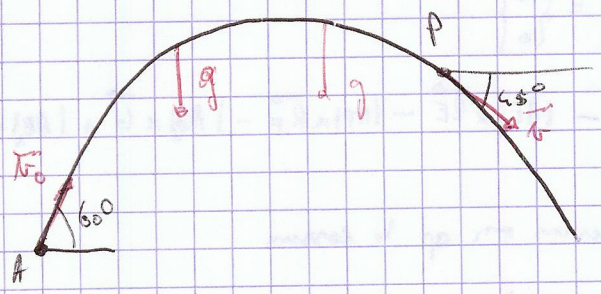
Oef 3.1



a een constante die relatief laag is positief
a een constante die relatief hoog is negatief

→ D

Oef 3.2



$\vec{v}(t) = 30 \cdot \cos\alpha \ \vec{e_x} + (-gt + 30 \sin\alpha)\ \vec{e_y}$

$= 30 \cdot \cos 60° \ \vec{e_x} + (30 \sin 60° - 10t)\ \vec{e_y}$

In p: $\frac{v_x}{v_y} = -1$

$\Rightarrow \frac{30 \cdot \cos 60°}{30 \sin 60° - 10t} = -1$

$\frac{30 \cdot \cos 60° + 30 \sin 60°}{10} = t$

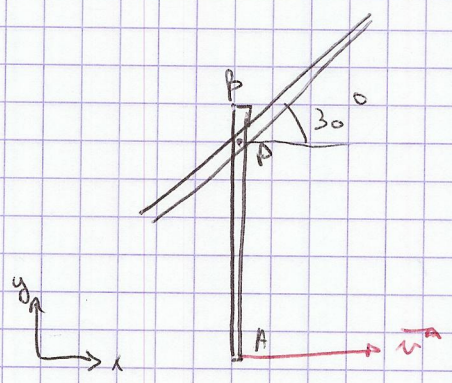
$t = 5{,}18$

$\Rightarrow \vec{v}(5{,}18) = 15 \frac{m}{s} \vec{e_x} - 15 \frac{m}{s} \vec{e_y}$

$\|v\| = 21{,}21$

Zie 3.10 voor verdere uitwerking

oef 3.3



$v_x = 3\,m/s$
$a_x = 2\,m/s^2$

* $\cos 30° = \frac{v_x}{v} \Rightarrow v = \frac{3\,m/s}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}\,m/s$

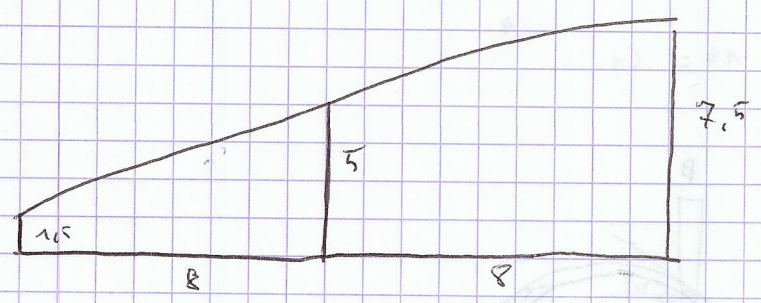
$= \frac{a_x}{a} \Rightarrow a = \frac{2\,m/s^2}{\frac{\sqrt{3}}{2}} = \frac{4\sqrt{3}}{3}\,m/s^2$

* $\sin 30° = \frac{v_y}{v} \Rightarrow v_y = 2\sqrt{3}\,m/s \cdot \frac{1}{2} = \sqrt{3}\,m/s$

$= \frac{a_y}{a} \Rightarrow a_y = \frac{4\sqrt{3}}{3}\,m/s^2 \cdot \frac{1}{2} = \frac{2\sqrt{3}}{3}\,m/s^2$

$\Rightarrow \vec{v} = \begin{Bmatrix} 3 \\ \sqrt{3} \end{Bmatrix} m/s$ en $\vec{a} = \begin{Bmatrix} 2 \\ \frac{2\sqrt{3}}{3} \end{Bmatrix} m/s^2$

oef 3.4



$\vec{a} = \begin{Bmatrix} 0 \\ -g \end{Bmatrix}$

$\vec{v} = \begin{Bmatrix} c_1 \\ -gt + c_2 \end{Bmatrix}$

$\vec{r} = \begin{Bmatrix} c_1 t + c_3 \\ -\frac{gt^2}{2} + c_2 t + c_4 \end{Bmatrix}$

$$\begin{cases} x(t) = C_1 t + C_3 \\ y(t) = \frac{-g t^2}{2} + C_2 t + C_4 \end{cases}$$

punten invullen:

$$\begin{cases} 0 = C_3 \\ 1{,}5 = C_4 \end{cases}$$

$$\begin{cases} 8 = C_1 t_1 + 0 \\ 5 = \frac{-g t_1^2}{2} + C_2 t_1 + 1{,}5 \end{cases}$$

$$\begin{cases} 16 = C_1 t_2 \\ 7{,}5 = \frac{-g t_2^2}{2} + C_2 t_2 + 1{,}5 \end{cases}$$

uitwerken:

$$\frac{8}{C_1} = t_1$$

$$\Rightarrow 5 = -5 \cdot \left(\frac{8}{C_1}\right)^2 + C_2 \cdot \frac{8}{C_1} + 1{,}5$$

$$C_2 = \frac{\left(5 - 1{,}5 + 5\left(\frac{8}{C_1}\right)^2\right) C_1}{8}$$

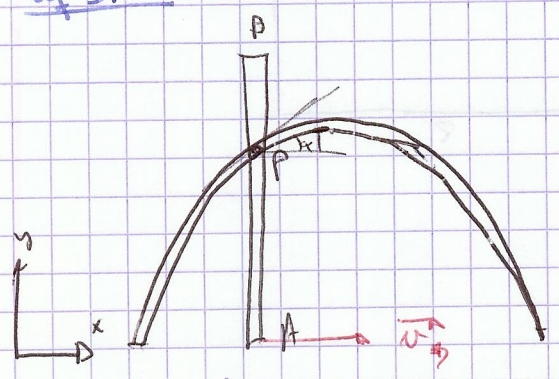
$$\Rightarrow 7{,}5 = -5\, t_2^2 + \frac{\left(3{,}5 + 5 \cdot \left(\frac{8}{C_1}\right)^2\right) C_1}{8} \cdot t_2 + 1{,}5$$

$$6{,}8 = -40\, t_2^2 + \left(3{,}5 C_1 + \frac{5 \cdot 64}{C_1}\right) \cdot t_2$$

$$t_2 = \frac{-2\left(3{,}5 C_1 + \frac{5 \cdot 64}{C_1}\right) \pm \sqrt{\left(3{,}5 C_1 + \frac{5 \cdot 64}{C_1}\right)^2 - 4 \cdot (-40)(-6{,}8)}}{2 \cdot (-40)}$$

$$\Rightarrow 16 = C_1.$$

oef 3.5



$$P = \begin{Bmatrix} 1(3 \\ \sin|1(3| \end{Bmatrix} m$$

$$v_x = 3\,m/s \Rightarrow x = 3t$$

$$y = \sin 3t$$

$$\Downarrow$$

$$v_y = 3\cos 3t$$

$$v_y = 3\cos x = 3\cos\frac{\pi}{3} = 2{,}835$$

$$\Rightarrow \vec{v} = \begin{Bmatrix} 3 \\ 2,85 \end{Bmatrix}$$

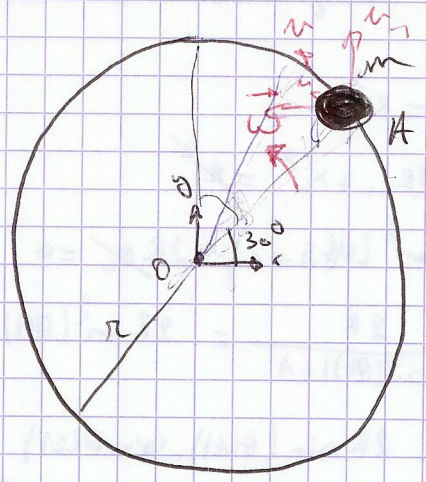
* $a_x = v_x' = 0 \ m/s^2$

$a_y = v_y' = -9 \sin 3t$

in $\frac{1}{3}$: $a_y = -9 . \sin x$

$= -9 . \sin \frac{1}{3} = -2,9 \ m/s^2$

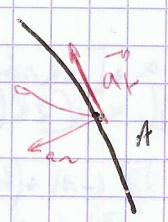
<u>ex 3.6</u>

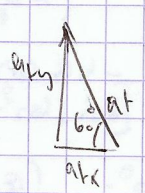


a) $\begin{cases} v_x = -(R \times \vec{\omega}) . \cos 60° & = -3888 \\ v_y = (R \times \vec{\omega}) . \sin 60° & = 6734 \end{cases}$

$\begin{cases} a_x = \vec{\omega} \times \vec{\omega} \times (\vec{R}) \cos 60° & = -8,08 \\ a_y = \vec{\omega} \times \vec{\omega} \times (\vec{R}) \sin 60° & = -4,67 \end{cases}$

b)





* $a_{tx} = -\cos 60° . a_t$

$= -50$

$\Rightarrow a_x = a_{tx} + a_{nx}$

$= -58,08$

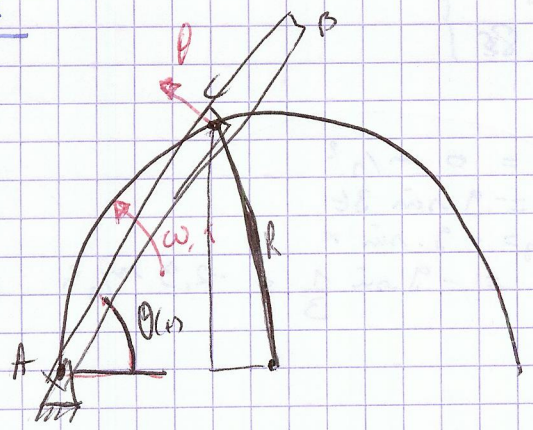
* $a_{ty} = \sin 60° . a_t$

$= 50\sqrt{3}$

$\Rightarrow a_y = a_{ty} + a_{ny}$

$= 84,94$

exf 3.7



a) $\tan(\theta(t)) = \frac{y_c}{x_c} \Rightarrow y_c = x_c \tan(\theta(t))$

$$y_c^2 + (R - x_c)^2 = R^2$$

$$x_c^2 \tan(\theta(t))^2 + \cancel{R^2} - 2Rx_c + x_c^2 = \cancel{R^2}$$

$$x_c^2 (\tan^2(\theta(t)) + 1) - 2R\cancel{x_c} = 0$$

$$\begin{cases} x_c = \frac{2R}{\tan^2(\theta(t)) + 1} = 2R\cos^2(\theta(t)) \\ y_c = 2R\sin(\theta(t)).\cos(\theta(t)) \end{cases}$$

$$* \Rightarrow \begin{cases} v_x = [2.R.2.\cos(\theta(t)).-\sin(\theta(t))].\omega(t) \\ v_y = [2R.\cos^2(\theta(t)) - 2R.\sin^2(\theta(t))].\omega(t) \end{cases}$$

$$\Rightarrow \text{bij } 60° : \begin{cases} v_x = -4.0,2.\cos(60°).\sin(60°).3 \\ v_y = [2.0,2.\cos^2(60°) - 2.0,2.\sin^2(60°)]3 \end{cases}$$

$$\Rightarrow \begin{cases} v_x = -1,04 \\ v_y = -0,6 \end{cases} \Rightarrow v = 1,2 \text{ m/s}$$

$$* \Rightarrow \begin{cases} a_x = -4R\cos(\theta).\sin(\theta).\alpha + [4R\sin^2\theta - 4R\cos^2(\theta)].\omega \\ a_y = [2R.\cos^2(\theta) - \sin^2(\theta)].\alpha + [2R(-2\cos(\theta).\sin\theta - 2\sin(\theta)\cos(\theta))]\omega \end{cases}$$

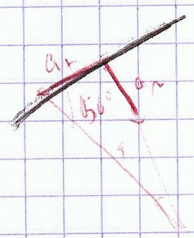
$$\Rightarrow \text{bij } 60° : \begin{cases} a_x = -4.0,2.\frac{1}{2}.\frac{\sqrt{3}}{2}.(-1) + (4.0,2.\frac{3}{4} - 4.0,2.\frac{1}{4}).3 \\ a_y = 2.0,2.(\frac{1}{4} - \frac{3}{4}).(-1) + 2.0,2.(-2.\frac{1}{2}.\frac{\sqrt{3}}{2} - 2.\frac{\sqrt{3}}{2}.\frac{1}{2}).3 \end{cases}$$

$$\Rightarrow \begin{cases} a_x = 3,95 \\ a_y = -6,04 \end{cases} \Rightarrow a = 7,21$$

b)

$a_t = -0,40 \text{ m/s}^2$

$a_n = 7,20 \text{ m/s}^2$



oef 3.8

$t=0 \Rightarrow v_0 = -2\,m/s$

$t=0$ tot 2 : $a = 3\,m/s^2$

$t=2$ tot ... : $a = -2\,m/s^2$

$v=0$?

