KATHOLIEKE UNIVERSITEIT LEUVEN

Notities Ingenieurswetenschappen - 1e Fase

mechanica I

oefeningen

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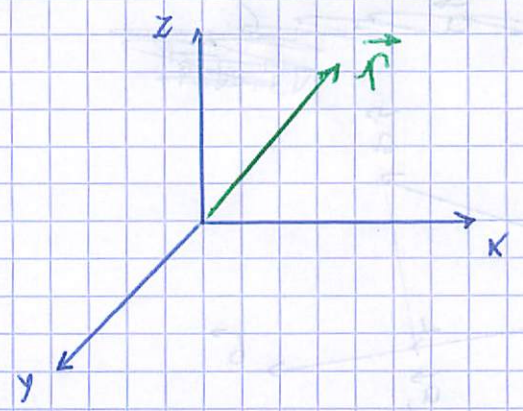
Faculteit Ingenieurswetenschappen

OEFENINGEN MECHANICA

Vectorrekenen

$\vec{a} = \vec{e}_x + \vec{e}_y + \vec{e}_z$; $\vec{b} = 3\vec{e}_x + 2\vec{e}_y + \vec{e}_z$

1) a) Geef richtingscosinussen.



$$\vec{e}_r = \cos\alpha . \vec{e}_x + \cos\beta . \vec{e}_y + \cos\gamma . \vec{e}_z$$

$$\cos\alpha = \frac{a_x}{\|\vec{a}\|}$$

$\vec{a}$: $\cos\alpha = \frac{1}{\sqrt{3}} = \cos\beta = \cos\gamma$

$\vec{b}$: - $\cos\alpha = \frac{3}{\sqrt{14}} = \frac{b_x\text{- coördinaat}}{\sqrt{b_x^2 + b_y^2 + b_z^2}}$

- $\cos\beta = \frac{2}{\sqrt{14}}$
- $\cos\gamma = \frac{1}{\sqrt{14}}$

b) $3\vec{a} - 2\vec{b} = 3\vec{e}_x + 3\vec{e}_y + 3\vec{e}_z - 6\vec{e}_x - 4\vec{e}_y - 2\vec{e}_z$

$= -3\vec{e}_x - \vec{e}_y + \vec{e}_z$

c) $\vec{a}.\vec{b} = 1.3 + 1.2 + 1.1 = 6$

d) $\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = \vec{a} \times \vec{b}$

$= 1.1.\vec{e}_x + \vec{e}_y . 1.3 + \vec{e}_z . 1.2$

$- 3.1.\vec{e}_z - 1.2.\vec{e}_x - \vec{e}_y . 1.1$

$= -\vec{e}_x + 2\vec{e}_y - \vec{e}_z$

e) Hoek tussen $\vec{a}$ en $\vec{b}$: $\cos\varepsilon = \frac{\vec{a}.\vec{b}}{\|\vec{a}\|.\|\vec{b}\|} = \frac{6}{\sqrt{3}.\sqrt{14}}$

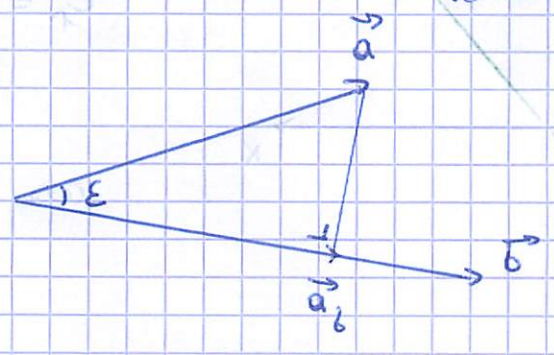
$\varepsilon = 22,2°$

f) $\dfrac{\vec{a} \times \vec{b}}{\|\vec{a} \times \vec{b}\|} = -\vec{e}_x + 2\vec{e}_y - \vec{e}_z \;/\; \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \dfrac{1}{\sqrt{6}} \left(-\vec{e}_x + 2\vec{e}_y - \vec{e}_z \right)$

~~g) $\vec{a}_b = \cos\varepsilon \cdot \vec{a} = \dfrac{6}{\sqrt{3} \cdot \sqrt{14}}$~~

g)



$\vec{a}_b = \|\vec{a}\| \cdot \cos\varepsilon \cdot \vec{e}_b = \|\vec{a}\| \cdot \cos\varepsilon \cdot \dfrac{\vec{b}}{\|\vec{b}\|}$

$= \cancel{\sqrt{3}} \cdot \dfrac{6}{\cancel{\sqrt{3}} \cdot \sqrt{14}} \cdot \left(3\vec{e}_x + 2\vec{e}_y + \vec{e}_z \right) \cdot \dfrac{1}{\sqrt{14}}$

$= \dfrac{3}{7} \left(3\vec{e}_x + 2\vec{e}_y + \vec{e}_z \right)$

e)

Ingesloten hoek ε tss $\vec{a} = \vec{e}_x + 2\vec{e}_y + 3\vec{e}_z$

en $\vec{b} = -3\vec{e}_x - 2\vec{e}_y - \vec{e}_z$.

$\cos\varepsilon = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \dfrac{-3 \cdot 1 + 2 \cdot (-2) + 3 \cdot (-1)}{\sqrt{1 + 2^2 + 3^2} \cdot \sqrt{(-3)^2 + (-2)^2 + (-1)^2}}$

$= \dfrac{-10}{\sqrt{14} \cdot \sqrt{14}} = \dfrac{-10}{14}$

$\hookrightarrow \varepsilon = 135{,}5^\circ$

3) $\vec{a}$: - lengte 7

- $a_x = 2$

- $a_y = 3$

$\hookrightarrow \|\vec{a}\| = \sqrt{2^2 + 3^2 + a_z^2} = 7$

$= 13 + a_z^2 = 49 \quad \Rightarrow a_z = \pm 6$

(vervolg oefn)

$$\cos\alpha = \frac{a_x}{\|\vec{a}\|} = \frac{2}{7} \longrightarrow \alpha = 73°$$

8) $\vec{G} = -G\cos(\pi-\alpha)\cdot\vec{e}_x - G\sin(\pi-\alpha)\cdot\vec{e}_y$

$= +\cos\alpha\cdot\vec{e}_x - \sin\alpha\cdot\vec{e}_y$

9) manier A

Als $\theta = 0°$ dan $\vec{v} = \underset{\overset{\|}{0}}{v\cdot\vec{e}_x} + \underset{\overset{\|}{v}}{v\cdot\vec{e}_y}$

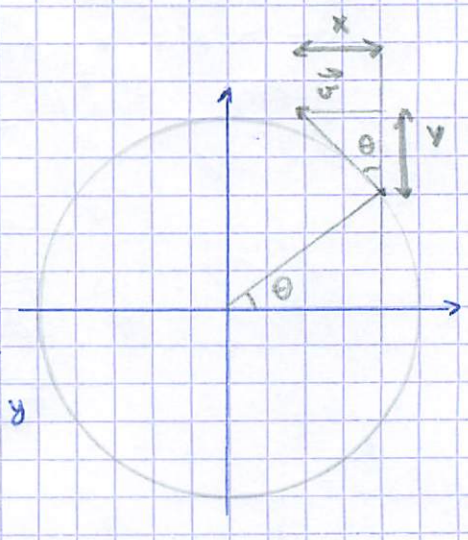
$\Rightarrow \vec{v} = -\sin\theta\cdot v\cdot\vec{e}_x + \cos\theta\cdot v\cdot\vec{e}_y$

$\vec{e}_x = \frac{\vec{x}}{\|\vec{x}\|}$

manier B

$\sin\theta = \frac{\|\vec{x}\|}{\|\vec{v}\|} = -\frac{\vec{x}}{v\cdot\vec{e}_x}$

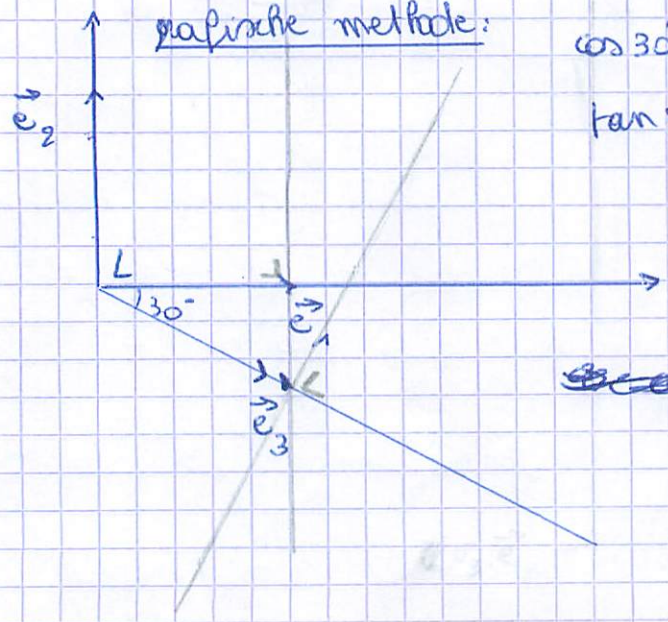
$\cos\theta = \frac{\|\vec{y}\|}{\|\vec{v}\|} = \frac{+\vec{y}}{v\cdot\vec{e}_y}$



$\vec{v} = \vec{x} + \vec{y}$

$\Rightarrow \vec{v} = -\sin\theta\cdot v\cdot\vec{e}_x + \cos\theta\cdot v\cdot\vec{e}_y$

10) grafische methode:



$\cos 30° = v_3\cdot\|\vec{e}_3\| = \|\vec{e}_1\|$

$\tan 30° = \frac{-v_2\cdot\|\vec{e}_2\|}{\|\vec{e}_1\|}$

$\rightarrow v_3 = \frac{2}{\sqrt{3}}$; $v_2 = -\frac{1}{\sqrt{3}}$

andere methode:

$$\vec{e}_1 + v_2 \cdot \vec{e}_2 = v_3 \cdot \vec{e}_3$$

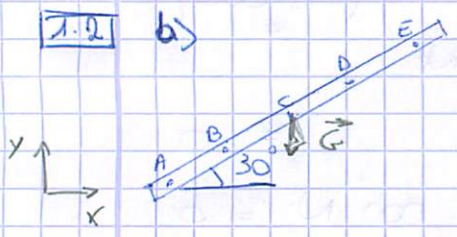
$$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + v_2 \cdot \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = v_3 \begin{Bmatrix} \cos(30°) \\ -\sin(30°) \end{Bmatrix}$$

$$\Rightarrow \quad 1 = v_3 \cdot \cos(30°)$$

$$v_2 = -v_3 \cdot \sin(30°)$$

statica van enkelvoudige lichamen

1.2 b)

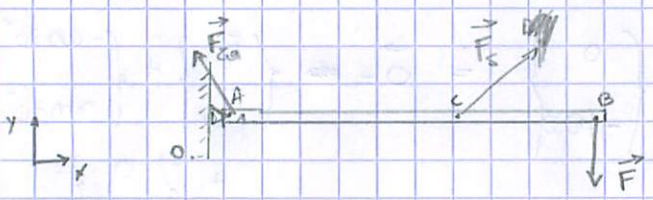


$$\vec{M}_A = \vec{r}_{AC} \cdot \vec{G} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \cos 30° \cdot 0{,}1 & \sin 30° \cdot 0{,}1 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$= 0{,}1\,kg \cdot 10\,\frac{N}{kg}$

$$= -0{,}1 \cdot \cos 30° \cdot \vec{e}_z$$

1.5 * vrijlichaamsdiagram



* - krachtenevenwicht:

$$\vec{F}_{za} + \vec{F}_s + \vec{F} = \vec{0}$$

$$\begin{Bmatrix} F_{za,x} \\ F_{za,y} \end{Bmatrix} + \begin{Bmatrix} F_s \cdot \cos 45° \\ F_s \cdot \sin 45° \end{Bmatrix} + \begin{Bmatrix} 0 \\ -50\,000\,N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

- momentenevenwicht:

$$\vec{r}_A \times \vec{F}_{za} + \vec{r}_C \times \vec{F}_S + \vec{r}_B \times \vec{F} = 0$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & 0 \\ F_{za,x} & F_{za,y} & 0 \end{vmatrix} = 0 \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & 0 & 0 \\ F_s \cdot \cos 45° & F_s \cdot \sin 45° & 0 \end{vmatrix} = 2 \cdot F_s \cdot \sin 45° \cdot \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & 0 & 0 \\ 0 & -50\,000 & 0 \end{vmatrix} = -150\,000\,N \cdot \vec{e}_z$$

$\Rightarrow$ ~~$F_s = 106 \cdot 10^3\,N$~~

$\Rightarrow$

$$\longrightarrow \quad 0 + 2F_0 \cdot \sin 45° \cdot \vec{e}_z - 150\,000\,N \cdot \vec{e}_z = 0$$

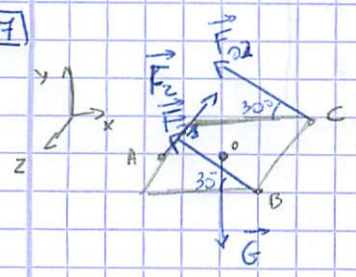
$$F_0 = 106\ kN \quad \Rightarrow \quad \vec{F}_0 = \begin{Bmatrix} 75\,000 \\ 75\,000 \\ 0 \end{Bmatrix} N$$

einsetzen in Kräftegleichgewicht

$$\Rightarrow \begin{cases} F_{2a,x} + F_0 \cos 45° + 0 = 0 \\ F_{2a,y} + F_0 \sin 45° - 50\,000\ N = 0 \end{cases}$$

$$\begin{cases} F_{2a,x} = -75000\ N \\ F_{2a,y} = -25000\ N \end{cases} \quad \Rightarrow \vec{F}_{2a} = \begin{Bmatrix} -75000 \\ -25\,000 \\ 0 \end{Bmatrix} N$$

$\boxed{1.7}$



Kräftegleichgewicht:

$$\vec{F}_2 + \vec{F}_{01} + \vec{F}_{02} + \vec{G} = \vec{0}$$

$$\begin{Bmatrix} F_{2x} \\ F_{2y} \end{Bmatrix} + \begin{Bmatrix} F_{01x} \\ F_{01y} \end{Bmatrix} + \begin{Bmatrix} F_{02x} \\ F_{02y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -1000 \end{Bmatrix} = \vec{0} = \begin{Bmatrix} F_{2x} \\ F_{2y} \end{Bmatrix} + \begin{Bmatrix} -\cos 30° F_{01} \\ \sin 30° \cdot F_{01} \end{Bmatrix} + \begin{Bmatrix} -\cos 30° F_{02} \\ \sin 30° F_{02} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -1000 \end{Bmatrix}$$

Momentengleichgewicht: (mittlerer Scharnier: ursprung)

$$\vec{r}_A \times \vec{F}_2 + \vec{r}_0 \times \vec{G} + \vec{r}_B \times \vec{F}_{01} + \vec{r}_C \times \vec{F}_{02} = 0$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}25m & 0 & 0 \\ 0 & -1000 & 0 \end{vmatrix} = -2{,}50m \cdot \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}5m & 0 & +0{,}25m \\ -\cos 30° \cdot F_{01} & \sin 30° \cdot F_{01} & 0 \end{vmatrix} = 0{,}25m \cdot \cos 30° \cdot F_{01} \cdot \vec{e}_y + 0{,}25\ m \cdot \sin 30° \cdot F_{01} \cdot \vec{e}_x + 0{,}5m \cdot \sin 30° \cdot F_{01} \cdot \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}5m & 0 & -0{,}25\ m \\ -\cos 30° \cdot F_{01} & \sin 30° \cdot F_{01} & 0 \end{vmatrix} = -0{,}25\ m \cdot \cos 30° \cdot F_{02} \cdot \vec{e}_y + 0{,}5m \cdot \sin 30° \cdot F_{02} \cdot \vec{e}_z - 0{,}25\ m \cdot \sin 30° \cdot F_{02} \cdot \vec{e}_x$$

$$\cancel{\Rightarrow -2{,}5m \cdot \vec{e}_z + (0{,}25\ m - 0{,}25\ m) \cdot \cos 30° \cdot F_{01}}$$

$$\Rightarrow \begin{cases} -0{,}25m \cdot \sin 30° \cdot F_{01} \cdot \vec{e}_x + 0{,}25\ m \cdot \sin 30° \cdot F_{02} \cdot \vec{e}_x \\ -2{,}50m \cdot \vec{e}_z + 0{,}25\ m \cdot \cos 30° \cdot F_{01} \cdot \vec{e}_y - 0{,}25\ m \cdot \cos 30° \cdot F_{01} \cdot \vec{e}_y \\ + 0{,}5\ m \cdot \sin 30° \cdot F_{01} \cdot \vec{e}_z + 0{,}5m \cdot \sin 30° \cdot F_{02} \cdot \vec{e}_z = 0 \end{cases}$$

deel a: $\boxed{F_{s1} = F_{s2}}$

$$-2{,}50\,m \cdot \vec{e}_z + 1\,m \cdot \sin 30^\circ \cdot F_{s1} \cdot \vec{e}_z = 0$$

$$\underline{F_{s1} = 500\,N = F_{s2}}$$

↓ invullen in krachtenevenwicht

$$\begin{Bmatrix} F_{2x} \\ F_{2y} \end{Bmatrix} + \begin{Bmatrix} -\cos 30^\circ F_{s1} \\ \sin 30^\circ F_{s1} \end{Bmatrix} + \begin{Bmatrix} -\cos 30^\circ F_{s2} \\ \sin 30^\circ F_{s2} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -1000 \end{Bmatrix} = 0$$

$$\underline{F_{2x}} = 2 \cdot \cos 30^\circ \cdot F_{s1} = \underline{866\,N}$$

$$\underline{F_{2y}} = 2 \cdot \sin 30^\circ \cdot F_{s1} = \underline{500\,N}$$

$$\Rightarrow \vec{F}_{s1} = \vec{F}_{s2} = \begin{Bmatrix} -433 \\ 250 \end{Bmatrix} N$$

$$\vec{F}_2 = \begin{Bmatrix} 866 \\ 500 \end{Bmatrix} N$$

deel b: $F_{s1} = F_{s2}$ MAAR momentenevenwicht verandert:

~~$-0{,}25\,m \cdot \sin 30^\circ \cdot F_{s1} \cdot \vec{e}_x + 0{,}25\,m \cdot \sin 30^\circ \cdot F_{s2} \cdot \vec{e}_x$~~
~~$-250\,m \cdot \vec{e}_z + 0{,}25\,m \cdot \cos 30^\circ \cdot F_{s1} \cdot \vec{e}_y - 0{,}25\,m \cdot \cos 30^\circ F_{s1} \cdot \vec{e}_y$~~
~~$+0{,}5\,m \cdot \sin 30^\circ \cdot F_{s1} \cdot \vec{e}_z + 0{,}5\,m \cdot \sin 30^\circ \cdot F_{s2} \cdot \vec{e}_z = 0$~~

* nieuwe F_{s1}-determinant:

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}25\,m & 0 & 0 \\ -\cos 30^\circ \cdot F_{s1} & \sin 30^\circ \cdot F_{s1} & 0 \end{vmatrix} = 0{,}25\,m \cdot \sin 30^\circ \cdot F_{s1} \cdot \vec{e}_z$$

* $$\vec{M} = \begin{Bmatrix} 0{,}25\,m \cdot \sin 30^\circ \cdot F_{s2} \\ +0{,}25\,m \cdot \cos 30^\circ \cdot F_{s2} \\ -250\,m + 0{,}5\,m \cdot \sin 30^\circ \cdot F_{s2} + 0{,}25\,m \cdot \sin 30^\circ \cdot F_{s1} \end{Bmatrix}$$

↓ F_{s1} en F_{s2} invullen om (-zwaartemoment) te vinden opdat staal in evenwicht blijft

~~F_{s1} ... 1000 N~~

$$\vec{M} = \begin{cases} 62,5 \text{ Nm } \vec{e}_x \\ +108,25 \text{ Nm} \cdot \vec{e}_y \\ -62,5 \text{ Nm} \cdot \vec{e}_z \end{cases} \rightarrow \vec{M}_{\cancel{2}} \cdot (-1) = \vec{M}_2$$

$$\Rightarrow \vec{M}_{2} = -62,5 \text{ Nm } \vec{e}_x - 108,25 \text{ Nm } \vec{e}_y - 62,5 \text{ Nm } \vec{e}_z$$

extra oefeningen MECHANICA (14.11.'09)

1.3 vrijlichaamsdiagram:

|AB| = 10 cm
|AC| = 15 cm

krachtenevenwicht:

$$\vec{F}_{2a} + \vec{G} + \vec{F}_0 = 0$$

$$\begin{Bmatrix} F_{2a,x} \\ F_{2a,y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_{0,y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -3N \end{Bmatrix} = 0$$

$$\Rightarrow F_{2a,x} = 0$$

$$F_{2a,y} + F_0 = 3N \quad (1)$$

momentenevenwicht (vanuit A)

$$\vec{r}_B \times \vec{G} + \vec{r}_C \times \vec{F}_0 = 0$$

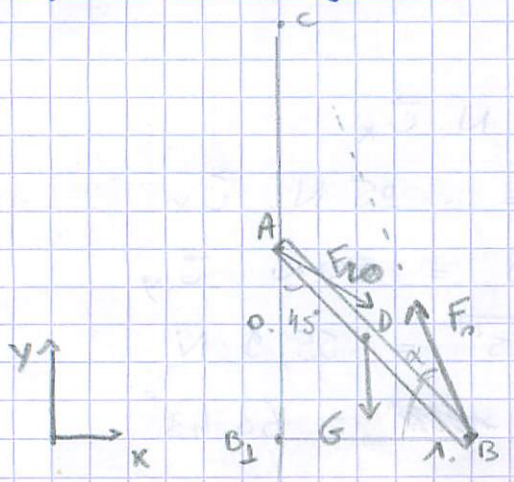
$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}1m & 0 & 0 \\ 0 & -3N & 0 \end{vmatrix} = -0{,}3\,Nm \cdot \vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}15m & 0 & 0 \\ 0 & F_0 & 0 \end{vmatrix} = 0{,}15 F_0\, m \cdot \vec{e}_z$$

$$\Rightarrow 0{,}15 F_0 = 0{,}3\,N$$

$$F_0 = 2N \cdot \vec{e}_y \quad (2)$$

$$(1)+(2) \Rightarrow F_{2a} = 1N \cdot \vec{e}_y$$

1.6 vrijlichaamsdiagram



momentenevenwicht (vanuit B)

$$\vec{r}_D \times \vec{G} + \vec{r}_A \times \vec{F}_{10} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -50N & 0 \\ \cos 45^\circ & \sin 45^\circ & 0 \end{vmatrix} = -50N \cdot \cos 45^\circ \cdot \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ F_{10,x} & F_{10,y} & 0 \\ 2\cos 45^\circ & 2\sin 45^\circ & 0 \end{vmatrix} = 2\cos 45^\circ \cdot F_{10,y} \cdot \vec{e}_z + 2\sin 45^\circ \cdot F_{10,x} \cdot \vec{e}_z$$

$$\Rightarrow \quad 2\cos 45^\circ \cdot F_{10,y} + 2\sin 45^\circ \cdot F_{10,x} = 50\,N \cdot \cos 45^\circ$$

$$2F_{10,y} + 2F_{10,x} = 50\,N$$

$$F_{10,y} + F_{10,x} = 25\,N$$

krachtenevenwicht

$$\vec{F}_{10} + \vec{G} + \vec{F}_0 = 0$$

$$\begin{Bmatrix} F_{10,x} \\ F_{10,y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -50N \end{Bmatrix} + \begin{Bmatrix} F_{0,x} \\ F_{0,y} \end{Bmatrix} = 0$$

! de contactkracht die lichaam 0 (wand) uitoefent op lichaam 1: doordat de wand glad is, bestaat deze kracht enkel uit een component die normaal staat op het contactoppervlak (zie H3 p 56)

$$\Rightarrow \quad F_{10} = F_{10,x} = 25\,N \cdot \vec{e}_x$$

$$F_{0,x} = -F_{10,x} = -25\,N \cdot \vec{e}_x$$

$$F_{0,y} = -(-50N) = 50N \cdot \vec{e}_y$$

$$\|F_0\| = \sqrt{50^2 + 25^2} = 55,9\,N$$

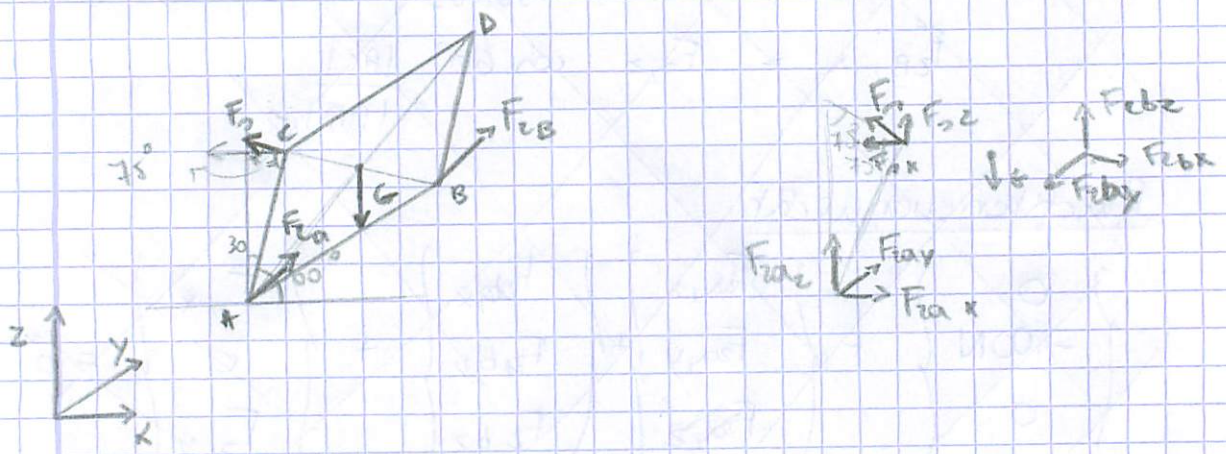
$$\alpha = \cos^{-1}\left(\frac{25}{55,9}\right) = 63,43^\circ$$

$$\tan\alpha = \frac{|CB_\perp|}{2 \cdot \sin 45^\circ} \Rightarrow |CB_\perp| = 2{,}82$$

$$\Rightarrow |AC| = |CB_\perp| - |AB_\perp| = 2{,}82 - 2 \cdot \cos 45^\circ$$

$$= 1{,}41 \text{ m}$$

$\boxed{1.8}$ vrijlichaamdiagram juiste methode



momentenevenwicht (vanuit A)

$$\vec{r}_G \times \vec{G} + \vec{r}_C \times \vec{F}_D + \vec{r}_B \times \vec{F}_{2B} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \cos 60^\circ \cdot \frac{1}{2} \cdot |AC| & \frac{1}{2} \cdot |AB| & \sin 60^\circ \cdot \frac{1}{2} \cdot |AC| \\ 0 & 0 & -400\text{N} \end{vmatrix} = 400\text{N} \cdot \frac{1}{2} \cdot |AB| \cdot \vec{e}_x + 400\text{N} \cdot \frac{1}{2} \cdot \cos 60^\circ \cdot |AC| \cdot \vec{e}_y$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \cos 60^\circ \cdot |AC| & 0 & \sin 60^\circ \cdot |AC| \\ -F_{D,x} & 0 & F_{D,z} \end{vmatrix} = -F_{D,z} \cdot \cos 60^\circ |AC| \cdot \vec{e}_y - F_{D,x} \cdot \sin 60^\circ |AC| \cdot \vec{e}_y$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & |AB| & 0 \\ F_{2B,x} & F_{2B,y} & F_{2B,z} \end{vmatrix} = -F_{2B,z} \cdot |AB| \cdot \vec{e}_x + F_{2B,x} \cdot |AB| \cdot \vec{e}_z$$

$$+ 200\text{N} \cdot \cos 60^\circ \cdot |AC| \cdot \vec{e}_y$$

$$\Rightarrow 200\text{ N} \cdot |AB| \cdot \vec{e}_x - F_{D,z} \cdot \cos 60^\circ |AC| \cdot \vec{e}_y + F_{D,x} \cdot \sin 60^\circ |AC| \cdot$$

$$-F_{2B,z} \cdot |AB| \cdot \vec{e}_x + F_{2B,x} \cdot |AB| \cdot \vec{e}_z = \vec{0}$$

$$\hookrightarrow \begin{Bmatrix} 200\,N \cdot |AB| \\ 200\,N \cdot \cos 60^\circ \cdot |AC| \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F_{3,x} \cdot \sin 60^\circ \cdot |AC| \\ F_{3,z} \cdot \cos 60^\circ \cdot |AC| \end{Bmatrix} + \begin{Bmatrix} -F_{2B,z} \cdot |AB| \\ 0 \\ F_{2B,x} \cdot |AB| \end{Bmatrix} = 0$$

$$\Rightarrow F_{2B,z} = 200\,N$$

$$F_{3,x} = -200\,N \cdot \frac{\cos 60^\circ}{\sin 60^\circ} = -115{,}47\,N$$

$$F_{2B,x} = F_{3,z} \cdot \cos 60^\circ \frac{|AC|}{|AB|}$$

<u>Bracken-Gleichgewicht</u>

$$\begin{Bmatrix} 0 \\ -400\,N \\ 0 \end{Bmatrix} + \begin{Bmatrix} F_{2a,x} \\ F_{2a,y} \\ F_{2a,z} \end{Bmatrix} + \begin{Bmatrix} F_{2b,x} \\ F_{2b,y} \\ F_{2b,z} \end{Bmatrix} + \begin{Bmatrix} F_{3,x} \\ 0 \\ F_{3,y} \end{Bmatrix} = 0$$

$$\hookrightarrow \begin{Bmatrix} 200\,N\,|AB| \\ 200\,N \cos 60^\circ |AC| \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -F_{3x} \sin 60^\circ |AC| - F_{3z} \cos 60^\circ |AC| \\ 0 \end{Bmatrix} + \begin{Bmatrix} -F_{2B,z}|AB| \\ 0 \\ F_{2b,x}|AB| \end{Bmatrix} = 0$$

$\rightarrow$ * $F_{2b,x} = 0$

* $F_{2b,z} = 200\,N$

* $200\,N \cdot \cos 60^\circ \cdot |AC| + F_3 \cdot |AC| (-\cos 15^\circ \sin 60^\circ - \sin 15^\circ \cos 60^\circ) = 0$

$\rightarrow F_3 = 103{,}5$

<u>Bracken-Gleichgewicht</u>

$$\begin{Bmatrix} 0 \\ 0 \\ -400\,N \end{Bmatrix} + \begin{Bmatrix} F_{2ax} \\ F_{2ay} \\ F_{2az} \end{Bmatrix} + \begin{Bmatrix} F_{2bx} \\ F_{2by} \\ F_{2bz} \end{Bmatrix} + \begin{Bmatrix} -F_{3,x} \\ 0 \\ F_{3x} \end{Bmatrix} = 0$$