$v = a.t + v_0$

* van 0 tot 2:

$v = 3.t - 2$

$\Downarrow$

$t = \frac{2}{3} \Rightarrow$ na $\frac{2}{3}$ seconde

* van 2 tot ... :

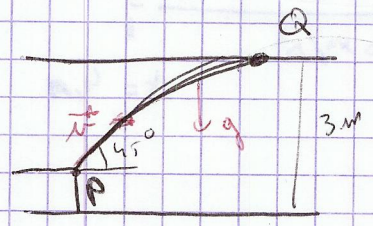
$v = -2.t + v_1$ met $v_1 = 6 - 2 = 4$

$v = -2.t + 4$

$\Downarrow$

$t = 2 \Rightarrow$ na 4 seconden

oef 3.9



* $\vec{r}(t) = \begin{Bmatrix} 0 \\ -\frac{g}{2}t^2 \end{Bmatrix} + \begin{Bmatrix} v_0 \cos\alpha\, t \\ v_0 \sin\alpha\, t \end{Bmatrix}$

$\Rightarrow 2 = -5t^2 + 7\sqrt{2}.\frac{\sqrt{2}}{2}.t$

$\Downarrow$

$5t^2 - 7t + 2 = 0$ $\qquad t = \frac{7 \pm \sqrt{49-40}}{10}$

$\Downarrow$

$t = 0{,}4$ of 1

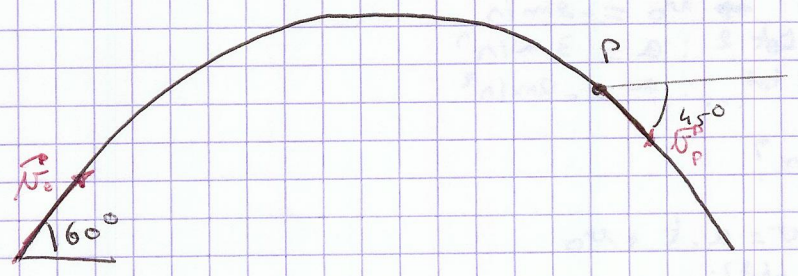
* x-component: $r_x = 7\sqrt{2}.\frac{\sqrt{2}}{2}.0{,}4$

$= 2{,}8$

* $|PQ| = \sqrt{2{,}8^2 + 2^2}$

$= 3{,}4$

cf 3.10



* $\vec{r}(t) = \begin{Bmatrix} 0 \\ -\frac{gt^2}{2} \end{Bmatrix} + \begin{Bmatrix} v_0 \cdot \cos\alpha \, t \\ v_0 \cdot \sin\alpha \, t \end{Bmatrix}$

* $\vec{v}(t) = \begin{Bmatrix} 0 \\ -gt \end{Bmatrix} + \begin{Bmatrix} v_0 \cdot \cos\alpha \\ v_0 \cdot \sin\alpha \end{Bmatrix}$

* $\frac{v_{px}}{v_{py}} = -1 \Rightarrow \frac{v_0 \cdot \cos\alpha}{v_0 \cdot \sin\alpha - gt} = -1$

$\frac{v_0 \cos\alpha + v_0 \sin\alpha}{g} = t$

$\Rightarrow \frac{30\,m/s \cdot \frac{1}{2} + 30\,\frac{m}{s} \cdot \frac{\sqrt{3}}{2}}{10\,m/s^2} = t$

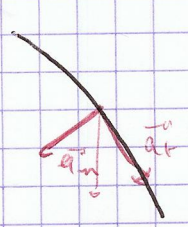
$t = 4{,}1$

a) $\begin{cases} v_x = 15\,m/s \\ v_y = -15\,m/s \end{cases}$

b) $t = 4{,}1\,s$

c) $\begin{cases} x = 15\,\frac{m}{s} \cdot 4{,}1\,s & = 61{,}5\,m \\ y = \frac{-10\,m/s^2 \cdot (4{,}1\,s)^2}{2} + 15\sqrt{3}\,\frac{m}{s}\,4{,}1\,s & = 22{,}5\,m \end{cases}$

d)

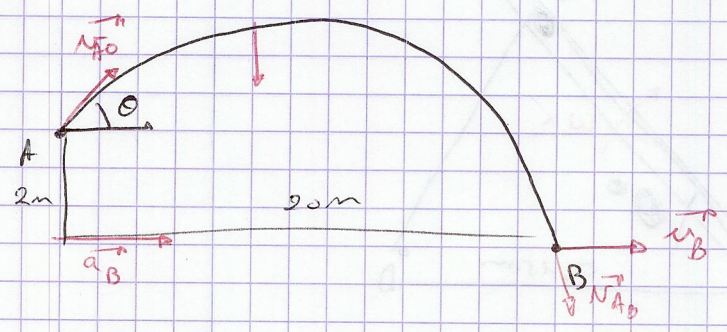


$a_n = a_t$

$2 a_n^2 = a^2$

$a_t = a_n = \sqrt{50} = 7{,}1\,m/s^2$

ex 3.11



* $\vec{r}(t) = \begin{Bmatrix} 0 \\ -g\frac{t^2}{2} \end{Bmatrix} + \begin{Bmatrix} v_0 \cos\theta . t \\ v_0 \sin\theta . t \end{Bmatrix}$

* $\vec{v}(t) = \begin{Bmatrix} 0 \\ -gt \end{Bmatrix} + \begin{Bmatrix} v_0 \cos\theta \\ v_0 \sin\theta \end{Bmatrix}$

$$\Rightarrow \begin{cases} 20 = v_0 . \cos\theta . t \longrightarrow t = \frac{20}{v_0 . \cos\theta} \\ -2 = -5t^2 + v_0 \sin\theta . t \end{cases}$$

$$\rightarrow -2 = -5\left(\frac{20}{v_0 \cos\theta}\right)^2 + \cancel{v_0} \sin\theta . \frac{20}{\cancel{v_0} \cos\theta}$$

$$\downarrow$$

$$-2 = -5 . \frac{400}{v_0^2 \cos^2\theta} + 20 . \tan\theta$$

$$-2v_0^2 = -5\frac{400}{\cos^2\theta} + 20 . v_0^2 \tan\theta$$

$$0 = 20 . \tan\theta . v_0^2 + 2 . v_0^2 - \frac{2000}{\cos^2\theta}$$

$$\downarrow$$

$$v_0 = \sqrt{\frac{2000}{\cos^2\theta(10\tan\theta + 1)}}$$

is minimal als $v_0' = 0$

$$v_0' = \frac{1}{2\sqrt{\frac{100}{\cos^2\theta(10\tan\theta+1)}}} \cdot \frac{-1000\left(-2\cos\theta\sin\theta(10\tan\theta+1) + \cos^2\theta \cdot \left(\frac{2}{\cos^2\theta}\right)\right)}{\left(\cos^2\theta(10\tan\theta+1)\right)^2}$$

$$= 0 \Leftrightarrow \theta = 42{,}14^\circ$$

$$\rightarrow v_0 = 13{,}6\,m/s \Rightarrow t = 1{,}98$$

* $a(t) = 0{,}5$

$v(t) = 0{,}5t + v_0$

$r(t) = \frac{0{,}5}{2}t^2 + v_0 t$

$$\Rightarrow r(3{,}54) = \frac{1}{4} . 3{,}54^2 + 2{,}23 . 3{,}54$$

$$= 11\,m$$

$$\rightarrow 5 = \frac{1}{2}t + v_0 \Rightarrow v_0 = 5 - \frac{1}{2}t \quad v_0 = 2{,}23$$

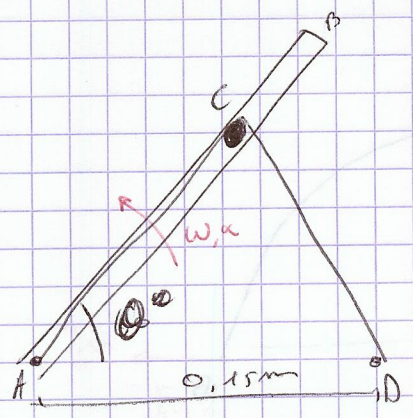
$$\rightarrow 20 = \frac{1}{4}t^2 + \left(5 - \frac{1}{2}t\right)t$$

$$20 = \frac{1}{4}t^2 + 5t - \frac{1}{2}t^2 \Rightarrow 0 = -\frac{1}{4}t^2 + 5t - 20$$

$$t = \frac{-5 \pm \sqrt{25 - 4 . -\frac{1}{4} . 20}}{-\frac{1}{2}}$$

$$= \frac{-5 \pm \sqrt{5}}{-2} = 3{,}54$$

oef 3.12



In ΔACD: $|AC| + |CD| = 0{,}30$ m

Cosinusregel: $|CD|^2 = |AC|^2 + |AD|^2 - 2.|AC|.|AD|.\cos 60°$

$$(0{,}30 - |AC|)^2 = |AC|^2 + 0{,}15^2 - 2.|AC|.0{,}15.\frac{1}{2}$$

$$0{,}30^2 - 0{,}6|AC| + \cancel{|AC|^2} = \cancel{|AC|^2} + 0{,}15^2 - 0{,}15|AC|$$

$$|AC| = \frac{0{,}09 - 0{,}0225}{0{,}45}$$

$$= 0{,}15$$

$$= |CD|$$

Algemeen: $|AC| = \dfrac{0{,}09 - 0{,}0225}{(0{,}6 - 0{,}3.\cos\theta)} = \dfrac{0{,}0675}{(0{,}6 - 0{,}3\cos\theta)}$

$$\begin{cases} x(\theta) = \dfrac{0{,}0675}{(0{,}6 - 0{,}3\cos\theta)} \cdot \cos\theta \\ y(\theta) = \dfrac{0{,}0675}{0{,}6 - 0{,}3.\cos\theta} \cdot \sin\theta \end{cases}$$

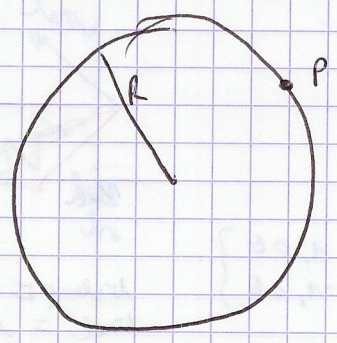
$$\begin{cases} v_x(\theta) = \dfrac{-0{,}0675\sin\theta.(0{,}6 - 0{,}3\cos\theta) - 0{,}0675.\cos\theta.0{,}3\sin\theta}{N^2} \cdot \omega \\ v_y(\theta) = \dfrac{0{,}0675\cos\theta(0{,}6 - 0{,}3\cos\theta) - 0{,}0675\sin\theta.0{,}3\sin\theta}{N^2} \cdot \omega \end{cases}$$

a) benodigd gegevens om optie kosten

b)

$$\begin{cases} a_t = a.\cos 30° \\ a_n = a.\cos 60° \end{cases}$$

oef 3.13



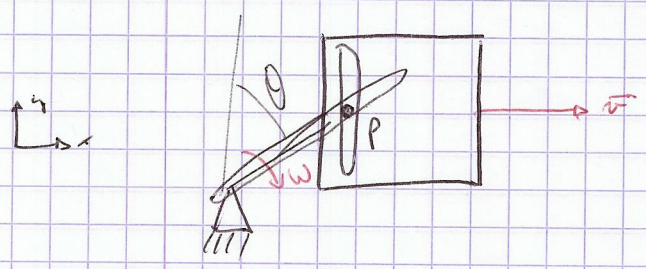
* $\theta(t) = 2t^2 + 4t^3$
  $\omega(t) = 4t + 12t^2$
  $\alpha(t) = 4 + 24t$

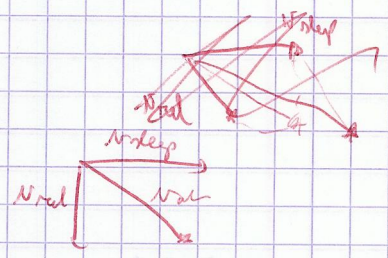
* $\vec{a_t} = \vec{\alpha} \times R$
  $= (4+24t) \times 0,5$
  $= (4+12) \cdot 0,5 = 8\ m/s^2$

* $\vec{a_n} = \vec{\omega} \times \vec{\omega} \times R$
  $= (4t+12t^2) \times (4t+12t^2) \times 0,5$
  $= (2+3) \times (2+3) \times 0,5 = \frac{25}{2}\ m/s^2$

4

oef 4.1





$v_{abs} = v_{sleep} + v_{rel}$

$v_{abs} = r\omega = 1,5\ m/s$

$v_{sleep} = v_{blok}$

v_{rel} = verticaal

$\begin{Bmatrix} 1,5 \cdot \cos 45° \\ 1,5 \cdot \sin 45° \end{Bmatrix}$

$\begin{Bmatrix} 1,06\ m/s \\ -1,06\ m/s \end{Bmatrix} \Rightarrow \vec{v}_{sleep} = 1,06\ m/s\ \vec{e_x}$

$\vec{v}_{rel} = -1,06\ m/s\ \vec{e_y}$

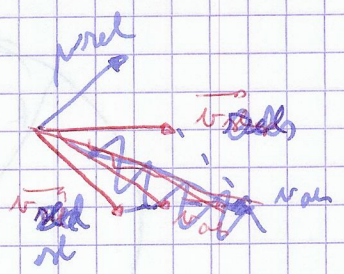
v_{sleep} = snelheid blok
v_{rel} = snelheid waarmee p omhoog of omlaag gaat in gleuf
v_{abs} = snelheid van p t.o.v. O = cirkelbeweging

oef 4.2

(tekening zie 4.1)

$v_{abs} = v_{rel} + v_{slep}$

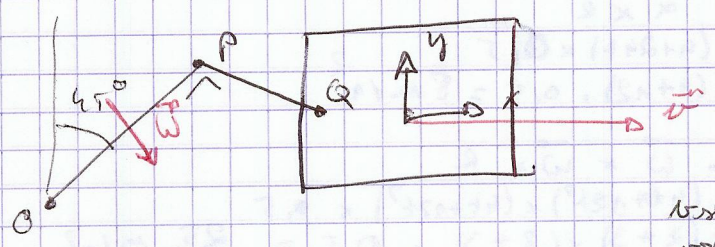
$v_{abs} = v_{blok}$, v_{rel}, $v_{slep} = r.\omega = 1,5$



$$\begin{Bmatrix} v_{abs} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1,06 \\ 1,06 \end{Bmatrix} + \begin{Bmatrix} 1,06 \\ -1,06 \end{Bmatrix}$$

v_{slep} = cirkelbeweging rond O
v_{rel} = volgens gleuf
$v_{abs} = v_{slep} + v_{rel}$

oef 4.3

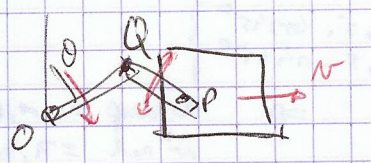


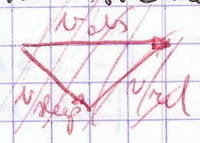
v_{slep} = snelheid blok
v_{rel} = cirkelbeweging rond Q
v_{abs} = cirkelbeweging rond O

$v_{abs} = v_{slep} + v_{rel}$
$v_{slep} = v_{blok}$

oef 4.4

v_{slep} = cirkelbeweging rond O
v_{rel} = cirkelbeweging rond Q
v_{abs} = snelheid blok

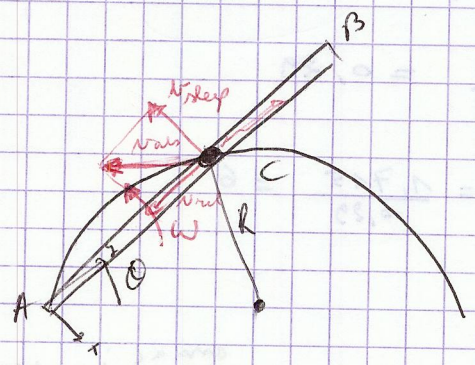




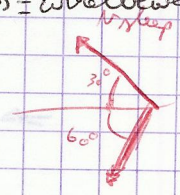
$\vec{v}_{abs} = \vec{v}_{slep} + \vec{v}_{rel}$

$$v_{slep} = \begin{Bmatrix} 0,6R.\frac{\sqrt{2}}{2} \\ 0,35\,4\,\frac{\sqrt{2}}{2} \end{Bmatrix} = \begin{Bmatrix} 0,75 \\ -1,3 \end{Bmatrix}$$

oef 4.5



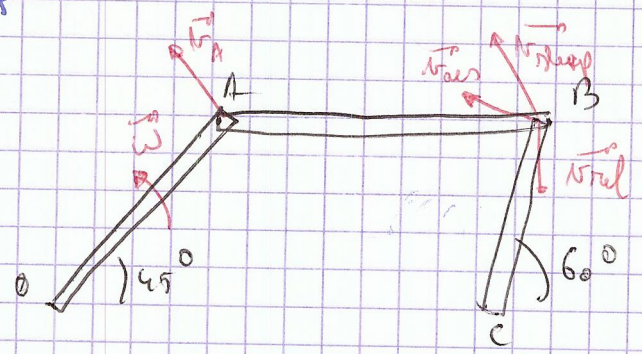
v_{sleep} = cirkelbeweging van C rond A
v_{rel} = volgen staaf AB
v_{abs} = cirkelbeweging rond O



$|AC| = 0,2$ (zie 3.7)

$$v_{sleep} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0,2.\cos 60° & 0,2.\sin 60° & 0 \\ 0 & 0 & 3 \end{vmatrix} = \vec{e_x}(0,3\sqrt{3}) - \vec{e_y}(0,3)$$

oef 4.6



$$\vec{v_{abs}} = \vec{v_{sleep}} + \vec{v_{rel}}$$

$$\begin{Bmatrix} -v_B \cdot \cos 30° \\ v_B \cdot \cos 30° \end{Bmatrix} = \begin{Bmatrix} -v_A \cdot \cos 45° \\ v_A \sin 45° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -v_{rel} \end{Bmatrix}$$

* $v_A = \omega \times |OA| = 0,35 \cdot 6 = 2,10$

$$\Rightarrow -v_B \cdot \cos 30^\circ = -v_A \cos 45^\circ$$

$$v_B = \frac{2,10 \cdot \cos 45^\circ}{\cos 30^\circ}$$

$$= 1,715$$

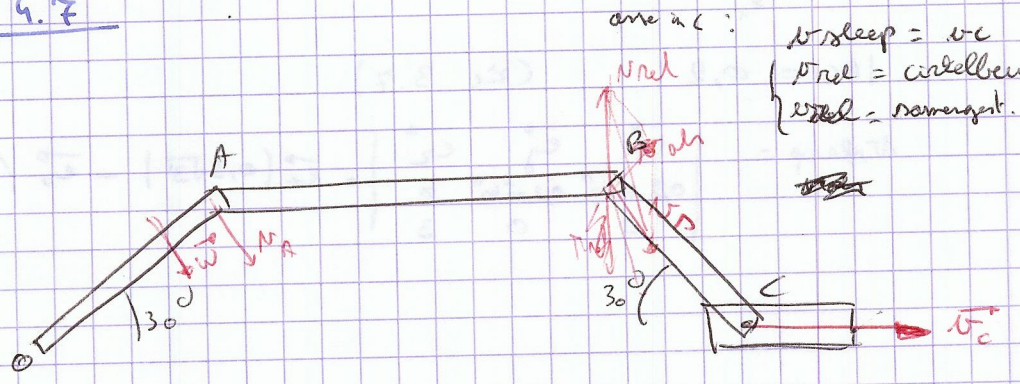
+ |CB| berekenen

$$\text{hoogte} = \sin 45^\circ \cdot 0,35$$
$$= 0,25$$

$$|CB| = \frac{0,25}{\sin 60^\circ} = 0,29$$

$$\Rightarrow \omega = \frac{v_B}{|CB|} = \frac{1,715}{0,29} = 6$$

oef 4.7



ave in C :

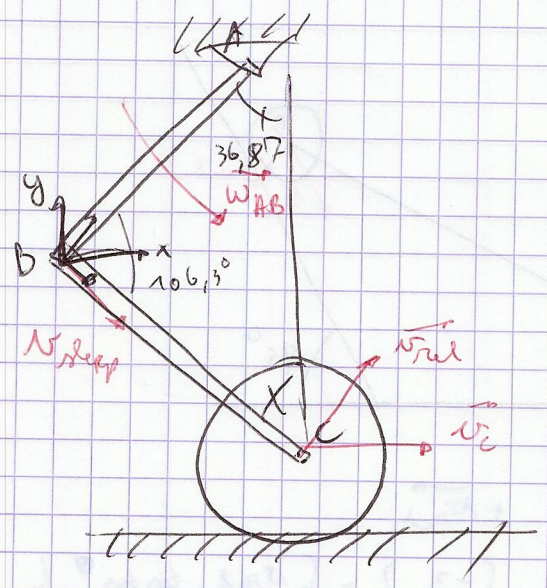
$v_{sleep} = v_C$

v_{rel} = cirkelbeweging rond C

v_{sleep} = samengest. beweging rond O en A

$$v_{sleep} = \begin{Bmatrix} 2,25 \\ -3,90 \end{Bmatrix}$$

oef 4.8



$$\vec{v}_C = \vec{v}_{slep} + \vec{v}_{rel}$$

$$r.\omega_C \qquad r.\omega_{AB} \qquad r.\omega_{BC}$$

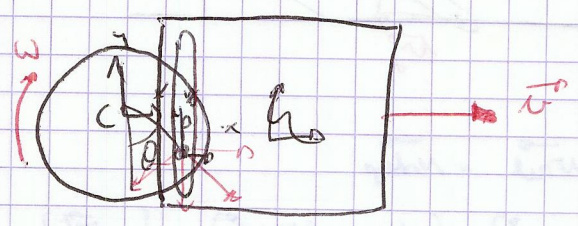
$$\begin{Bmatrix} v_C \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0,8 \\ -0,6 \end{Bmatrix} + \begin{Bmatrix} -0,4\,\omega_{BC} \\ -0,3\,\omega_{BC} \end{Bmatrix}$$

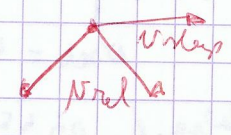
$\Rightarrow \omega_{BC} = -2$

$\Rightarrow v_C = 1,6$

$\Rightarrow \omega_C = 8$

oef 4.9



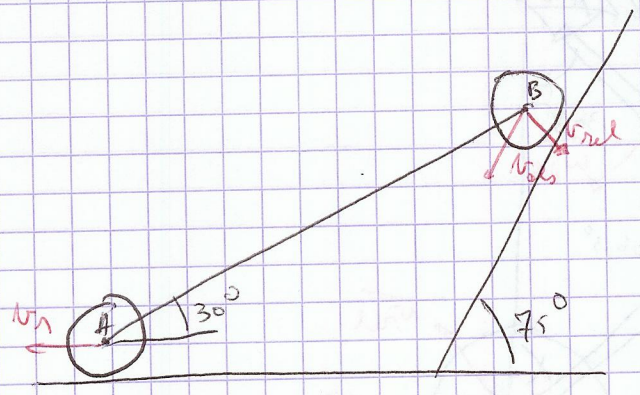


$$\vec{v}_{abs} = \vec{v}_{slep} + \vec{v}_{rel}$$

$$\begin{Bmatrix} v_{abs} \cos 60° \\ v_{abs} \sin 60° \end{Bmatrix} = \begin{Bmatrix} 0,6 \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot -\cos 30° \\ v_{rel} \cdot -\sin 30° \end{Bmatrix}$$

$\Rightarrow v_{abs} = 0,45$

ex 4.10



* $\vec{v}_{abs} = \vec{v}_{rel} + \vec{v}_{rel}$

$$\begin{Bmatrix} -v_{abs} \cdot \cos 75° \\ -v_{abs} \cdot \sin 75° \end{Bmatrix} = \begin{Bmatrix} -3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 60° \\ v_{rel} \cdot -\sin 60° \end{Bmatrix}$$

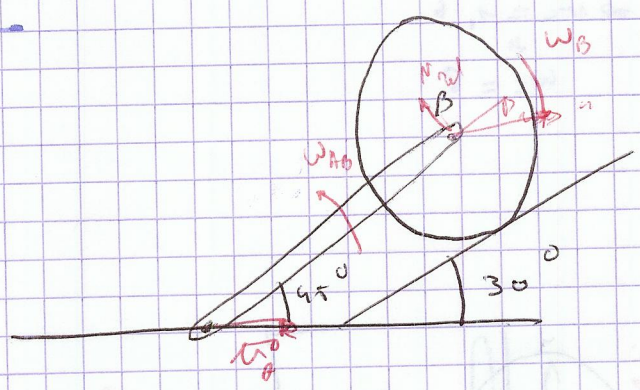
$\Rightarrow v_{rel} = 4,098$

$\Rightarrow v_{abs} = 3,67$

* $v_{abs} = r \cdot \omega = 0,20 \cdot \omega$

$\Rightarrow \omega = 18,37$

ex 4.11



$\vec{v}_{abs} = \vec{v}_{rel} + \vec{v}_{rel_{eq}}$

$$\begin{Bmatrix} v_{abs} \cdot \cos 30° \\ v_{abs} \cdot \sin 30° \end{Bmatrix} = \begin{Bmatrix} v_{rel} \cdot \cos 45° \\ v_{rel} \cdot \sin 45° \end{Bmatrix} + \begin{Bmatrix} \vec{v}_A \\ 0 \end{Bmatrix}$$

$v_{abs} = r \cdot \omega_B$ $\qquad$ $v_{rel} = \omega_{AB} \cdot l$

$\Rightarrow v_{abs} = \omega_{AB} \cdot l \cdot \dfrac{\sin 45°}{\sin 30°}$

$\Rightarrow \omega_{AB} \cdot l \cdot \dfrac{\sin 45°}{\sin 30°} \cdot \cos 30° = \omega_{AB} \cdot l \cdot \cos 45° + v_A$

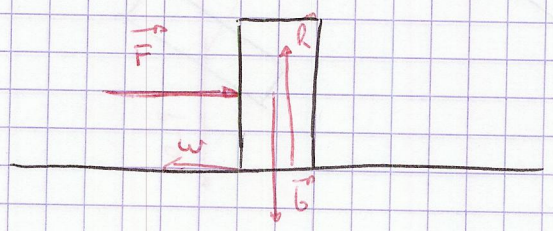
$$\omega_{AB} = \frac{v_A}{l\left(\frac{\sin 45°}{\sin 30°} \cdot \cos 30° - \cos 45°\right)} = \frac{v_A}{l \cdot 0,5176}$$

Ex 4.12

cf 2.13

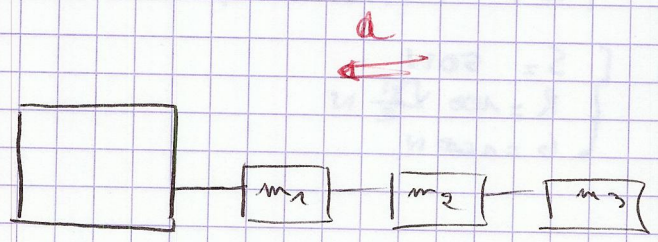
5

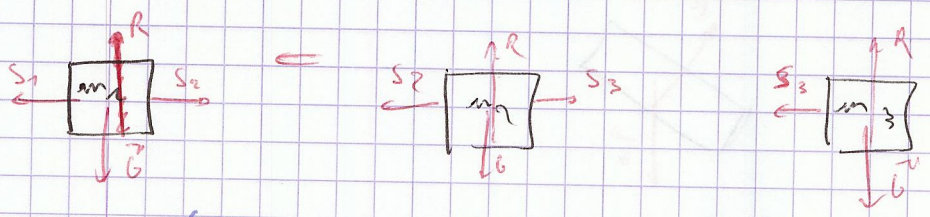
oef 5.1



De kist beweegt niet → W = 20 N

oef 5.2





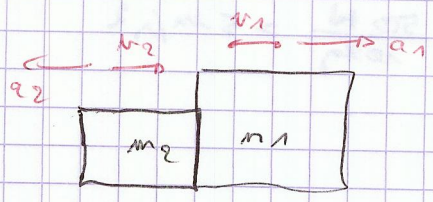
$$\begin{cases} -S_1 + S_2 = m \cdot a \\ -S_2 + S_3 = m \cdot a \\ -S_3 = m \cdot a \end{cases}$$

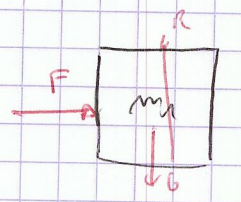
→ $-S_1 = 1200 + 3800 = 5000\,N$

→ $-S_2 = 1800 + 2000\,N \Rightarrow S_2 = -3800\,N$

→ $S_3 = -2000\,N$

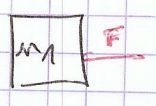
oef 5.3





$F = m \cdot a$

$F = 1670\,kg \cdot 100\,\frac{N}{kg} = 167\,000\,N$



$F = m \cdot a$

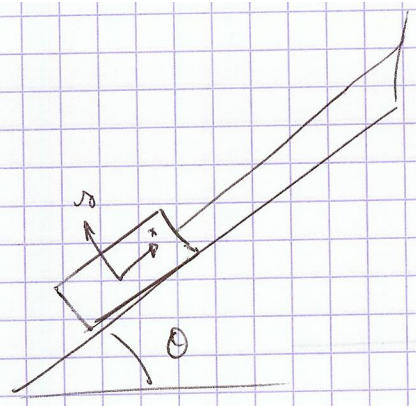
$870\,kg \cdot a_2 = 1670\,kg \cdot 10\,g$

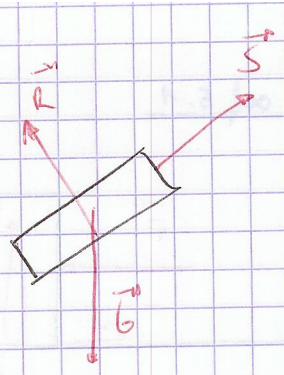
$a_2 = \frac{1670}{870}\,a_1$

⇒ a1 beter berekenen

Déf 5.4

a)



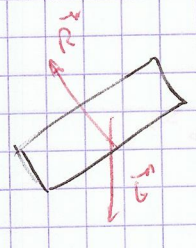


$$\vec{S} + \vec{R} + \vec{G} = \vec{0}$$

$$\begin{Bmatrix} S \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} -G \cdot \cos 60° \\ -G \cdot \sin 60° \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} S = 50\,N \\ R = 100 \frac{\sqrt{3}}{2}\,N \\ G = 100\,N \end{cases}$$

b)

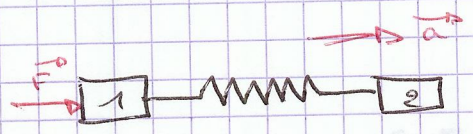


$$\vec{R} + \vec{G} = m \cdot \vec{a}$$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} -G \cos 60° \\ -G \sin 60° \end{Bmatrix} = \begin{Bmatrix} -m \cdot a \\ 0 \end{Bmatrix}$$

$$a = \frac{50\,N}{10\,kg} = 5\,m/s^2$$

Déf 5.5



* $F = m \cdot a$

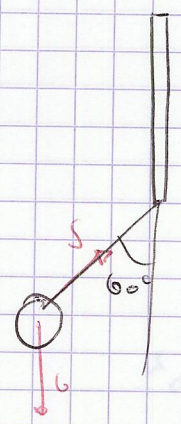
$100\,N = 10\,kg \cdot a \Rightarrow a = 10\,m/s^2$