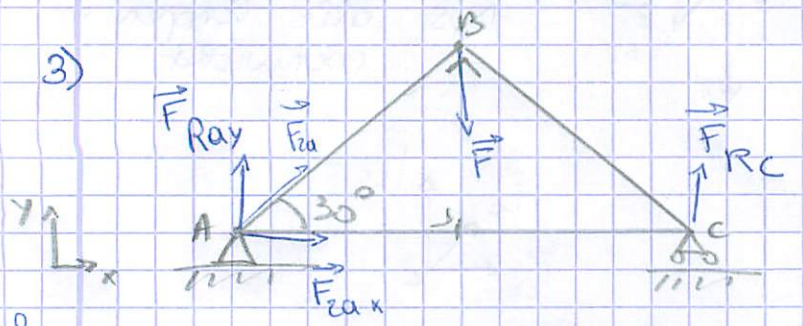
$\Rightarrow F_{2ax} + F_{2bx} - F_{3x} = 0 \rightarrow F_{2ax} = 100\,N$

$F_{2ay} = -F_{2by}$

$F_{2az} - 400\,N + F_{2bz} + F_{3z} = 0 \rightarrow F_{2az} = 173\,N$

STATICA VAN SAMENGESTELDE LICHAMEN

3)



$Ax = \cos 30° \cdot AB$
$= \cos 30° \cdot \cos 30° \, AC$

* geheel:

momentenevenwicht:

↳ vanuit punt a:

$$(\vec{r}_c - \vec{r}_a) \times \vec{F}_{zc} + (\vec{r}_b - \vec{r}_a) \times \vec{F} = \vec{0}$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ |AC| & 0 & 0 \\ 0 & F_{zc} & 0 \end{vmatrix} = |AC| \cdot F_{zc} \cdot \vec{e}_z$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \cos^2 30° \, AC & \cos 30° \cdot \sin 30° \cdot AC & 0 \\ -8000\,N & 0 & 0 \end{vmatrix} = -8000\,N \cdot \cos^2 30° \cdot |AC| \cdot \vec{e}_z$$

$$\Rightarrow F_{zc} \cdot |AC| \cdot \vec{e}_z = 8000\,N \cdot \cos^2 30° \cdot |AC| \, \vec{e}_z$$

$$F_{zc} = 6000\,N$$

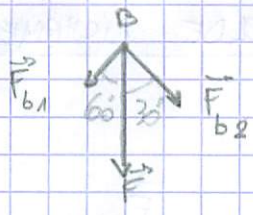
krachtenevenwicht:

$$\vec{F} + \vec{F}_{zc} + \vec{F}_{zay} + \vec{F}_{zax} = \vec{0}$$

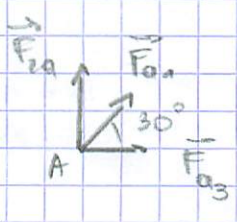
$$\Rightarrow \vec{F}_{zax} = 0$$

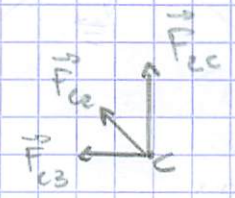
$$\vec{F}_{zay} = 2000\,N$$

* Krachten :



constructie in evenwicht
DUS alle krachten in evenwicht





KNOOP B: $\vec{F} + \vec{F}_{b1} + \vec{F}_{b2} = \vec{0}$

↳ $\cos 60° = \dfrac{\vec{F}_{b1}}{\vec{F}} \Rightarrow \vec{F}_{b1} = 4000\ N$

↳ $\cos 30° = \dfrac{F_{b2}}{\vec{F}} \Rightarrow \vec{F}_{b2} = 6928\ N$

KNOOP A: $\vec{F}_{2a} + \vec{F}_{a1} + \vec{F}_{a3} = \vec{0}$

$\vec{F}_{a1} = -4000\ N = -F_{b1}$

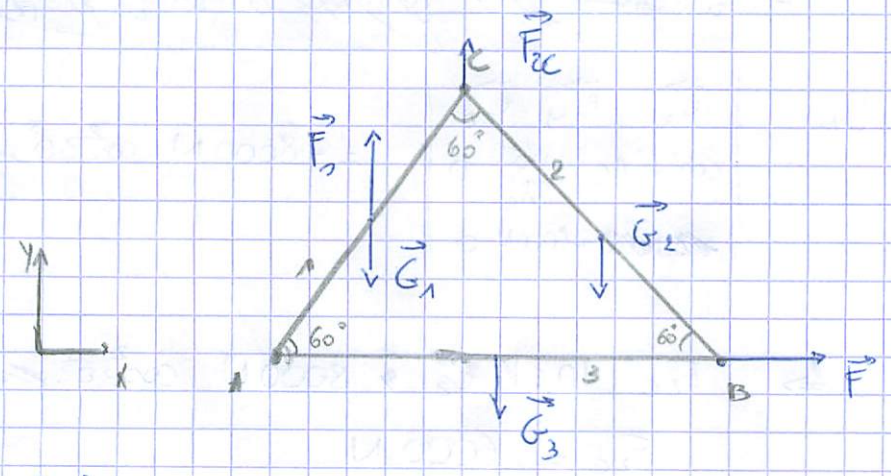
$F_{a3} = \cos 30° \cdot F_{a1}$

$= -3464\ N$

KNOOP C: $F_{c2} = -F_{b2} = -6928\ N$

$F_{c3} = -F_{a3} = 3464\ N$

6)



gekeerd.

* Krachtenevenwicht:

$$\vec{F}_{2c} + \vec{F}_1 + \vec{G}_1 + \vec{G}_2 + \vec{G}_3 + \vec{F} = 0$$

* momentenevenwicht, aanuit C:

$$(\vec{r}_{mAC} - \vec{r}_C) \times \vec{F}_D + (\vec{r}_{mAC} - \vec{r}_C) \times \vec{G}_1$$

$= 0$ want F (AB,C)

$$+ (\vec{r}_{mAB} - \vec{r}_C) \times \vec{G}_3 + (\vec{r}_{mBC} - \vec{r}_C) \times \vec{G}_2$$

$$+ (\vec{r}_B - \vec{r}_C) \times \vec{F} = 0$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ +\frac{\cos 60°}{2} & -\frac{\sin 60°}{2} & 0 \\ 0 & F_D & 0 \end{vmatrix} = -F_D \cdot \frac{\cos 60°}{2} \cdot \vec{e}_z$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -\frac{\sin 60°}{2} & -\frac{\cos 60°}{2} & 0 \\ 0 & -G_1 & 0 \end{vmatrix} = +G_1 \cdot \frac{\sin 60°}{2} \cdot \vec{e}_z$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \sin 60° & -\frac{\cos 60°}{2} & 0 \\ 0 & -G_2 & 0 \end{vmatrix} = -G_2 \cdot \frac{\sin 60°}{2} \cdot \vec{e}_z$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ +\cos 60° & -\sin 60° & 0 \\ F & 0 & 0 \end{vmatrix} = +F \cdot \sin 60° \cdot \vec{e}_z$$

$$\Rightarrow -\frac{1}{4} F_D \cdot \cancel{\vec{e}_z} + \cancel{\frac{\sqrt{3}}{4} \cdot \overset{200N}{G_1} \cdot \vec{e}_z} - \cancel{\frac{\sqrt{3}}{4} \overset{200N}{G_2} \cdot \vec{e}_z}$$

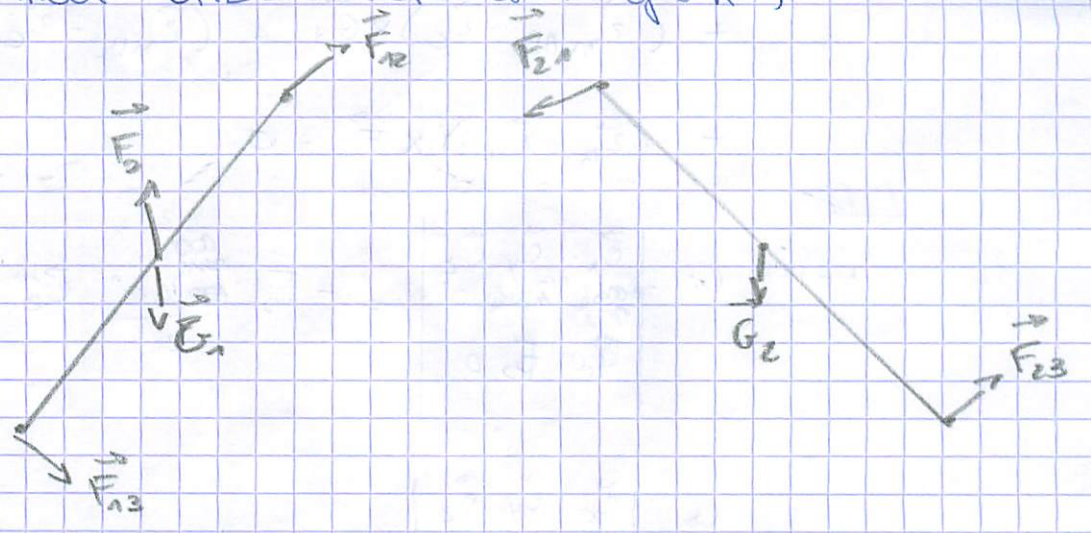
$$+ \frac{\sqrt{3}}{2} \cdot F \cdot \cancel{\vec{e}_z} = 0$$

$$\hookrightarrow \frac{1}{4} F_D = \frac{\sqrt{3}}{2} F$$

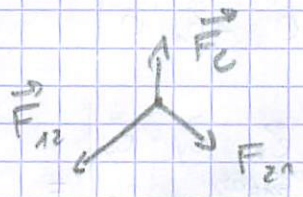
$$F_D = 2\sqrt{3}\, F \Rightarrow \begin{cases} F_{2cx} = -F \\ F_{2cy} = -2\sqrt{3}F + 600N \end{cases}$$

* <u>staven en kangen</u>:

(als we enkel staven vrij maken krijgen we meer onbekenden dan vgl'n)



STAAF 1 STAAF 2

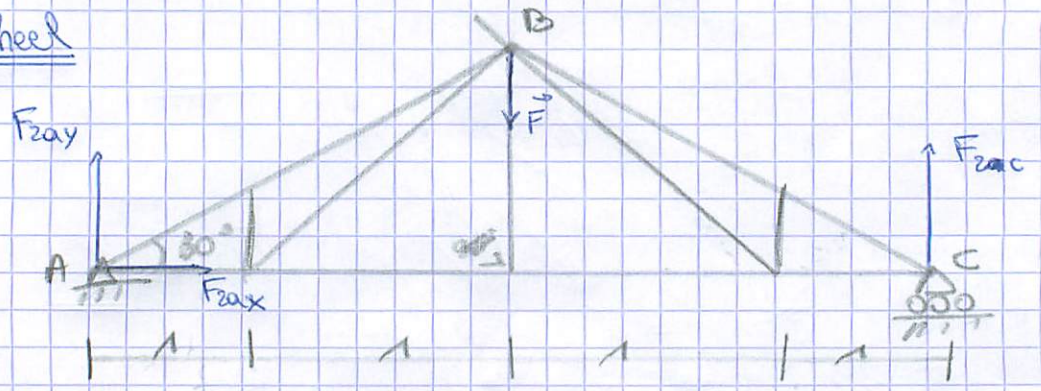


KNOOPPUNT C

↳ momenten- en krachtenvgl'en uitwerken

7)

<u>geheel</u>



<u>momentenevenwicht:</u> (aanuit a)

$$(\vec{r}_B - \vec{r}_a) \times \vec{F} + (\vec{r}_c - \vec{r}_a) \times \vec{F}_{zc} = 0$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & \tan 30° \cdot 2 & 0 \\ 0 & -F & 0 \end{vmatrix} = -2 \cdot F \cdot \vec{e}_z$$

$$\hookrightarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 4 & 0 & 0 \\ 0 & F_{zc} & 0 \end{vmatrix} = 4 F_{zc} \cdot \vec{e}_z$$

$$\Rightarrow \quad 2F = 4 F_{zc}$$

$$F = 2 F_{zc} \quad \Rightarrow \quad F_{zc} = 5000 \text{ N}$$

<u>krachten evenwicht:</u>

$$\vec{F}_{zay} + \vec{F}_{zax} + \vec{F} + \vec{F}_{zc} = \vec{0}$$

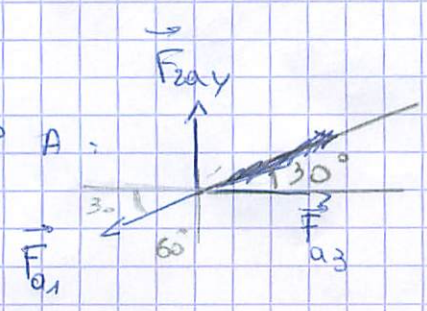
$$\Rightarrow \quad F_{zay} = 5000 \text{ N}$$

$$F_{zax} = 0 \text{ N}$$

trekkracht = pos duwkracht = neg

<u>knopen:</u>

* KNOOP A:



$$\vec{F}_{a1} = -\frac{F_{zay}}{\cos 60°}$$

$$= -10000 \text{ N}$$

$$F_{a3} = \sin 60° \cdot F_{a1} = \sqrt{3} \cdot 5000 \text{ N}$$

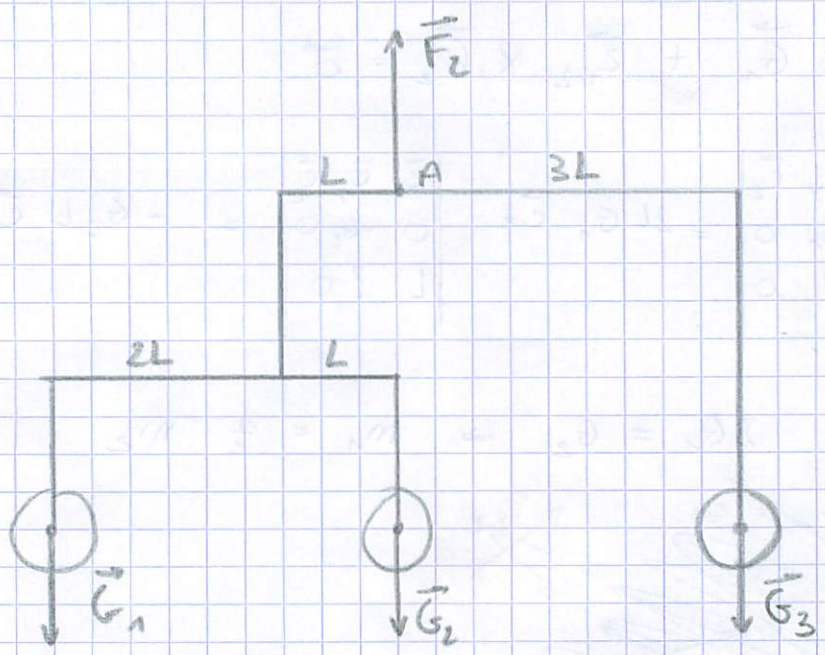
* $F_{G_1} = F_{C_2}$

11)

extra oefeningen MECHANICA (14.11.09)

(TOTAAL)

$\boxed{2.1}$ vrijlichaamsdiagram:



krachtenevenwicht:

$$\vec{G}_1 + \vec{G}_2 + \vec{G}_3 + \vec{F}_2 = \vec{0}$$

momentenevenwicht: (zond A)

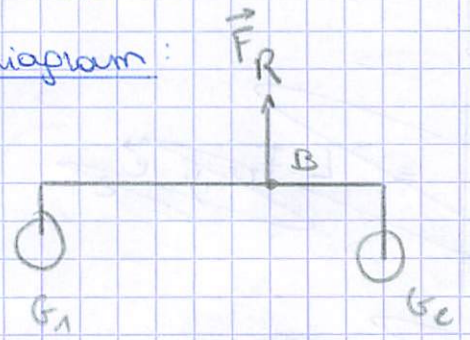
$$\vec{G}_1 \times \vec{z}_{G_1} + \vec{G}_3 \times \vec{z}_{G_3} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -G_1 & 0 \\ -3L & ? & 0 \end{vmatrix} = 3L \cdot G_1 \cdot \vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -G_3 & 0 \\ 3L & ? & 0 \end{vmatrix} = -3L \cdot G_3 \cdot \vec{e}_z$$

$$\Rightarrow G_1 = G_3 \quad \rightarrow \quad m_1 = m_3$$

(ENKEL M_1 en M_2)

vrijlichaamsdiagram:



Kraftgleichgewicht

$$\vec{G}_1 + \vec{G}_2 + \vec{F}_2 = \vec{0}$$

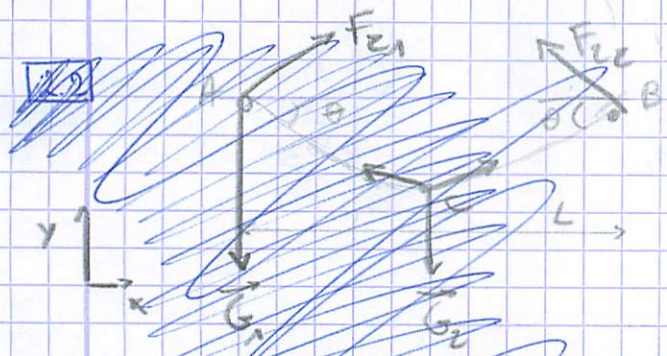
Momentengleichgewicht

$$\vec{r}_{G_1} \times \vec{G}_1 + \vec{r}_{G_2} \times \vec{G}_2 = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -G_1 & 0 \\ -2L & ? & 0 \end{vmatrix} = 2L\,G_1\,\vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -G_2 & 0 \\ L & ? & 0 \end{vmatrix} = -G_2 L\,\vec{e}_z$$

$$\Rightarrow \quad 2G_1 = G_2 \quad \Rightarrow \quad m_1 = \frac{1}{2} \cdot m_2$$

~~2.2~~



~~Kraftgleichgewicht~~

$$\begin{Bmatrix} 0 \\ -G_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_2 \end{Bmatrix} + \begin{Bmatrix} F_{21,x} \\ F_{21,y} \end{Bmatrix} + \begin{Bmatrix} F_{22,x} \\ F_{22,y} \end{Bmatrix} = 0$$

~~Momentengleichgewicht~~ (bzgl A)

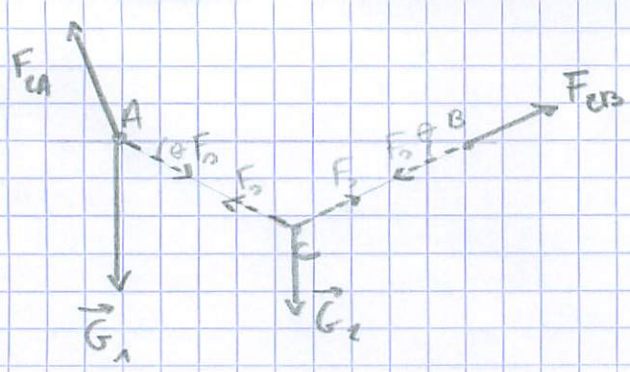
$$\vec{r}_{AC} \times \vec{G}_2 + \vec{r}_{AB} \times F_{22} = 0$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -340\,N & 0 \\ \frac{1}{2}L & ? & 0 \end{vmatrix} = -340\,N \cdot \frac{1}{2} L\,\vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ F_{22,x} & F_{22,y} & 0 \\ L & 0 & 0 \end{vmatrix} = L \cdot F_{22,y}\,\vec{e}_z$$

$$\Rightarrow \quad F_{22,y} = 170\,N$$

2.2

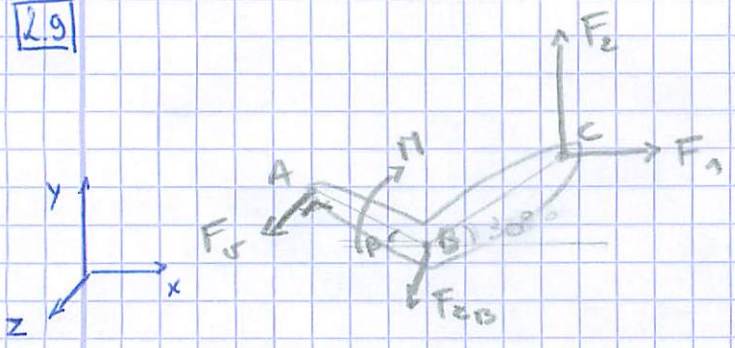


* $F_0 = \vec{G}_1 = 980\,N$
* krachtenevenwicht in punt C:

$$2 \cdot \sin\theta \cdot \vec{F}_0 = \vec{G}_2$$

$$\theta = \sin^{-1}\left(\frac{340}{2 \cdot 980}\right) = 10°$$

2.9



<u>momentenevenwicht</u> (rond B)

$$\vec{r}_{AB} \times \vec{F}_V + \vec{r}_{BC} \times \vec{F}_1 + \vec{r}_{BC} \times \vec{F}_2 + \vec{M} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -0{,}2 \cdot \cos\beta & +0{,}2 \cdot \sin\beta & 0 \\ \cos(90°-\beta) \cdot F_V & -\sin(90°-\beta) \cdot F_V & 0 \end{vmatrix} = 0{,}2 \cdot \cos\beta \cdot \sin(90°-\beta) \cdot F_V \cdot \vec{e}_z + 0{,}2 \cdot \sin\beta \cdot \cos(90°-\beta) \cdot F_V \cdot \vec{e}_z$$

$$= 0{,}2 \cdot \cos^2\beta \cdot F_V \cdot \vec{e}_z - 0{,}2 \cdot \sin^2\beta \cdot F_V \cdot \vec{e}_z$$

$$= 0{,}2 \cdot F_V \cdot \text{———} \cdot \vec{e}_z$$

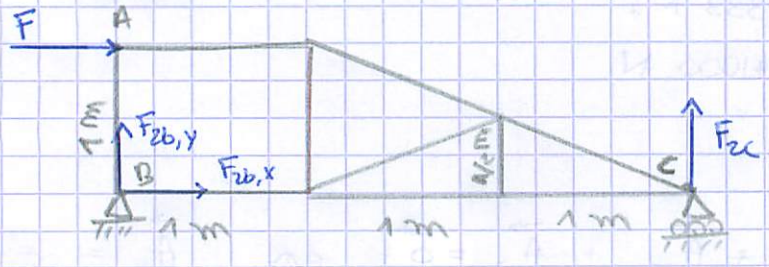
$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}5 \cdot \cos 30° & 0{,}5 \cdot \sin 30° & 0 \\ F_1 & F_2 & 0 \end{vmatrix} = 1732\,N\,\vec{e}_z - 125\,N \cdot \vec{e}_z$$

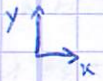
$$0{,}2 \cdot F_V \cdot \text{———}\ \vec{e}_z + 1607\,N\,\vec{e}_z - 500\,Nm \cdot \vec{e}_z = \vec{0}$$

$$F_V = 5535\,N$$

extra oefeningen MECHANICA (15.11.09)

2.10 geheel





momentenevenwicht (rond B)

$$\vec{r}_{AB} \times \vec{F} + \vec{r}_{BC} \cdot \vec{F}_{2c} = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ F & 0 & 0 \end{vmatrix} = -F \cdot \vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & 0 & 0 \\ 0 & F_{2c} & 0 \end{vmatrix} = 3F_{2c} \cdot \vec{e}_z$$

$$\Rightarrow F_{2c} = +333\ N \cdot \vec{e}_y$$

krachtenevenwicht

$$\vec{F} + \vec{F}_{2b} + \vec{F}_{2c} = \vec{0}$$

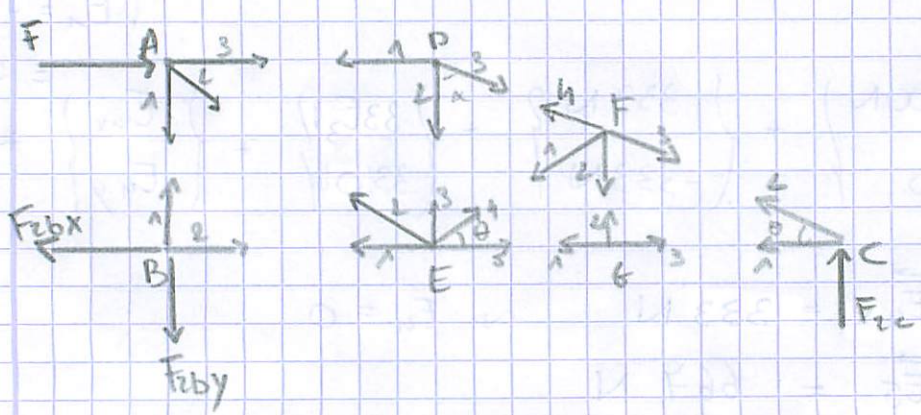
$$\begin{Bmatrix} 1000\ N \\ 0 \end{Bmatrix} + \begin{Bmatrix} F_{2b,x} \\ F_{2b,y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 333\ N \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow F_{2b,x} = -1000\ N$$

$$F_{2b,y} = -333\ N$$

Proper (constructie in evenwicht dus proper in evenwicht)

$\alpha = 63{,}4°$
$\theta = 26{,}6°$



Pivot B

$$\vec{F}_{2bx} + \vec{F}_{2by} + \vec{B}_1 + \vec{B}_2 = \vec{0}$$

$$\Rightarrow \vec{B}_1 = +333 \text{ N},$$
$$\vec{B}_2 = +1000 \text{ N}$$

Pivot A

* $\vec{F} + \vec{A}_1 + \vec{A}_2 + \vec{A}_3 = \vec{0}$ en $\vec{A}_1 = -\vec{B}_1$
$= -333 \text{ N}$

* $\vec{A}_{2y} = -\vec{A}_1 = +333 \text{ N} \rightarrow \vec{A}_2 = \frac{+333 \text{ N}}{\cos 45°} = +471 \text{ N}$
* $\vec{A}_{2x} + \vec{A}_3 = -\vec{F} = -1000 \text{ N}$

$$\Rightarrow \vec{A}_3 = -667 \text{ N}$$

Pivot C

* $\vec{F}_{2c} + \vec{C}_1 + \vec{C}_2 = \vec{0}$ en $\tan\theta = \frac{1}{2}$
$\Rightarrow \theta = 26{,}6°$

$$\begin{Bmatrix} 0 \\ F_{2c} \end{Bmatrix} + \begin{Bmatrix} C_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} C_{2x} \\ C_{2y} \end{Bmatrix} = \vec{0}$$

$$\Rightarrow \vec{C}_{2y} = -333 \text{ N}$$
$$\vec{C}_2 = 745 \text{ N}$$
$$\vec{C}_1 = -666 \text{ N} = -\vec{C}_{2x}$$

Pivot E (zie eerst Pivot D)

* $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \vec{E}_5 = \vec{0}$ met
$\vec{E}_3 = -\vec{D}_2 = 333 \text{ N}$
$E_2 = A_2 = 471 \text{ N}$
$\vec{E}_1 = -\vec{B}_2 = -1000 \text{ N}$

$$\hookrightarrow \begin{Bmatrix} -1000 \text{ N} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 333 \text{ N} \\ -333 \text{ N} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 333 \text{ N} \end{Bmatrix} + \begin{Bmatrix} E_{4x} \\ E_{4y} \end{Bmatrix} + \begin{Bmatrix} E_5 \\ 0 \end{Bmatrix} = 0$$

$$\Rightarrow \vec{E}_{4y} = 0 \text{ N} \rightsquigarrow E_4 = 0$$
$$\vec{E}_5 = 667 \text{ N}$$

Knoop D

$$\begin{Bmatrix} 667\,N \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ D_2 \end{Bmatrix} + \begin{Bmatrix} D_{3x} \\ D_{3y} \end{Bmatrix} = 0$$

$$\Rightarrow \vec{D}_1 = -\vec{A}_3 = 667\,N = -\vec{D}_{3x}$$

$$\Rightarrow D_3 = \cancel{\sin(63{,}4°)}\ \frac{D_{3x}}{\sin(63{,}4°)} = 745\,N$$

$$\vec{D}_2 = -333\,N$$

Knoop F (eerst Knoop G)

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

$$\Rightarrow \vec{F}_3 = -\vec{F}_4$$

met
$$\begin{cases} \vec{F}_1 = -\vec{E}_4 = \vec{0} \\ \vec{F}_2 = -\vec{G}_2 = \vec{0} \\ \vec{F}_3 = -\vec{C}_2 = 745\,N \\ F_4 = D_3 = 745\,N \end{cases}$$

Knoop G

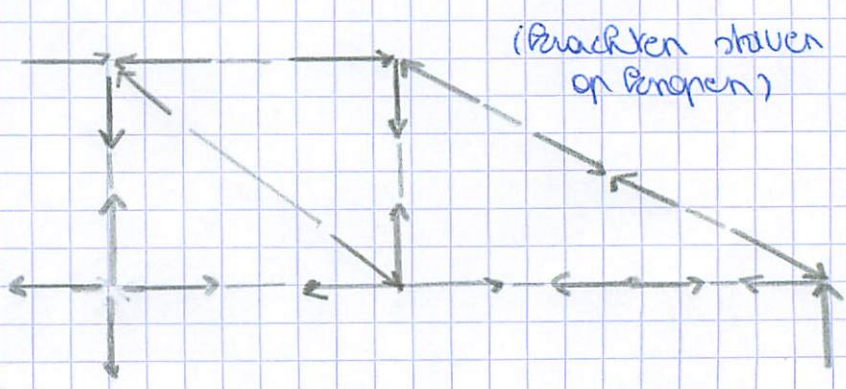
$$\vec{G}_1 + \vec{G}_2 + \vec{G}_3 = \vec{0}$$

$$\Rightarrow \vec{G}_2 = \vec{0}$$

met
$$\begin{cases} \vec{G}_3 = -\vec{C}_1 = 666\,N \\ \vec{G}_1 = -\vec{E}_5 = -666\,N \end{cases}$$

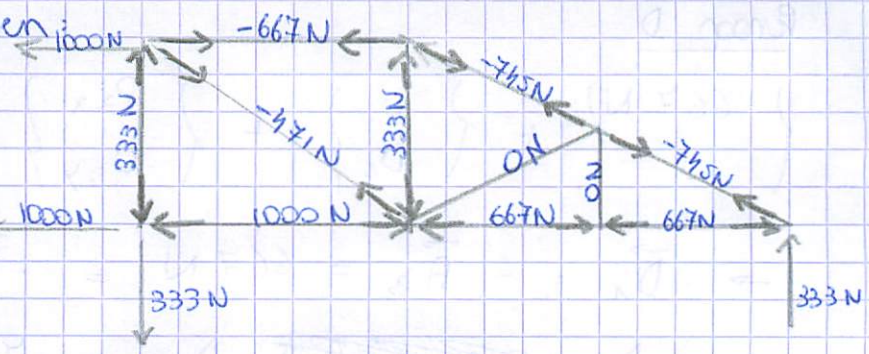
$\Longrightarrow$ samenvatting:

* Krachten:



(krachten staven op knopen)

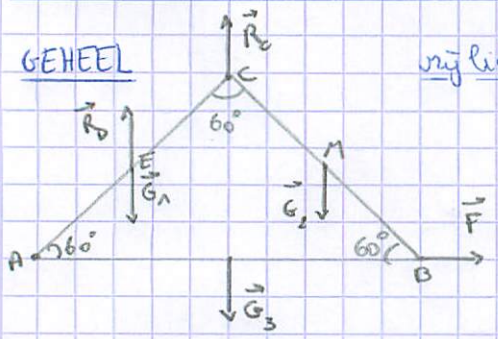
* shown


1000N
-667N
-745N
333N
-471N
333N
0N
0N
-745N
1000N
1000N
667N
667N
333N
333N

extra oefn mechanica

$\boxed{2.6}$ GEHEEL

vrijlichaamsdiagram



momentenevenwicht (aanuit C)

$$\vec{r}_{CE} \times (\vec{R}_D + \vec{G}_1) + \vec{r}_{CM} \times \vec{G}_2 + \vec{r}_{CB} \times \vec{F} = 0$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 \\ 0 & R_D + G_1 & 0 \end{vmatrix} = \frac{-R_D - G_1}{4} \vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 \\ 0 & G_2 & 0 \end{vmatrix} = \frac{G_2}{4} \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ F & 0 & 0 \end{vmatrix} = \frac{\sqrt{3}}{2} F \vec{e}_z$$

$$\Rightarrow R_D = -G_1 + G_2 + \sqrt{3} \cdot 2 F = 2\sqrt{3} F$$

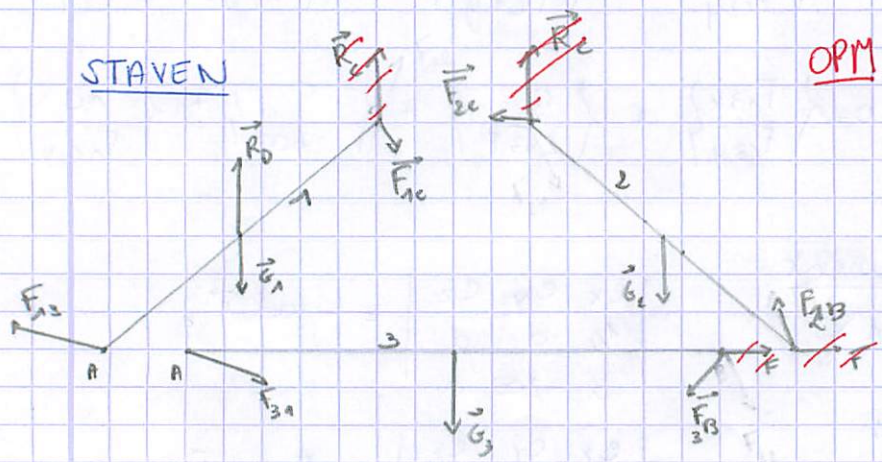
krachtenevenwicht

$$\begin{Bmatrix} 0 \\ R_D \end{Bmatrix} + \begin{Bmatrix} R_{Cx} \\ R_{Cy} \end{Bmatrix} + \begin{Bmatrix} 0 \\ +G_1 \end{Bmatrix} + \begin{Bmatrix} 0 \\ G_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ G_3 \end{Bmatrix} + \begin{Bmatrix} F \\ 0 \end{Bmatrix} = 0$$

$$\rightarrow R_{Cx} = -F$$

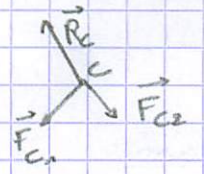
$$R_{Cy} = -2\sqrt{3} F + 600 \text{ N}$$

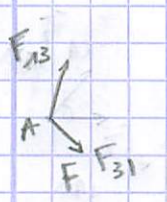
STAVEN

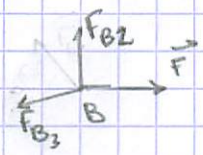


OPM: externe krachten grijpen aan in knoppunten, niet op staven zelf!

KNOPEN







~~momentenevenwicht staaf 1 (gericht A):~~

$$\cancel{\vec{r}_{AE} \times (\vec{R_D} + \vec{G_1}) + \vec{r}_{AC} \times \vec{F}_{12} = \vec{0}}$$

$$\cancel{\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{1}{4} & \frac{\sqrt{3}}{4} & 0 \\ 0 & 2\sqrt{3}F - 200 & 0 \end{vmatrix} = \frac{\sqrt{3}}{2} - 50\ \vec{e}_z \quad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ F_{12x} & F_{12y} & 0 \end{vmatrix} = \left(\frac{F_{12y}}{2} - \frac{\sqrt{3}}{2} F_{12x}\right) \vec{e}_z}$$

$$\cancel{\rightarrow \frac{F_{12y}}{2} - \frac{\sqrt{3}}{2} F_{12x} = \frac{\sqrt{3}}{2} - 50}$$

Krachtenevenwicht

* knoop A $\rightarrow$ $\vec{F}_{13} = -\vec{F}_{31}$
* staaf 3 $\rightarrow$ $\begin{Bmatrix} F_{31x} \\ F_{31y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -200 \end{Bmatrix} + \begin{Bmatrix} F_{3bx} \\ F_{3by} \end{Bmatrix} + \begin{Bmatrix} F \\ 0 \end{Bmatrix} = 0$
* knoop B $\rightarrow$ $\begin{Bmatrix} F_{b2x} \\ F_{b2y} \end{Bmatrix} + \begin{Bmatrix} F_{b3x} \\ F_{b3y} \end{Bmatrix} + \begin{Bmatrix} F \\ 0 \end{Bmatrix} = 0$
* staaf 2 $\rightarrow$ $0 = \begin{Bmatrix} F_{2bx} \\ F_{2by} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -200 \end{Bmatrix} + \begin{Bmatrix} F \\ 0 \end{Bmatrix} + \begin{Bmatrix} F_{2cx} \\ F_{2cy} \end{Bmatrix} + \begin{Bmatrix} -F \\ -2\sqrt{3}F + 600 \end{Bmatrix}$
* knoop C $\rightarrow$ $\begin{Bmatrix} F_{c2x} \\ F_{c2y} \end{Bmatrix} + \begin{Bmatrix} F_{c1x} \\ F_{c1y} \end{Bmatrix} + \begin{Bmatrix} -F \\ -2\sqrt{3} + 600 \end{Bmatrix} = 0$

verkeerd omdat: MFO
ik mag zomaar
niet, verkeerd
? geen -teken

* staaf 1 $\rightarrow$ $0 = \begin{Bmatrix} F_{13x} \\ F_{13y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 2\sqrt{3}F \end{Bmatrix} + \begin{Bmatrix} 0 \\ -600 \end{Bmatrix} + \begin{Bmatrix} F_{1cx} \\ F_{1cy} \end{Bmatrix} + \begin{Bmatrix} -F \\ -2\sqrt{3} + 600 \end{Bmatrix}$

momentenevenwicht

staaf 3 (zorg A)

$$\begin{vmatrix} e_x & e_y & e_z \\ 1/2 & 0 & 0 \\ 0 & -200 & 0 \end{vmatrix} = -100\ \vec{e}_z$$

$\rightarrow F_{3by} = 100N$

$$\begin{vmatrix} e_x & e_y & e_z \\ 1 & 0 & 0 \\ F_{3bx} + F & F_{3by} & 0 \end{vmatrix} = F_{3by}\ \vec{e}_z$$

vervolg oef $\boxed{2.6}$

* knoop A

krachtenevenwicht $\rightarrow$ $F_{13} = -F_{31}$

* staaf 3

- krachtenevenwicht :

$$\begin{Bmatrix} F_{31x} \\ F_{31y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -200 \end{Bmatrix} + \begin{Bmatrix} F_{3bx} \\ F_{3by} \end{Bmatrix} = 0$$

- momentenevenwicht rov A :

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{1}{2} & 0 & 0 \\ 0 & -200 & 0 \end{vmatrix} = -100\ \vec{e}_z \qquad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ F_{3bx} & F_{3by} & 0 \end{vmatrix} = F_{3by}\ \vec{e}_z$$

$\rightarrow$ $F_{3by} = +100\ N$

$F_{31y} = 100\ N$ of $F_{13y} = -100\ N$

$F_{31x} = -F_{3bx}$

* staaf 1

- momentenevenwicht tov C :

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ F_{13x} & F_{13y} & 0 \end{vmatrix} = \left(-\frac{F_{13y}}{2} + \frac{\sqrt{3}}{2} F_{13x} \right) \vec{e}_z$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -\frac{1}{4} & -\frac{\sqrt{3}}{4} & 0 \\ 0 & R_D + G & 0 \end{vmatrix} = -\frac{1}{4} \left(+2\sqrt{3} F - 200\ N \right) \vec{e}_z$$

$\Rightarrow$ $50N + \frac{\sqrt{3}}{2} F_{13x} - \frac{\sqrt{3}}{2} F + 50\ N = 0$

$\rightarrow$ $F_{13x} = -\frac{200}{\sqrt{3}} + F$

dus $F_{31x} = \frac{200}{\sqrt{3}} - F = -F_{3bx}$

$F_{3bx} = -\frac{200}{\sqrt{3}} + F$

- krachtenevenwicht:

$$\begin{Bmatrix} F_{13x} \\ F_{13y} \end{Bmatrix} + \begin{Bmatrix} 0 \\ R_D \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_1 \end{Bmatrix} + \begin{Bmatrix} F_{1cx} \\ F_{1cy} \end{Bmatrix} = 0$$

$$\Rightarrow F_{1cx} = -F + \frac{200}{\sqrt{3}}$$

$$F_{1cy} = 300 - 2\sqrt{3}F$$

* Knoop c

krachtenevenwicht:

$$\begin{Bmatrix} F_{c1x} \\ F_{c1y} \end{Bmatrix} + \begin{Bmatrix} F_{c2x} \\ F_{c2y} \end{Bmatrix} + \begin{Bmatrix} F_{2cx} \\ F_{2cy} \end{Bmatrix} = 0$$

$$\rightarrow F_{c2x} = F - F + \frac{200}{\sqrt{3}}$$

$$\text{maw } F_{2cx} = \frac{-200}{\sqrt{3}} \text{ N}$$

$$F_{c2y} = 2\sqrt{3}F - 600 + 300 - 2\sqrt{3}F$$

$$\text{maw } F_{2cy} = 300 \text{ N}$$

* Knoop B

krachtenevenwicht:

$$\begin{Bmatrix} F_{B2x} \\ F_{b2y} \end{Bmatrix} + \begin{Bmatrix} F_{b3x} \\ F_{b3y} \end{Bmatrix} + \begin{Bmatrix} F \\ 0 \end{Bmatrix} = 0$$

$$\rightarrow F_{b2x} = \frac{-200}{\sqrt{3}} + F - F$$

$$\text{maw } F_{2bx} = \frac{200}{\sqrt{3}} \text{ N}$$

$$F_{b2y} = -100 \text{ N}$$

$$\text{maw } F_{2by} = 100 \text{ N}$$

KINEMATICA VAN EEN PUNT

3.3

* $a_x = 2 \ m/s^2$

$v_x = 3 \ m/s$

* $\cos 30° = \dfrac{a_x}{a} \Rightarrow a = \dfrac{a_x}{\cos 30°} = \dfrac{2}{\frac{\sqrt{3}}{2}} = \dfrac{4}{\sqrt{3}} \ \dfrac{m}{s^2}$

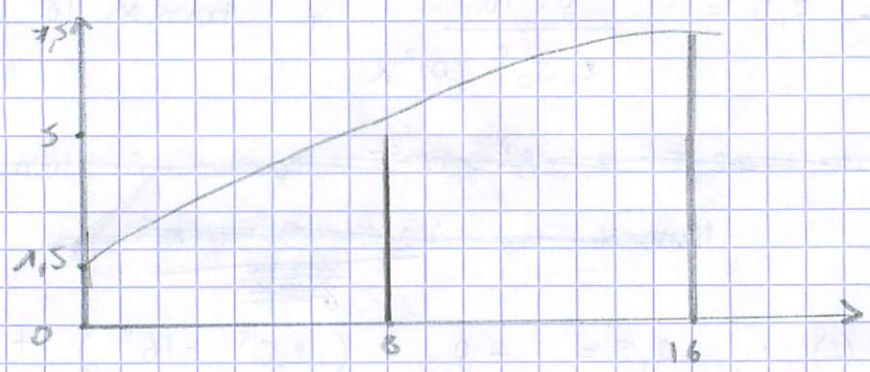
$= \dfrac{v_x}{v} \Rightarrow v = \dfrac{v_x}{\cos 30°} = \dfrac{3}{\frac{\sqrt{3}}{2}} = 2.\sqrt{3} \ \dfrac{m}{s}$

* $\sin 30° = \dfrac{a_y}{a} \Rightarrow a_y = a.\sin 30° = \dfrac{4}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{2}{\sqrt{3}} \ \dfrac{m}{s^2}$

$= \dfrac{v_y}{v} \Rightarrow v_y = v.\sin 30° = 2\sqrt{3} \cdot \dfrac{1}{2} = \sqrt{3} \ \dfrac{m}{s}$

$\Rightarrow \vec{v}_\uparrow = \begin{Bmatrix} 3 \\ \sqrt{3} \end{Bmatrix} \qquad \vec{a}_\uparrow = \begin{Bmatrix} 2 \\ \frac{2}{\sqrt{3}} \end{Bmatrix}$

3.4



$(\vec{r}(t) = r_x(t) . \vec{e}_x + \vec{r}_y(t) . \vec{e}_y)$

* $\vec{a}(t) = \begin{Bmatrix} 0 \\ -g \end{Bmatrix} = \dfrac{d\vec{v}}{dt}$

* $\vec{v}(t) = \int \vec{a}(t)\,dt = \int \begin{Bmatrix} 0 \\ -g \end{Bmatrix} dt$

$\hookrightarrow \vec{v}_x(t) = v_0 . \cos\alpha$

$\vec{v}_y(t) = -g.t + v_0 . \sin\alpha$

* $\vec{r}(t) = \int \vec{v}(t)\,dt = \int \begin{Bmatrix} v_0 . \cos\alpha \\ -gt + v_0 . \sin\alpha \end{Bmatrix} dt$

$\hookrightarrow \vec{r}_x(t) = v_0 . \cos\alpha \ . t$

$\vec{r}_y(t) = \dfrac{-gt^2}{2} + v_0 . \sin\alpha \ t + 1,5$

~~punten invullen~~

$$* \quad t = \frac{\vec{z}_x(t)}{v_0 \cdot \cos\alpha}$$

↓ invullen in $\vec{z}_y(t)$

$$\vec{z}_y(t) = \frac{-g \cdot \vec{z}_x(t)^2}{2 \cdot v_0^2 \cdot \cos^2\alpha} + \frac{\cancel{v_0} \cdot \sin\alpha \cdot \vec{z}_x(t)}{\cancel{v_0} \cdot \cos\alpha} + 1{,}5$$

* invullen van punten $(8;5)$ en $(16;7{,}5)$:

$$\begin{cases} 5 = \dfrac{-g \cdot 8^2}{2 \cdot v_0^2 \cdot \cos^2\alpha} + \tan\alpha \cdot 8 + 1{,}5 & (1) \\ 7{,}5 = \dfrac{-g \cdot 16^2}{2 \cdot v_0^2 \cdot \cos^2\alpha} + \tan\alpha \cdot 16 + 1{,}5 & (2) \end{cases}$$

$$\cancel{2{,}5 = 8^2 - 16^2 + \tan\alpha \cdot 8 - \tan\alpha \cdot 16}$$

$$\cancel{\tan\alpha = \frac{2{,}5 - 8^2 + 16^2}{8 - 16}}$$

↳ 2. (1) − (2) : $2{,}5 = \dfrac{-g}{2 v_0^2 \cdot \cos^2\alpha}\left(2 \cdot 8^2 - 16^2\right) + 1{,}5$

$$v_0 = \sqrt{\frac{-g}{2\cos^2\alpha}\left(2 \cdot 8^2 - 16^2\right)} = \sqrt{\frac{640}{\cos^2\alpha}}$$

$$= \frac{\sqrt{640}}{\cos\alpha}$$

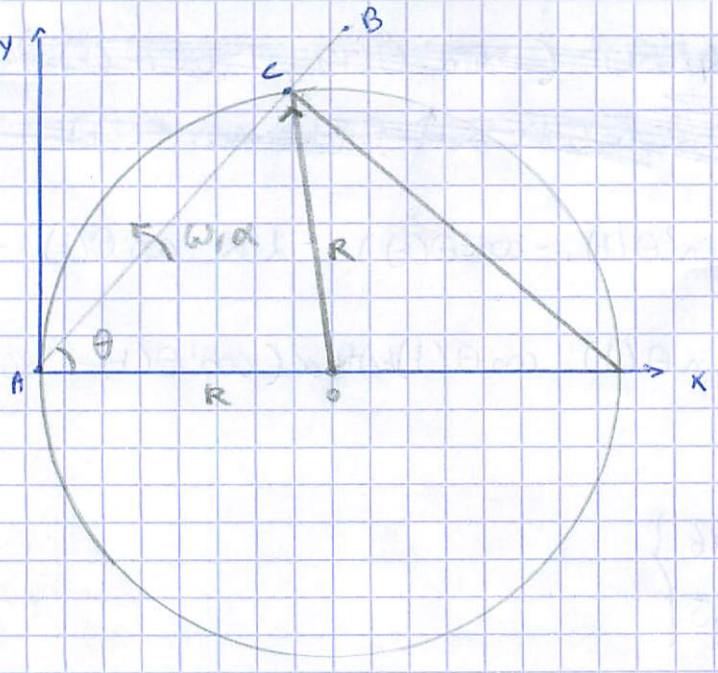
invullen in (2):

$$7{,}5 = \frac{-g \cdot \cancel{16^2}^{\,2}}{2 \cdot \frac{640}{\cancel{\cos^2\alpha}} \cancel{\cos^2\alpha}} + \tan\alpha \cdot 16 + 1{,}5$$

$$\tan^{-1}\alpha = 26{,}56^\circ = \alpha$$

$$v_0 = 28{,}1 \ m/s$$

$\boxed{3.7}$



$\omega = 3\,rad/s$

$\alpha = -1\,rad/s^2$

als $\theta = 60°$

$R = 20\,cm$

gr: $\vec{r}(t) = |AC| \cdot \cos\theta(t)\,\vec{e}_x + |AC| \cdot \sin\theta(t)\,\vec{e}_y$

$\cos\theta = \dfrac{|AC|}{2R}$ $\boxed{\theta = \omega \cdot t}$ $\boxed{\omega'(t) = \alpha(t)}$

$\vec{r}(t) = 2R \cdot \cos^2\theta(t)\,\vec{e}_x + 2R \cdot \cos\theta(t)\sin\theta(t)\,\vec{e}_y$

$\vec{v}(t) = -2R \cdot 2 \cdot \cos\theta(t) \cdot \sin\theta(t)\,\vec{e}_x \cdot \omega(t) +$
$2R \cdot \omega(t)\sin^2\theta(t) \cdot \vec{e}_y - 2R\,\omega(t)\cos^2\theta(t)\,\vec{e}_y$
$= 2R\,\omega(t)\left[-2\cos\theta(t) \cdot \sin\theta(t) \cdot \vec{e}_x - \sin^2\theta(t) \cdot \vec{e}_y + \cos^2\theta(t) \cdot \vec{e}_y\right]$

$\omega(t)$ afleiden → ook kan met α! (kettingregel)

$\vec{a}(t) = 2R\,\omega^2(t)\left[+2\sin^2\theta(t) \cdot \vec{e}_x - 2\cos^2\theta \cdot \vec{e}_x - 2\sin\theta(t) \cdot \cos\theta(t)\,\vec{e}_y - 2\sin\theta(t) \cdot \cos\theta(t) \cdot \vec{e}_y\right]$
$+ 2R\alpha \cdot \left[-2\cos\theta(t) \cdot \sin\theta(t) \cdot \vec{e}_x - \sin^2\theta(t)\,\vec{e}_y + \cos^2\theta(t) \cdot \vec{e}_y\right]$
$= 2R\,\omega^2\left[2\sin^2\theta(t) \cdot \vec{e}_x - 2\cos^2\theta \cdot \vec{e}_x - 4\sin\theta(t) \cdot \cos\theta(t)\,\vec{e}_y\right]$
$+ 2R\alpha \cdot \left[-2 \cdot \cos\theta(t) \cdot \sin\theta(t) \cdot \vec{e}_x - \sin^2\theta(t)\,\vec{e}_y + \cos^2\theta(t) \cdot \vec{e}_y\right]$

$\vec{v}_C = \begin{Bmatrix} -4R \cdot \omega \cdot \cos\theta(t) \cdot \sin\theta(t) \\ 2R\omega \cdot (\sin^2\theta(t) - \cos^2\theta(t)) \end{Bmatrix} = \begin{Bmatrix} -1{,}039 \\ -0{,}6 \end{Bmatrix}$

$$\cancel{* \vec{a}_c = \begin{Bmatrix} 4R\omega^2 (\sin^2\theta(t) - \cos^2\theta(t)) \\ -8R\omega^2 \sin\theta(t)\cos\theta(t) \end{Bmatrix} = \begin{Bmatrix} 3,6 \\ -6,235 \end{Bmatrix}}$$

$$* \vec{a}_c = \begin{Bmatrix} 2R\left[\omega^2(\sin^2\theta(t) - \cos^2\theta(t)) - \alpha \cos\theta(t) \sin\theta(t)\right] \\ 2R\left[4\omega^2 \sin\theta(t) \cos\theta(t) + \alpha(\cos^2\theta(t) - \sin^2\theta(t))\right] \end{Bmatrix}$$

$$= \begin{Bmatrix} 3,946 \\ -6,035 \end{Bmatrix}$$

$$* \; a_n = \frac{v^2}{\rho} = \dots$$

$$\begin{cases} v^2 = v_x^2 + v_y^2 \\ \rho = R \end{cases}$$

$$a_t = \frac{dv}{dt} = \sqrt{(-4R\omega(t)\cos\theta(t)\sin\theta(t))^2 + (-2R\omega(t)(\sin^2\theta(t) + \cos^2\theta(t)))^2}$$

$$= \frac{d\sqrt{v_x^2 + v_y^2}}{dt}$$

$$= \dots$$

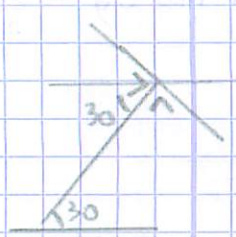
Extra Oefeningen MECHANICA (15. 11. '09)

3.6 a) $\vec{a} = a_t \cdot \vec{e}_t + a_2 \cdot \vec{e}_n$

$$= \frac{dv}{dt} \cdot \vec{e}_t + \frac{v^2}{\rho} \cdot \vec{e}_n$$

$$v = \omega \cdot R = 4776 \ \frac{m}{s}$$

$$\Rightarrow \vec{a} = \frac{v^2}{\rho}\vec{e}_n = \frac{v^2}{R} \cdot \vec{e}_n = 9,33 \ \frac{m}{s^2} \cdot \vec{e}_n$$



$$\vec{a} = 9,33 \ \frac{m}{s^2} \left(-\cos 30° \cdot \vec{e}_x - \sin 30° \cdot \vec{e}_y\right)$$

$$= -8,08 \ \frac{m}{s^2} \cdot \vec{e}_x - 4,67 \cdot \frac{m}{s^2} \cdot \vec{e}_y$$

b) $\vec{a} = -8,08 \ \frac{m}{s^2} \cdot \vec{e}_x - 4,67 \cdot \frac{m}{s^2} \cdot \vec{e}_y + 100 \ \frac{m}{s^2} \left(-\cos 60° \cdot \vec{e}_x + \sin 60° \cdot \vec{e}_y\right)$

$$= -58,08 \ \frac{m}{s^2} \cdot \vec{e}_x + 81,93 \cdot \frac{m}{s^2} \cdot \vec{e}_y$$

3.5 * <u>helling bepalen</u>:

$y = \sin x \rightarrow y' = +\cos x$

$\Rightarrow$ helling op $x = \frac{1}{3}$: $y'(\frac{1}{3}) = \cos \frac{1}{3} = 0,944$

$= \tan \alpha$

$\hookrightarrow \alpha = \tan^{-1}(0,944) = 43,4°$

* $\cos 43,4° = \frac{v_x}{v} \rightarrow v = 4,13 \ \frac{m}{s}$

$$= \begin{Bmatrix} 3 \\ 2,835 \end{Bmatrix} \frac{m}{s}$$

* <u>andere methode</u>

- $y = \sin x$
  $v_x = 3 \rightarrow x = 3t$ $\Big\}$ $y = \sin 3t$ of $v_y = 3 \cos 3t$
  $= 3 \cos x = 3 \cos \frac{1}{3}$
  $= 2,835$
- $a_x = v_x' = 0$ en $a_y = v_y' = -9 \sin 3t = -9 \sin x = -2,94$

$\boxed{3.9}$ * snelheid $\vec{v} = 7 \cdot \frac{m}{s} \cdot \vec{e}_x + 7 \frac{m}{s} \cdot \vec{e}_y$

versnelling $\vec{a} = -10 \frac{m}{s^2} \cdot \vec{e}_y$

* $\vec{a}(t) = -10 \frac{m}{s^2} \cdot \vec{e}_y$

$\vec{v}(t) = \int \vec{a}(t) = -10 \frac{m}{s^2} \cdot \vec{e}_y \cdot t + \frac{7m}{s} \cdot \vec{e}_x + 7\frac{m}{s} \cdot \vec{e}_y$

$\vec{r}(t) = \int v(t) = -5 \frac{m}{s^2} \cdot t^2 \cdot \vec{e}_y + 7 \frac{m}{s} \cdot t \cdot \vec{e}_y + 1 m \cdot \vec{e}_y + 7\frac{m}{s} \cdot t \cdot \vec{e}_x$

* $Q = (3, x)$

$\rightarrow \quad -5t^2 + 7t + 1 = 3$

$-5t^2 + 7t - 2 = 0$

$D = 9 = 3^2$

$t_1 = \frac{-3-7}{-10} = 1 s$

$t_2 = \frac{3-7}{-10} = \frac{2}{5} s$

$\Rightarrow$ x-positie: $r_x(t) = 7 \frac{m}{s} \cdot t$

$= \frac{14}{5} m$

* $|PQ| = \sqrt{(3-1)^2 + (\frac{14}{5} - 0)^2} = 3{,}44 \ m$

$\boxed{3.11}$ * baan bomaat: $\vec{r}(t) = -5t^2 \cdot \vec{e}_y + \cos\theta \cdot v_0 \cdot t \cdot \vec{e}_x + \sin\theta \cdot v_0 \cdot t \cdot \vec{e}_y + 2 \cdot \vec{e}_y$

$\hookrightarrow 20 \cdot \vec{e}_x = -5t^2 \cdot \vec{e}_y + \cos\theta \cdot v_0 \cdot t \cdot \vec{e}_x + \sin\theta \cdot v_0 \cdot t \vec{e}_y + 2\vec{e}_y$

$$\begin{Bmatrix} 0 \\ -5t^2 \end{Bmatrix} + \begin{Bmatrix} \cos\theta \cdot v_0 \cdot t \\ \sin\theta \cdot v_0 \cdot t \end{Bmatrix} + \begin{Bmatrix} -20 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad t = \frac{20}{\cos\theta \cdot v_0}$$

$$\Rightarrow \quad -5\left(\frac{20}{\cos\theta \cdot v_0}\right)^2 + \frac{\sin\theta \cdot \cancel{v_0} \cdot 20}{\cos\theta \cdot \cancel{v_0}} + 2 = 0$$

$$\frac{-2000}{\cos^2\theta \cdot v_0^2} + \tan\theta \cdot 20 + 2 = 0$$

$$-2000 + \tan\theta \cdot \cos^2\theta \cdot 20 \cdot v_0^2 + 2 \cdot \cos^2\theta \cdot v_0^2 = 0$$

$$v_0 = \sqrt{\frac{2000}{2\cos^2\theta\,(10\tan\theta + 1)}}$$

$* \; v_0$ minimaal $\Leftrightarrow \frac{v_0^2}{1000}$ minimaal

$$\Rightarrow \frac{1}{\cos^2\theta\,(10\tan\theta + 1)} \text{ minimaal}$$

$$\Updownarrow$$

afgeleide $= 0$

$$0 = -\left[2\cos\theta \cdot \sin\theta\,(10\tan\theta + 1) + \frac{\cos^2\theta \cdot 10 \cdot (\cos^2\theta + \sin^2\theta)}{\cos^2\theta}\right]$$

$$0 = \sin(2\theta)\,(10\tan\theta + 1) + 10\cos(2\theta)$$

$$\tan(2\theta)\,(10\tan\theta + 1) = -10$$

v_0 minimaal als $v_0' = 0$

$$\Rightarrow \quad 0 = \frac{1}{2\sqrt{\frac{2000}{2\cos^2\theta\,(10\tan\theta + 1)}}} \cdot \frac{-2000\,(4\cos\theta\sin\theta\,(10\tan\theta + 1) + 20)}{(2\cos^2\theta\,(10\tan\theta + 1))^2}$$

$$\Leftrightarrow \quad -2\sin 2\theta\,(10\tan\theta + 1) + 20 = 0$$

$$\theta = 42{,}14^\circ$$

$$\Rightarrow v_0 = 13{,}45 \ \frac{m}{s}$$

* vgl professor:

$$a(t) = 0,5 \ \frac{m}{s^2}$$

$$v(t) = 0,5\,t + v_0$$

$$x(t) = \frac{0,5}{2} t^2 + v_0 t$$

* invullen: 18 km/u na 20 m

$$\rightarrow \begin{cases} 5 = 0,5\,t + v_0 \\ 20 = \frac{0,5}{2} t^2 + v_0 t \end{cases} \rightarrow 20 = \frac{0,5}{2} \cdot t^2 + (5 - 0,5\,t)\,t$$

$$0 = -\frac{1}{4} t^2 + 5t - 20$$

$$t = \frac{+\sqrt{5} - 5}{-\frac{1}{2}}$$

$$\Rightarrow t = 3,6\ s \quad \Rightarrow v_0 = 3,19 \ \frac{m}{s}$$

* tomaat bereikt piek na bepaalde tijdsduur:

$$t = \frac{20}{\cos\theta \cdot v_0} = 2\ s$$

* positie prof: $t = 2 = 4 - 2$ $\rightarrow$

$\Rightarrow$ <u>antwoord</u>:

extra oefn mechanica (24.01.'10)

$\boxed{3.2}$ $v(t) = 30 \cos 60 \cdot \vec{e}_x + \left(\frac{-9{,}81}{2} t + 30 \sin 60\right) \cdot \vec{e}_y$