* $F + F_r = m \cdot a$

$100\,N - 20\,\frac{N}{m}(1 - l) = 9\,kg \cdot 10\,\frac{m}{s^2}$

$1 - l = \frac{-10\,N\,m}{-20\,N} = +\frac{1}{2}\,m \Rightarrow l = \frac{1}{2}\,m$

ex 5.6



$$S + G = m \cdot a$$

$$\begin{Bmatrix} S \cdot \cos 30^\circ \\ S \cdot \sin 30^\circ \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} m \cdot a \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} S \cos 30^\circ = m \cdot a \\ S \sin 30^\circ = G \end{cases} \Rightarrow S = \frac{50}{\frac{1}{2}} = 100\,N$$

$$\Rightarrow 100\,N \cdot \frac{\sqrt{3}}{2} = 5\,kg \cdot a$$

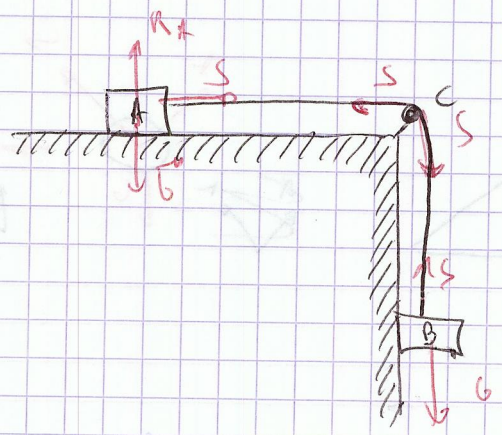
$$a = 10\sqrt{3}\ m/s^2$$

$$\omega^2 R = 10\sqrt{3}$$

$$\omega = \sqrt{\frac{10\sqrt{3}}{0{,}2 \cdot \cos 30^\circ}}$$

$$= 10$$

ex 5.7



$$\left.\begin{array}{l} \text{In A: } S = m \cdot a \Rightarrow a = \frac{S}{m_1} \\ \text{In B: } G - S = m \cdot a \Rightarrow a = \frac{G-S}{m_2} \end{array}\right\} \Rightarrow \frac{S}{m_1} = \frac{G-S}{m_2}$$

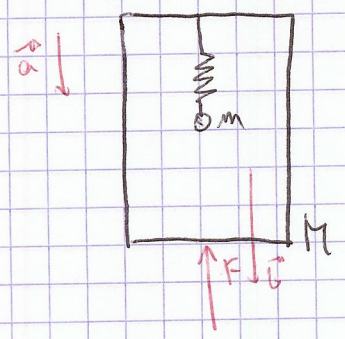
$$m_2 S = 2m_2(10m_2 - S)$$

$$3\cancel{m_2} S = 20\cancel{m_2}$$

$$S = 6{,}66\,m_2$$

$$\Rightarrow 6{,}66\,\cancel{m_2} = 2\cancel{m_2} \cdot a \Rightarrow a = 3{,}33$$

ex 5.8



* $F - G = m \cdot a$

$\frac{400N - 100N}{100} = a \Rightarrow a = -6 m/s^2$

* $F_v - G = m \cdot a$

$F_v = 1kg \cdot -6 m/s^2 + 10 = 4N$

$F_v = k(l - l_0)$ $\rightarrow$ $10N = k(0,25 - 0,20) \Rightarrow k = 200 \frac{N}{m}$

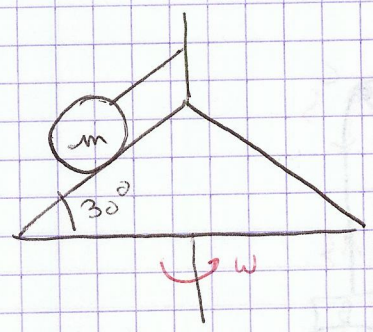
En repos : $F_v = G \Rightarrow F_v = 10N$

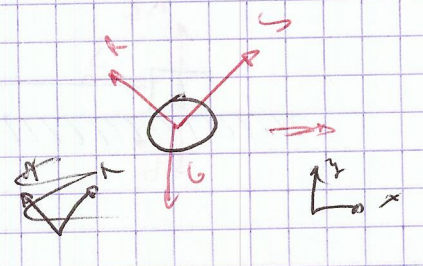
$\Longrightarrow$ $F_v = k(l - l_0)$

$4N = 200 \frac{N}{m} (l - 0,2)$

$\frac{4}{200} m + 0,2 = l \Rightarrow l = 0,22 m$

ex 5.9





a) * $\vec{R} + \vec{S} + \vec{G} = \vec{0}$

$\begin{Bmatrix} -R \cdot \cos 60° \\ R \cdot \sin 60° \end{Bmatrix} + \begin{Bmatrix} S \cdot \cos 30° \\ S \cdot \sin 30° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} m \cdot a \\ 0 \end{Bmatrix}$

$\rightarrow \begin{cases} -R = \frac{m \cdot a - S \cos 30°}{\cos 60°} \\ R = \frac{-S \sin 30° + G}{\sin 60°} \end{cases}$

$$\Rightarrow R = \frac{0{,}75 \cdot 0{,}5 \cdot \cos 30^\circ \cdot 4^2 - S \cdot \cos 30^\circ}{\cos 60^\circ}$$

$$= \frac{3\sqrt{3} - \frac{\sqrt{3}}{2} \cdot S}{\frac{1}{2}} = 6\sqrt{3} - \sqrt{3}\, S$$

$$\Rightarrow 6\sqrt{3} - \sqrt{3}\, S = \frac{-\frac{S}{2} + \frac{30}{4}}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{18}{2} - \frac{3}{2} S = -\frac{S}{2} + \frac{30}{4}$$

$$-S = \frac{30}{4} - \frac{36}{4} = -\frac{6}{4}$$

$$S = \frac{3}{2}$$

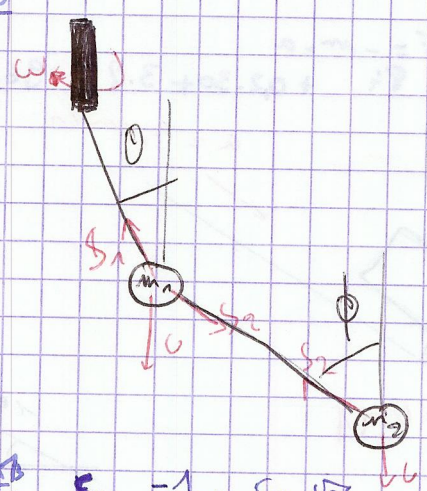
b) Bal kont los als $S_y = 6$

$$F_s = \frac{7{,}5}{\sin 30^\circ} = 15\ N$$

$$\Sigma F_n = m \cdot a = m \cdot R \cdot \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{15 \cdot \cos 30^\circ}{0{,}5 \cdot \cos 30^\circ \cdot 0{,}75}} = 6{,}32\ rad/s^2$$

oef 5.10



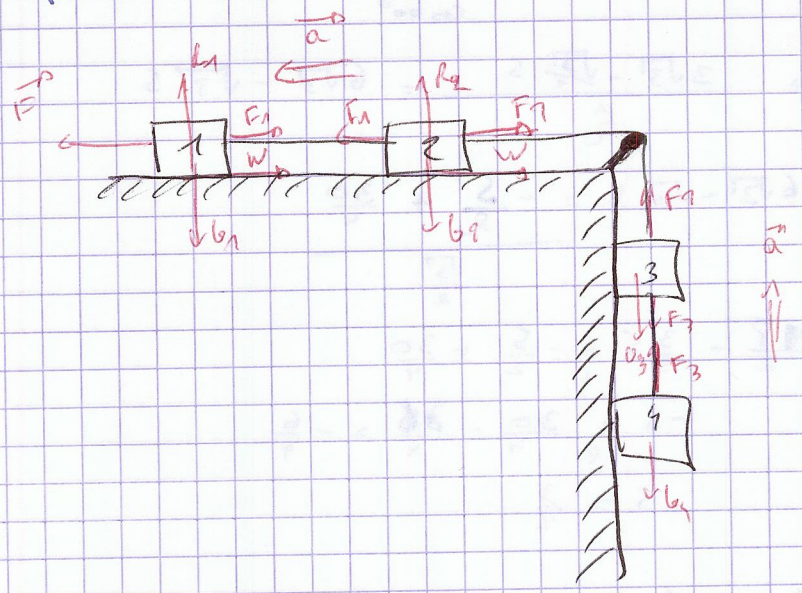
$$\begin{cases} m_1 \cdot \frac{1}{2} \cdot \omega_1^2 \cdot \frac{\sqrt{3}}{2} = S_1 \cdot \frac{-1}{2} + S_2 \cdot \frac{\sqrt{3}}{2} \\ m_2 \cdot \frac{1}{2} \cdot \omega_2^2 \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = S_2 \cdot \frac{\sqrt{3}}{2} \end{cases}$$

$\omega_1 = \omega_2$ en $\begin{cases} S_2 = \frac{2}{\sqrt{2}} \cdot 10\, m_2 \\ S_1 = \frac{2}{\sqrt{3}} (10\, m_2 + 10\, m_1) \end{cases}$

$$\Rightarrow \omega = 4{,}07$$

$$\Rightarrow \frac{m_1}{m_2} = 2{,}19$$

oef 5.11



* Blok 4: $F_3 - G_4 = m \cdot a$
  $F_3 = 3 \cdot 2 + 30 = 36\,N$

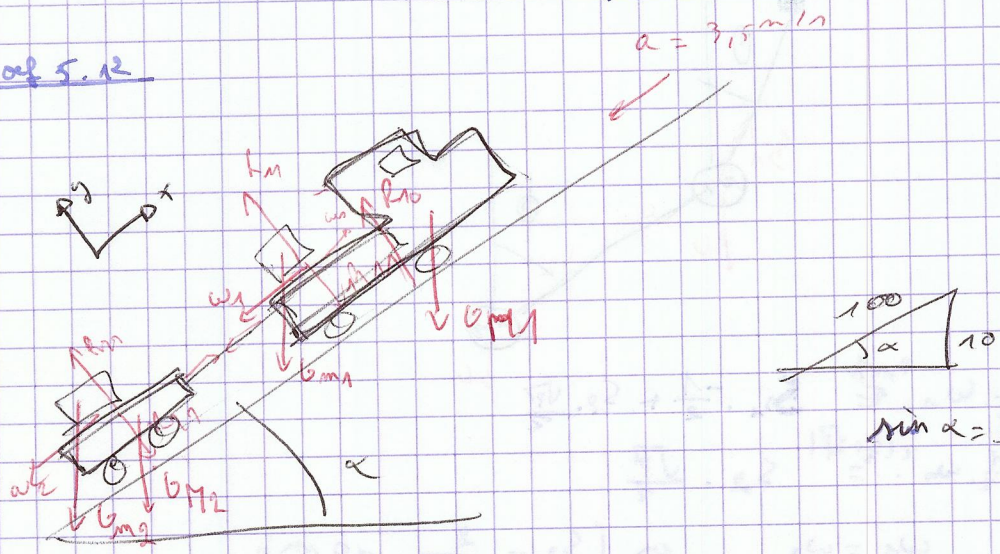
* Blok 3: $F_2 - F_3 - G_3 = m \cdot a$
  $F_2 = 3 \cdot 2 + 30 + 36 = 72\,N$

* Blok 2: $F_2 - F_1 + W_2 = -m \cdot a$

  $F_1 = 72 + 0{,}2 \cdot 30 + 3 \cdot 2 = 84\,N$

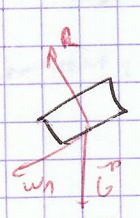
* Blok 3: $F_1 + W_1 - F = -m \cdot a$
  $F = 84 + 0{,}2 \cdot 30 + 3 \cdot 2 = 96\,N$

oef 5.12



$\sin\alpha = \frac{10}{100} \Rightarrow \alpha = 5{,}71°$

a) *



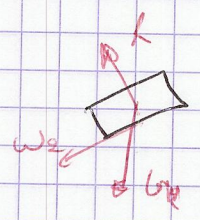
* $\vec{R} + \vec{G} + \vec{W_1} = m \cdot \vec{a}$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} -G \cdot \cos 84{,}29° \\ -G \cdot \sin 84{,}29° \end{Bmatrix} + \begin{Bmatrix} -W_1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -m \cdot 3{,}5 \\ 0 \end{Bmatrix}$$

$\Rightarrow R = 1000\,kg \cdot 10\,\frac{N}{kg} \cdot \sin 84{,}29^\circ$
$= 9950{,}4\,N$

$\Rightarrow W = R \cdot \mu$
$= 9950{,}4 \cdot 0{,}2$
$= 1990\,N$

a)



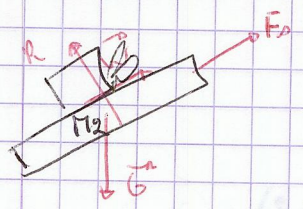
$\vec{R} + \vec{G_2} + \vec{W_1} = m \cdot \vec{a}$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} -G \cdot \cos 84{,}29^\circ \\ -G \cdot \sin 84{,}29^\circ \end{Bmatrix} + \begin{Bmatrix} -W_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -m \cdot 3{,}5 \\ 0 \end{Bmatrix}$$

$\Rightarrow R = 9950{,}4\,N$

$\Rightarrow W_2 = -10000\,N \cdot \cos 84{,}29^\circ + 1000 \cdot 3{,}5\,N$
$= 2505\,N$

b)

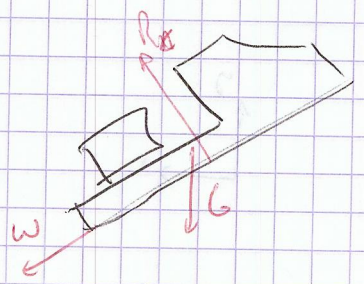


$\vec{R} + \vec{W} + \vec{F_1} + \vec{G} = m \cdot \vec{a}$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -G \cos\alpha \\ -G \sin\alpha \end{Bmatrix} = \begin{Bmatrix} -m \cdot 3{,}5 \\ 0 \end{Bmatrix}$$

$\Rightarrow F_1 = 20000\,N \cdot \cos 84{,}29^\circ - 2000\,kg \cdot \frac{3{,}5\,m}{s^2}$
$= -5010$
$\rightarrow$ druckkraft

c)



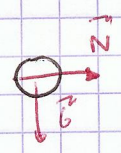
$R + G + W = m \cdot a$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} -G \cdot \cos\alpha \\ -G \cdot \sin\alpha \end{Bmatrix} + \begin{Bmatrix} -W \\ 0 \end{Bmatrix} = \begin{Bmatrix} -3{,}5 \cdot m \\ 0 \end{Bmatrix}$$

$\Rightarrow$

⑥

oef 6.1



* $\sum_{i=1}^{n} \vec{N_i} = \Delta\vec{p}$

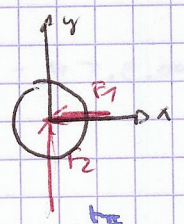
$= v_{II} \cdot m$

$= 40 \cdot 0{,}06 = 2{,}4$

* $\Delta\vec{N} = \vec{F} \cdot \Delta t$

$F = \frac{2{,}4}{5 \cdot 10^{-3}} = 480\,N$

oef 6.2



* $\vec{N} = \int_{t_I}^{t_{II}} \vec{F}\, dt$

$= -\vec{e_x}(4+2+2) + \vec{e_y}(1+1+2)$

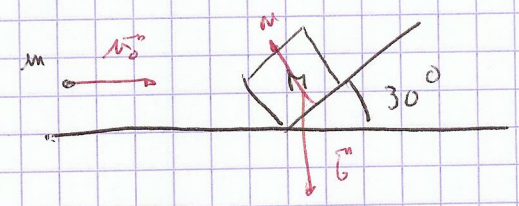
$= -8\,Ns\,\vec{e_x} + 4\,Ns\,\vec{e_y}$

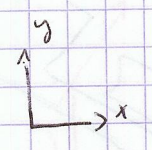
* $\vec{J} = \vec{p_{II}} - \vec{p_I}$

$-8\,Ns\,\vec{e_x} + 4\,Ns\,\vec{e_y} = 0{,}5\,kg\,v_{II} - 0{,}5\,kg \cdot 10\,\frac{m}{s}\,\vec{e_x}$

$v_{II} = -6\,Ns\,\vec{e_x} + 8\,Ns\,\vec{e_y}$

oef 6.3





* $\vec{N} = \vec{p_{II}} - \vec{p_I}$

$\begin{Bmatrix} -N \cdot \cos 60^\circ \\ N \cdot \sin 60^\circ \end{Bmatrix} = \begin{Bmatrix} (m+M) \cdot v_{II} \cdot \cos 30^\circ \\ (m+M) \cdot v_{II} \cdot \sin 30^\circ \end{Bmatrix} - \begin{Bmatrix} m \cdot v_I \\ 0 \end{Bmatrix}$

$\Rightarrow \begin{cases} -N \cdot \cos 60^\circ = 2{,}03 \cdot v_{II} \cdot \cos 30^\circ - 0{,}03 \cdot 100 \\ N \sin 60^\circ = 2{,}03\, v_{II} \cdot \sin 30^\circ \end{cases}$

Gewoon gelijkstellen om op te lossen

oef 6.4

* $\vec{N} = \int_{t_I}^{t_{II}} \vec{F}\, dt$

$= \frac{0{,}04 \cdot 6}{2} = 0{,}12\ kNs = -120\ Ns$

* $\vec{N} = \vec{p}_{II} - \vec{p}_{I}$

$-120\,N = 5 \cdot v_{II} - 5 \cdot 20$

$v_{II} = -4\ \frac{m}{s}$

oef 6.5

* $N = 0$

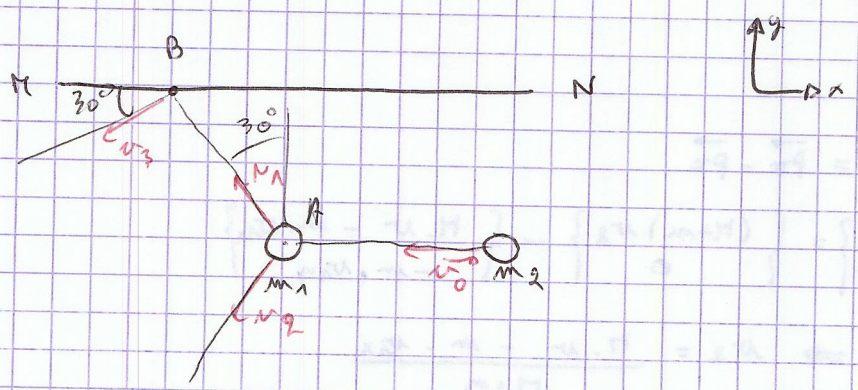
$p_{II} = p_{I}$

$\begin{Bmatrix} m_A \cdot v_A \\ m_B \cdot v_B \end{Bmatrix} = \begin{Bmatrix} (m_A + m_B) \cdot v \cdot \cos 30^\circ \\ (m_A + m_B) \cdot v \cdot \sin 30^\circ \end{Bmatrix}$

$\Rightarrow \begin{cases} 1{,}2\, m_B\, v_A = 2{,}2\, m_B \cdot v \cdot \cos 30^\circ \\ m_B\, v_B = 2{,}2\, m_B \cdot v \cdot \sin 30^\circ \end{cases}$

$\Rightarrow \frac{v_A}{v_B} = \frac{\frac{2{,}2\, v \cdot \cos 30^\circ}{1{,}2}}{2{,}2\, v \cdot \sin 30^\circ} = \frac{5 \cdot \cos 30^\circ}{6 \cdot \sin 30^\circ} = 1{,}44$

oef 6.6



Oplossing 1: behoud v. impuls

$p_{II} = p_{I}$

$\begin{Bmatrix} -v_{1x} \cdot m_1 \sin 30^\circ - v_{2x} \cdot m_2 \\ v_1 \cdot m_1 \cos 30^\circ - v_{2y} \cdot m_2 \end{Bmatrix} = \begin{Bmatrix} -v_0 \cdot m_2 \\ 0 \end{Bmatrix}$

$\Rightarrow \begin{cases} v_{2x} = \frac{-2{,}5 \cdot 0{,}020 \sin 30^\circ + 20 \cdot 0{,}005}{0{,}005} \\ v_{2y} = \frac{2{,}5 \cdot 0{,}020 \cos 30^\circ}{0{,}005} \end{cases}$

$\Rightarrow \begin{cases} v_{2x} = -15 \\ v_{2y} = 8{,}66 \end{cases}$

$\Rightarrow v_2 = -15\ \frac{m}{s}\, \vec{e}_x + 8{,}66\ \frac{m}{s}\, \vec{e}_y$

* $N_2 = \vec{p_{II,2}} - \vec{p_{I,2}}$

$N = m_2 \cdot v_2 - m_2 \cdot v_2$

$= \begin{cases} 0{,}005 \cdot (-15) \\ 0{,}005 \cdot (-8{,}66) \end{cases} - 0{,}005 (-20)$

$= \begin{cases} 0{,}025\ N_1 \\ -0{,}433\ N_1 \end{cases}$

* $\vec{N_1} = \vec{p_{II,1}} - \vec{p_{I,1}}$

$N = \begin{cases} 0{,}02 \cdot 2{,}5 \cdot \sin 30^\circ \\ 0{,}02 \cdot 2{,}5 \cdot \cos 30^\circ \end{cases}$

$= \begin{cases} 0{,}025\ N_1 \\ 0{,}0433\ N_1 \end{cases}$

Botsing 2: $\vec{N_3} = \vec{p_{II}} - \vec{p_I}$

$\begin{Bmatrix} 0 \\ -N \end{Bmatrix} = \begin{Bmatrix} m_1 \cdot v_3 \cdot \cos 30^\circ \\ m_1 \cdot v_3 \cdot \sin 30^\circ \end{Bmatrix} - \begin{Bmatrix} m_1 \cdot v_1 \cdot \sin 30^\circ \\ m_1 \cdot v_1 \cdot \cos 30^\circ \end{Bmatrix}$

$\Rightarrow v_3 = \dfrac{\cancel{m_1} \cdot v_1 \cdot \sin 30^\circ}{\cancel{m_1} \cdot \cos 30^\circ} = 1{,}44$

$\Rightarrow \vec{v_3} = \begin{cases} -1{,}44 \cdot \cos 30^\circ \\ -1{,}44 \cdot \sin 30^\circ \end{cases} = \begin{cases} -1{,}25\ m/s \\ -0{,}72\ m/s \end{cases}$

$\Rightarrow N_3 = 0{,}057$

<u>oef 6.7</u>

a) $\vec{N} = \vec{p_{II}} - \vec{p_I}$

$\begin{Bmatrix} 0 \\ N \end{Bmatrix} = \begin{Bmatrix} (M+m)\, v_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} M \cdot v - m \cdot v_{1x} \\ -m \cdot v_{1y} \end{Bmatrix}$

$\Rightarrow v_2 = \dfrac{M \cdot v - m \cdot v_{1x}}{M+m}$

$= \dfrac{70 \cdot 4 - 3 \cdot 70}{73} = \dfrac{70}{73} = 0{,}96\ m/s$

b) De vlot vergroot, want v_{1y} stijgt

c) Als $v_2 = 0 \Rightarrow M \cdot v = m \cdot v_{1x}$

$\dfrac{v_{1x}}{v} = \dfrac{M}{m}$

$= \dfrac{70}{3}$

$= 23{,}33$

oef 6.8

a) $\begin{Bmatrix} 0 \\ N \end{Bmatrix} = \begin{Bmatrix} 105 . v_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 15 . 3 . \cos 30^0 \\ 15 . 3 . \sin 30^0 \end{Bmatrix}$

$\Rightarrow v_2 = \frac{45 . \cos 30^0}{105} = 0,374 \ m/s$

b) $\begin{Bmatrix} 0 \\ N \end{Bmatrix} = \begin{Bmatrix} 105 . v_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 3 . \cos 30^0 . 15 - v_2 . 15 \\ 0 \end{Bmatrix}$

$\Rightarrow v_2 = \frac{15 . 3 . \cos 30^0}{120} = 0,325 \ m/s$

oef 6.9

Situatie 1: $\begin{Bmatrix} 0 \\ N \end{Bmatrix} = \begin{Bmatrix} 380 . v_2 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 80 . (3 - v_2) \\ 0 \end{Bmatrix}$

$\Rightarrow v_2 = \frac{240}{460} = 0,52 \ m/s$

Situatie 2: $\begin{Bmatrix} 0 \\ N \end{Bmatrix} = \begin{Bmatrix} 300 . v_3 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 80 (3 - v_3) - 380 . 0,52 \\ 0 \end{Bmatrix}$

$\Rightarrow v_3 = \frac{437,6}{380} = 1,15 \ m/s$

oef 6.10

Situatie 1: $\vec{p}_I \underset{\downarrow}{=} \vec{p}_{II}$

$0 \underset{\downarrow}{=} M . v - 4M . v_1$

$v_1 = \frac{1}{4} \ m/s$

$\Rightarrow v_{als} = -\frac{1}{4} + 1 = 0,75 \ m/s \qquad \Rightarrow t = \frac{5}{0,75} = 6,67 \ s$

Situatie 2: $\vec{p}_I \underset{\downarrow}{=} \vec{p}_{II}$

$0 \underset{\downarrow}{=} M . v - 6M . v_1$

$v_1 = \frac{1}{6} \frac{m}{s}$

$\Rightarrow v_{als} = -\frac{1}{6} + 1 = 0,83 \ m/s \qquad \Rightarrow t = \frac{5}{0,83} = 6 \ s$

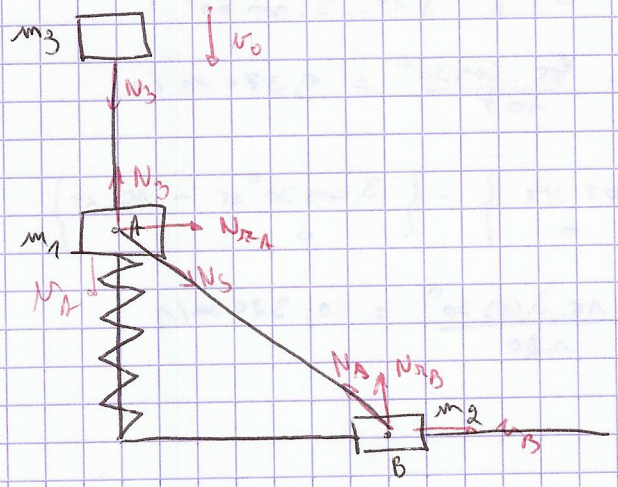
Situatie 3: $\vec{p}_I \underset{\downarrow}{=} \vec{p}_{II}$

$0 \underset{\downarrow}{=} M . v - 2M . v - 6M \ v_1$

$v_1 = -\frac{1}{6}$

$\Rightarrow v_{als} = \frac{1}{6} + 1 = 1,17 \ m/s \qquad \Rightarrow t = \frac{5}{1,17} = 4,29 \ s$

oef 6.11



* Massa 3: $\vec{N} = \vec{p_{II}} - \vec{p_I}$

$$\begin{Bmatrix} 0 \\ -N_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -25 \cdot 10 \end{Bmatrix} - \begin{Bmatrix} 0 \\ -25 \cdot v_A \end{Bmatrix}$$

$$\Rightarrow N_3 = 250 - 25 v_A$$

* Massa 1: $\vec{N} = \vec{p_{II}} - \vec{p_I}$

$$\begin{Bmatrix} 0 \\ N_3 \end{Bmatrix} + \begin{Bmatrix} N_{rA} \\ 0 \end{Bmatrix} + \begin{Bmatrix} N_5 \cdot \cos 60^\circ \\ -N_5 \cdot \sin 60^\circ \end{Bmatrix} = \begin{Bmatrix} 0 \\ 25 \cdot v_A \end{Bmatrix} - \begin{Bmatrix} 0 \\ 25 \cdot 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} N_{rA} + N_5 \cdot \cos 60^\circ = 0 \\ N_3 - N_5 \sin 60^\circ = 25 \cdot v_A \end{cases}$$

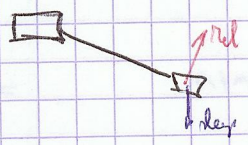
* Massa 2: $\vec{N} = \vec{p_{II}} - \vec{p_I}$

$$\begin{Bmatrix} -N_5 \cdot \cos 60^\circ \\ N_5 \cdot \sin 60^\circ \end{Bmatrix} + \begin{Bmatrix} 0 \\ N_{rB} \end{Bmatrix} = \begin{Bmatrix} 25 \cdot v_B \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} -N_5 \cdot \cos 60^\circ = 25 v_B \\ N_5 \cdot \sin 60^\circ + N_{rB} = 0 \end{cases}$$

* Samengestelde beweging:

B roteert rond ↑ transleert verticaal
rel ↓ slep

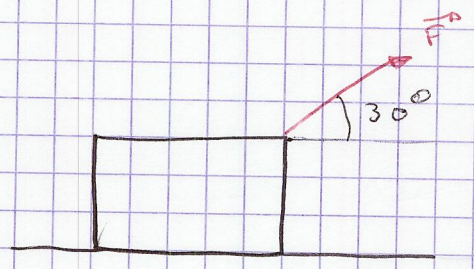


$$\Rightarrow \begin{Bmatrix} v_B \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -v_A \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 30^\circ \\ v_{rel} \cdot \sin 30^\circ \end{Bmatrix}$$

$$\Rightarrow \frac{v_A}{v_B} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow v_B = \sqrt{3} \cdot A$$

6 onbekenden, 6 vgl ⇒ genoeg gegevens om opl. te lossen

oef 6.12



a) $U = \int_{r=0}^{r=40} F \, dr$

$= 25 . \cos 30° . 40$

$= 866 \, J$

b) $U = \Delta V + \Delta T$

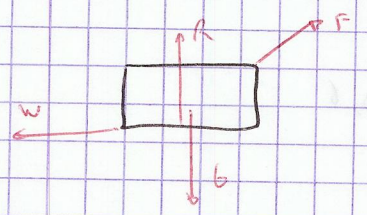
$\Delta V = m . g . \Delta h = 0$

$\Rightarrow U = \Delta T = 866 \, J$

c) $\Delta T = \frac{m . v^2}{2}$

$\Rightarrow v = \sqrt{\frac{866 . 2}{25}} = 8,32 \, m/s^2$

d)



* $R = G - F . \sin 30° = 237,5$
* $w = \mu . R = 21,375$

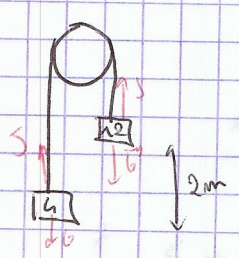
$\Rightarrow \Delta T = 25 . \cos 30° . 40 - 21,375 . 40$