$\rightarrow \frac{v_y}{v_x} = -1$

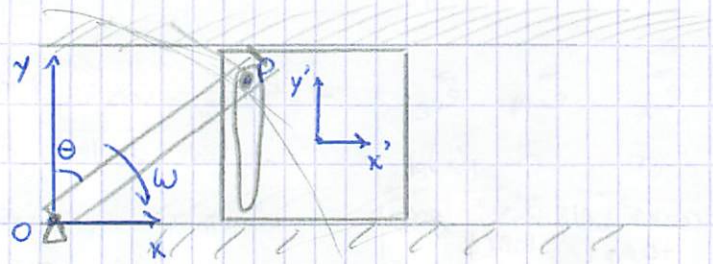
of max $t = 8{,}35$

$\Rightarrow \vec{v}(8{,}35) = 15 \frac{m}{s} \vec{e}_x - 15 \frac{m}{s} \vec{e}_y$

$\|\vec{v}\| = 21 \frac{m}{s}$

oefenzitting 4: kinematica van een lichaam

4.1



translatie in y-richting

* $\vec{v}_{abs} = \vec{v}_{sleep} + \vec{v}_{rel}$

↳ $\vec{v}_1 = \vec{v}_0'$ → translatie in x-richting

↳ $\vec{v}_2 = \vec{\omega} \times \vec{r}_q$ → geen relatie assenkruis ⇒ $\vec{v}_2 = 0$

↳ ! punt waar v_{abs}

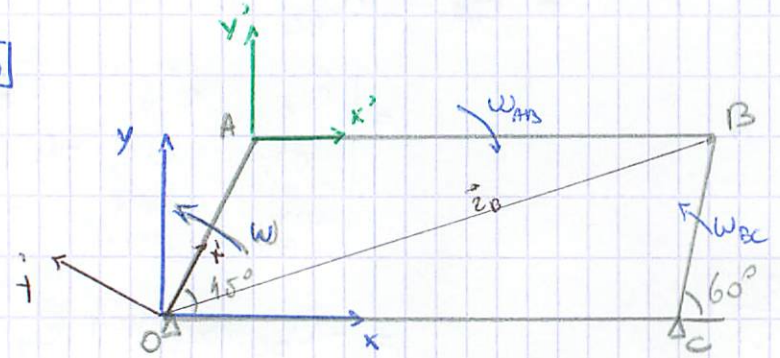
* $$\vec{v}_{abs} = \vec{\omega} \times \vec{OP} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega \\ \sin\theta \cdot |OP| & \cos\theta \cdot |OP| & 0 \end{vmatrix} = \omega \cdot \cos\theta \cdot |OP| \cdot \vec{e}_x - \omega \cdot \sin\theta \, |OP| \cdot \vec{e}_y$$

$$= 3\,\frac{rad}{s} \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\,m \cdot \vec{e}_x - 3\,\frac{rad}{s} \cdot \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\,m \cdot \vec{e}_y$$

$$= \frac{3\sqrt{2}}{4}\,\frac{m}{s}\,\vec{e}_x - \frac{3\sqrt{2}}{4}\,\frac{m}{s} \cdot \vec{e}_y$$

* $\vec{v}_{sleep}$ = x-component van $\vec{v}_{abs}$ = $\frac{3\sqrt{2}}{4}\,\frac{m}{s} \cdot \vec{e}_x$
* $\vec{v}_{rel}$ = y-component van $\vec{v}_{abs}$ = $-\frac{3\sqrt{2}}{4}\,\frac{m}{s} \cdot \vec{e}_y$

4.6



1 mogelijkheden van bewegend assenstelsel: <u>roterend</u> / <u>translerend</u>

↓

makkelijker

$$\vec{v}_{sleep} = \vec{OA} \times \vec{\omega} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega \\ \cos 45^\circ \cdot |OA| & \sin 45^\circ \cdot |OA| & 0 \end{vmatrix} = -\omega \cdot \sin 45^\circ \cdot |OA| \cdot \vec{e}_x + \omega \cdot \cos 45^\circ \cdot |OA| \cdot \vec{e}_y$$

$$= 6\,\frac{rad}{s} \cdot \frac{\sqrt{2}}{2} \cdot 0{,}35\,m \cdot \vec{e}_x + \ldots = -1{,}48\,\frac{m}{s}\,\vec{e}_x + 1{,}48\,\frac{m}{s} \cdot \vec{e}_y$$

$$\vec{v}_{rel} = \vec{\omega}_{AB} \times \vec{AB} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega_{AB} \\ 0{,}60 & 0 & 0 \end{vmatrix} = -\omega_{AB} \cdot 0{,}60 \cdot \vec{e}_y$$

$$\vec{v}_{abs} = \vec{\omega}_{BC} \times \vec{BC} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_{BC} \\ \cos 60^\circ \cdot |BC| & \sin 60^\circ \cdot |BC| & 0 \end{vmatrix} = \omega_{BC} \cdot \cos 60^\circ \cdot |BC| \cdot \vec{e}_y - \omega_{BC} \cdot \sin 60^\circ |BC| \, \vec{e}_x$$

met $\sin 60^\circ |BC| = \sin 45^\circ |OA|$

$\Rightarrow |BC| = 0{,}2857$

$\vec{v}_{abs} = \vec{v}_{sleep} + \vec{v}_{rel}$

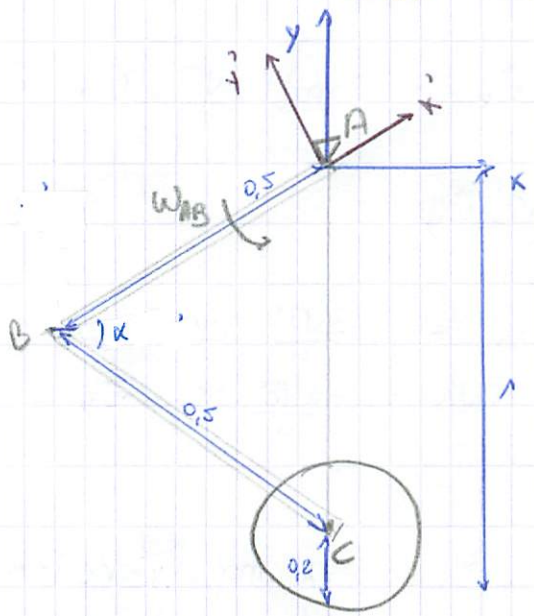
(enkel x-componenten)

$$\Rightarrow -\omega_{BC} \cdot \sin 60^\circ |BC| \cdot \vec{e}_x = -\omega_{OA} \cdot \sin 45^\circ \cdot |OA| \cdot \vec{e}_x$$

$$-\omega_{BC} \cdot \cancel{\sin 45^\circ} \cancel{|OA|} \cdot \cancel{\vec{e}_x} = -\omega_{OA} \cdot \cancel{\sin 45^\circ} \cdot \cancel{|OA|} \cdot \cancel{\vec{e}_x}$$

$$\omega_{BC} = \omega_{OA} = 6 \ \frac{\text{rad}}{s}$$

$\boxed{4.8}$



$$\alpha = 180^\circ - \angle \cos^{-1}\left(\frac{0{,}4}{0{,}5}\right)$$

$$= 106{,}26^\circ$$

$$\otimes \ \vec{v}_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_{AB} \\ 0 & -0{,}8 & 0 \end{vmatrix}$$

$$= +0{,}8\,\omega_{AB} \cdot \vec{e}_x$$

$$\otimes \ \vec{v}_{rel} = \vec{\omega}_{BC} \cdot \vec{BC}$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega_{BC} \\ \cos\alpha \cdot |BC| & -\sin\alpha \cdot |BC| & 0 \end{vmatrix}$$

$$= -\omega_{BC} \cdot \cos\alpha \cdot |BC| \cdot \vec{e}_y - \omega_{BC} \cdot \sin\alpha \cdot |BC| \cdot \vec{e}_x$$

$$\otimes \ v_{abs} = v_C \cdot \vec{e}_x$$

$\rightarrow \omega_{BC} \cdot \cos\alpha \cdot |BC| = 0$

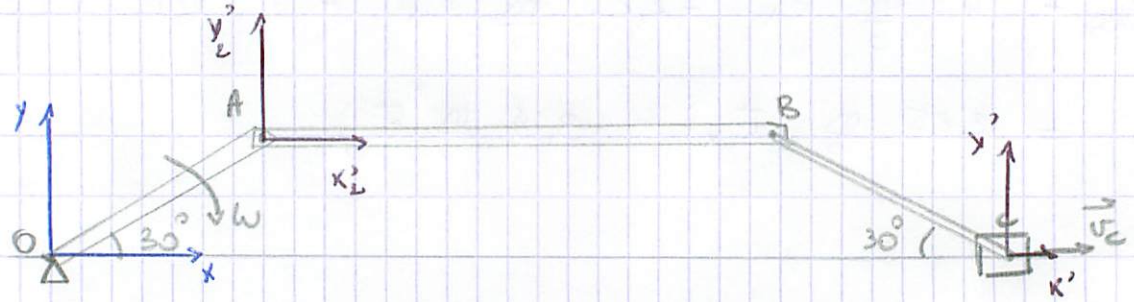
$\Rightarrow \omega_{BC} = 0$

$\hookrightarrow \vec{v}_{abs} = \vec{v}_{sleep} + \vec{v}_{rel} = 0{,}8\ \omega_{AB} \cdot \vec{e}_x + 0$

$= 1{,}6 \cdot \frac{m}{s} \cdot \vec{e}_x = v_C$

$\rightarrow \omega_C = \frac{v_C}{R} = \frac{1{,}6\ \frac{m}{s}}{0{,}2\ m} = 8\ \frac{rad}{s}$

$\boxed{4.7}$



$\boxed{\text{TEGENWIJZERZIN} = \omega \text{ POSITIEF}}$

$\vec{v}_{sleep,1} = v_C = 4{,}5\ \frac{m}{s} \cdot \vec{e}_x$

$\vec{v}_{rel,1} = \vec{\omega}_{BC} \times \vec{BC} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_{BC} \\ -\cos 30° \cdot 0{,}3 & \sin 30° \cdot 0{,}3 & 0 \end{vmatrix} = -\omega_{BC} \cdot 0{,}15 \cdot \vec{e}_x - \omega_{BC} \cdot \sqrt{3} \cdot 0{,}15 \cdot \vec{e}_y$

$\vec{v}_{sleep,2} = \vec{\omega}_{OA} \times \vec{OA} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega_{OA} \\ \cos 30° \cdot |OA| & \sin 30° \cdot |OA| & 0 \end{vmatrix}$

$= -\omega_{OA} \cdot \cos 30° \cdot |OA|\ \vec{e}_y + \omega_{OA} \cdot \sin 30° \cdot |OA| \cdot \vec{e}_x$

$= -15\ \frac{rad}{s} \cdot \frac{\sqrt{3}}{2} \cdot 0{,}3\ m\ \vec{e}_y + 15\ \frac{rad}{s} \cdot \frac{1}{2} \cdot 0{,}3\ m \cdot \vec{e}_x$

$= -3{,}897\ \frac{m}{s}\ \vec{e}_y + 2{,}25\ \frac{m}{s} \cdot \vec{e}_x$

$\vec{v}_{rel,2} = \vec{\omega}_{AB} \cdot \vec{AB} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega_{AB} \\ 0{,}4 & 0 & 0 \end{vmatrix} = -\omega_{AB} \cdot 0{,}4\ \vec{e}_y$

$$\vec{v}_{abs} = \vec{v}_{rel,1} + \vec{v}_{sleep,1} = \vec{v}_{rel,2} + \vec{v}_{sleep,2}$$

$$\Rightarrow \quad 4{,}5 \ \frac{m}{s} \cdot \vec{e}_x - \omega_{BC} \cdot 0{,}15 \cdot \vec{e}_x - \omega_{BC} \cdot \sqrt{3} \cdot 0{,}15 \cdot \vec{e}_y$$
$$= -3{,}897 \ \frac{m}{s} \cdot \vec{e}_y + 2{,}25 \ \frac{m}{s} \cdot \vec{e}_x - \omega_{AB} \cdot 0{,}4 \ \vec{e}_y$$

$$\hookrightarrow \left(4{,}5 \ \frac{m}{s} - \omega_{BC} \cdot 0{,}15\right) \cdot \vec{e}_x = 2{,}25 \ \frac{m}{s} \cdot \vec{e}_x$$

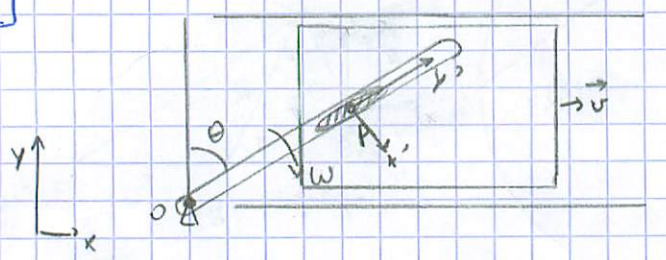
$$\omega_{BC} = 15 \ \frac{rad}{s}$$
$$(\rho = 15 \cdot 0{,}15)$$

$$\Rightarrow \vec{v}_{abs} = 4{,}5 \ \frac{m}{s} \cdot \vec{e}_x - 2{,}25 \cdot \frac{m}{s} \cdot \vec{e}_x - 15 \cdot 0{,}15 \cdot \sqrt{3} \cdot \vec{e}_y$$

$$= 2{,}25 \ \frac{m}{s} \cdot \vec{e}_x - 3{,}9 \ \frac{m}{s} \ \vec{e}_y$$

extra oefeningen mechanica 14.12.09

4.2



zel asverschuiving op roterende vast stamp

* $\vec{v}_{abs} = \vec{v}_{rel} + \vec{v}_{sleep}$

$\vec{v}_{abs}$ ↓ translatie in x-richting

$\vec{v}_{rel}$ ↓ rotatie + translatie

↳ rotatie rond O

* $\vec{v}_{sleep} = \vec{\omega} \times \vec{OP} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega \\ 0{,}5m.\sin\theta & 0{,}5m.\cos\theta & 0 \end{vmatrix} = \omega.0{,}5m.\cos\theta.\vec{e}_x - \omega.0{,}5m.\sin\theta.\vec{e}_y$

$= \frac{3}{4}\sqrt{2}.\vec{e}_x - \frac{3}{4}.\sqrt{2}.\vec{e}_y$

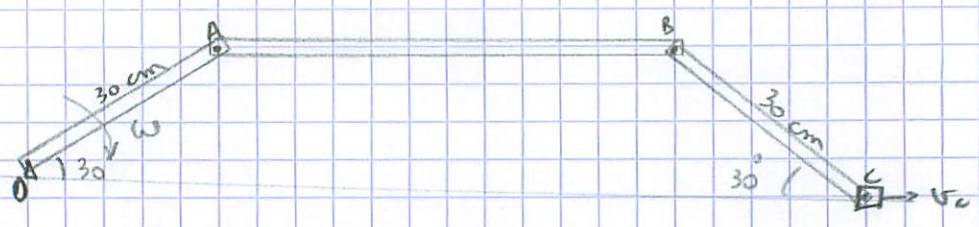
* $\vec{v}_{rel} \Rightarrow$ - y-richting : $\vec{v}_{abs,y} = 0 \Rightarrow \vec{v}_{rel,y} = -\vec{v}_{sleep,y}$

- x-richting ?

↳ $\vec{v}_{rel} \perp \vec{v}_{sleep}$ want P schuift naar boven/beneden in plenk

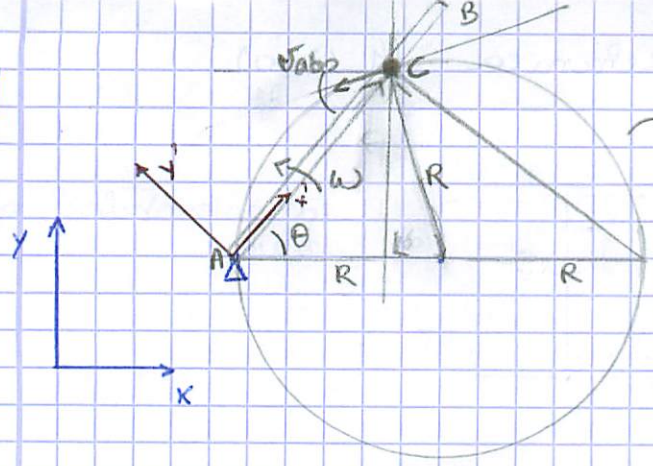
$(\theta = 45°) \Rightarrow$

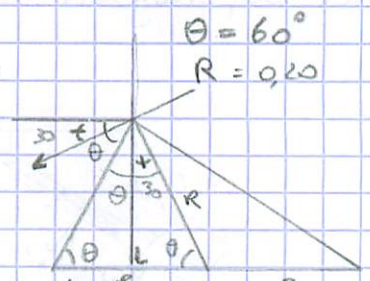
4.7



zie notities

$\boxed{4.5}$





(vpeljeveben v 60° ⇒ poljubno)

⇓

$|AC| = R$

$$v_{abs} = v_{sleep} + v_{rel}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\underset{\substack{\parallel \\ -v_a \cdot \cos 30}}{v_{a,x}} / \underset{\substack{\parallel \\ -v_a \cdot \sin 30}}{v_{a,y}} \qquad \omega \times |AC| \qquad \begin{Bmatrix} -v_2 \cdot e_x \\ 0 \end{Bmatrix} \longrightarrow \begin{Bmatrix} -v_2 \cdot \cos 60^\circ \cdot e_x \\ -v_2 \cdot \sin 60^\circ \cdot e_y \end{Bmatrix}$$

$$\downarrow$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega \\ |AC| \cdot \cos\theta & |AC| \cdot \sin\theta & 0 \end{vmatrix} = -\omega \cdot |AC| \cdot \sin\theta \cdot \vec{e}_x + \omega \cdot |AC| \cdot \cos\theta \cdot \vec{e}_y \qquad (|AC| = R)$$

$$\Rightarrow \quad v_{abs} = v_{sleep} + v_{rel}$$

$$\begin{Bmatrix} -v_a \cdot \cos 30 \\ -v_a \cdot \sin 30 \end{Bmatrix} = \begin{Bmatrix} -\omega \cdot R \cdot \sin 60 \\ \omega \cdot R \cos 60 \end{Bmatrix} + \begin{Bmatrix} -v_2 \cdot \cos 60 \\ -v_2 \cdot \sin 60 \end{Bmatrix}$$

$\rightarrow$

Kinematica van een lichaam (voorbereiding examen)

$\boxed{4.1}$ $v_{abs} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -3 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \end{vmatrix} = +\frac{3}{4}\sqrt{2}\,\vec{e}_x - \frac{3}{4}\sqrt{2}\,\vec{e}_y$

$v_{sleep} = \frac{3}{4}\sqrt{2}\,\vec{e}_x$

$v_{rel} = -\frac{3}{4}\sqrt{2}\,\vec{e}_y$

$\boxed{4.2}$ $v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -3 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \end{vmatrix} = \frac{3}{4}\sqrt{2}\,\vec{e}_x - \frac{3}{4}\sqrt{2}\,\vec{e}_y$

~~$v_{abs} = \frac{3}{4}\sqrt{2}\,\vec{e}_x$~~

~~$v_{rel} =$~~

$v_{abs} \rightarrow$ enkel x-richting

$v_{rel} \rightarrow$ hoek bekend!

$\Rightarrow \begin{Bmatrix} v_{rel} \cdot \cos 45° \\ v_{rel} \cdot \sin 45° \end{Bmatrix}$

$* \begin{Bmatrix} v_{abs} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \frac{3}{4}\sqrt{2} \\ -\frac{3}{4}\sqrt{2} \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 45° \\ v_{rel} \cdot \sin 45° \end{Bmatrix}$

$\Rightarrow v_{rel} = \frac{3}{2}\,\frac{m}{s} \rightarrow \vec{v}_{rel} = \begin{Bmatrix} \frac{3}{4}\sqrt{2} \\ \frac{3}{4}\sqrt{2} \end{Bmatrix}$

$\rightarrow \vec{v}_{abs} = \begin{Bmatrix} \frac{6}{4}\sqrt{2} \\ 0 \end{Bmatrix}$

$\boxed{4.3}$ $* \; v_{abs} = v_{sleep} + v_{rel}$

$* \; v_{abs} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -3 \\ \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 \end{vmatrix} = \frac{3}{4}\sqrt{2}\,\vec{e}_x - \frac{3}{4}\sqrt{2}\,\vec{e}_y$

$* \; v_{sleep} \rightarrow$ enkel x-richting

$v_{rel} \rightarrow$ cirkelbeweging van P $\rightarrow$ hoek bekend!

$\hookrightarrow \begin{Bmatrix} \frac{3}{4}\sqrt{2} \\ -\frac{3}{4}\sqrt{2} \end{Bmatrix} = \begin{Bmatrix} v_{sleep} \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 45° \\ v_{rel} \cdot \sin 45° \end{Bmatrix}$

$$\Rightarrow \vec{v}_{rel} = \begin{Bmatrix} -\frac{3}{4}\sqrt{2} \\ -\frac{3}{4}\sqrt{2} \end{Bmatrix}$$

$$\vec{v}_{sleep} = \begin{Bmatrix} \frac{2}{3}\sqrt{2} \\ 0 \end{Bmatrix}$$

$\boxed{4.4}$ * $v_{abs} = v_{sleep} + v_{rel}$

$$* \; v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -3 \\ 0{,}618 \cdot \frac{\sqrt{2}}{2} & 0{,}354 \cdot \frac{\sqrt{2}}{2} & 0 \end{vmatrix} = 0{,}75 \frac{m}{s}\vec{e}_x - 1{,}3 \frac{m}{s}\vec{e}_y$$

* v_{abs} → enkel x-richting

v_{rel} → hoek bekend!

$$\rightarrow \begin{Bmatrix} v_{abs} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0{,}75 \\ -1{,}3 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 45° \\ v_{rel} \sin 45° \end{Bmatrix}$$

$$\Rightarrow \vec{v}_{rel} = \begin{Bmatrix} 1{,}3 \\ 1{,}3 \end{Bmatrix}$$

$$\vec{v}_{abs} = \begin{Bmatrix} 2{,}05 \\ 0 \end{Bmatrix}$$

$\boxed{4.5}$ * $v_{abs} = v_{sleep} + v_{rel}$ $\qquad |AC| = 0{,}2$

$$* \; v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & 3 \\ \frac{1}{2} \cdot 0{,}2 & \frac{\sqrt{3}}{2} \cdot 0{,}2 & 0 \end{vmatrix} = -3 \cdot \sqrt{3} \cdot 0{,}1 \frac{m}{s}\vec{e}_x + 0{,}3 \frac{m}{s}\vec{e}_y$$

$$= -0{,}52 \frac{m}{s}\vec{e}_x + 0{,}3 \frac{m}{s}\vec{e}_y$$

* v_{rel} → beweegt in staaf ⇒ enkel y-component

v_{abs} → hoek bekend!

oppelet: enkel y'-component bekend

$$\hookrightarrow \begin{Bmatrix} v_{abs} \cdot \cos 60° \\ v_{abs} \sin 60° \end{Bmatrix} = \begin{Bmatrix} -0{,}52 \\ 0{,}3 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \\ 0 \end{Bmatrix}$$

* $v_{rel} \rightarrow$ enkel y'-component

$v_{abs} \rightarrow$ raakend aan cirkel

$$\Rightarrow \vec{v}_{rel} = \begin{Bmatrix} 0 \\ -v_{rel} \cdot y' \end{Bmatrix} = \begin{Bmatrix} -v_{rel} \cdot \cos 60^\circ \\ -v_{rel} \cdot \sin 60^\circ \end{Bmatrix}$$

$$\vec{v}_{abs} = \begin{Bmatrix} -v_{abs} \cdot \cos 30^\circ \\ -v_{abs} \cdot \sin 30^\circ \end{Bmatrix}$$

$$\hookrightarrow v_{abs} = v_{rel} + v_{sleep}$$

of maw: $$\begin{Bmatrix} -v_{abs} \cos 30^\circ \\ -v_{abs} \sin 30^\circ \end{Bmatrix} = \begin{Bmatrix} -0{,}52 \\ 0{,}3 \end{Bmatrix} + \begin{Bmatrix} -v_{rel} \cdot \cos 60^\circ \\ -v_{rel} \cdot \sin 60^\circ \end{Bmatrix} \begin{matrix} (1) \\ (2) \end{matrix}$$

$$\cancel{\Rightarrow v_{abs} = 0{,}6 + v_{rel}} \quad (\rightarrow \cancel{(1)^2 + (2)^2})$$

dubbelprod.

$$\Rightarrow \cancel{-(0{,}6 + v_{rel}) \cdot \cos 30^\circ = -0{,}52 - v_{rel} \cdot \cos 60^\circ}$$

$$\cancel{\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\right) v_{rel} = -0{,}52 + 0{,}6 \cdot \frac{\sqrt{3}}{2}}$$

$$\cancel{v_{rel} =}$$

$$\hookrightarrow v_{abs} = \frac{0{,}52}{\cos 30^\circ} + \frac{v_{rel} \cdot \cos 60^\circ}{\cos 30^\circ}$$

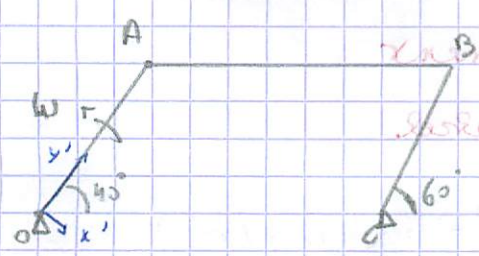
$$\Rightarrow -\frac{0{,}52}{\cos 30^\circ} \cdot \sin 30^\circ - \frac{\sin 30^\circ \cos 60^\circ}{\cos 30^\circ} \cdot v_{rel} = 0{,}3 - v_{rel} \cdot \sin 60^\circ$$

$$\rightarrow v_{rel} = -1{,}04 \ \frac{m}{s}$$

$$\Leftrightarrow v_{abs} = 1{,}2 \ \frac{m}{s}$$

$$\Rightarrow \vec{v}_{abs} = \begin{Bmatrix} -1{,}04 \\ -0{,}6 \end{Bmatrix}$$

4.6



verplaatsingen, snelheden + versn. x
stelsel naar knooppunt + ω

snelheid B berekenen

$* \; v_{abs} = v_{sleep} + v_{rel}$

→ v_{abs} raakend aan cirkel rond C

$$\rightarrow v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & 6 \\ 0{,}35 \cdot \cos 45° + 0{,}6 & 0{,}35 \sin 45° & 0 \end{vmatrix} = -1{,}48 \, \frac{m}{s} \vec{e}_x + 5{,}08 \, \frac{m}{s} \vec{e}_y$$

→ v_{rel} = cirkelbeweging van B rond A
⇒ enkel y-component

kopieerdeel

$$\rightsquigarrow v_{abs} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_z \\ 0{,}14 & 0{,}247 & \end{vmatrix} \qquad (|BC| = 0{,}285)$$

$$= -0{,}247 \, \omega_z \, \vec{e}_x + 0{,}14 \, \omega_z \, \vec{e}_y$$

$$* \quad \begin{Bmatrix} -0{,}247 \cdot \omega_z \\ 0{,}14 \cdot \omega_z \end{Bmatrix} = \begin{Bmatrix} -1{,}48 \\ 5{,}08 \end{Bmatrix} + \begin{Bmatrix} 0 \\ v_{rel} \end{Bmatrix}$$

$\Rightarrow \omega_z = 6$ rad/s

$\boxed{4.7}$ * bewegend assenstelsel : translerend in C

↳ $v_{abs_1} = v_{rel_1} + v_{sleep_1}$

→ v_{rel_1} = cirkelbeweging rond C

→ v_{sleep_1} = translatie van C

$\Rightarrow \vec{v}_{sleep_1} = \begin{Bmatrix} 4,5 \\ 0 \end{Bmatrix}$

$$\vec{v}_{rel_1} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_2 \\ -0,3.\cos 30° & 0,3 \sin 30° & 0 \end{vmatrix} = -0,3 \sin 30° \, \omega_2 \, \vec{e}_x - 0,3 \cos 30° \, \omega_2 \, \vec{e}_y$$

2 bewegende assenstelsels !

* bewegend assenstelsel : roterend rond O

$v_{abs_2} = v_{rel_2} + v_{sleep_2}$

→ v_{rel_2} = cirkelbeweging rond A ⇒ enkel y-component

$$\rightarrow v_{sleep_2} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -15 \\ 0,3.\cos 30° & 0,3.\sin 30° & 0 \end{vmatrix} = 2,25 \, \vec{e}_x - 3,9 . \vec{e}_y$$

$\Rightarrow v_{abs_1} = v_{abs_2} \Leftrightarrow v_{sleep_1} + v_{rel_1} = v_{sleep_2} + v_{rel_2}$

$$\hookrightarrow \begin{Bmatrix} -0,3 . \sin 30° \, \omega_2 \\ -0,3 \cos 30° \, \omega_2 \end{Bmatrix} + \begin{Bmatrix} 4,5 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -v_{rel_2} \end{Bmatrix} + \begin{Bmatrix} 2,25 \\ -3,9 \end{Bmatrix}$$

→ $\omega_2 = 15$ rad/s

→ $v_{abs} = 2,25 \, \frac{m}{s} \, \vec{e}_x - 3,89 \, \frac{m}{s} \, \vec{e}_y$

$\boxed{4.8}$ * translerend aanentrekel in B : extra : $A\hat{B}C = 106,3°$

$v_{abs,C} = v_{rel,C} + v_{sleep,C}$

$$\rightarrow v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & c \\ 0,5 \cdot \sin 36,87° & -0,5 \cdot \cos 36,87° & 0 \end{vmatrix} = 0,8\,\vec{e}_x - 0,6\,\vec{e}_y$$

$\rightarrow v_{abs}$: enkel x-component

$\rightarrow v_{rel}$: cirkelbeweging rond B

$$\hookrightarrow v_{rel} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_z \\ -0,5 \sin 36,87° & 0,5 \cos 36,87° & 0 \end{vmatrix} = -0,4\,\omega_z\,\vec{e}_x - 0,3\,\omega_z\,\vec{e}_y$$

$$\Rightarrow \begin{Bmatrix} v_{abs} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0,8 \\ -0,6 \end{Bmatrix} + \begin{Bmatrix} -0,4\,\omega_z \\ -0,3\,\omega_z \end{Bmatrix}$$

$\rightarrow \omega_z = -2$

$$\Rightarrow v_{abs} = \begin{Bmatrix} 1,6 \\ 0 \end{Bmatrix}$$

* $v = \omega \cdot R \Rightarrow \omega_{wiel} = \frac{1,6}{0,2} = 8$ rad/s

* $\omega_{BC} = \omega_{AB} + \omega_z = 0$

→ kan rechtstreeks gevonden worden mbv translerend aanentrekel in A

$\boxed{4.9}$ * translerend assenstelsel in P :

↳ $v_{abs} = v_{rel} + v_{sleep}$

bepaling v_{blok} rel tov C

$$* \; v_{sleep} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega \\ 0{,}1 \cdot \sin 60^\circ & -0{,}1 \cdot \cos 60^\circ & 0 \end{vmatrix} \qquad \text{met } \omega = \frac{v}{R} = \frac{0{,}6}{0{,}2} = 3 \frac{rad}{s}$$

$$\Rightarrow \quad = -0{,}15 \frac{m}{s} \vec{e}_x - 0{,}21 \frac{m}{s} \vec{e}_y$$

* v_{rel} ⇒ enkel y-component

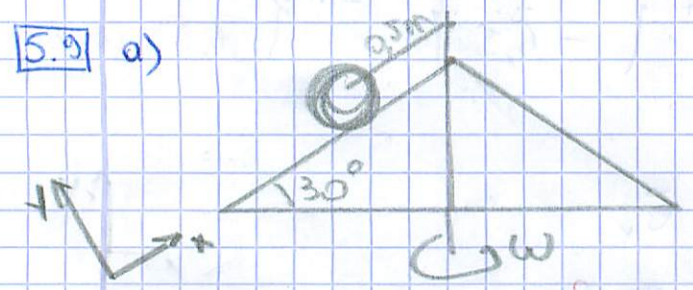
* v_{abs} ⇒ enkel x-component

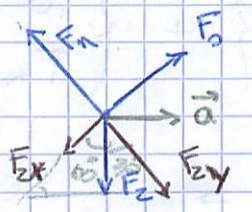
$$\Rightarrow v_{blok,rel} = -0{,}15 \frac{m}{s}$$

$$v_{blok} = 0{,}45 \frac{m}{s} = 0{,}6 \frac{m}{s} - 0{,}15 \frac{m}{s}$$

Oefenzitting 5 : Postulaten van Newton

5.9 a)





$$\Sigma \vec{F} = m \cdot \vec{a}$$

$$\rightarrow \vec{a} = \vec{a}_n \cdot \vec{e}_n + \cancel{\vec{a}_t \cdot \vec{e}_t}$$

$$= \frac{v^2}{R} \cdot \vec{e}_n$$

$$= \frac{\omega^2 R^2}{R} \cdot \vec{e}_n = \omega^2 \cdot R \cdot \cos 30° \cdot \vec{e}_n = 6{,}93 \text{ N} \cdot \vec{e}_n$$

$$\rightarrow \vec{F}_2 = -0{,}750 \text{ kg} \cdot 10 \frac{\text{N}}{\text{kg}} \cdot \vec{e}_y = -7{,}5 \text{ N} \cdot \vec{e}_y$$

$$\rightarrow \vec{F}_{2x} = F_2 \cdot \cos 60° = -3{,}75 \text{ N} \cdot \vec{e}_x$$

$$\vec{F}_{2y} = F_2 \cdot \cos 30° = -6{,}5 \text{ N} \cdot \vec{e}_y$$

$$\cancel{\Rightarrow \vec{F}_n + \vec{F}_{2t} = m \cdot \vec{a}}$$

$$\cancel{\vec{F}_n = m \cdot \vec{a} + \vec{F}_{2t}}$$

$$\cancel{= 3{,}75 \text{ N} \cdot \vec{e}_t + 6 \text{ N} \cdot \vec{e}_t = 9{,}75 \text{ N} \cdot \vec{e}_t}$$

$$\cancel{= \frac{6{,}93}{\sin 30°} \text{ N} \cdot \vec{e}_t + \frac{6{,}93}{\cos 30°} \text{ N} \cdot \vec{e}_n +}$$

$$\cancel{\Rightarrow \vec{F}_n + \vec{F}_2 + \vec{F}_n = m \cdot \vec{a}}$$

$$\cancel{\Rightarrow \vec{F}_n = m\vec{a} - \vec{F}_n - \vec{F}_2 = 5{,}20 \vec{e}_x - \vec{F}_n - \vec{F}_{2x}}$$

$$\cancel{= -\vec{F}_{2y} = 5{,}20 \cdot \vec{e}_x + 6{,}5 \vec{e}_y - \vec{F}_n - n}$$

$$\cancel{=}$$

$$\vec{F}_s + \vec{F}_n + \vec{F}_z = m.\vec{a}$$

$$\begin{Bmatrix} F_s \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_n \end{Bmatrix} + \begin{Bmatrix} -3,75 \\ -6,5 \end{Bmatrix} = m. \begin{Bmatrix} 6,93 . \sin 60° \\ 6,93 . \cos 60° \end{Bmatrix}$$

$$\Rightarrow \vec{F}_s = 8,25 \text{ N} . \vec{e}_x$$

$$\vec{F}_n = 9,10 \text{ N} . \vec{e}_y \; ?$$

b) $m . \vec{a}^2 . \cos 60° = F_{z,y}$

$$\omega = \sqrt{\frac{F_{zy}}{m \cos(60) . R . \cos(30)}} = 6,32 \frac{\text{rad}}{s}$$

$\boxed{5.5}$

geheel:

$$\vec{F} = m . \vec{a} \rightarrow \frac{100 \text{ N}}{10 \text{ kg}} = \vec{a} = 10 \frac{m}{s^2}$$

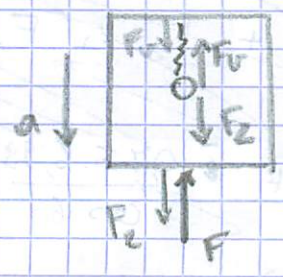
blok 1

$$\vec{F}_v + \vec{F} = m . \vec{a} \Rightarrow \vec{F}_v = 90 \text{ N} - 100 \text{ N} = -10 \text{ N}$$

veer

$$F_v = k . \Delta x \Rightarrow \Delta x = \left| \frac{-10 \text{ N}}{20 \frac{N}{m}} \right| = 0,5 \text{ m}$$

$\boxed{5.8}$



geheel

$$\vec{F} = m \cdot \vec{a} \rightarrow \vec{F}_{z,tot} + \vec{F} = m_{tot} \vec{a}$$

$$\Rightarrow \vec{a} = -6 \ \frac{m}{s^2}$$

blokje

$$\vec{F} = m \cdot \vec{a} \rightarrow \vec{F}_{z,1} + \vec{F}_v = m_1 \cdot \vec{a}$$

$$\Rightarrow \vec{F}_v = \vec{F}_{z1} - m_1 \cdot \vec{a}$$

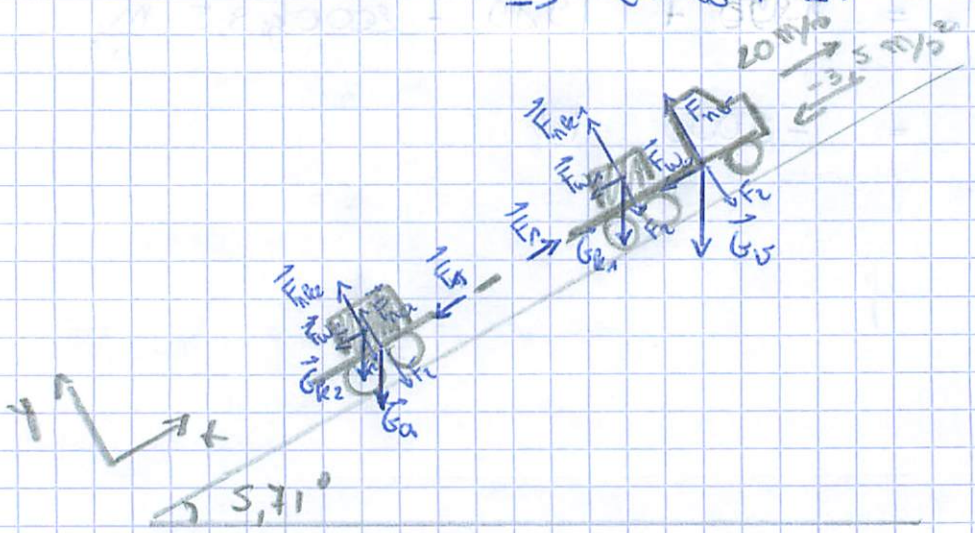
$$= 10 \ N - 6 \ N = 4 \ N$$

veer

- kbepalen: $\vec{F}_v = k \cdot \Delta x = \vec{F}_{z,1} \Rightarrow \frac{10 \ N}{0{,}05 \ m} = k = 200 \ \frac{N}{m}$
- uitrekking: $\vec{F}_v = k \cdot \Delta x \Rightarrow \Delta x = \frac{4 \ N}{200 \ \frac{N}{m}} = 0{,}02 \ m$

$$\Rightarrow l = l_0 + \Delta x = 0{,}22 \ m$$

$\boxed{5.12}$



OPM:

a) $\vec{F}_{nk1} + \vec{F}_{w1} + \vec{G}_{k1} = m_{k1} \cdot \vec{a}$

$$\begin{Bmatrix} 0 \\ F_{nk1} \end{Bmatrix} + \begin{Bmatrix} -F_{nk1} \cdot \mu_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -\cos(84{,}29°) \cdot \vec{G}_{k1} \\ -\sin(84{,}29°) \cdot \vec{G}_{k1} \end{Bmatrix} = m_{k1} \begin{Bmatrix} -3{,}5 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \vec{F}_{nk1} = +9950{,}4 \ N \cdot \vec{e}_y$$

$$\vec{F}_{w1} = -1990 \cdot N \cdot \vec{e}_x$$

$$\vec{F}_{nk_2} + \vec{F}_{w_2} + \vec{G}_{k_2} = m_{k_2} \cdot \vec{a}$$

$$\begin{Bmatrix} 0 \\ F_{nk_2} \end{Bmatrix} + \begin{Bmatrix} -F_{nk_2} \cdot \mu_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -\cos(84{,}29°)\, \vec{G}_{k_2} \\ -\sin(84{,}29°)\, \vec{G}_{k_2} \end{Bmatrix} = m_{k_2} \begin{Bmatrix} -3{,}5 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \vec{F}_{nk_2} = 9950{,}4\ N \cdot \vec{e}_y$$

$$\vec{F}_w = -3{,}5 \cdot 1000\,kg + (\cos 84{,}29°) \cdot 10000\ N$$

$$= -2505\ N$$

b) ~~$\vec{F}_0 + \vec{F}_{w_2} + \vec{F}_{w_0} + \cos 84{,}29° \cdot g(m_{k_1} + m_{k_2}) = m \cdot \vec{a}$~~

~~$\vec{F}_0 = -6000\,kg \cdot 3{,}5\,\frac{m}{s^2} + 1990\,N - \vec{F}_{w_0} + 5970\,N$~~

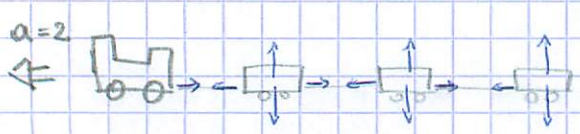
AAN: $\vec{F}_0 + \vec{F}_{w_2}$ (→ interne kracht, dus weglaten!) $+ \cos 84{,}29° \cdot g(m_{k_2} + m_{aan}) = m \cdot \vec{a}$

$$\vec{F}_0 = \cancel{2500} + 1989 - 2000\,kg\, 3{,}5\,\frac{m}{s^2}$$

$$= -5000\ N$$

extra oefeningen (25.01.'10)

5.2 $a = 2$



laatste wagon

$F = m \cdot a$

$F_{s_3} = 1000\ kg \cdot 2\ \frac{m}{s} = 2000\ N$

voorlaatste wagon

$F = m \cdot a$

$F_{s_2} - F_{s_3} = 900\ kg \cdot 2\ \frac{m}{s}$

$F_{s_2} = 3800\ N$

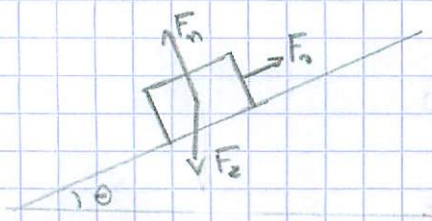
eerste wagon

$F = m \cdot a$

$F_{s_1} - F_{s_2} = 600\ kg \cdot 2\ \frac{m}{s}$

$F_{s_1} = 5000\ N$

5.4 a)



$F_z = 100\ N$

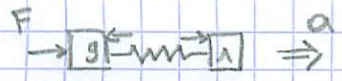
$F_n = 86{,}6\ N = \cos 30° \cdot 100\ N$

$F_s = 50\ N = \sin 30° \cdot 100\ N$

b) $F = m \cdot a$

$a = \frac{50\ N}{10\ kg} = 5\ \frac{m}{s^2}$

5.5



* $F = m \cdot a$

$\rightarrow \quad a_1 = \dfrac{100\,N - F_s}{9}$

$a_2 = \dfrac{F_s}{1}$

$\rightsquigarrow \quad a_1 = a_2 \quad \Rightarrow \quad \dfrac{100}{9} - \dfrac{F_s}{9} = F_s$

$F_s = 10\,N$

* $F_s = k(l_0 - l)$

$\rightarrow \quad l = \frac{1}{2}\,m$

5.6

OPM: centripetale kracht bestaat niet!
→ enkel versnelling!

* $F_{2,y} = G$

$\Rightarrow \quad F_2 = \dfrac{F_{2y}}{\sin 30°} = \dfrac{G}{\sin 30°} = 100\,N$

~~$F_c = \cos 30° \cdot 100\,N = 86{,}6\,N$~~

~~* $F_c = m \cdot a = m \cdot \omega^2 R$~~

~~$\omega = \sqrt{\dfrac{F_c}{m \cdot R}} = \sqrt{\dfrac{86{,}6\,N}{5 \cdot 0{,}2 \cdot \sin 60°}}$~~

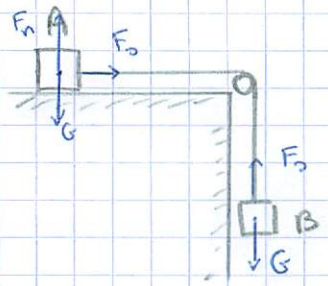
~~$= 10\ rad/s$~~

* $F_2 = \cos 30° \; 100\,N = 86{,}6\,N$

$\rightarrow \quad F_2 = m \cdot a_c = m\,\omega^2 R$

$\omega = \sqrt{\dfrac{F_2}{mR}} = \sqrt{\dfrac{86{,}6\,N}{5 \cdot 0{,}2 \cdot \sin 60°}} = 10\ rad/s$

5.7



* $F = m.a$

$\rightarrow \quad a_1 = \dfrac{F_s}{m_1}$

$a_2 = \dfrac{G - F_s}{m_2}$

$a_1 = a_2 \Rightarrow \dfrac{F_s}{2m_b} = \dfrac{10\,m_b - F_s}{m_b}$

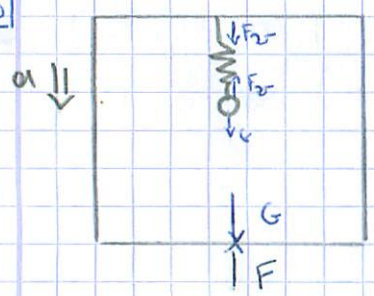
$\dfrac{3F_s}{2m_b} = +10$

$F_s = 6{,}66 . m_b$

opgelet voor volgende: $G - F_s$ ipv $F_s - G$ want $G > F_s$

$\Rightarrow a = \dfrac{6{,}66\ m_b}{2\ m_b} = 3{,}33\ \dfrac{m}{s^2}$

5.8



* geheel:

$F = m . a$

$a = \dfrac{F - G}{m} = \dfrac{400\ N - 1000\ N}{100\ kg} = -6\ \dfrac{m}{s^2}$

* bal:

$F = m . a$

$F_v - G = m . a$

$F_v = 1\,kg(-6\ \dfrac{m}{s^2}) + 10\ N$

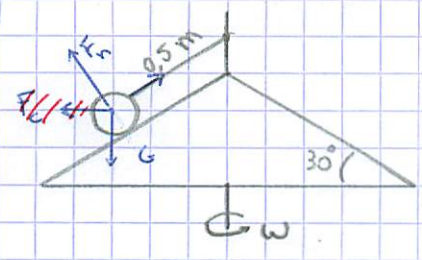
$= 4\ N$

* bepaling k

$F_v = k\,(l_1 - l_0) \rightarrow k = \dfrac{10\ N}{(0{,}25 - 0{,}2)\,m} = 200\ \dfrac{N}{m}$

* lengte: $\Rightarrow F_v = k\,(l_1 - l_0)$ of maw $l_1 = \dfrac{4}{200} + 0{,}20 = 0{,}22$

5.9



~~a)~~ $\cancel{F_c = m \cdot \omega^2 R}$

$\cancel{= 0{,}75 \cdot 4^2 \cdot 0{,}5 \cdot \cos 30°}$

$\cancel{= 5{,}19\ N}$

$\cancel{F_c = 0}$, ~~want nog niet van kegel~~ !

F_c bestaat niet

~~* G = 7,5 N~~

~~* $F_n = \cos 30° \cdot 7{,}5\ N = 6{,}5\ N$~~

~~* $F_s = \cos 60° \cdot 7{,}5\ N = 3{,}75\ N$~~

rekening houden met versnelling!

a) * $\vec{G} = -7{,}5\ N \cdot \vec{e}_y$

* $\vec{F} = m \cdot \vec{a}$

$$\hookrightarrow \begin{Bmatrix} 0 \\ -7{,}5\ N \end{Bmatrix} + \begin{Bmatrix} -\cos 60° \cdot F_n \\ \sin 60° \cdot F_n \end{Bmatrix} + \begin{Bmatrix} \cos 30° \cdot F_s \\ \sin 30° \cdot F_s \end{Bmatrix} = m \begin{Bmatrix} \omega^2 \cdot R \\ 0 \end{Bmatrix}$$

$$\hookrightarrow F_n = \frac{7{,}5 - \sin 30° \cdot F_s}{\sin 60°}$$

$$\Rightarrow -\cos 60° \left(\frac{7{,}5 - \sin 30° \cdot F_s}{\sin 60°} \right) + \cos 30°\ F_s = 0{,}75 \cdot 4^2 \cdot 0{,}5 \cdot \cos 30°$$

$$\Leftrightarrow F_s = \frac{9{,}52}{1{,}15} = 8{,}28\ N$$

$$F_n = 3{,}87\ N$$

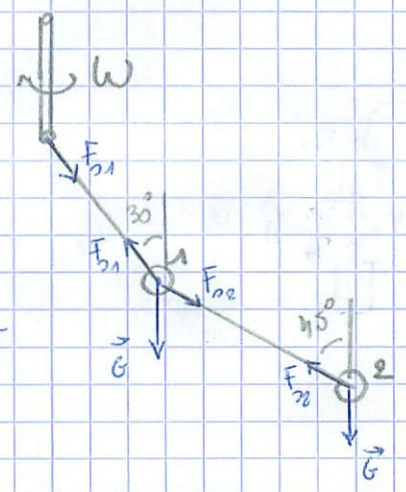
b) bal komt los wanneer $F_n = 0$ en dus $F_{s,y} = G$

$$\Leftrightarrow F_s = \frac{7{,}5}{\sin 30°} = 15\ N$$

of maw $F_c = m \cdot a = m \cdot \omega^2 \cdot R$

<u>dus</u> : $\omega = \sqrt{\dfrac{15 \cdot \cos 30°}{0{,}5 \cdot \cos 30° \cdot 0{,}75}} = 6{,}32\ \dfrac{m}{s^2}$

5.10



$* \; F = m \cdot a = m R \cdot \omega^2$

$$\rightarrow \omega_2^2 = \frac{F_{02} \cdot \frac{\sqrt{2}}{2}}{m_2 \cdot \frac{(1+\sqrt{2})}{4}}$$

$$\omega_1^2 = \frac{-F_{02} \cdot \frac{\sqrt{2}}{2} + F_{01} \cdot \frac{1}{2}}{m_1 \cdot \frac{1}{4}}$$

$\rightsquigarrow \; \omega_1 = \omega_2$ en $\quad F_{02} = \frac{2}{\sqrt{2}} \cdot 10 \, m_2$

$F_{01} = \frac{2}{\sqrt{3}} \left(10 \, m_2 + 10 \, m_1 \right)$

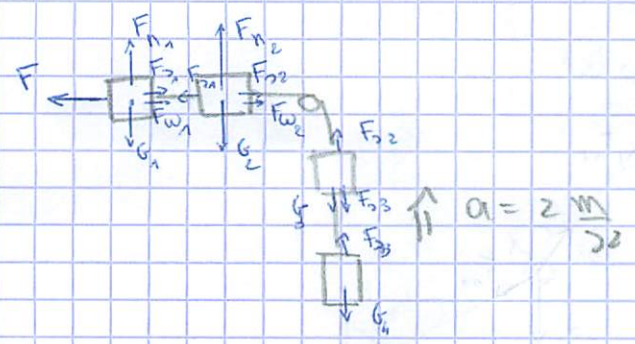
$$\Rightarrow \omega = \frac{10 \, \cancel{m_2} \, \frac{\sqrt{2}}{2} \, \frac{2}{\sqrt{2}}}{\cancel{m_2} \, \frac{(1+\sqrt{2})}{4}} = 4{,}07 \text{ rad/s}$$

$$\hookrightarrow \frac{-\frac{\sqrt{2}}{2} \, 10 \, m_2 \, \frac{2}{\sqrt{2}} + \frac{1}{2} \cdot \frac{2}{\sqrt{3}} \, 10 \left(m_2 + m_1 \right)}{m_1 \, \frac{1}{4}} = 4{,}14^2$$

$$\left(-10 + \frac{10}{\sqrt{3}} \right) m_2 = m_1 \left(\frac{10}{(1+\sqrt{2})} - \frac{10}{\sqrt{3}} \right)$$

$$\Rightarrow \frac{m_1}{m_2} = 2{,}59$$

5.11



bloc 4

* $\vec{G}_4 = 30\,N$
* $F = m \cdot a \Rightarrow (F_{\partial 3} - G) = 3\,kg \cdot 2 \frac{m}{s^2}$

$F_{\partial 3} = 36\,N$

bloc 3

* $F_{\partial 3} = 36\,N$
* $G = 30\,N$
* $F = m \cdot a \Rightarrow (F_{\partial 2} - G - F_{\partial 3}) = 3\,kg \cdot 2 \frac{m}{s^2}$

$F_{\partial 2} = 72\,N$

bloc 2

* $F_{n_1} = F_{n_2} = G_1 = G_2 = 30\,N$
* $F_{\omega_1} = F_{\omega_2} = F_n \cdot \mu_d = 6\,N$
* $F = m \cdot a \Rightarrow (F_{\partial 1} - F_{\partial 2} - F_{\omega}) = 3\,kg \cdot 2 \frac{m}{s^2}$

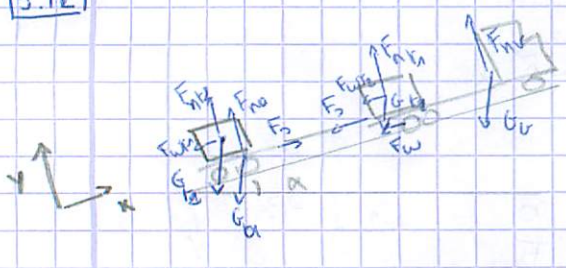
$F_{\partial 1} = 84\,N$

bloc 1

* $F = m \cdot a \Rightarrow (F - F_{\partial 1} - F_{\omega}) = 3\,kg \cdot 2 \frac{m}{s^2}$

$F = 96\,N$

5.12



berekening α

$$\alpha = \sin^{-1}\left(\frac{10}{100}\right) = 5{,}73°$$

aanhangwagen

eerst van apart !

$G_{t_2} + G_a = 20\ kN$

$F_{nt_2} = 9{,}94\ kN$

$F_{wt_2} \leq F_{nt_2} \cdot \mu_2$

$F = m \cdot a \Rightarrow F_{wt_1} + (G_{t_2} + G_a) \cdot \sin 5{,}73° + F_3 = 2 \tan(-3{,}5)\ \frac{m}{s^2}$

Kist 2

* $\vec{G}_{t_1} = 10\ kN = -9{,}94\ kN \cdot \vec{e}_y - 1\ kN\ \vec{e}_x$
* $\vec{F} = m \cdot \vec{a}$

$$\rightarrow \begin{Bmatrix} -1 \\ -9{,}95 \end{Bmatrix} + \begin{Bmatrix} -F_{wt_1} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_{nt_1} \end{Bmatrix} = 1 \begin{Bmatrix} -3{,}5 \\ 0 \end{Bmatrix}$$

$\Rightarrow \vec{F}_{nt_1} = 9{,}95\ kN\ \vec{e}_y$

$\vec{F}_{wt_1} = -2{,}5\ kN\ \vec{e}_x = -F_{nt_1} \cdot \mu_2\ \vec{e}_x$

Kist 1

* $\vec{G}_{t_2} = 10\ kN = -9{,}94\ kN \cdot \vec{e}_y - 1\ kN \cdot \vec{e}_x$
* $\vec{F} = m \cdot \vec{a}$

$$\rightarrow \begin{Bmatrix} -1 \\ -9{,}95 \end{Bmatrix} + \begin{Bmatrix} -F_{nt_2} \cdot \mu_d \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_{nt_2} \end{Bmatrix} = 1 \begin{Bmatrix} -3{,}5 + a_{t_2} \\ 0 \end{Bmatrix}$$

$\Rightarrow \vec{F}_{nt_2} = 9{,}95\ kN\ \vec{e}_y$

$\vec{F}_{wt_2} = -F_{nt_2} \cdot \mu_1 = -1{,}99\ kN\ \vec{e}_x$

$\vec{a}_{t_2} = 0{,}51\ \frac{m}{s^2}$

aanhangwagen

* $\vec{G} = -19{,}88\ kN.\ \vec{e}_y - 2\ kN\ \vec{e}_x$

* $\vec{F} = m.\vec{a}$

↳ $\begin{Bmatrix} -2 \\ -19{,}88 \end{Bmatrix} + \begin{Bmatrix} F_D \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ F_{D_{y}} \end{Bmatrix} = 2 \begin{Bmatrix} -3{,}5 \\ 0 \end{Bmatrix}$

! Krachtenmuur

$\Rightarrow F_D = -5\ kN$

...

oefenzitting 6 : impulswet - energiewet

THEORIE :

- $\sum_{i=1}^{n} \vec{N}_i = (\vec{p}_{II} - \vec{p}_{I})$

  impuls — verschil in bewegingshoeveelheid

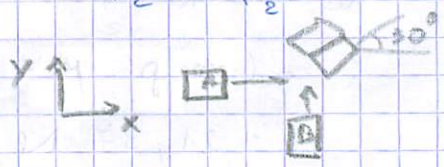
- $\vec{p} = \sum_{i=1}^{n} m_i \cdot v_i$

- $d\vec{N} = \vec{F} dt$

- $U = \int_K \vec{F} d\vec{r}$

- $V_1 + T_1 = V_2 + T_2$

OEFN:



$\boxed{6.5}$ wet van behoud van impuls: (geen externe stoot)

$$\rightarrow m_A \vec{v}_{A,1} + m_B \cdot \vec{v}_{B,1} = m_A \cdot \vec{v}_{A,2} + m_B \cdot \vec{v}_{B,2}$$

$$1,2\, m_B \cdot v_A + m_B \cdot v_B = 2,2\, m_B \cdot (\cos 30° \cdot v + \sin 30° \cdot v)$$

$$\Rightarrow \begin{cases} 1,2\, m_B \cdot v_A = 2,2\, m_B \cdot \cos 30° \, v & (1) \\ m_B \cdot v_B = 2,2\, m_B \cdot \sin 30° \, v & (2) \end{cases}$$

$$\frac{(1)}{(2)} \rightarrow \frac{v_A}{v_B} = \frac{\cot 30°}{1,2} = 1,44$$

$\boxed{6.13}$ gep: $F = 25\ N$

$\Delta x = 40\ m$

$m = 25\ kg$

hoek $= 30°$

gevr: a) U

b) E_{kin}

c) v

d) v (wrz vrijwarp)

opl:

a) $U = \int_{\vec{r}_I}^{\vec{r}_{II}} F \cdot d\vec{r}$

$= 25\,N \cdot \cos 30^\circ \cdot 40\,m$

$= 866\,J$

b) $T + V = U$

$\Rightarrow T = U - 866\,J$

c) $T = \frac{mv^2}{2} \Rightarrow v = \sqrt{\frac{2T}{m}} = 8,32\,\frac{m}{s}$

d) $\vec{F}_w = F_n \cdot \mu = (F_z - \sin 30^\circ \cdot F)\,\mu$

$= (-\sin 30^\circ \cdot 25\,N + 25\,kg \cdot 10\,\frac{N}{kg})\,0,09$

$= 21,37\,N$

$\vec{F}_{res} = \cos 30^\circ \cdot 25\,N - 21,37\,N = 0,28\,N$

$\Rightarrow U = \vec{F}_{res,cst} \cdot \Delta x = 11,22\,J$

$\hookrightarrow v = \sqrt{\frac{2U}{m}} = 0,94\,\frac{m}{s}$

$\boxed{6.23}$ gep: $m_1 = 20\,kg$

$m_2 = 5\,kg$

$R = 0,3\,m \longleftrightarrow v = 4\,\frac{m}{s}$ tov slede

$k = 1000\,N/m$

gevr: Δx_{veer}

opl: ~~* behoud van impuls: $\vec{p}_I = \vec{p}_{II}$~~

~~$0 = v_s \cdot m_s + v_b \cdot m_b$~~

~~$v_{b,0} = -4\,v_{s,0}$~~

~~$\sqrt{\frac{2U}{m}}$~~

behoud van energie

* toestand I : $E_{pot,veer}$

toestand II : $E_{kin,slee} + E_{kin,bal} + E_{pot,bal}$

$$E_{kin,slee} = \frac{m_s \cdot v_s^2}{2} = 1,6 \text{ J}$$

$$E_{kin,bal} = \frac{m_b \, v_{b,abs}^2}{2} = 36,48 \text{ J}$$

$$E_{pot,bal} = m_b g \cdot h = 15 \text{ J}$$

$$E_{pot,veer} = \frac{k \cdot \Delta x^2}{2} = 1000 \cdot \Delta x^2 \text{ J}$$

* <u>behoud van impuls</u> :

tangent aanlikken

$$\underbrace{\vec{v}_{b,zel,1} \cdot \cos 60^\circ + \vec{v}_{slee}}_{p_I} = \underbrace{\vec{v}_{b,abs,1}}_{p_{II}} \qquad \vec{p}_I = \vec{p}_{II}$$

$$\hookrightarrow \vec{v}_{b,abs,1} \cdot m_b + \vec{v}_s \cdot m_s = 0 \quad \text{(behoud van impuls)}$$

$$(\vec{v}_{b,zel,1} \cdot \cos 60^\circ + \vec{v}_s) \cdot m_b + \vec{v}_s \cdot m_s = 0$$

$$\Rightarrow \vec{v}_s = \frac{-\vec{v}_{b,zel,1} \cdot \cos 60^\circ \cdot m_b}{m_b + m_s} = -0,4 \ \frac{m}{s}$$

$$\left(\text{en dus } \vec{v}_{b,abs,x} = -\frac{\vec{v}_s \cdot m_s}{m_b} = +1,6 \ \frac{m}{s}\right)$$

$$\hookrightarrow v_{abs,b} = v_{sleep} + v_{zel} = \begin{Bmatrix} -0,4 \\ 0 \end{Bmatrix} + \begin{Bmatrix} \cos 60^\circ \cdot 4 \\ \sin 60^\circ \cdot 4 \end{Bmatrix} = 3,82$$