$= 11,03 \, J$

* $\Delta T = \frac{m v^2}{2}$

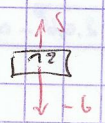
$\Rightarrow v = \sqrt{\frac{11,03 . 2}{25}} = 0,94 \, m/s$

oef 6.13



*

$\Rightarrow S = 40 N$

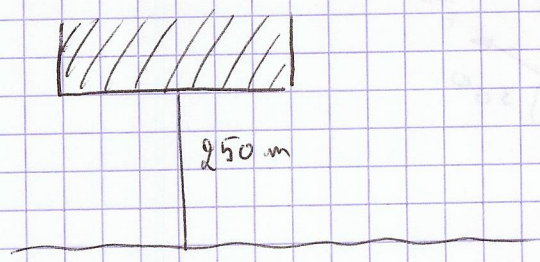
* 

$\Rightarrow F_{res} = 80 N$

* $U = \int_0^8 F \, dr = 160$

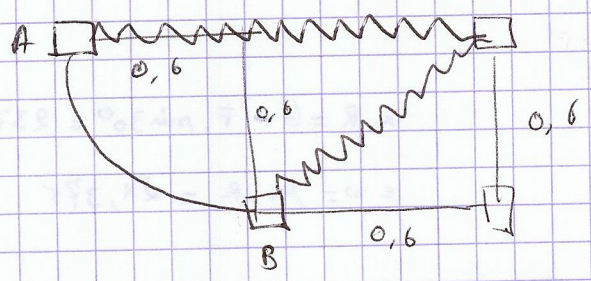
* $160 = \frac{m v^2}{2} \Rightarrow v = \sqrt{\frac{320}{16}} = 4,47$

oef 6.14



$$E_{pot} = \frac{\overset{2}{\cancel{4}}\cdot 10^{6}\cdot \cancel{10^{3}}\cdot 250\cdot \cancel{10}}{2\cdot \cancel{10^{4}}} = 5\cdot 10^{8}\ \frac{J}{s}$$

oef 6.15



Behoud van energie:

- In A: pot. gr. + pot. veer

$$E_1 = m\cdot g\cdot h + \frac{k\,(x-l_0)^2}{2}$$

$$= 3\cdot 10\cdot 0{,}6 + \frac{350\,(1{,}2-0{,}6)^2}{2}$$

$$= 81\ J$$

- In B: pot. veer + kin

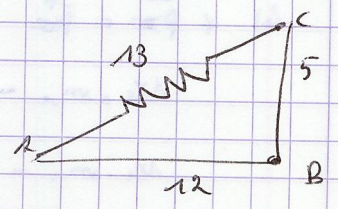
$$E_2 = \frac{k\,(x-l_0)^2}{2} + \frac{m\cdot v^2}{2}$$

$$= \frac{350\,(\sqrt{2\cdot 0{,}6^2}-0{,}6)^2}{2} + \frac{3\cdot v^2}{2}$$

- $E_1 = E_2 \Rightarrow 81\,J = 175\cdot 0{,}25 + \frac{3}{2}v^2$

$$v^2 = 46{,}8 \Rightarrow v = 6{,}84$$

oef 6.16



Behoud van energie:

* In A: pot. veer

$$E_1 = \frac{k(x-l_0)^2}{2}$$

$$= \frac{800\,(13-0{,}04)^2}{2}$$

$$= 400 \cdot 0{,}09^2 = 3{,}24\ J$$

* In B: pot. veer + kin.

$$E_2 = \frac{k(x-l_0)^2}{2} + \frac{m v^2}{2}$$

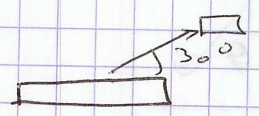
$$= \frac{800\,(0{,}05-0{,}04)^2}{2} + \frac{0{,}1 \cdot v^2}{2}$$

$$= 0{,}04 + 0{,}05\, v^2$$

* $E_1 = E_2 \Rightarrow 3{,}24 = 0{,}04 + 0{,}05\, v^2$

$$v = \sqrt{\frac{3{,}20}{0{,}05}} = 8\ m/s$$

oef 6.17



Behoud van impuls en behoud van energie

* $\vec{p_{II}} = \vec{p_I}$

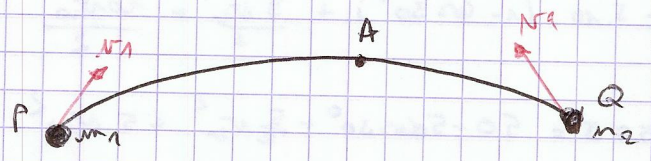
$$-105 \cdot v_k = 15 \cdot \cos 30^\circ \cdot v_2 \Rightarrow v_2 = 8{,}08\ v_k$$

* $\Delta U = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$

$$1505 = \frac{105 \cdot v_k^2}{2} + \frac{15\, v_2^2}{2} \Rightarrow v_k = 0{,}526\ m/s$$

$$v_2 = +4{,}25\ m/s$$

oef 6.18



a) Gelijkbedig, ze daar evenlang van P tot A als Q tot A, dus zonder verloren energie evenlang van A tot P als A tot Q.

b) * $\vec{p}_{x} = \vec{p}_{x}$

$$v_1 \cdot m_1 - v_2 \cdot m_2 = -m_1 \cdot \frac{v_1}{2} + m_2 v_x$$

$$v_1 \cdot m_1 - \frac{v_1}{\cancel{2}} \cdot \cancel{2} m_1 = -\frac{v_1}{2} m_1 + 2 m_1 \cdot v_x$$

$$\frac{v_1 \cancel{m_1}}{2 \cdot 2 \cancel{m_1}} = v_x$$

$$v_x = \frac{v_1}{4}$$

$\Rightarrow$ gelijktijdig wordt $v_2 = \frac{v_1}{2}$ in enkele gevar

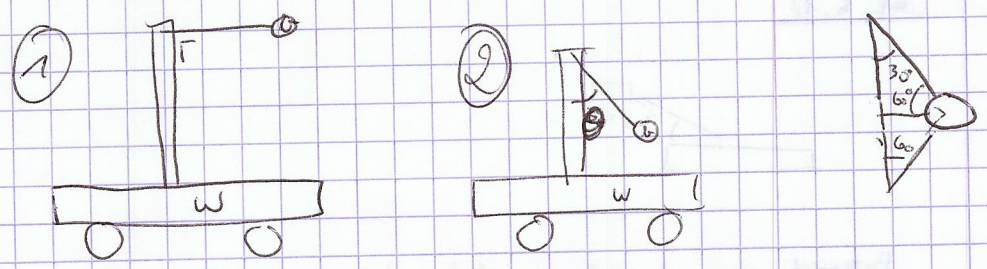
* verloren energie:

$$\Delta E = \left[\frac{m_1 v_1^2}{2} + \frac{2 m_1 \left(\frac{v_1}{2}\right)^2}{2}\right] - \left[\frac{m_1 \cdot \left(\frac{v_1}{2}\right)^2}{2} + \frac{2 m_1 \left(\frac{v_1}{4}\right)^2}{2}\right]$$

$$= \frac{m_1 v_1^2}{2} + \frac{m_1 v_1^2}{4} - \frac{m_1 \cdot v_1^2}{8} - \frac{m_1 v_1^2}{16}$$

$$= \frac{m_1 v_1^2 (8+4-2-1)}{16} = \frac{9}{16} m_1 v_1^2$$

oef 6.19



* Behoud van energie:

Bij 1 : pot. gr.

$$E_1 = m \cdot g \cdot h$$
$$= 5 \cdot 10 \cdot 1 = 50 \text{ J}$$

Bij 2 : pot. gr. + kin. b + kin w

$$E_2 = m \cdot g \cdot h + \frac{m_b v_b^2}{2} + \frac{m_w v_w^2}{2}$$

$$= 5 \cdot 10 \cdot (1 - \cos 30^\circ) + \frac{5 v_b^2}{2} + \frac{10 v_w^2}{2}$$

$\Rightarrow E_1 = E_2 \Rightarrow 50 \text{ J} = 50 - 5 \cos 30^\circ + \frac{5}{2} v_b^2 + 5 v_w^2$

* Behoud van impuls:

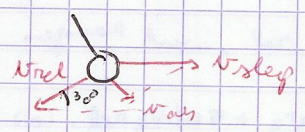
$$\vec{p_I} = \vec{p_{II}}$$

$$0 = m_w \cdot v_w - v_{br,x} \cdot m_b$$

* samengestelde beweging:

B draait rond A

A translaat naar rechts



$$\Rightarrow v_{abs,x} = v_{sleep} + v_{rel}$$

$$\parallel$$

$$v_{b,x} = v_w + v_{rel,x}$$

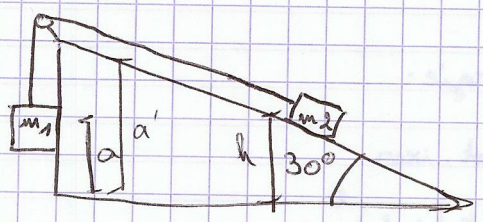
* cos-regel: $v_{b,x}^2 = v_{rel}^2 + v_w^2 - 2\, v_{rel} \cdot v_w \cdot \cos 30°$

$$\Rightarrow \begin{cases} 50 = 50 - 50 \cdot \cos 30° + 2{,}5\, v_b^2 + 5\, v_w^2 \\ v_b \cdot 5 = 10 \cdot v_w \\ v_b^2 = v_{rel}^2 + v_w^2 - 2 \cdot v_{rel} \cdot v_w \cdot \cos 30° \end{cases}$$

$\Rightarrow$ 3 onbekenden, 3 vgl: genoeg gegevens om op te lossen

oef 6.20

Behoud van energie:



* Situatie 1: pot. gr.

$$E_1 = m_1 \cdot a \cdot g + m_2 \cdot h \cdot g$$
$$= 200a + 500$$

* Situatie 2: pot. gr + kin.

$$E_2 = m_1 \cdot a' \cdot g + \frac{m_1 \cdot \vec{v_1}^2}{2} + \frac{m_2 \cdot \vec{v_2}^2}{2} + \frac{m_3 \cdot \vec{v_3}^2}{2}$$

* $E_1 = E_2 \Rightarrow 200a + 500 = 200a' + \frac{20}{2} v_1^2 + \frac{50}{2} v_2^2 + \frac{100}{2} v_3^2$

$$0 = 200(a' - a) - 500 + 10 v_1^2 + 25 v_2^2 + 50 v_3^2$$

Behoud van impuls:

* $\vec{p_I} = \vec{p_{II}}$

$$\Downarrow$$

$$0 = m_1 \vec{v_1} + m_2 \vec{v_2} + m_3 \vec{v_3}$$

* $\vec{v_1} = \begin{Bmatrix} -v_3 \\ v_{1,rel} \end{Bmatrix} = -v_3 \vec{e_x} + v_{1,rel} \vec{e_y}$

$\vec{v_2} = \begin{Bmatrix} v_{2,rel} \cdot \cos 30^\circ - v_3 \\ -v_{2,rel} \sin 30^\circ \end{Bmatrix} = (v_{2,rel} \cdot \cos 30^\circ - v_3) \vec{e_x} - v_{2,rel} \cdot \sin 30^\circ \vec{e_y}$

$v_{1\,rel} = v_{2\,rel}$, want hangen aan 1 touw

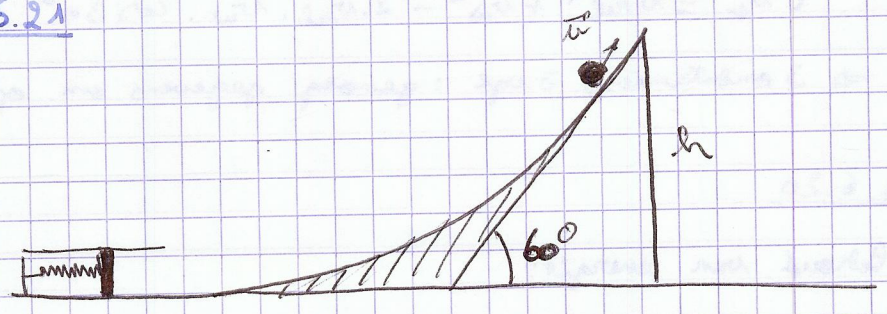
$$\Rightarrow \quad 0 = -20 v_3 + 50(v_{2\,rel} \cdot \cos 30^\circ - v_3) - 100 v_3$$
$$0 = -170 v_3 + 25\sqrt{3}\, v_{2\,rel}$$

* $a' - a = 2$, want afstand x die m_2 aflegt tot op de grond:

$$x = \frac{h}{\sin 30^\circ} = 2$$

Genoeg gegevens om op te lossen

oef 6.21



Behoud van energie:

Situatie 1: pot. veer

$$E_1 = \frac{k(x - l_0)^2}{2}$$
$$= \frac{2000(\Delta L)^2}{2} = 1000 (\Delta L)^2$$

Situatie 2: pot. gr. + kin

$$E_2 = m \cdot g \cdot h + \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$$
$$= 5 \cdot 10 \cdot 0{,}3 + \frac{20 \cdot v_1^2}{2} + \frac{5 \cdot v_2^2}{2}$$

* $v_2 = \begin{Bmatrix} -v_1 + 4 \cdot \cos 60^\circ \\ 0 + 4 \cdot \sin 60^\circ \end{Bmatrix}$

$$= \begin{Bmatrix} 2 - v_1 \\ 2\sqrt{3} \end{Bmatrix}$$

Behoud van impuls:

$$\vec{p_I} = \vec{p_{II}}$$

$$0 = m_2 . v_2 - m_1 . v_1$$

$$0 = 5 . (2 - v_1) - 20 . v_1$$

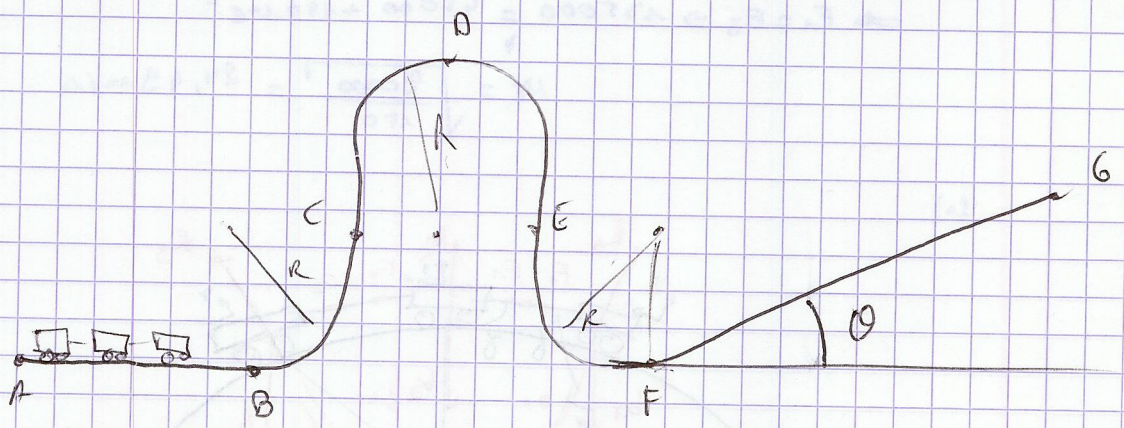
$$\Rightarrow \quad v_1 = \frac{10}{25} = 0,4 \ m/s$$

$$E_1 = E_2 \Rightarrow 1000 (\Delta L)^2 = 15 + 10 . 0,4^2 + 2,5 . \left[(1,6)^2 + (2\sqrt{3})^2 \right]$$

$$\Delta L = \sqrt{\frac{53}{1000}}$$

$$= 0,23$$

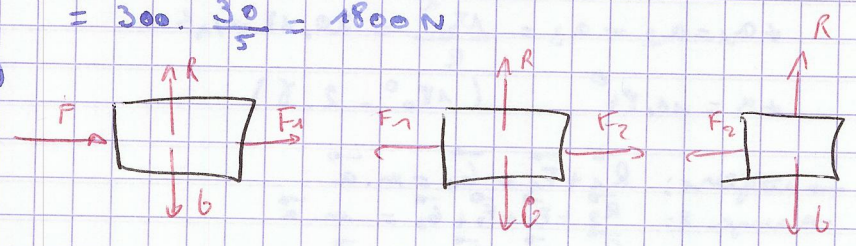
oef 6.22



1) a) $F = m . a$

$$= 300 . \frac{30}{5} = 1800 \ N$$

b)



massa 1: $F + F_1 = m . a$

$$1800 \ N + F_1 = 100 . 6$$

$$F_1 = -1200 \ N \rightarrow \text{drukkracht}$$

massa 2: $-F_1 + F_2 = m . a$

$$1200 N + F_2 = 100 . 6$$

$$F_2 = -600 N \rightarrow \text{drukkracht}$$

2) a) Behoud van energie:

* In B: kin

$$E_1 = \frac{300 \cdot 30^2}{2} = 135\,000 \text{ J}$$

* In D: pot. gr. + kin

$$E_2 = 300 \cdot 10 \cdot 30 + \frac{300\, v_D^2}{2} = 90\,000 + 150\, v_D^2$$

* In E: pot. gr. + kin

$$E_3 = 300 \cdot 10 \cdot 15 + \frac{300\, v_E^2}{2} = 45000 + 150\, v_E^2$$

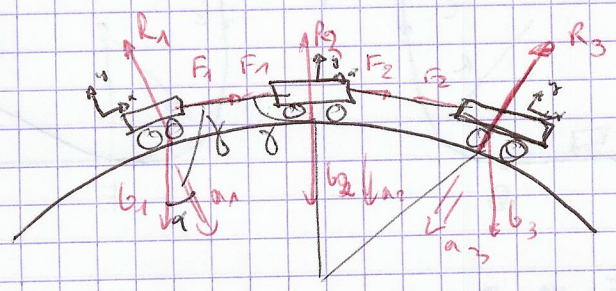
$\Rightarrow E_1 = E_2 \Rightarrow 135000 = 90\,000 + 150 v_D^2$

$$v_D = \sqrt{\frac{45000}{150}} = 17{,}32 \text{ m/s}$$

$\Rightarrow E_1 = E_3 \Rightarrow 135000 = 45000 + 150 v_E^2$

$$v_E = \sqrt{\frac{90000}{150}} = 24{,}49 \text{ m/s}$$

b)



* $a_1 = a_2 = a_3 = \frac{v_D^2}{R} = 20{,}18 \text{ m/s}^2$

* $\alpha = 11{,}84^\circ$ $\quad (18_0^\circ - 2 \cdot \gamma)$

→ wagen 1: $\vec{R_1} + \vec{F_1} + \vec{G_1} = m \cdot \vec{a}$
→ wagen 2: $\vec{R_2} + \vec{F_1} + \vec{F_2} + \vec{G_2} = m \cdot \vec{a}$
→ wagen 3: $\vec{R_3} + \vec{F_2} + \vec{G_3} = m \cdot \vec{a}$

$\Rightarrow R_1 = -1038 \text{ N}$, $R_2 = -1018 \text{ N}$, $R_3 = -1038 \text{ N}$

3) $U_n = V_G - V_F + T_G - T_F$

* In F: kin.

$$E_1 = \frac{300 \cdot 30^2}{2} = 135000 \text{ J} = T_F$$

* In G: pot. gr

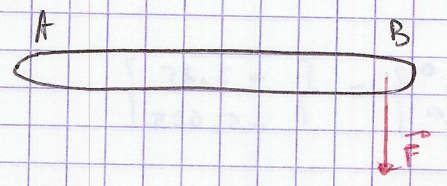
$$E_2 = 300 \cdot 10 \cdot 60 \cdot \sin 10^\circ = 31260 \text{ J} = V_G$$

$\Rightarrow U_n = 31260 \text{ J} - 135000 \text{ J} = -103740 \text{ J}$

* $F = \frac{U_n}{60} \Rightarrow F = -1729 \text{ N}$

7

oef 7.1



* translatie:

$$\sum_{i=1}^{n} F_i = m \cdot a_c \longrightarrow a = 75 \ \frac{m}{s^2}$$

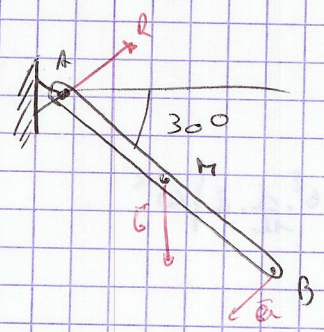
* rotatie:

$$\vec{M_c} = I_c \cdot \vec{\alpha} \Rightarrow \vec{M_c} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0,3 & 0 & 0 \\ 0 & -150 & 0 \end{vmatrix} = -45 \ Nm \ \vec{e_z}$$

$$I_c = \frac{1}{12} m L^2 \Rightarrow I_c = \frac{1}{12} \cdot 2 \cdot 0,6^2 = 0,06$$

$$\Rightarrow \alpha = \frac{-45}{0,06} = -750 \ rad/s^2 \ \vec{e_z}$$

oef 7.2



* translatie:

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = \begin{Bmatrix} -m \cdot a_x \\ -m \ a_y \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -2mg \end{Bmatrix} = \begin{Bmatrix} -2a \cdot \cos 60° \\ -2 \cdot a \cdot \sin 60° \end{Bmatrix}$$

* rotatie:

$$\vec{M_O} = I_O \cdot \vec{\alpha} \Rightarrow M_O = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 0,3\cos 30° & -0,3\sin 30° & 0 \\ 0 & -20 & 0 \end{vmatrix} = -3\sqrt{3} \ Nm \ \vec{e_z}$$

$$I_O = I_c + r_{OC}^2 \cdot m = \frac{1}{12} \cdot 2 \cdot 0,6^2 + 0,3^2 \cdot 2 = 0,24$$

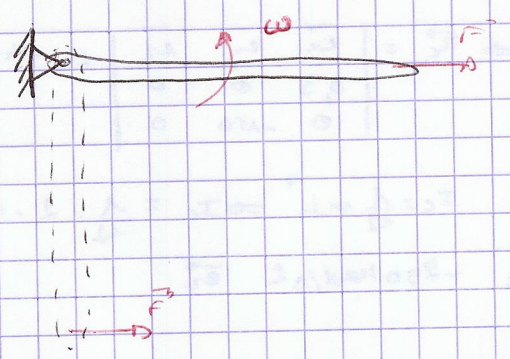
$$\Rightarrow \alpha = \frac{-3\sqrt{3}}{0,24} = -21,65 \ rad/s^2$$

* $a_t = \alpha . R$
$a_n = \omega^2 . R$

$$\Rightarrow \vec{a} = \begin{Bmatrix} \alpha . R . \cos 60^\circ \\ \alpha . R . \sin 60^\circ \end{Bmatrix} = \begin{Bmatrix} -3,25 \\ -5,625 \end{Bmatrix}$$

$$\Rightarrow R = \begin{Bmatrix} -6,5 \\ 8,75 \end{Bmatrix} N$$

oef 7.3



Behoud van energie:

* $U = 0,6 . 150 = 90 J$

* $\Delta E = m . g . h + \frac{1}{2} I_0 \omega^2$

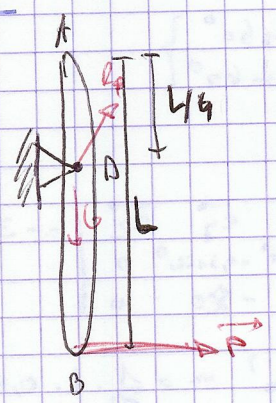
$$= 2 . 10 . 0,3 + \frac{1}{2}\left(0,3^2 . 2 + 0,6^2 . \frac{1}{12} . 2\right)\omega^2$$

$$\Rightarrow 90 = 6 + 0,12 \omega^2$$

$$\omega = \sqrt{700} = 26,46$$

* $v = \omega . R = 26,46 . 0,3 = 7,94 \ m/s$

oef 7.4



* $\vec{R} + \vec{G} + \vec{P} = m . \vec{a}$

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} P \\ 0 \end{Bmatrix} = \begin{Bmatrix} m . a \\ 0 \end{Bmatrix}$$

want B rechtsonder

* $\vec{M}_D = P . \frac{3}{4} L$

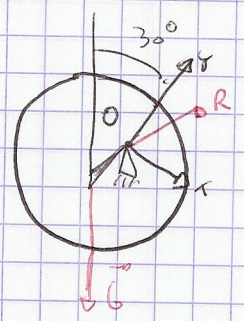
$I_D = I_c + r^2 m$

$= \frac{1}{12} m L^2 + \left(\frac{L}{4}\right)^2 m$

$$\Rightarrow M_D = I_D . \alpha \Rightarrow \alpha = \frac{\frac{3}{4} L . P}{\frac{1}{12} L^2 + \frac{1}{16} L^2} = 5,14 \frac{P}{L}$$

$$\Rightarrow R_x = 5,14 \frac{P}{L} . \frac{L}{4} - P = 0,29 \quad \Rightarrow \vec{R} = 0,29 \vec{e_x} + G \vec{e_y}$$

ref 7.5



* $\vec{R} + \vec{G} = m \cdot \vec{a}$

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} G \cdot \cos 60° \\ -G \cdot \sin 60° \end{Bmatrix} = \begin{Bmatrix} m a_x \\ m a_y \end{Bmatrix}$$

* $M_O = I_O \cdot \alpha$

$$M_O = 0{,}05 \cdot G \cos 60°$$
$$I_O = I_C + r^2 m$$
$$= \frac{1}{4} \cdot 20 \cdot 0{,}1^2 + 0{,}05^2 \cdot 20$$

$$\Rightarrow \quad \alpha = \frac{0{,}05 \cdot 200 \cdot \frac{1}{2}}{\frac{1}{4} \cdot 20 \cdot 0{,}1^2 + 0{,}05^2 \cdot 20} = \frac{5}{0{,}15} = 33{,}3 \text{ rad/s}^2$$

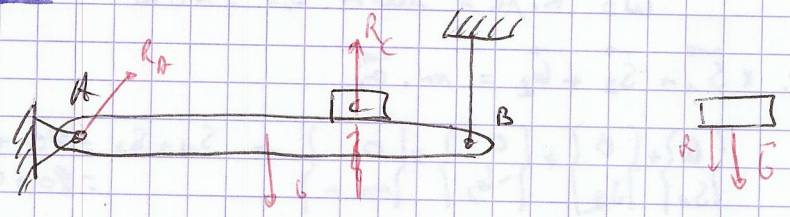
* $a_x = R \cdot \alpha = 0{,}05 \cdot 33{,}3$

$$\Rightarrow \quad R_x = 20 \cdot 0{,}05 \cdot 33{,}3 - 200 \cdot \cos 60°$$
$$= -66{,}67$$

* $a_y = R \cdot \omega^2 = 0{,}05 \cdot 3^2$

$$\Rightarrow \quad R_y = 20 \cdot 0{,}05 \cdot 3^2 + 200 \cdot \sin 60°$$
$$= 182{,}2$$

ref 7.6



a) $|AC| = \frac{l}{2}$

* $\vec{R_A} + \vec{G} + \vec{R_C} = m \cdot \vec{a}$

$$\begin{Bmatrix} R_{Ax} \\ R_{Ay} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} R_C \\ 0 \end{Bmatrix} = \begin{Bmatrix} m \cdot a_x \\ m \cdot a_y \end{Bmatrix}$$

* $\vec{M_O} = I_O \cdot \vec{\alpha}$

$$\vec{M_O} = -\frac{l}{2} \times \vec{G} + \frac{l}{2} \times R_C$$
$$I_O = \frac{1}{12} \cdot l^2 \cdot 15 + \left(\frac{1}{2} l\right)^2 \cdot 15 = \frac{15 l^2}{12} + \frac{15 l^2}{4} = 5 l^2$$

$$\Rightarrow \alpha = \frac{M_O}{I_O} = \frac{-75\ell + \frac{\ell}{2} R_C}{5\ell^2}$$

$$\alpha = \frac{-75 + R_C}{10}$$

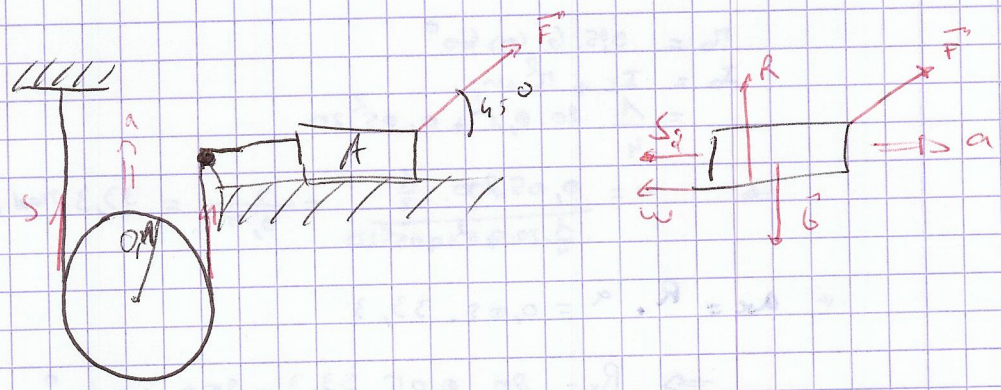
* $-R_C - G = -m \cdot a$

$$R_C = 50 - 5 \cdot \underset{\ell \cdot \alpha}{a}$$

$$\Rightarrow \quad a = \frac{-75 + 50 - 5 \cdot \frac{\ell}{2} a}{10}$$

Genoeg gegevens om op te lossen

oef 77



* In A: * $\vec{R} + \vec{G} + \vec{F} + \vec{S_2} + \vec{W} = m \cdot \vec{a_A}$

$$\begin{Bmatrix} 0 \\ R \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} F \cdot \cos 45° \\ F \cdot \sin 45° \end{Bmatrix} + \begin{Bmatrix} -S_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -W \\ 0 \end{Bmatrix} = \begin{Bmatrix} m \cdot a \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad R = 200\,N - 100\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 100\,N$$

$$W = R \cdot \mu = 100\,N \cdot 0{,}1 = 10\,N$$

* In C: * $\vec{S_1} + \vec{S_2} + \vec{G_2} = m \cdot \vec{a_C}$

$$\begin{Bmatrix} 0 \\ S_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ S_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ m \cdot a \end{Bmatrix} \Rightarrow S_1 + S_2 = 80 + 8 \cdot a_C$$

$$= 80 + 0{,}8\alpha$$

* $\vec{M_C} = I_C \cdot \alpha$

$$I_C = \frac{1}{2} 8 \cdot 0{,}1^2 = 0{,}04$$

$$M_C = 0{,}1 S_2 - 0{,}1 S_1 = 0{,}1(S_2 - S_1)$$

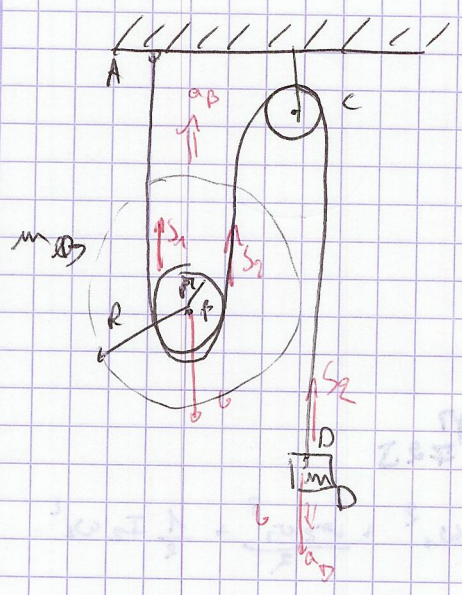
$$\Rightarrow \quad 0{,}1(S_2 - S_1) = 0{,}04\alpha$$