OPGELET: hoek is niet meer 60°

(behoud van energie)

$$\Longrightarrow \Delta x = \sqrt{\frac{(1,6 + 36,48 + 15)\text{ J}}{1000}} = 0,23 \text{ m}$$

$\boxed{6.21}$ <u>gegeven</u> :

$m_w = 10$ kg

$m_b = 5$ kg

$\ell = 1$ m

$\theta = 90^\circ$

<u>gevr</u> : bij $\theta = 30^\circ \rightarrow v_w$ en v_b

<u>opl</u> : * <u>behoud van energie</u> :

toestand I : E_{pot,b_I}

toestand II : $E_{kin,b} + E_{kin,w} + E_{pot,b_{II}}$

- $E_{pot,b_I} = m \cdot g \cdot h = 10 \frac{N}{kg} \cdot 5\,kg \cdot 1\,m = 50\,J$

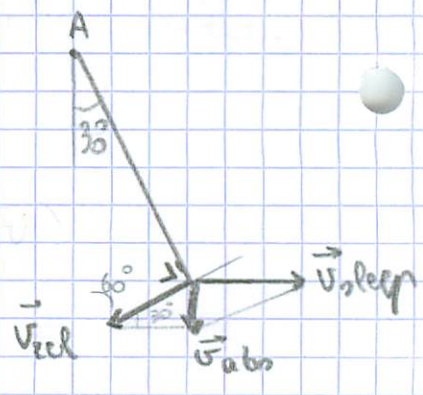
- $E_{pot,b_{II}} = mgh = 10 \frac{N}{kg} \cdot 5\,kg \cdot (1 - 1 \cdot \cos 30^\circ) = 6,7\,J$

- $E_{kin,b} = \frac{m \cdot v_{abs,b}^2}{2}$

- $E_{kin,w} = \frac{m \cdot v_{abs,w}^2}{2}$

<u>behoud van impuls</u>

$\vec{p}_I = \vec{p}_{II}$ (in x-richting)



$\Rightarrow \quad 0 = \vec{v}_w \cdot m_w + \vec{v}_{b,x}\, m_b$

$\vec{v}_{b,x} = \vec{v}_{sleep,x} + \vec{v}_{rel,x}$

$= \vec{v}_w + \vec{v}_{rel,x}$

↳ volgens cos-regel:

$\vec{v}_{abs}^{\,2} = \vec{v}_{rel}^{\,2} + \vec{v}_{sleep}^{\,2} - 2\,\vec{v}_{rel} \cdot \vec{v}_{sleep} \cdot \cos(30^\circ)$

$= \vec{v}_{rel}^{\,2} + \vec{v}_w^{\,2} - 2\,\vec{v}_{rel} \cdot \vec{v}_w \cdot \cos(30^\circ)$

$\Rightarrow$ 3 vgl'en : $\begin{cases} \text{behoud van energie} \\ \text{behoud van impuls} \\ \text{cos-regel} \end{cases}$

↳ $\begin{cases} 50\,J = 6,7\,J + 2,5\, v_{abs,b}^2 + 5\, v_{abs,w}^2 \\ 0 = 5 \cdot \vec{v}_{abs,b} + 10\, \vec{v}_{abs,w} \rightarrow v_{abs,w} = -2\, v_{abs,b} \\ v_{abs,b}^2 = v_{rel,b}^2 + v_{abs,w}^2 - 2\, v_{rel,b} \cdot v_{abs,w} \cdot \cos(30^\circ) \end{cases}$

$\Rightarrow \quad 50\,J = 6,7\,J + 2,5\, v_{abs,b}^2 + 20\, v_{abs,b}^2$

$\rightarrow \vec{v}_{abs,b} = 1,92 \frac{m}{s}$

$\rightarrow v_{rel,b}^2 - 1,92 \cdot \cos(30^\circ)\, v_{rel,b} - 3,7 = 0$

$D = 7,68$

$v_{rel,b} = \frac{6,65 \pm 7,68}{2}$

$= 7,17$

extra oefeningen : impulswet - energiewet (25.01.'10)

$\boxed{6.1}$ * $\sum_{i=1}^{n} \vec{N}_i = \Delta \vec{p}$

$= v_{II} \cdot m_{II} = 2{,}4 \ \frac{kg\,m}{s}$

* $\Delta \vec{N} = \vec{F} \, \Delta t$

$\Rightarrow F = 480 \ N$

$\boxed{6.2}$ * $\vec{N} = \int_{t_I}^{t_{II}} \vec{F} \, dt$

$= -8 \ Ns \, \vec{e}_x + 4 \ Ns \, \vec{e}_y$

* $\vec{N} = \vec{p}_{II} - \vec{p}_{I}$

$-8 \ Ns \, \vec{e}_x + 4 \ Ns \, \vec{e}_y = v_{II} \cdot 0{,}5 \ kg - 0{,}5 \ kg \left(10 \ \frac{m}{s} \ \vec{e}_x \right)$

$\rightarrow v_{II} = -6 \ \frac{m}{s} \vec{e}_x + 8 \ \frac{m}{s} \ \vec{e}_y$

$\boxed{6.3}$

CLUE

geen behoud van impuls

* $\vec{p}_{II} - \vec{p}_{I} = \vec{N}_{bodem}$ ~~behoud van impuls!~~

$\rightarrow$ ~~$\frac{0{,}03 \ kg \cdot 100 \ \frac{m}{s}}{\cos 30°} \vec{e}_x$~~ ~~$\frac{0{,}03 \ kg \cdot 100 \ \frac{m}{s}}{\sin 30°} \vec{e}_y$~~

~~$= 2{,}03 \ kg \cdot v_2 \cdot \vec{e}_x$~~

~~$\Rightarrow v_2 = 1{,}71 \ \frac{m}{s} \cdot \vec{e}_x$~~

$\rightarrow \begin{Bmatrix} v_2 \cdot M_{tot} \cdot \cos 30° \\ v_2 \ M_{tot} \cdot \sin 30° \end{Bmatrix} - \begin{Bmatrix} m \cdot v_0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -N \cdot \cos 60° \\ N \cdot \sin 60° \end{Bmatrix}$

$\Rightarrow N = v_2 \cdot 2{,}03 \cdot \frac{\sqrt{3}}{3}$

$\Leftrightarrow v_2 \left(2{,}03 \cdot \frac{\sqrt{3}}{2} + 2{,}03 \frac{\sqrt{3}}{6} \right) = 0{,}03 \cdot 100 \ \frac{m}{s} \Rightarrow v_2 = 1{,}28 \ \frac{m}{s} = \begin{Bmatrix} 1{,}1 \\ 0{,}64 \end{Bmatrix} \frac{m}{s}$

$\boxed{6.4}$ * $\vec{N} = \int_{t_1}^{t_2} \vec{F}\,dt$

$= \frac{0{,}04 \cdot 6}{2} = 0{,}12\ kNs$

$= -120\ Ns$

* $\sum N = \Delta \vec{p}$

$-120\ Ns = 5kg\ v_2 - 5kg \cdot 20\ \frac{m}{s}$

$\Rightarrow v_2 = -4\ \frac{m}{s}$

$\boxed{6.5}$ behoud van impuls (geen externe stoten)

$\rightarrow \vec{p}_I = \vec{p}_{II}$

$1{,}2\ v_A \cdot m_B\ \vec{e}_x + v_B \cdot m_B \cdot \vec{e}_y = 2{,}2\ m_B\ v \cdot \cos 30° \cdot \vec{e}_x + 2{,}2 \cdot m_B \cdot v \sin 30° \cdot \vec{e}_y$

$\Leftrightarrow \begin{Bmatrix} 1{,}2\ v_A \\ v_B \end{Bmatrix} = \begin{Bmatrix} 2{,}2\ v \cos 30° \\ 2{,}2\ v \sin 30° \end{Bmatrix}$

$\rightarrow \frac{v_A}{v_B} = \frac{\cot 30°}{1{,}2} = 1{,}44$

$\boxed{6.6}$ <u>1e botsing (in A)</u>

→ geen externe stoten dus behoud van impuls:

$\vec{p}_I = \vec{p}_{II}$

$\begin{Bmatrix} -v_0 m_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} -m_1 v_1 \sin 30° - m_2 v_{2,x} \\ m_1 v_1 \cos 30° - m_2 \cdot v_{2,y} \end{Bmatrix} \begin{matrix} (1) \\ (2) \end{matrix}$

$\cancel{0{,}02 \cdot 9{,}5\ \frac{m}{s} \cdot \cos 30° = 0{,}02\ v_2 \cos\theta}$

$\cancel{v_2 \cos\theta = 8{,}66\ \frac{m}{s}}$

$\cancel{(1)\ -20\ \frac{m}{s} \cdot 0{,}005 = -0{,}02 \cdot v_2 \sin\theta}$

$\cancel{\theta = 52{,}9°}$

$\cancel{\Rightarrow v_2 = 14{,}12\ \frac{m}{s}}$

$\Rightarrow v_{2,x} = -15 \frac{m}{s} \vec{e}_x - 8,66 \frac{m}{s} \vec{e}_x$

<u>berekening interne stoten</u>

1) $p_{II,1} - p_{I,1} = N_1$

$\rightarrow \vec{N}_1 = -0,025 \, Ns \, \vec{e}_x + 0,043 \, Ns \, \vec{e}_y$

2) $p_{I,2} - p_{II,2} = N_2$

$\rightarrow \vec{N}_2 = 0,025 \, Ns \, \vec{e}_x - 0,0433 \, Ns \, \vec{e}_y$

<u>2e botsing (in B)</u>

extreme stoot van de muur!

$\hookrightarrow \vec{p}_{II} - \vec{p}_{I} = \vec{N}$

$$\begin{Bmatrix} -v_3 m_1 \cdot \cos 30° \\ -v_3 m_1 \sin 30° \end{Bmatrix} - \begin{Bmatrix} -m_1 v_1 \sin 30° \\ m_1 v_1 \cos 30° \end{Bmatrix} = \begin{Bmatrix} 0 \\ N \end{Bmatrix}$$

$\rightarrow v_3 = v_1 \tan 30° = 1,44 \frac{m}{s}$

$\hookrightarrow \vec{v}_3 = -1,25 \frac{m}{s} \cdot \vec{e}_x - 0,72 \frac{m}{s} \cdot \vec{e}_y$

$\vec{N} = -0,0057 \, Ns$

$\boxed{6.7}$ a) externe stoot van bodem!

$\vec{p}_{II} - \vec{p}_{I} = \vec{N}$

$$\begin{Bmatrix} M \cdot v - m v_1 \\ -m v_{1,y} \end{Bmatrix} - \begin{Bmatrix} (M+m) v_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ N \end{Bmatrix}$$

$$\Rightarrow v_2 = \frac{70 \, kg \cdot 4 \frac{m}{s} - 3 \, kg \cdot 70 \frac{m}{s}}{70 \, kg + 3 \, kg} = 0,96 \frac{m}{s}$$

b) stoot van bodem wordt groter

c) Dan: $Mv - mv_1 = 0$ of $\frac{v_1}{v} = \frac{M}{m} = 23,33$

6.8

a) externe kracht van bodem.

uit de Kano!

$$\begin{Bmatrix} 15\,kg \cdot 3\,\frac{m}{s} \cdot \cos 30 \\ 15\,kg \cdot 3\,\frac{m}{s} \sin 30 \end{Bmatrix} - \begin{Bmatrix} v_2 \cdot 105\,kg \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ N \end{Bmatrix}$$

$$\Rightarrow v_2 = 0{,}433\,\frac{m}{s}$$

b)

$$\begin{Bmatrix} 15\,kg \cdot v_1 \\ 15\,kg \cdot v_1 \end{Bmatrix} - \begin{Bmatrix} v_2 \cdot 105\,kg \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ N \end{Bmatrix}$$

$$v_1 + v_2 = 3\,\frac{m}{s}\cos 30° \Leftrightarrow v_1 = 3 \cdot \cos 30° - v_2$$

$$\Rightarrow v_2 = 0{,}325$$

! 6.9 <u>1e man duikt</u> (behoud van impuls)

$$p_{II} = p_I$$

beweging duiker relatief!

$$\begin{Bmatrix} -380 \cdot v_1 + 80 \cdot (3 - v_1) \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow v_1 = 0{,}52\,\frac{m}{s}$$

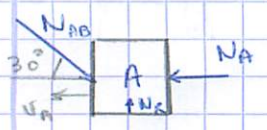
<u>2e man duikt</u> (behoud van impuls)

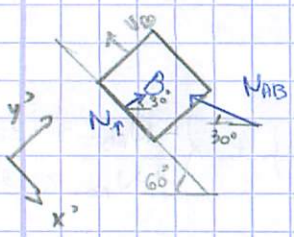
$$p_{IV} = p_{III}$$

$$\begin{Bmatrix} -300 \cdot v_2 + 80\,(3 - v_2) \\ 0 \end{Bmatrix} = \begin{Bmatrix} -380 \cdot 0{,}52 \\ 0 \end{Bmatrix}$$

$$\Rightarrow v_2 = 1{,}15\,\frac{m}{s}$$

6.10 * vrijmaken van puntmassa's en stoken aanbrengen:
belangrijk: alles apart (ook diepere die bekeren!)





blok C

* $\vec{r}_{II} - \vec{r}_{I} = N_A$

$$\rightarrow \begin{Bmatrix} -2v_A \\ 0 \end{Bmatrix} - \begin{Bmatrix} -2 \cdot 3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} N_A \\ 0 \end{Bmatrix}$$

$$\Rightarrow N_A = 6 - 2v_A$$

* blok A

$$\begin{Bmatrix} -N_A \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ N_R \end{Bmatrix} + \begin{Bmatrix} +N_{AB} \cdot \cos 30° \\ -N_{AB} \cdot \sin 30° \end{Bmatrix} = \begin{Bmatrix} -2v_A \\ 0 \end{Bmatrix} \quad (*)$$

* blok B

$$\begin{Bmatrix} -N_{AB} \cdot \cos 30° \\ -N_{AB} \cdot \sin 30° \end{Bmatrix} + \begin{Bmatrix} 0 \\ N_R \end{Bmatrix} = \begin{Bmatrix} -2v_B \\ 0 \end{Bmatrix}$$

$$(*) \Rightarrow -6 + 2v_A + 2v_B = -2v_A$$

* zamengestelde beweging van B:

absolut → hoek bekend
sleep → v_A
relatief → cirkelbeweging rond A

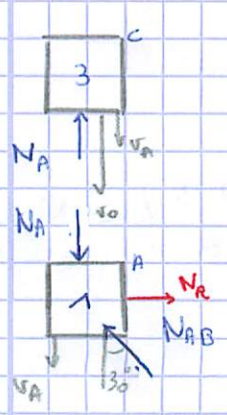
$$\Rightarrow \begin{Bmatrix} -v_{abs} \cdot \cos 60° \\ v_{abs} \cdot \sin 60° \end{Bmatrix} = \begin{Bmatrix} -v_A \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cdot \cos 60° \\ v_{rel} \sin 60° \end{Bmatrix}$$

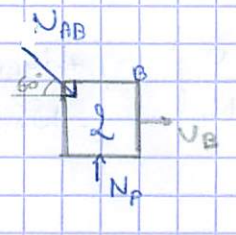
$$\begin{cases} v_{abs} = v_B \\ -2\cos 60° \, v_B = -v_A \end{cases}$$

$$v_B = v_A$$

$$\Rightarrow v_A = v_B = 1 \ \frac{m}{s}$$

6.12 * vrijmaken van massa's en stoten aanbrengen:





blok c

* $\vec{p}_{II} - \vec{p}_{I} = \vec{N}$

$\hookrightarrow \begin{Bmatrix} -25 \cdot v_A \\ 0 \end{Bmatrix}_2 - \begin{Bmatrix} -25 \cdot 10 \\ 0 \end{Bmatrix}_2 = \begin{Bmatrix} N_A \\ 0 \end{Bmatrix}_2$

$\Rightarrow N_A = 250 - 25 v_A$

* blok A

$$\begin{Bmatrix} 0 \\ -N_A \end{Bmatrix} + \begin{Bmatrix} N_R \\ 0 \end{Bmatrix} + \begin{Bmatrix} -N_{AB} \cdot \sin 30° \\ +N_{AB} \cdot \cos 30° \end{Bmatrix} = \begin{Bmatrix} \overset{0}{\cancel{-25 v_A}} \\ \cancel{0}\ -25 v_A \end{Bmatrix}$$

* blok B

$$\begin{Bmatrix} +N_{AB} \cdot \cos 60° \\ -N_{AB} \cdot \sin 60° \end{Bmatrix} + \begin{Bmatrix} 0 \\ N_P \end{Bmatrix} = \begin{Bmatrix} 25 v_B \\ 0 \end{Bmatrix}$$

$\Rightarrow N_A = 250 - 25 v_A$

$25 v_A + N_{AB} \cdot \cos 30° = 250 - 25 v_A$

$25 v_B \cdot \dfrac{\cos 30°}{\cos 60°} = 250 - 50 v_A$

* samengestelde beweging (van B)

$\begin{cases} \text{absolut} \rightarrow \text{enkel x-component} \\ \text{sleep} \rightarrow v_A \\ \text{relatief} \rightarrow \text{cirkelbeweging rond A} \end{cases}$

$$\rightarrow \begin{Bmatrix} v_B \\ 0 \end{Bmatrix} = \begin{Bmatrix} v_{rel} \cdot \cos \overset{30}{\cancel{60}} \\ v_{rel} \cdot \sin \overset{30}{\cancel{60}} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -v_A \end{Bmatrix}$$

$$\frac{v_A}{v_B} = \tan \overset{30}{\cancel{60}} \rightarrow v_A = \frac{1}{\sqrt{3}} v_B$$

$$\rightarrow 25 \sqrt{3} v_A \underbrace{\frac{\frac{\sqrt{3}}{2}}{2}}_{\frac{1}{2}} = 250 - 50 v_A$$

$75 \sqrt{3}\, v_B = 250 \rightarrow \begin{cases} v_B = 2 \cdot \sqrt{3}\ \frac{m}{s} \\ v_A = 2\ \frac{m}{s} \end{cases} \rightarrow$

$N_R = 86{,}6$ Ns
$N_A = 200$ Ns
$N_{AB} = 173{,}2$ Ns
$N_P = 150$ Ns

6.13 a) $U = F \cdot \Delta x$

$= 25\,N \cdot \cos 30° \cdot 40\,m$

$= 866\,J$

OPM: normaalkracht niet altijd gelijk aan F_z
→ let op andere krachten!

b) $E_{kin} = U = 866\,J$

c) $E_{kin} = \frac{m v^2}{2} \Rightarrow v = 8{,}32\,\frac{m}{s}$

d) $\vec{F_n} = \vec{R} = \vec{G} - F \cdot \sin 30° = 237{,}5\,N$

$\vec{F_w} = \vec{R} \cdot \mu_d = 21{,}375\,N$

$\Rightarrow F_{z,x} = 0{,}275$

$\Rightarrow v_2 = \sqrt{\frac{2}{m} \cdot F_{zx} \cdot \Delta x} = 0{,}94\,\frac{m}{s}$

6.14

OPM: Pakketje blok paat omhoog, waardoor zijn kin. energie ook stijgt!

* $F_o = 40\,N$, $G = -120\,N$ } $F_z = -80\,N\,\vec{e}_y$

* $\Delta U = \Delta T = 160\,J = \frac{16\,kg\,v^2}{2}$

$\Rightarrow v_c = 4{,}47\,\frac{m}{s}$

6.16 behoud van energie

* situatie 1: potentiële gravitatie-energie en potentiële energie ow veerkracht

$$\Rightarrow E_1 = V = m.gh + \frac{k}{2}(x-l_0)^2$$
$$= 18 + 63$$
$$= 81\ J$$

* situatie 2: potentiële energie (veer) en kinetische energie

$$\Rightarrow E_2 = V + T = \frac{mv^2}{2} + \frac{k}{2}(x-l_0)^2 = E_1$$

$$\hookrightarrow v = \sqrt{\frac{2}{m}\left(81J - \frac{350}{2}\left(\sqrt{0,6^2+0,6^2} - 0,6\right)\right)} = 6,84\ \frac{m}{s}$$

6.17 behoud van energie

* situatie 1: potentiële energie ow veer

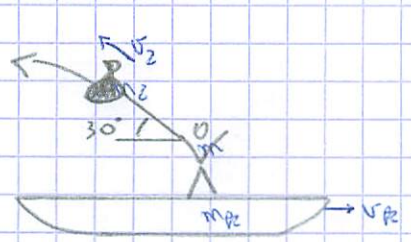
$$\Rightarrow E_1 = V = \frac{k}{2}(x-l_0)^2$$
$$= 400\left(\sqrt{0,12^2+0,05^2} - 0,04\right)^2$$
$$= 3,24\ J$$

* situatie 2: potentiële energie ow veer en kinetische energie

$$\Rightarrow E_2 = V + T = \frac{k}{2}(x-l_0)^2 + \frac{mv^2}{2} = E_1$$

$$\hookrightarrow v = \sqrt{\frac{2}{0,1kg}\left(3,24 - 400\,(0,05-0,04)^2\right)} = 8\ \frac{m}{s}$$

6.18



! behoud impuls + behoud energie

* behoud van impuls :

$$\vec{p}_{II} - \vec{p}_{I} = 0$$

$$-m_2 \cdot v_2 + (m_{pk} + m) \cdot v_{pk} = 0 \quad (1)$$

* behoud van energie :

$$\Delta U = \Delta T = \frac{(m_{pk} + m) \cdot v_{pk}^2}{2} + \frac{m_2 \cdot v_2^2}{2} \quad (2)$$

$\rightarrow$ (1) : $+15\, v_2 \cos 30 = 105\, v_{pk} = 0$

$$v_2 = \frac{-2}{\sqrt{3}} 7\, v_{pk} = 8{,}08\, v_{pk}$$

$\rightarrow$ (2) : $150\, J = \frac{105\, v_{pk}^2}{2} + \frac{15 \cdot 65{,}5\, v_{pk}^2}{2}$

$$\Rightarrow v_{pk} = 0{,}526 \ \frac{m}{s}$$

$$v_2 = -4{,}25 \ \frac{m}{s}$$

$\boxed{6.19}$ a) <u>behoud van energie</u>

Als er geen energie verloren gaat wordt bij de botsing enkel de snelheidsvectoren omgedraaid

$\rightarrow$ ze komen gelijkrijdig aan

b) * <u>behoud van impuls</u>

$$\vec{p}_{II} = \vec{p}_{I}$$

$$-\frac{v_1}{2} \cdot m_1 + v_x \cdot 2 m_1 = v_1 m_1 - \frac{v_1}{2} \cdot 2 m_1$$

$$\Rightarrow v_x = \frac{v_1}{4}$$

$\rightarrow$ komen opnieuw gelijkrijdig aan

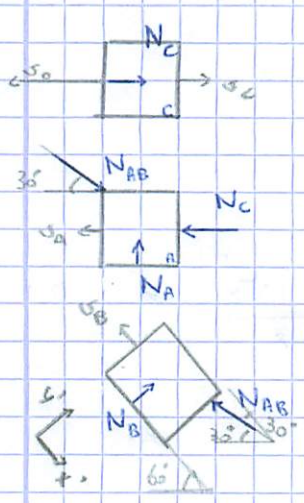
* <u>hoeveelheid energie verloren</u>

$$\Delta E = \left[\frac{m_1 v_1^2}{2} + \frac{2 m_1 \left(\frac{v_1}{2}\right)^2}{2}\right] - \left[\frac{m_1 \left(\frac{v_1}{2}\right)^2}{2} + \frac{2 m_1 \left(\frac{v_1}{4}\right)^2}{2}\right]$$

$$= \frac{3}{4} m_1 v_1^2 - \frac{3}{16} m_1 v_1^2 = \frac{9}{16} m_1 v_1^2$$

! $\boxed{6.20}$ * vrijmaken van puntmassa's en stoten aanbrengen:

IMPULS



blok C

$$\vec{p}_{II} - \vec{p}_{I} = N_C$$

$$\begin{Bmatrix} 2 \cdot v_C \\ 0 \end{Bmatrix} - \begin{Bmatrix} -2 \cdot 3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} N_C \\ 0 \end{Bmatrix}$$

$$\Rightarrow N_C = 2v_C + 6 \quad (1)$$

blok A

$$\sum \vec{N}_i = \vec{p}_{II} - \vec{p}_{I}$$

$$\begin{Bmatrix} N_{AB} \cdot \cos 30° \\ -N_{AB} \cdot \sin 30° \end{Bmatrix} + \begin{Bmatrix} 0 \\ N_A \end{Bmatrix} + \begin{Bmatrix} -N_C \\ 0 \end{Bmatrix} = \begin{Bmatrix} 2v_A \\ 0 \end{Bmatrix}$$

blok B (in x'y')

$$\sum \vec{N}_i = \vec{p}_{II} - \vec{p}_{I}$$

$$\begin{Bmatrix} 0 \\ N_B \end{Bmatrix} + \begin{Bmatrix} -N_{AB} \cos 30° \\ -N_{AB} \cdot \sin 30° \end{Bmatrix} = \begin{Bmatrix} -2v_B \\ 0 \end{Bmatrix}$$

$$(1) \Rightarrow 2v_B + 2v_A = 2v_C + 6 \xrightarrow{(2)} v_C = 2v_A - 3 \quad (4)$$

* samengestelde beweging (van v_B)
  → verband tussen v_A en v_B

$$\rightsquigarrow \begin{cases} v_{abs} = v_B \rightarrow \text{hoek} \\ v_{sleep} = v_A \\ v_{rel} = \text{cirkelbeweging van B rond A} \end{cases}$$

$$\Rightarrow \begin{Bmatrix} -v_B \cdot \cos 60° \\ v_B \cdot \sin 60° \end{Bmatrix} = \begin{Bmatrix} -v_A \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{rel} \cos 60° \\ v_{rel} \cdot \sin 60° \end{Bmatrix}$$

$$\Rightarrow v_B = v_A \quad (2)$$

* behoud van energie: (Rumerinck)

$$\Delta E = \frac{m_C v_0^2}{2} - \frac{m_C \cdot v_C^2}{2} - \frac{m_A \cdot v_A^2}{2} - \frac{m_B \cdot v_B^2}{2} = 0$$

$v_A = v_B$

$$\Rightarrow 9 - v_C^2 = 2v_A^2 \quad (3)$$

(4)

$$9 - (2v_A - 3)^2 = 2v_A^2 \longrightarrow -4v_A^2 + 12v_A = 2v_A^2$$

$$\Rightarrow v_A = 2\,\frac{m}{s} \longrightarrow v_B = 2\,\frac{m}{s} \text{ en } v_C = 1\,\frac{m}{s}$$

!! $\boxed{6.21}$ <u>behoud van energie</u>

→ potentiële energie wordt omgezet in kinetische energie

$$m_2 g h_1 = m_2 \cdot g h_2 + \frac{m_2 v_2^2}{2} + \frac{m_1 v_1^2}{2}$$

$$5kg\ 10\frac{N}{kg}\ 1\ m = 5kg \cdot 10\frac{N}{kg}(1 - 1\cos 30°)\ m + \frac{5kg\ v_2^2}{2} + \frac{10kg\ v_1^2}{2}$$

$$\Rightarrow v_2^2 + 2v_1^2 = 10\sqrt{3} \quad (1)$$

<u>behoud van impuls</u>

→ geen externe stoten in x-richting

oppelet : relatieve v_2 raken aan cirkel rond A

→ absolute v_2 : andere hoek!

$$\vec{p}_{II,x} - \vec{p}_{I,x} = 0$$

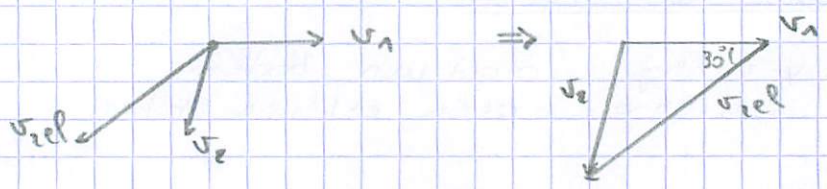
$$m_2 v_2 \cos\alpha - m_1 v_1 = 0$$

$$v_2 \cdot \cos\alpha = 2v_1 \quad \text{en} \quad v_2 \cdot \cos\alpha - v_{2el}\cos 30° = -v_1$$

$$\Rightarrow v_{2el} \cdot \cos 30° = 3v_1 \quad \text{of} \quad v_{2el} = \frac{2}{\cancel{3}}\sqrt{3}\, v_1 \quad (2)$$

<u>samengestelde beweging</u>

translerend bewegend aanstekelsel in A



$$v_2^2 = v_1^2 + v_{2el}^2 - 2 v_1 v_{2el} \cos 30°$$

(1) en (2)

$$10\sqrt{3} - 2v_1^2 = v_1^2 + 12 v_1^2 - 6 v_1^2$$

$$10\sqrt{3} = 9 v_1^2$$

$$v_1 = 1{,}39\ \frac{m}{s}$$

$$\Rightarrow v_{2el} = 4{,}8\ \frac{m}{s}$$

$$v_2 = -2{,}77\ \frac{m}{s}\ \vec{e}_x - 2{,}41\ \frac{m}{s}\ \vec{e}_y$$

!! $\boxed{622}$ * behoud van energie !!

E_{pot} van m_2 → E_{kin} van m_1, m_2 en m_3 en E_{pot} van m_1

↳ aanvankelijk: $m_2 g \cdot h_1 = 500\ J$

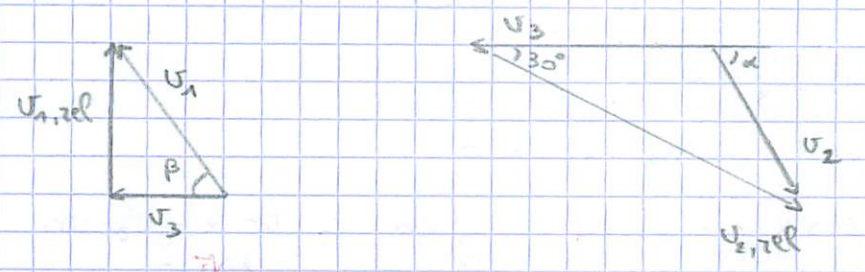
situatie 2: $m_2 g h_2 + \frac{v_1^2 m_1}{2} + \frac{v_2^2 m_2}{2} + \frac{v_3^2 m_3}{2} = E_1$

en $h_2 \cdot \sin 30° = h_1$

$\Rightarrow \quad 500 = 400 + 10 v_1^2 + 25 v_2^2 + 50 v_3^2$

$20 = 2 v_1^2 + 5 v_2^2 + 10 v_3^2 \qquad (1)$

* samengestelde beweging



$v_2^2 = v_3^2 + v_{2,rel}^2 - 2 v_3 \cdot v_{2,rel} \cdot \cos 30° \qquad (2)$

en ~~$\|v_1\| = \|v_2\|$~~ (verbonden met touw) en $v_1^2 = v_{1,rel}^2 + v_3^2$

$v_{1rel} = v_{2rel}$

* behoud van impuls

{ in y-richting: stoot van bodem
{ in x-richting: geen externe stoten

$\rightarrow -m_1 v_1 \cdot \cos\beta + v_2 \cdot m_2 \cdot \cos\alpha - m_3 \cdot v_3 = 0$

$\cos\beta = \frac{v_3}{v_1}$ en $v_{2,rel} \cdot \cos 30° - v_2 \cdot \cos\alpha = + v_3$

$-m_1 \cdot v_3 + m_2 (v_{2,rel} \cdot \cos 30° - v_3) - m_3 v_3 = 0$

$170 v_3 = 50 \cdot \frac{\sqrt{3}}{2} \cdot v_{2,rel}$

$v_{2,rel} = \frac{34}{5\sqrt{3}} v_3 \qquad (3)$

* (2) ⇝ $v_2^2 = v_3^2 + \frac{34^2}{25 \cdot 3} v_3^2 - 2 \cdot \frac{\sqrt{3}}{2} \frac{34}{5\sqrt{3}} v_3^2$

$v_3^2 = 0{,}104\, v_2^2$ of $v_2^2 = 9{,}61\, v_3^2$

* (1) ⇝ $20 = 2 v_1^2 + 5 v_2^2 + 10 v_3^2$

$$\Rightarrow \quad 20 = 2\left(v_{1,rel}^2 + v_3^2\right) + 5 \cdot 9{,}61\, v_3^2 + 10\, v_3^2$$

$$20 = 2\left(\left(\frac{34}{5\sqrt{3}}\right)^2 v_3^2 + v_3^2\right) + 5 \cdot 9{,}61\, v_3^2 + 10\, v_3^2$$

$$\rightarrow \quad v_3 = 0{,}47 \ \frac{m}{s}$$

$$* \quad \vec{v}_3 = -0{,}47 \ \frac{m}{s} \cdot \vec{e}_x$$

$$\vec{v}_1 = -0{,}47 \ \frac{m}{s} \cdot \vec{e}_x + 1{,}84 \ \frac{m}{s} \cdot \vec{e}_y$$

$$\vec{v}_2 = 1{,}12 \ \frac{m}{s} \cdot \vec{e}_x - 0{,}92 \ \frac{m}{s} \cdot \vec{e}_y$$

$\boxed{6.23}$ <u>behoud van energie</u>

<u>situatie 1</u>: $+\frac{k}{2}(x - l_0)^2 = E_1$

<u>situatie 2</u>: $\frac{m_1 \cdot v_1^2}{2} + \frac{m_2\, v_2^2}{2} + m_2 \cdot g \cdot h = E_2$

$$\rightarrow \quad E_1 = E_2 \iff 1000\,(x - l_0)^2 = 15\ J + 10\, v_1^2 + 2{,}5\, v_2^2$$

<u>behoud van impuls</u>

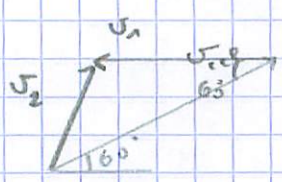
→ geen externe impulsen in x-richting!
WANT helling is deel van systeem

$$\Rightarrow \quad -m_1 \cdot v_1 + m_2 \cdot v_{2,x} = 0$$

$$\rightsquigarrow (1) \quad -20\, v_1 - 5\, v_1 = -2{,}5\, v_{2,rel} \Rightarrow v_1 = 0{,}3 \ \frac{m}{s}$$

<u>samengestelde beweging</u>

$$\begin{Bmatrix} v_{2,x} \\ v_{2,y} \end{Bmatrix} = \begin{Bmatrix} -v_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} v_{2,rel} \cdot \cos 60^\circ \\ v_{2,rel} \cdot \sin 60^\circ \end{Bmatrix} \quad (1)$$



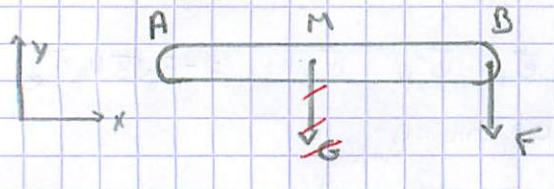
$$\rightsquigarrow \quad v_2^2 = v_1^2 + v_{2,rel}^2 - 2\, v_1 \cdot v_{2,rel} \cdot \cos 60^\circ$$

$$\rightsquigarrow \quad v_2 = 3{,}86 \ \frac{m}{s}$$

$$\Rightarrow \quad x - l_0 = \text{indrukking} = 0{,}23\ m$$

extra oefeningen : vlakke dynamica (26.01.'10)

7.1



OPM : horizontaal vlak !

~~G = 20 N~~
F = 150 N

translatie :

$$\sum \vec{F}_i = m\,\vec{a}_c \quad \Rightarrow \quad \vec{a}_c = -7{,}5\ \frac{m}{s^2}$$

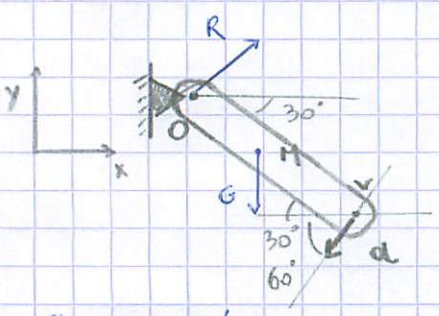
rotatie

$$\vec{M}_c = I_c\,\vec{\alpha} \quad \Rightarrow \quad \vec{M}_c = \vec{r}_{MB} \times \vec{F} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}3 & 0 & 0 \\ 0 & -150 & 0 \end{vmatrix} = -45\ Nm\ \vec{e}_z$$

$$I_c = \frac{1}{12}\ 2\,kg\,(0{,}6\ m)^2 = 0{,}06$$

$$\Rightarrow \vec{\alpha} = -750\ rad/s^2\ \vec{e}_z$$

7.2



G = 20 N

translatie

$$\sum \vec{F}_i = m\,\vec{a}_c$$

$$\begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G \end{Bmatrix} = m \cdot \begin{Bmatrix} -a_{c,x} \\ -a_{c,y} \end{Bmatrix}$$

met a_c loodrecht op cirkel rond O

$$\Rightarrow \begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -20 \end{Bmatrix} = 2\,kg \begin{Bmatrix} -a_c \cdot \cos 60^\circ \\ -a_c \sin 60^\circ \end{Bmatrix}$$

rotatie

$\vec{M}_o = I_o \cdot \vec{\alpha}$

! slechtcoor → $\vec{M}_o = \vec{r}_{OM} \times \vec{G} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}3 \cdot \cos 30° & -0{,}3 \sin 30° & 0 \\ 0 & -10 & 0 \end{vmatrix} = -3\sqrt{3}\, Nm\, \vec{e}_z$

$I_o = I_c + m\, r_{oc}^2 = \frac{1}{12} \cdot 2kg \cdot 0{,}6^2 + 2kg \cdot 0{,}3^2$

$= 0{,}24 \ kg \cdot m^2$

$\rightarrow \vec{\alpha} = -\frac{25 \cdot \sqrt{3}}{2} \ rad/s^2 \ \vec{e}_z$

berekening versnelling

in een cirkelbeweging geldt: $a_t = \alpha \cdot R (= -a_c)$

$a_n = \omega^2 \cdot R$

$\rightarrow \vec{a} = \begin{Bmatrix} -3{,}25 \\ -5{,}625 \end{Bmatrix} \frac{m}{s^2}$

berekening krachten

$\vec{R} = \begin{Bmatrix} -6{,}5 \\ 8{,}75 \end{Bmatrix} N$

$\boxed{7.3}$ behoud van energie

→ geleverde arbeid = verschil in energie

$\rightsquigarrow U = \int_a^b \vec{F} \cdot d\vec{r}$ OPM!: enkel horizontale verplaatsing in rekening brengen, want $\vec{F} \cdot \vec{r}_y = 0$

$= 0{,}6 \ m \cdot 150 \ N$

$= 90 \ J$

$\Delta E = m \cdot g \cdot h + \frac{1}{2} I_O \omega^2$ met $I_O = I_c + m \cdot r_{OM}^2$

$= 0{,}24 \ kg \ m^2$

$= 2kg \cdot 10 \frac{N}{kg} \cdot 0{,}3 \ m + \frac{1}{2} \cdot 0{,}24 \ kg \ m^2 \cdot \omega^2$

$= 6 \ J + 0{,}12 \ \omega^2$

$\rightarrow \omega = \sqrt{700} = 26{,}45 \ rad/s \ \vec{e}_z$

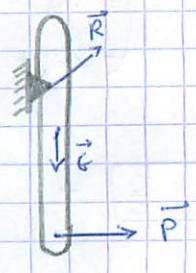
$v = \omega \cdot R = \sqrt{700} \cdot 0{,}3 = 7{,}9 \frac{m}{s} \ \vec{e}_y$

$\boxed{7.4}$ * $\vec{M}_o = I_o \vec{\alpha}$ rotation

$$\rightarrow \quad \vec{M}_o = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -\frac{3}{4}L & 0 \\ P & 0 & 0 \end{vmatrix} = \frac{3}{4} L P \vec{e}_z$$

$$I_o = I_c + r_{DM}^2 \cdot m^2$$
$$= \frac{1}{12} L^2 \cdot 1\,kg + 1\,kg \left(\frac{1}{4} L\right)^2$$
$$= \frac{7}{48} L^2$$

$$\Rightarrow \vec{\alpha} = \frac{\vec{M}_o}{I_o} = \frac{3}{4} \cdot \frac{48}{7} \frac{LP}{L^2} \vec{e}_z = \frac{36}{7} \frac{P}{L} \cdot \vec{e}_z$$



* translation

$$\sum \vec{F}_i = m \cdot \vec{a}_c$$

$$\begin{Bmatrix} P \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_x \\ R_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -10 \end{Bmatrix} = 1\,kg \begin{Bmatrix} a_x \\ a_y \end{Bmatrix}$$

mit $a_x = a_t = \alpha \cdot R = \frac{36}{7} \frac{P}{L} \cdot \frac{1}{4} L = \frac{9}{7} P$

$a_y = a_n = \omega^2 R = 0$

$$\Rightarrow \vec{R} = \begin{Bmatrix} 9/7\, P \\ 10 \end{Bmatrix} N = \begin{Bmatrix} 2/7\, P \\ 10 \end{Bmatrix} N$$

$\boxed{7.5}$ rotation

$$\vec{M}_o = I_o \cdot \vec{\alpha} \quad \rightarrow \quad \vec{M}_o = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & -0{,}05 & 0 \\ +200 \cdot \sin 30^\circ & -200 \cdot \cos 30^\circ & 0 \end{vmatrix} = 5 \cdot \vec{e}_z$$

$$\Rightarrow \vec{\alpha} = 33{,}33 \frac{rad}{s^2} \vec{e}_z$$

(Steiner!)

$$I_o = \frac{20\,kg \cdot 0{,}10^2\,m^2}{2} + 20\,kg \cdot 0{,}05^2\,m^2$$
$$= 0{,}15\,kg\,m^2$$

translation

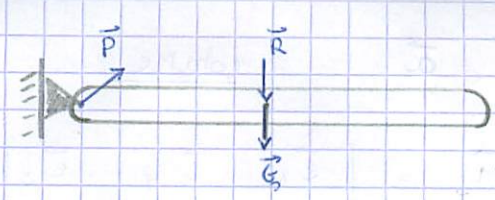
$$\sum \vec{F}_i = m \cdot \vec{a}$$

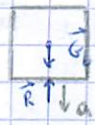
$$\begin{Bmatrix} +200 \cdot \sin 30^\circ \\ -200 \cdot \cos 30^\circ \end{Bmatrix} + \begin{Bmatrix} R_x \\ R_y \end{Bmatrix} = 20\,kg \begin{Bmatrix} a_x \\ a_y \end{Bmatrix} = 20\,kg \begin{Bmatrix} \alpha \cdot R \\ \omega^2 R \end{Bmatrix}$$

$$\Rightarrow \vec{R} = \begin{Bmatrix} -66{,}66 \\ 182{,}2 \end{Bmatrix} N$$

7.6

(situatie A:)





$\vec{M} = -\frac{\ell}{2} G_s - R \cdot r$

+ Rotatie:

$\vec{M}_o = I_o \cdot \vec{\alpha}$ met $a_t = \alpha \cdot R$ en $I_o = \frac{1}{12} \cdot 15 kg \cdot \ell^2 + 15 kg \frac{\ell^2}{4}$

$= 5\ell^2$

+ Translatie

$\Sigma \vec{F_i} = m \cdot a_c$

$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -R \end{Bmatrix} + \begin{Bmatrix} 0 \\ -G_s \end{Bmatrix} = m \begin{Bmatrix} a_x \\ a_t \end{Bmatrix}$$

↳ enkel nodig voor berekening P_y

$\Rightarrow$
- $\alpha = (-75\ell - R \cdot r)/5\ell^2$ (rotatie)
- $(P_y - R - 150 = 7{,}5\ell$ (translatie)) upl niet nodig
- $-50 + R = -5a \Leftrightarrow R = 50 - 5 \cdot r\alpha$ (vrijlichaams-diagram) $F = m \cdot a$

+ $\boxed{r = \frac{\ell}{2}}$ $\rightarrow$ $\alpha = (+75\ell + R \cdot \frac{\ell}{2}) / 5\ell^2$

$\Rightarrow R = 50 - \frac{5\ell}{2} \frac{\ell(+75 + R/2)}{5\ell^2}$

$\frac{3}{4}R = 12{,}5 \rightarrow R = 10\ N$

$\Rightarrow \alpha = \frac{16}{\ell} \frac{rad}{s^2}$

+ $\boxed{r > \frac{\ell}{2}}$ $\rightarrow$ $\alpha = (75\ell + R\frac{2}{3}\ell) / 5\ell^2$

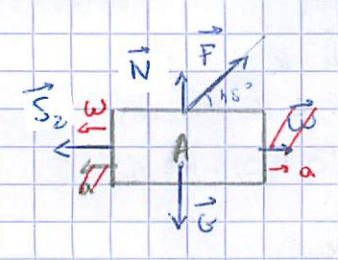
$\rightarrow R = 50 - 5 \cdot \frac{2\ell}{3} \frac{1}{5\ell^2} \ell (75 + R\frac{2}{3})$

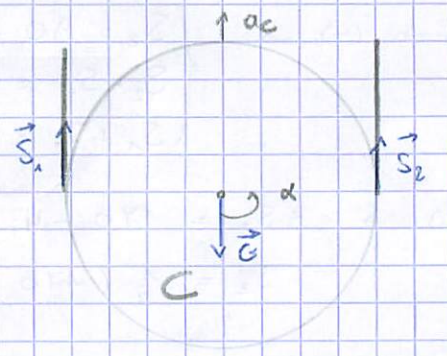
! niet mogelijk

$\Rightarrow \frac{13}{9}R = 0$ of $R = 0$

$\Rightarrow \alpha = \frac{15}{\ell} \frac{rad}{s^2}$

7.7





<u>translatie van A</u>

$$\sum \vec{F}_i = m \cdot \vec{a}$$

$$\begin{Bmatrix} 0 \\ -G \end{Bmatrix} + \begin{Bmatrix} 0 \\ N \end{Bmatrix} + \begin{Bmatrix} F\cos 45° \\ F\sin 45° \end{Bmatrix} + \begin{Bmatrix} -\omega \\ 0 \end{Bmatrix} + \begin{Bmatrix} -S_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} +m\, a_A \\ 0 \end{Bmatrix}$$

$$\rightarrow N = G - F\sin 45° = 200\,N - 100\sqrt{2}\,\frac{\sqrt{2}}{2}\,N$$

$$= 100\,N$$

$$\omega = N \cdot \mu = 100\,N \cdot 0{,}1 = 10\,N$$

$$\Rightarrow S_2 = 90\,N - 20\,a_A \quad (1)$$

<u>OPM</u>: cilinder/schijf zonder: $\frac{1}{2} m \cdot r^2$

<u>rotatie van C</u>

$$\vec{M}_C = I_C \cdot \alpha \quad \rightarrow \quad I_C = \frac{1}{2} \cdot 8\,kg \cdot (0{,}1\,m)^2 = 0{,}04\,kg\,m^2$$

$$\vec{M}_C = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -0{,}1 & 0 & 0 \\ 0 & S_1 & 0 \end{vmatrix} + \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0{,}1 & 0 & 0 \\ 0 & S_2 & 0 \end{vmatrix}$$

$$= 0{,}1\,(S_2 - S_1)\,\vec{e}_z$$

$$\rightarrow 0{,}1\,(S_2 - S_1) = 0{,}04\,\alpha \quad (2)$$

<u>translatie van C</u>

$$\sum \vec{F}_i = m \cdot a$$

$$\rightarrow S_1 + S_2 - G = +\,m \cdot a$$

$$S_1 + S_2 = 80 + 8 \cdot a = 80 + 0{,}8\,\alpha \quad (3)$$

<u>verband tussen a_A en α</u>

A en C verbonden door touw

→ als C naar boven wil wordt het touw langs beide kanten korter of maar dubbel zo snel

$$\Rightarrow a_A = 2\,2\,\alpha \quad (4)$$

OF gemakkelijker: $a_A = 2\,a_C$

(1), (2), (3) en (4) $\rightarrow$ $\begin{cases} S_2 = 90 - 4\alpha & (a) \\ S_2 - S_1 = 0,4\alpha & (b) \\ S_1 + S_2 = 80 + 0,8\alpha & (c) \end{cases}$

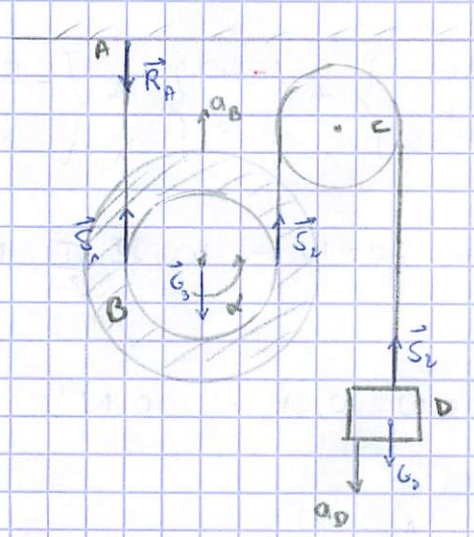
(a) + (b) + (c) $\rightarrow$ $3 S_2 = 170 - 2,8\alpha$

$S_2 = \frac{1}{3}(170 - 2,8\alpha)$

$\Rightarrow 170 - 2,8\alpha = 270 - 12\alpha$

$-100 = -9,2\alpha \rightarrow \alpha = 10,87 \text{ rad/s}^2$

$\boxed{7.8}$



Rotation B

$\vec{M}_B = I_B \cdot \alpha \rightarrow I_B = 0,2 \text{ kg m}^2$ (donné)

$$\vec{M}_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ -0,1 & 0 & 0 \\ 0 & S_1 & 0 \end{vmatrix} + \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0,1 & 0 & 0 \\ 0 & S_2 & 0 \end{vmatrix}$$

$= 0,1 (S_2 - S_1) \text{ Nm } \vec{e}_z$

$\rightarrow \alpha = \frac{1}{2}(S_2 - S_1)$ (1)

Translation B

$\sum \vec{F}_i = m \cdot a$

$S_1 + S_2 - G_B = m \cdot a_B$

$S_1 + S_2 = 20 + 2a_B$ (2)

Translation D

$\sum \vec{F}_i = m \cdot a$

$S_2 - G_D = -m \cdot a_D$

$S_2 = 20 - 2a_D$ (3)

verbanden mekreden

* $a_B = r\,\alpha = 0{,}1\,\alpha$ (4)

* $a_D = 2a_B$

* $\Leftrightarrow$ $a_D = 0{,}2\,\alpha$ (5)

$$\Rightarrow \begin{cases} \alpha = \frac{1}{2}(S_2 - S_1) & (a) \\ 20 + 0{,}2\,\alpha = S_1 + S_2 & (b) \\ 20 - 0{,}4\,\alpha = S_2 & (c) \end{cases}$$

$$2(a) + (b) \rightarrow \begin{cases} 2{,}2\,\alpha + 20 = 2S_2 \\ 20 - 0{,}4\,\alpha = S_2 \end{cases}$$

$$\Rightarrow \quad 3\alpha - 20 = 0$$

$$\alpha = \frac{20}{3} \text{ rad/s}^2$$

$$\rightarrow \quad a_D = -1{,}33 \ \frac{m}{s^2}\,\vec{e}_y \left(= \frac{20}{3} \cdot 0{,}2\right)$$

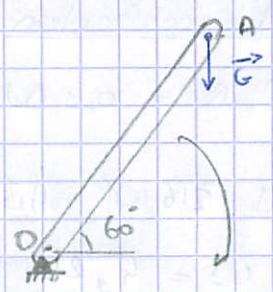
$$F_A = R_A = S_1 = 0{,}6\,\alpha = -4\,N\,\vec{e}_y$$

ex 2ur d.d. : M96

Hoofdstuk 7

$\boxed{7.9}$ behoud van energie

$(E_{pot} \rightarrow E_{kin})$



situatie 1 :

$$V_1 = m.g.h = 60\,kg \cdot 10\,\frac{N}{kg}\,(1\,m \cdot \sin 60° + 1)$$

$$= (300\sqrt{3} + 600)\,J$$

situatie 2 :

$$T_2 = \frac{1}{2} I_O\,\omega^2 \quad \rightarrow \quad I_O = I_C + m.z_{OA}^2$$

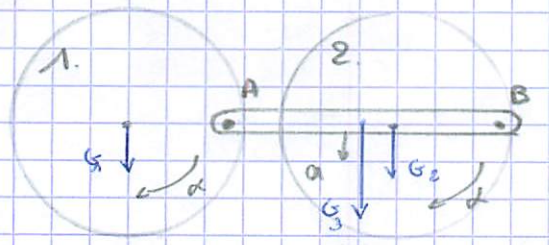
$$= \frac{1}{\cancel{4}\,2}\,60\,kg \cdot \left(\frac{1}{2}\right)^2 + 60\,kg \cdot 1^2$$

$$= \frac{17}{\cancel{16}\,8}\,60\,kg\,m^2$$

$$\Rightarrow \quad V_1 = T_2$$

of maw : $\omega = 5{,}76$ rad/s $\Leftrightarrow$ $v = \omega . R = 5{,}76\ \frac{m}{s}$

7.10 maximale hoeksnelheid wordt bereikt wanneer A en B zich zo dicht mogelijk bij de grond bevinden

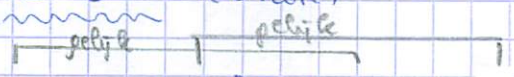


<u>behoud van energie</u>

<u>situatie 1</u> : (E_{pot})

$$V = m_3 \cdot g \cdot R = 0{,}3 \cdot 24\,kg \cdot 10\,\frac{N}{kg} = 72\,J$$

<u>situatie 2</u> : (E_{kin})



$$T = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} m_3 v_3^2 \left(\text{met } I_1 = I_2 = \frac{16\,kg \cdot 0{,}3^2}{2} = 0{,}72\right)$$

$$= 0{,}72\,\omega^2 + 16\,v^2 + 12\,v_3^2$$

$$\Rightarrow V = T \Leftrightarrow 72\,J = 0{,}72\,\omega^2 + 16\,v^2 + 12\,v_3^2 \quad (1)$$

<u>verband snelheden</u>

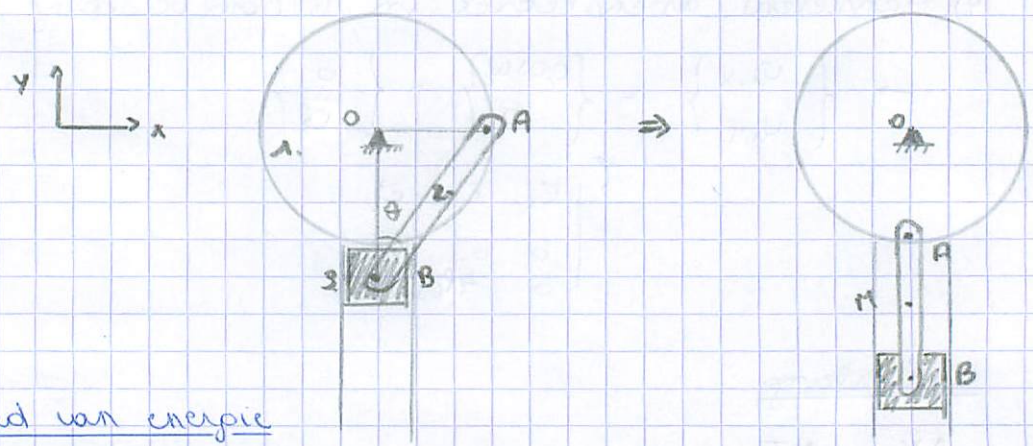
<u>OPM</u>: v_3 is nul in uiterste stand!

$$v = \omega \cdot R = 0{,}3\,\omega \quad (2)$$

$$(1) \text{ en } (2) \Rightarrow 72\,J = 0{,}72\,\omega^2 + 1{,}44\,\omega^2$$

$$\text{of } \omega = 5{,}77\ rad/s$$

7.11



behoud van energie

situatie 1 : * $V_1 = m_2 . h_1 g + m_3 h_1 g$

$= -0,1.10.\frac{0,05}{2}\cos\theta - 0,05.10.0,1\cos\theta$ met $\sin\theta = \frac{0,03}{0,05}$

$= -0,02 - 0,04 = -0,06$ J

situatie 2 : * $V_2 = m_2 . h_2 g + m_3 h_2 g$

$= -0,1.10.0,055 - 0,1.10.0,08$

$= -0,135$ J

* $T_2 = T_① + T_② + T_③$

$\rightarrow T_① = \frac{m_1 v_1^2}{2} + \frac{I_1 \omega_1^2}{2}$

met $I_1 = \frac{0,1 \text{ kg}.(0,03 \text{ m})^2}{2} = 4,5.10^{-5}$ kg m²

$\rightarrow T_② = \frac{m_2 v_2^2}{2} + \frac{I_2 \omega_2^2}{2}$

met $I_2 = \frac{0,1 \text{ kg}.(0,05 \text{ m})^2}{12} = 2,08.10^{-5}$ kg m²

$\rightarrow T_③ = \frac{m_3 v_3^2}{2}$

verbanden tussen snelheden

1) $v_3 = 0$ want uiterste stand

2) cilinder 1 : $v_1 = \omega_1 . r = 0,03\,\omega_1$

3) translerend assentrekel in A (om beweging M te beschrijven)

$\rightarrow \begin{Bmatrix} +v_{2,x} \\ v_{2,y} \end{Bmatrix} = \begin{Bmatrix} -0,03\,\omega_1 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0,025\,\omega_2 \\ 0 \end{Bmatrix}$

$\begin{Bmatrix} \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -\omega_1 \\ 0 & -0,03 & 0 \end{vmatrix} (v_{2,leep}) \end{Bmatrix} \quad \begin{Bmatrix} \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & \omega_2 \\ 0 & -0,025 & 0 \end{vmatrix} (v_{2,rel}) \end{Bmatrix}$

4) Hamlerend aansteekset in B (om beweging A te berekenen)

$$\rightarrow \begin{Bmatrix} v_{1,x} \\ v_{1,y} \end{Bmatrix} = \begin{Bmatrix} 0{,}05\omega_2 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ v_3 \end{Bmatrix}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \omega_2 \\ 0 & -0{,}05 & 0 \end{vmatrix}$$

<u>uitwerking</u>

$\Delta V = \Delta T$

$\rightarrow \Delta T = 0{,}075 \text{ J} =$

$T_{①} = 0{,}05\, v_1^2 + 2{,}25 \cdot 10^{-5}\, \omega_1^2$ $\quad (v_1 = 0{,}03 \cdot \omega_1)$

$T_{②} = 0{,}05\, v_2^2 + 1{,}04 \cdot 10^{-5}\, \omega_2^2$ $\quad \left(\begin{array}{l} v_2 = -0{,}03\,\omega_1 + 0{,}025\,\omega_2 \\ \omega_1 = \frac{5}{3}\,\omega_2 \end{array}\right)$

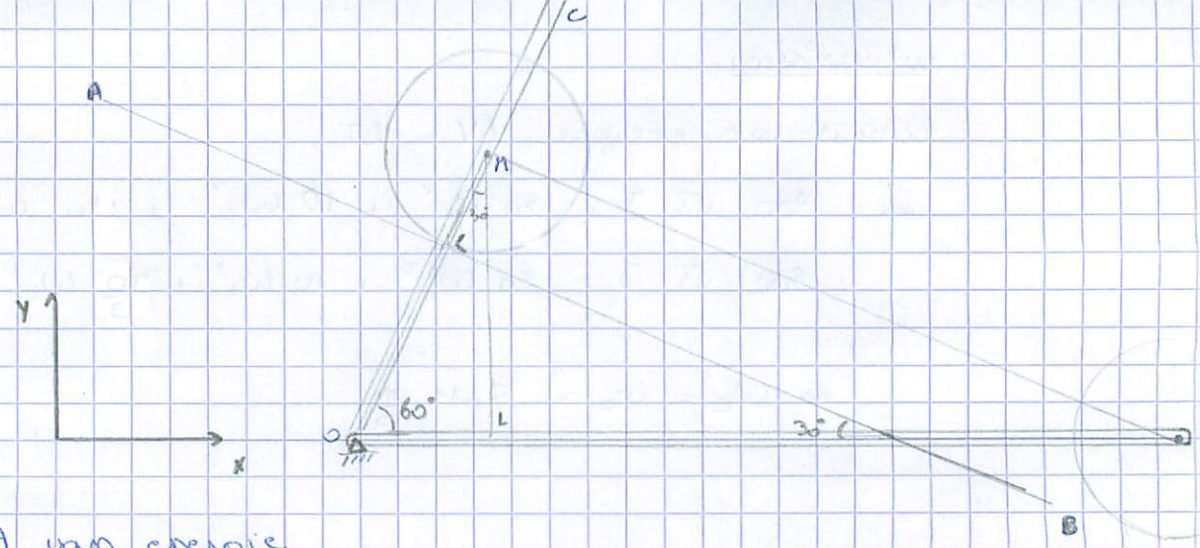
$T_{③} = 0$

$$\Leftrightarrow 0{,}075 \text{ J} = 4{,}5 \cdot 10^{-5}\, \omega_1^2 + 2{,}25 \cdot 10^{-5}\, \omega_1^2 + 0{,}05\left(-0{,}03\,\omega_1 + 0{,}025 \cdot \frac{5}{3} \cdot \omega_1\right)^2 + 1{,}04 \cdot 10^{-5} \cdot \left(\frac{5}{3}\right)^2 \omega_1^2$$