* $a_A = 2 \cdot a_C$, want cirkel gaat onderweg langs 2 touwen

$$\Rightarrow \begin{cases} S_2 = 90 - 20 a_A \\ S_2 - S_1 = 0{,}4\alpha \\ S_1 + S_2 = 80 + 0{,}8\alpha \\ a = R \cdot \alpha \end{cases}$$

Genoeg gegevens om op te lossen

oef 7.8



* In B: $M_B = I_B \cdot \alpha$

$I_B = 0{,}2\ kg\,m^2$ (gegeven)

$M_B = 0{,}1 S_2 - 0{,}1 S_1 = 0{,}1 (S_2 - S_1)$

$\Rightarrow \frac{(S_2 - S_1) \cdot 0{,}1}{0{,}2} = \alpha$

$2\alpha = S_2 - S_1$

* $\vec{S_1} + \vec{S_2} + \vec{G_B} = m \cdot \vec{a}$

$S_1 + S_2 - 20 = 2 \cdot a_B$

$\Rightarrow S_1 + S_2 = 20 + 2 a_B$

* In D: $\vec{S_2} + \vec{G_D} = m \cdot \vec{a}$

$S_2 - 20 = -2 \cdot a_D$

$\Rightarrow S_2 = 20 - 2 \cdot a_D$

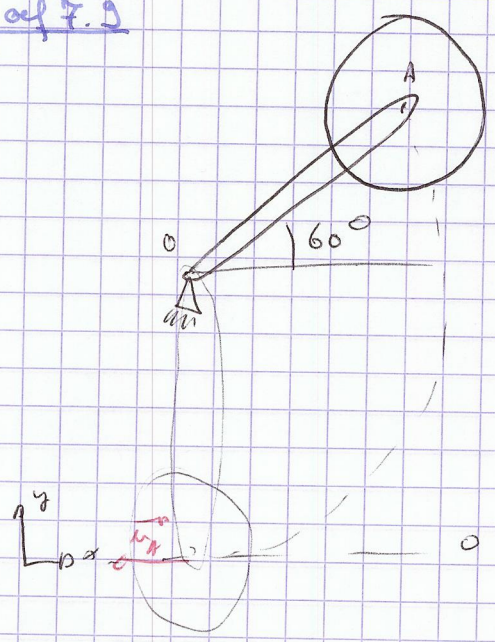
* $a_D = 2 \cdot a_B$, want katrol 2 touwen

$a_B = r \cdot \alpha = 0{,}1 \alpha$

$\Rightarrow a_D = 0{,}2 \alpha$

Genoeg gegevens om op te lossen

oef 7.9



Behoud van energie:

* Situatie 1: pot. gr.

$E_1 = m \cdot g \cdot h$

$= 60 \cdot 10 \cdot (1 + 1 \cdot \sin 60°)$

$= 1120\ J$

* Situatie 2: kin.

$E_2 = \frac{m v^2}{2} + \frac{I \cdot \omega^2}{2}$ $(v = R \cdot \omega)$

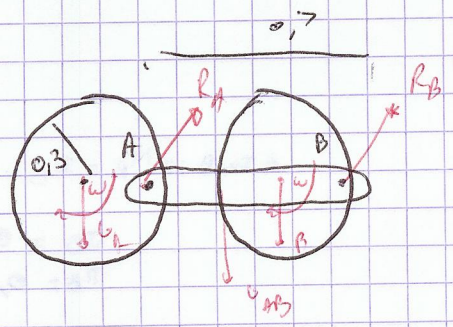
$= \frac{60}{2} \cdot \omega^2 + \frac{\frac{1}{3} \cdot 60 \cdot 1^2}{2} \cdot \omega^2$

* $E_1 = E_2 \Rightarrow 1120 = 30\omega^2 + 15\omega^2$

$\omega^2 = \sqrt{\frac{1120 \cdot 4^{1/2}}{135}} = 5{,}76$

$\Rightarrow v_A = 5{,}76\ \frac{m}{s}$

oef 7.10



Behoud van energie

* Situatie 1: pot. enr

$$E_1 = 24 \cdot 10 \cdot 0,3 = 72\,J$$

* Situatie 2: $\frac{m_1 v_1^2}{2} + \frac{1}{2} I_1 \omega_1^2 + \frac{m_2 v_2^2}{2} + \frac{1}{2} I_2 \omega_2^2$

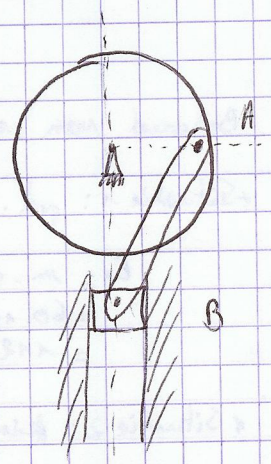
$$E_2 = 0,72\,\omega^2 + 16\,\omega^2$$

* $v = R \cdot \omega = 0,3 \cdot \omega$

$$\Rightarrow 72 = 0,72\,\omega^2 + 16 \cdot 0,09 \cdot \omega^2$$

$$\omega^2 = \frac{72}{2,16} \Rightarrow \omega = 5,77$$

oef 7.11

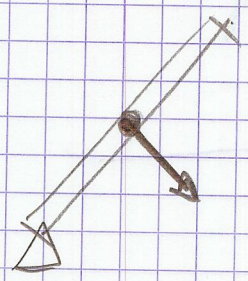


$$\frac{2 \times 10^6 \times 10^3 \cdot 250 \cdot 10}{2 \times 10^4} \frac{J}{s} = 5 \times 10^8 \frac{J}{s}$$

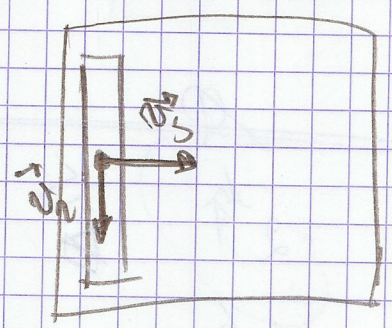
$$E_k = \sum \frac{m_i v_i^2}{2} = \sum_{cil\,1} + \sum_{cil\,2} + \sum_{stang}$$

$$E_{k,tr} + E_{k,rot}$$

4.1



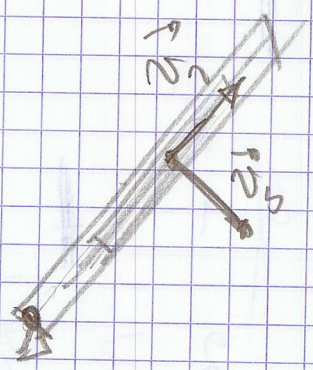
$\vec{v}_p = \vec{v}_p$



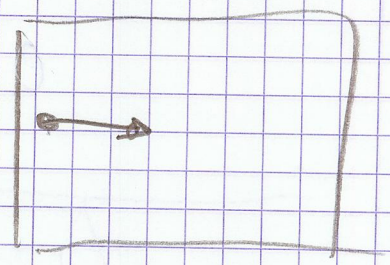
P schuift in gleuf

gleuf schuift in rechte

4.2

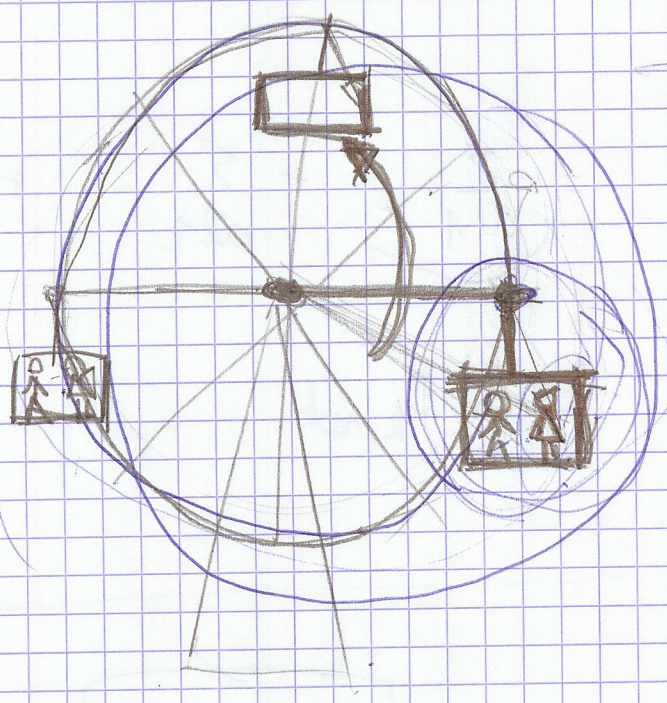


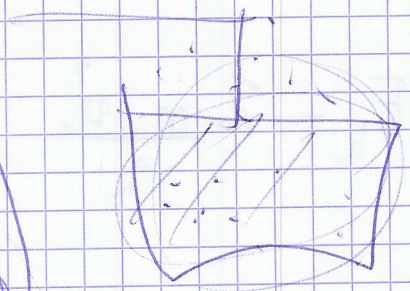
$\vec{v}_p = \vec{v}_p$



P glijdt in gleuf

gleuf roteert.





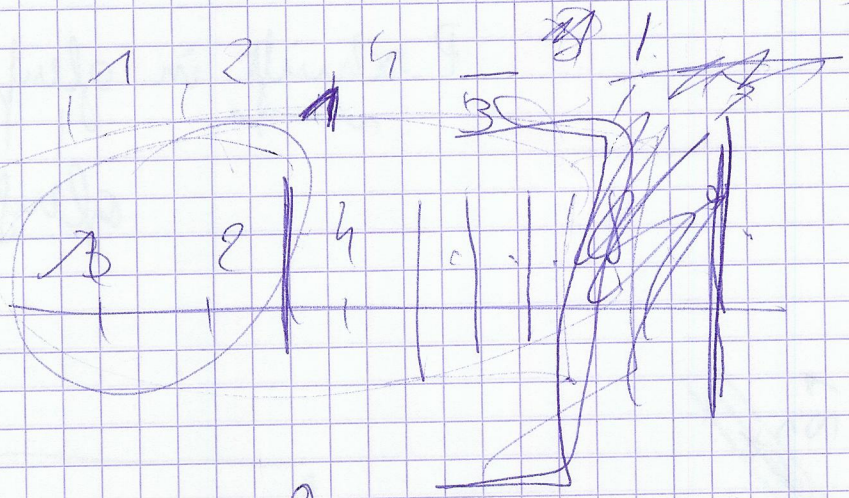
≤ 3

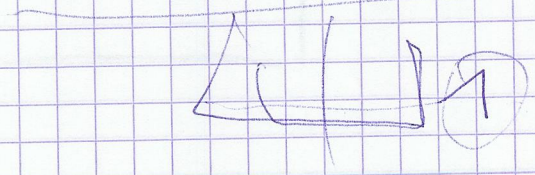
$m + n$

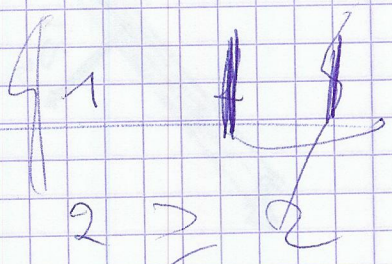
qwone entry

"parity entry"

$2^5 + 4$







Behoud van impuls:

* $\vec{p}_{\text{voor}} = \vec{p}_{\text{na}}$

$$0 = m_1 . \vec{v}_1 + m_2 . \vec{v}_2 + m_3 . \vec{v}_3$$

* $\vec{v}_1 = \begin{Bmatrix} -v_3 \\ v_{1\,rel} \end{Bmatrix} \Rightarrow \vec{v}_1 = -v_3 \vec{e_x} + v_{1\,rel} . \vec{e_y}$

$\vec{v}_2 = \begin{Bmatrix} v_{2\,rel} . \cos 30^\circ - v_3 \\ -v_{2\,rel} . \sin 30^\circ \end{Bmatrix} \Rightarrow \vec{v}_2 = (v_{2\,rel} \cos 30^\circ - v_3) \vec{e_x} - v_{2\,rel} \sin 30^\circ \vec{e_y}$

$\Rightarrow \quad 0 = -20 . v_3 + 50 (v_{2\,rel} . \cos 30^\circ - v_3) - 100 v_3$

$\Rightarrow \quad 0 = -170 v_3 + 25\sqrt{3}\, v_{2\,rel}$

* $a' - a = 2$, want afstand x die m_2 aflegt tot op de grond:

$$x = \frac{h}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

* $v_{1\,rel} = v_{2\,rel}$, want hangen aan 1 touw

Genoeg gegevens om op te lossen

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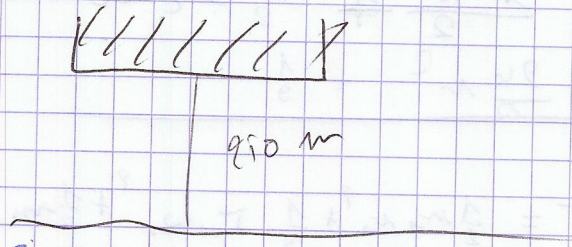
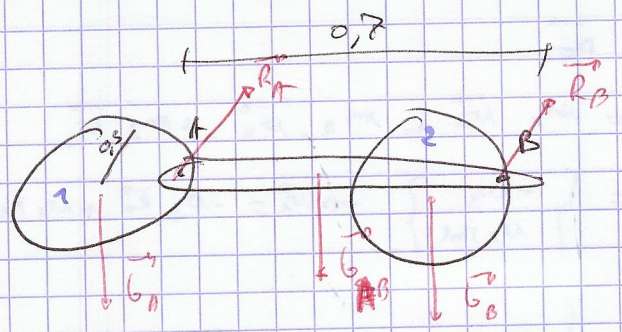
oef 6.14



* $P = F . v$

$$v = \frac{4\,000\,000\ m^3}{200\,000\ s} = 200 \frac{m^3}{s}$$

7,10



$pot = m \cdot g \cdot h$
$= 24 \cdot 10 \cdot 0,3 = 72\ J$

$Kin = \frac{m v^2}{2} + \frac{1}{2} I_c \omega^2$

$72 = \frac{24}{2} v^2 + \frac{1}{2}$

Ou 2 : $T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 \omega_2^2$

$\cancel{+ \frac{1}{2} m_1 v_1^2}$ $\left(I_1 = I_2 = \frac{16 \cdot 0,3^2}{2} = 0,72 \right)$

$= 0,72 \omega^2 + 16 v^2$

$72 = 0,72 \omega^2 + 16 v^2$ $\quad v = R \cdot \omega = 0,3 \omega$

x