?

$$= 1{,}032 \cdot 10^{-4}\, \omega_1^2$$

7.12



<u>behoud van energie</u>

situatie 1 : $V = m_s g h_1 + m_c g h_1$

$= 20\,kg \; 10\,\frac{N}{kg} \cdot \frac{6\,m}{2} \cdot \sin 60° + 40\,kg \; 10\,\frac{N}{kg} \cdot \frac{6\,m}{2} \cdot \cos 30° \cdot \cos$

$= 900 \cdot \sqrt{3} \; J$

situatie 2 : $T = T_c + T_s$

$\rightarrow T_c = \frac{m_c v_c^2}{2} + \frac{I_c \omega_c^2}{2}$

met $I_c = \frac{40\,kg \cdot (1\,m)^2}{2} = 20\,kg\,m^2$

$\rightarrow T_s = \frac{m_s v_s^2}{2} + \frac{I_s \omega_s^2}{2} = \frac{I_{O,s}\,\omega_{O,s}^2}{2}$

met $I_{O,c} = I_c + m r_{OH}^2 = \frac{20\,kg\,(6\,m)^2}{12} + 20\,kg \cdot 3^2 = 240\,kg\,m$

<u>verbanden snelheden</u> (samengestelde beweging)

1) $v_c = \omega_c \cdot r_c = \omega_c \cdot 1\,m = \omega_c$

2) draaiend assenstelsel in O voor beschrijving M :

$$\begin{Bmatrix} v_c \cdot \cos 30° \\ -v_c \sin 30° \end{Bmatrix} = \begin{Bmatrix} 0 \\ v_{rel} \cdot \vec{e}_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -6\,\omega_{O,s} \end{Bmatrix}$$

$\begin{Bmatrix} v_{rel} \cdot \vec{e}_y \\ -v_{rel} \cdot \vec{e}_x \end{Bmatrix}$ $\quad \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\omega_{O,s} \\ 6 & 0 & 0 \end{vmatrix}$

$$\Rightarrow \begin{Bmatrix} v_c \cos 30° \\ -v_c \sin 30° \end{Bmatrix} = \begin{Bmatrix} v_{rel} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -6\,\omega_{O,s} \end{Bmatrix}$$

$\rightarrow v_c = 12\,\omega_{O,s}$

of maw $\omega_c = 12\,\omega_{O,s}$

uitwerking

behoud van energie $\Delta V = \Delta T$

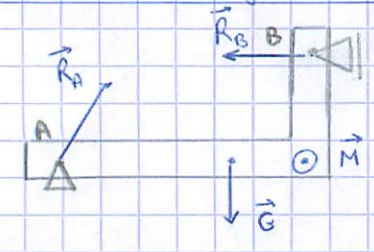
$$\Rightarrow \quad 900 \cdot \sqrt{3}\ J = 20\ v_c^2 + 10\ \omega_c^2 + 120\ \omega_{0,0}^2$$

$$900\ \sqrt{3}\ J = 80\ \omega_c^2 + 10\ \omega_c^2 + \frac{120}{12^2}\ \omega_c^2$$

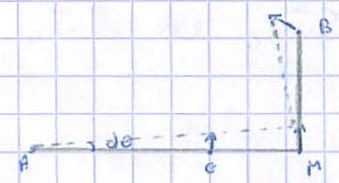
$$\Rightarrow \omega_c = v_c = 7{,}11\ \frac{m}{s}$$

extra oefeningen (virtuele arbeid) (27.01.'10)

8.1



draaibeweging rond A ($d\vec{\theta} = d\theta \cdot \vec{e}_z$)



$$\left.\begin{aligned} \delta\vec{u}_A &= 0 \\ \delta\vec{u}_G &= \tfrac{2}{3} b \cdot \delta\theta\, \vec{e}_y \\ \delta\vec{u}_M &= b \cdot \delta\theta \cdot \vec{e}_y \\ \delta\vec{u}_B &= b \cdot d\theta\, \vec{e}_y - R\, d\theta \cdot \vec{e}_x \end{aligned}\right\} \text{verplaatsingsvectoren}$$

$$\rightarrow \delta V = \vec{R}_A\, \delta\vec{u}_A + \vec{G} \cdot \delta\vec{u}_G + \vec{R}_B \cdot \delta\vec{u}_B + \vec{M} \cdot \delta\vec{\theta} = 0$$

$$\Leftrightarrow -\frac{2}{3} b\, G\, \delta\theta - R \cdot R_B\, \delta\theta + M\, \delta\theta = 0$$

of maw: $\vec{R}_B = \dfrac{M - \frac{2}{3} b\, G}{R} = \dfrac{10 - \frac{2}{3} \cdot 500}{0{,}5}$

$$= -647\,N \cdot \vec{e}_x$$

verticale verplaatsing δu

$$\delta\vec{u}_A = \delta\vec{u}_G = \delta\vec{u}_M = \delta\vec{u}_B = \delta u \cdot \vec{e}_y$$

$$\rightarrow \delta V = R_{ay} - G = 0$$

$$\vec{R}_{ay} = G = 500\,N\, \vec{e}_y$$

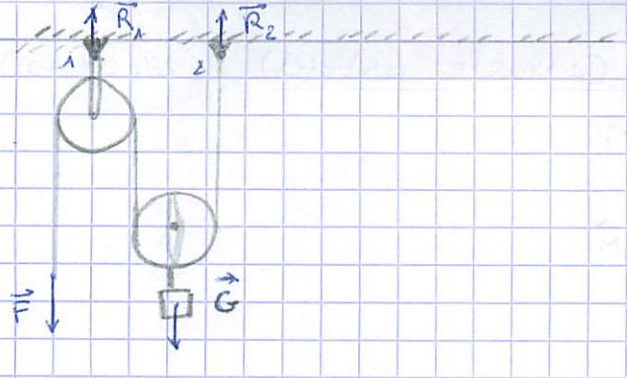
Horizontale verplaatsing δu

$$\delta\vec{u}_A = \delta\vec{u}_G = \delta\vec{u}_M = \delta\vec{u}_B = \delta u \cdot \vec{e}_x$$

$$\rightarrow \delta V = R_{ax} - R_b = 0$$

$$\vec{R}_{ay} = 647\,N\, \vec{e}_x$$

8.2



<u>verticale verplaatsing</u>

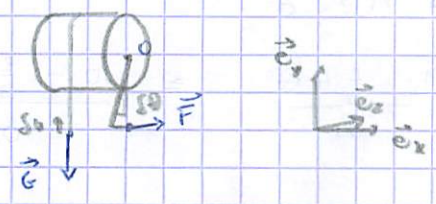
$\delta\vec{u}_1 = \delta\vec{u}_2 = 0$ en $\delta\vec{u}_F = 2\,\delta\vec{u}_G$ en $\delta\vec{u}_G = \delta u \cdot \vec{e}_y$

$\rightarrow \delta V = \vec{F} \cdot \delta\vec{u}_F + \vec{G} \cdot \delta\vec{u}_G$

$= 2F\,\delta u + G\,\delta u = 0$

$\Leftrightarrow \vec{F} = \frac{\vec{G}}{2} = -250\ N \cdot \vec{e}_y$

8.3



<u>virtuele verplaatsing</u> ($\delta\theta$ van cilinder)

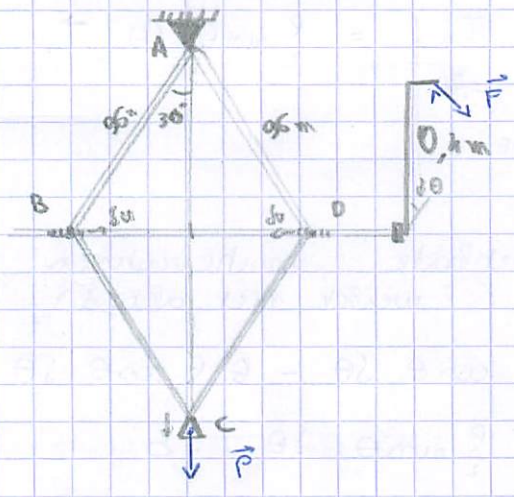
$\delta u_F = \delta\vec{\theta}_0 \times \vec{r}_d = 0{,}36 \cdot \delta\theta\ \vec{e}_z$

$\delta u_G = \delta\vec{\theta}_0 \times \vec{r}_2 = 0{,}02\,\delta\theta \cdot \vec{e}_y$

$\rightarrow \delta V = 0{,}02\,G + 0{,}36\,F = 0$

$\Leftrightarrow F = \frac{0{,}02}{0{,}36} \cdot 450\ N = 25\ N$

8.4



<u>virtuele arbeid</u> $\delta\vec{\theta}$

* $\delta\vec{u}_F = \ell \cdot \delta\vec{\theta} \; (= 0{,}4\,\delta\theta)$

* $\delta\vec{u}_{B,1} = \dfrac{R\,\delta\theta}{2\pi}\,\vec{e}_x = a\cos\alpha\,\delta\alpha\,\vec{e}_x \Leftrightarrow \delta\alpha = \dfrac{R}{2\pi\, a\cos\alpha}\,\delta\theta$

$$\rightarrow \delta\vec{u}_B = \frac{R\,\delta\theta}{2\pi}\,\vec{e}_x - a\sin\alpha\,\delta\alpha\,\vec{e}_y$$

$$= \frac{R\,\delta\theta}{2\pi}\,\vec{e}_x - \frac{\tan\alpha\, R\,\delta\theta}{2\pi}\,\vec{e}_y$$

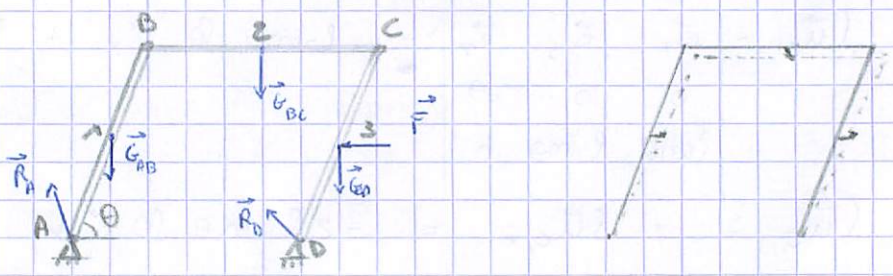
* $\delta\vec{u}_C = 2\cdot\delta\vec{u}_{B,y} = -\dfrac{\tan\alpha\, R\,\delta\theta}{\pi}\,\vec{e}_y$

$$\Rightarrow \delta V = \vec{F}\cdot\delta\vec{u}_F + \vec{P}\cdot\delta\vec{u}_C = 0$$

$$\rightsquigarrow F\ell\,\delta\theta - \frac{P\cdot\tan\alpha\cdot R}{\pi}\,\delta\theta = 0$$

$$\Leftrightarrow P = \frac{F\ell\pi}{R\cdot\tan\alpha} = 870{,}6\ \text{N}$$

8.5



<u>virtuele arbeid</u> → verplaatsing: $\delta\theta$

$$\delta\vec{u}_1 = \frac{\ell}{2}\sin\theta\cdot\delta\theta\,\vec{e}_x - \frac{\ell}{2}\cos\theta\,\delta\theta\,\vec{e}_y \Leftarrow \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\delta\theta \\ \frac{\ell}{2}\cos\theta & \frac{\ell}{2}\sin\theta & 0 \end{vmatrix}$$

$$\delta\vec{u}_2 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ \ell\cos\theta & \ell\sin\theta & 0 \end{vmatrix} = -\ell\cos\theta\,\delta\theta\,\vec{e}_y + \ell\sin\theta\,\vec{e}_x$$

$$\delta\vec{u}_3 = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\delta\theta \\ \frac{l}{2}\cos\theta & \frac{l}{2}\sin\theta & 0 \end{vmatrix} = \frac{l}{2}\sin\theta\,\delta\theta\,\vec{e}_x - \frac{l}{2}\cos\theta\,\delta\theta\,\vec{e}_y$$

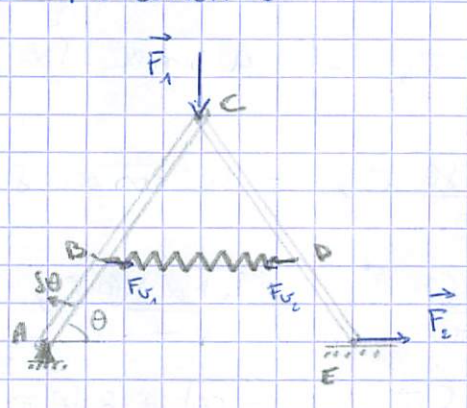
<u>OPM</u>: andere methode: positievectoren afleiden!
(! werkt niet altijd)

$$\rightarrow \delta V = -G\frac{l}{2}\cos\theta\,\delta\theta - G\,l\cos\theta\,\delta\theta - G\frac{l}{2}\cos\theta\,\delta\theta + F\frac{l}{2}\sin\theta\,\delta\theta = 0$$

$$\Rightarrow \quad F\frac{l}{2}\sin\theta = 2\,G\,l\cos\theta$$

$$F = 4\,G\,\mathrm{cotan}\,\theta$$

$\boxed{8.6}$



<u>virtuele arbeid</u> → $\delta\theta$

* virtuele verplaatsingen:

$$\delta\vec{u}_A = 0$$

$$\delta\vec{u}_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ \frac{1}{3}l\cos\theta & \frac{1}{3}l\sin\theta & 0 \end{vmatrix} = -\frac{1}{3}l\sin\theta\,\delta\theta\,\vec{e}_x + \frac{1}{3}l\cos\theta\,\delta\theta\,\vec{e}_y$$

$$\delta\vec{u}_C = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ l\cos\theta & l\sin\theta & 0 \end{vmatrix} = -l\sin\theta\,\delta\theta\,\vec{e}_x + l\cos\theta\,\delta\theta\,\vec{e}_y$$

$$\delta\vec{u}_E = 2\,\delta\vec{u}_{C,x} = -2l\sin\theta\,\delta\theta\,\vec{e}_x$$

$$\delta\vec{u}_D = \delta\vec{u}_E + \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & -\delta\theta \\ -\frac{l}{3}\cos\theta & \frac{l}{3}\sin\theta & 0 \end{vmatrix} = -\frac{5}{3}l\sin\theta\,\delta\theta\,\vec{e}_x + \frac{l}{3}\cos\theta\,\delta\theta\,\vec{e}_y$$

↳ relatieve beweging D ten E

* virtuele arbeid:

$$\delta V = \vec{F}_z\, \delta\vec{u}_E + \vec{F}_A\, \delta\vec{u}_C + \vec{F}_{v_A}\, \delta\vec{u}_B + \vec{F}_{v_C}\, \delta\vec{u}_D = 0$$

$$\hookrightarrow \quad 1000\,N \cdot \vec{e}_x \cdot -0{,}06\,m \sin\theta\, \vec{e}_x - 2000\,N\, \vec{e}_y \cdot 0{,}03 \cos\theta\, \delta\theta\, \vec{e}_y$$

$$+ F_v \cdot \vec{e}_x (-0{,}01) \sin\theta\, \delta\theta - F_v \cdot \vec{e}_x (-0{,}05) \sin\theta\, \delta\theta = 0$$

$$\Rightarrow -60 \sin\theta - 60 \cos\theta + 0{,}04\, F_v \sin\theta = 0 \quad (1)$$

* verband F_v en θ:

$$\cancel{F_v = k(l - l_0) \quad \text{met } l = \vec{u}_{D,x} - \vec{u}_{B,x}}$$

$$\cancel{= +0{,}04 \sin\theta\, \delta\theta}$$

$$\cancel{\hookrightarrow F_v = 300\,(0{,}15 - 0{,}04 \sin\theta)}$$

$$F_v = k\,(l - l_0) \quad \text{met } l = 2\,(\cos\theta \cdot 0{,}2)$$

$$F_v = \overset{30\,000}{\cancel{300}}\,(2\,(\cos\theta \cdot 0{,}2) - 0{,}15)$$

$$= 12\,000 \cos\theta - 4500 \quad (2)$$

$$\overset{(1) \text{ en } (2)}{\Longrightarrow} \quad 0{,}04\, F_v \sin\theta = 60 \sin\theta + 60 \cos\theta$$

$$F_v = 1500 + 1500 \cot\theta$$

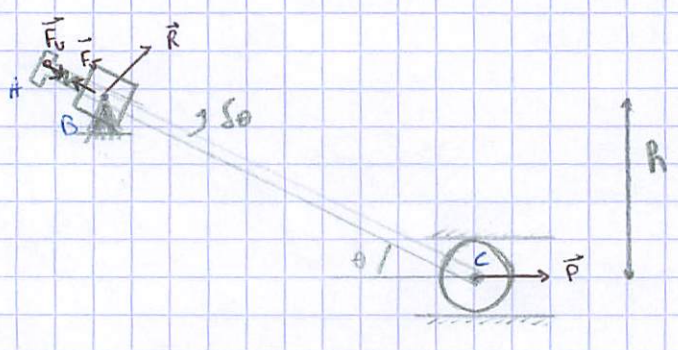
$$12\,000 \cos\theta - 4500 = 1500 + 1500 \cot\theta$$

$$12000 \cos\theta - 1500 \cot\theta - 6000 = 0$$

$$\times \frac{\sin\theta}{\cos\theta} \Big\downarrow$$

$$8 \sin\theta - 4 \tan\theta - 1 = 0$$

? $\boxed{8.7}$



* virtuele verplaatsingen: ($\delta\theta$ rond B)

$$\delta\vec{u}_B = 0$$

$$\delta\vec{u}_A = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ -x \cdot \cos\theta & x \sin\theta & 0 \end{vmatrix} = -x \sin\theta\, \delta\theta\, \vec{e}_x - x \cos\theta\, \delta\theta\, \vec{e}_y$$

$$\delta \vec{u}_c = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ \cot\theta . R & -R & 0 \end{vmatrix} = R\,\delta\theta\,\vec{e}_x + R\cot\theta\,\delta\theta\,.\,\vec{e}_y$$

x uitrucke arbeid:

$$\delta V = -x F_{v_x} \sin\theta - x F_{v_y} \cos\theta + RP = 0$$

$$F_v = k(l - l_0) = kx$$

$$\rightarrow x = l - l_0 \quad \text{met} \quad l_0 = |AC| - R$$

$$l = |AC| . - |BC|$$

$$\hookrightarrow \frac{R}{\sin\theta}$$

$$= \frac{R}{\sin\theta} - R$$

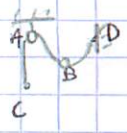
$$F_{vx} = F_v . \cos\theta = R\cot\theta - R\cos\theta$$

$$F_{vy} = -F_v . \sin\theta = R\sin\theta - R$$

$$\cancel{\hookrightarrow -|AC|^2(\sin\theta - 1)^2 k\sin\theta + |AC|^2(\sin\theta - 1)^2 k\cos\theta + Rl = 0}$$

enzoverder

$\boxed{8.8}$ opgelet! : speciale oefening



* beschouw virtuele verplaatsing (draaiing) rond A over $\delta\theta$:
(bewegend assemblsel in A draaiend!)

$$\delta\vec{u}_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ l\cdot\cos\theta & l\sin\theta & 0 \end{vmatrix} + \begin{Bmatrix} \Delta l \cdot \cos\theta \\ \Delta l \cdot \sin\theta \end{Bmatrix}$$

hoeveelheid touw weg

$$\begin{Bmatrix} 0 \\ \delta u_B \end{Bmatrix} = \begin{Bmatrix} -l\sin\theta\,\delta\theta \\ l\cos\theta\,\delta\theta \end{Bmatrix} + \begin{Bmatrix} \Delta l\cos\theta \\ \Delta l\sin\theta \end{Bmatrix}$$

B beweegt recht naar boven

$$\rightarrow \quad l\sin\theta\,\delta\theta = \Delta l\cos\theta$$

$$\Delta l = l\tan\theta\,\delta\theta$$

$\delta\vec{u}_C = 2\,\Delta l$ WANT zelfde verplaatsing kan ook bekeken worden rond D i.p.v. A

$$\rightarrow \quad \delta\vec{u}_C = 2\Delta l = 2l\tan\theta\,\delta\theta$$

* uitwerking:

$$\delta V = \delta\vec{u}_B\cdot\vec{G}_B + \delta\vec{u}_C\,\vec{G}_C = 0$$

$$340\,l\cos\theta\,\delta\theta + 340\,l\tan\theta\sin\theta + 980\cdot 2l\tan\theta\,d\theta = 0$$

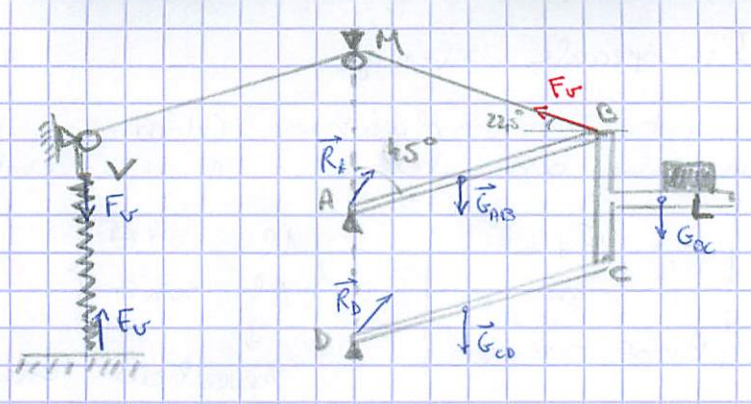
$$\rightarrow \quad 340\cos^2\theta + 340\sin^2\theta + 980\cdot 2\sin\theta = 0$$

$$\sin\theta = \frac{340}{1960}$$

of maw $\theta = 10°$

8.9

OPM: veerkracht loopt door rond w!



bepaling Δl

$$|MB|^2 = |AM|^2 + |AB|^2 - 2\,|AM||AB| \cos 45°$$

$$= 4a^2 + 4a^2 - 2 \cdot 4a^2 \cdot \frac{\sqrt{2}}{2}$$

$$= 4a^2 (2 - \sqrt{2})$$

$$\rightarrow |MB| = 2a \sqrt{2-\sqrt{2}} = \Delta l$$

virtuele arbeid

* virtuele verplaatsingen $\rightarrow \delta\vec{\theta}$

$$\delta\vec{G}_{AB} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ \frac{\sqrt{2}}{2}a & \frac{\sqrt{2}}{2}a & 0 \end{vmatrix} = -\frac{\sqrt{2}}{2} a\, \delta\theta\, \vec{e}_x + \frac{\sqrt{2}}{2} a\, \delta\theta\, \vec{e}_y$$

$$\delta\vec{G}_{AB} = \delta\vec{G}_{CD}$$

$$\delta\vec{G}_{BC} = \delta\vec{u}_B = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 0 & \delta\theta \\ \sqrt{2}a & \sqrt{2}a & 0 \end{vmatrix} = -\sqrt{2}\, a\, \delta\theta\, \vec{e}_x + \sqrt{2}\, a\, \delta\theta\, \vec{e}_y$$

$$\delta\vec{u}_A = \delta\vec{u}_D = 0$$

$$\cancel{\delta\vec{u}_B = \|\delta\vec{u}_B\| = -2a\,\delta\theta\,\vec{e}_y}$$

* uitwerking met virtuele arbeid:

$$\delta V = \delta\vec{u}_A \cdot \vec{R}_A + \delta\vec{u}_D \cdot \vec{R}_D + \delta\vec{G}_{AB} \cdot \vec{G}_{AB} + \delta\vec{G}_{CD} \cdot \vec{G}_{CD} + \delta\vec{G}_{BC} \cdot \vec{G}_{BC} + \delta\vec{u}_B \cdot \vec{F}_v = 0$$

$$\rightarrow 0 = -2\sqrt{2}\, a \cdot G - \frac{\sqrt{2}}{2} a G - \frac{\sqrt{2}}{2} a G + \sqrt{2}\, a \cdot F_v \cdot \cos(22{,}5°) + \sqrt{2}\, a\, F_v \sin(22{,}5°)$$

of nog: $\sqrt{2} F_v (\cos(22{,}5) + \sin(22{,}5)) = 3\sqrt{2}\, G$

$F_v = k \cdot \Delta l$

$$k = \frac{3G}{2a\sqrt{2-\sqrt{2}} \cdot (\cos(22{,}5) + \sin(22{,}5))}$$

$$= \frac{3G}{2a}$$

opmerkingen

* tegenwijzerzin = moment positief !

* moment : * afstand vanaf punt waarrond we moment berekenen
  * oppelet voor volgorde : $-\vec{r} \times \vec{F}$: } zelfde volgorde in det
  $-\vec{\omega} \times \vec{r}$:

  → oppelet voor teken dus !

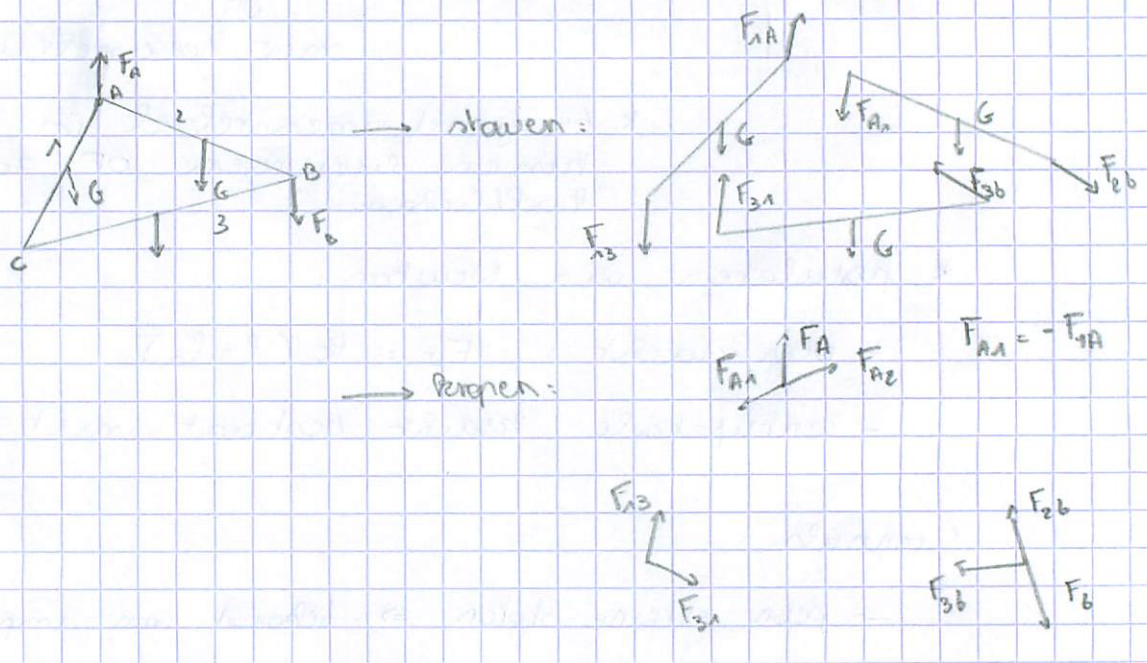
* stavenconstructies en vakwerken

  → externe krachten niet grijpen in vrijgemaakte staven !

  - stavenconstructie : zowel knopen als staven kunnen vrijgemaakt worden om de krachten te bepalen

  ↳ externe krachten grijpen aan op knopen !

vb



oppelet ! - staafkrachten niet in het algemeen gelijk volgens staven

- momentenevenwichten ook nodig voor bepaling

- als er geen externe kracht aangrijpt (zoals in punt C) dan zijn de krachten uitgeoefend door de 2 staven op het knooppunt wel eksaars tegengestelde MAAR anders niet

- vakwerken : enkel vrijmaken van knopen zinvol, want op staven grijpen telkens enkel 2 tegengestelde krachten aan

→ staafkrachten : vb F_{1A}
knoopkrachten : vb F_{A1}

ALS knopen naar staaf worden getrokken, dan trekkracht met positief teken (staafkracht)

* Kinematica vc punt:

- belangrijk: $a_t = \alpha . R$ en $a_n = \omega^2 . R$
- slingerbeweging: $\omega = \sqrt{g/R}$ en periode $= \frac{\omega}{2\pi}$

$\rightarrow \vec{r}(t) = R \sin\theta(t) . \vec{e}_x - R \cos\theta(t) . \vec{e}_y$

* Kinematica van een lichaam:

OPM: - soms meerdere bewegende assenstelsels gebruiken
- vaaral zeer belangrijk:
  * snelheid in cirkelbeweging raakend aan baan! → hoek zoeken
    OF
    mbv hoeksnelheid
  * bewegend assenstelsel in verbinding tussen lichamen OF roterend volgens "hoofd lichaam"

* postulaten van Newton:

- veerkracht: $F_v = k(l - l_0)$
- centripetale kracht bestaat niet! enkel vermelling!

* impuls:

→ geen externe stoten ⇒ behoud van impuls

→ externe stoten ⇒ verschil in impuls = stoot
$(\vec{p}_{II} - \vec{p}_{I} = N)$

↳ soms: best systeem vrijmaken in onderdelen en stoten aanbrengen
→ dan: $\sum \vec{N}_i = \vec{p}_{II} - \vec{p}_{I}$

* energie:

→ behoud van energie!

~ geen arbeid ⇒ $\Delta E = 0$
arbeid ⇒ $\Delta E = \Delta U$
met ΔE = verschil in potentiële en kinetische energie

* <u>opmerkingen</u> (vervolg)

* dynamica : 2 soorten oefeningen :

1) snelheid / hoeksnelheid gevraagd → behoud van energie !

⇝ oppelet : - zowel translatie als rotatie is mogelijk
- rotatie + translatie equivalent met rotatie rond uitwendig punt
- verbanden tussen snelheden via samengestelde beweging !

2) versnelling / hoekversnelling gevraagd

$$\rightarrow \begin{cases} \sum \vec{F}_i = m \cdot \vec{a} \\ \vec{M}_i = I \cdot \alpha \end{cases}$$

⇝ oppelet : - wanneer I niet rond massacentrum
→ regel van Steiner
- in een cirkelbeweging : !

$$\begin{cases} a_t = \alpha \cdot R \\ a_n = \omega^2 \cdot R \end{cases}$$

⇝ vertrekkende uit rust betekent $\omega = 0$

- momentenevenwicht steeds te berekenen rond het punt waarrond je het traagheidsmoment berekend

* virtuele arbeid :

- translatie → zelfde als statisch evenwicht in x- en y-richting

- rotatie ($\delta\vec{\theta}$) → verbanden tussen translaties en rotaties mbv determinanten én eventueel samengestelde beweging

<u>andere mogelijkheid</u> : plaatsvectoren afleiden