Multitask learning or coaching

```
#> Loading required package: glmnet
#> Loading required package: Matrix
#> Loaded glmnet 4.1-8
```

Multitask learning consists of data X with two or more responses $y_1, \dots, y_j$. We usually assume that there is shared signal across the responses, and that performance can be improved by jointly fitting models for the responses.

Pretraining is a natural choice for multitask learning – it is a method to pass information between models. The overview for our approach is to:

1. fit a multi-response Gaussian model,
2. extract the support (shared across responses) and offsets (one for each response), and
3. fit a model for each response, using the shared support and appropriate offset.

Importantly, in Step 1, we will use regularization so that the multi-response Gaussian model is forced to choose the same support for all responses $y_1, \dots, y_j$. This encourages learning *across* all responses in the first stage; in the second stage, then, we can find features that are specific to each individual response y_k.

We will illustrate this with simulated data with two Gaussian responses; the two responses share the first 5 features, and they each have 5 features of their own. The two responses are quite related, with Pearson correlation around 0.5.

```r
set.seed(1234)

n = 1000; ntrain = 500;
p = 500
sigma = 2

x = matrix(rnorm(n*p), n, p)
beta1 = c(rep(1, 5), rep(0.5, 5), rep(0, p - 10))
beta2 = c(rep(1, 5), rep(0, 5), rep(0.5, 5), rep(0, p - 15))

mu = cbind(x %*% beta1, x %*% beta2)
y  = cbind(mu[, 1] + sigma * rnorm(n),
           mu[, 2] + sigma * rnorm(n))
cat("SNR for the two tasks:", round(diag(var(mu)/var(y-mu)), 2))
#> SNR for the two tasks: 1.6 1.44

xtest = x[-(1:ntrain), ]
ytest = y[-(1:ntrain), ]

x = x[1:ntrain, ]
y = y[1:ntrain, ]

# Define training folds
nfolds = 5
foldid = sample(rep(1:nfolds, trunc(nrow(x)/nfolds)+1))[1:nrow(x)]

cat("Correlation between two tasks:", cor(y[, 1], y[, 2]))
```

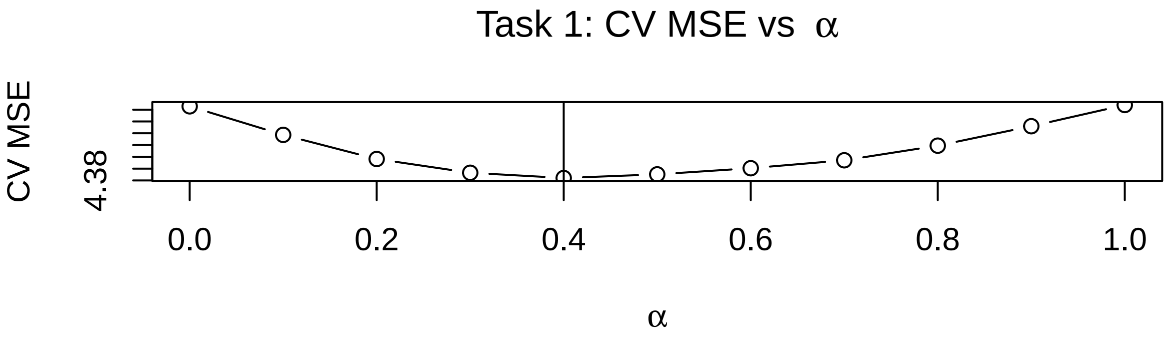
```
#> Correlation between two tasks: 0.5218575
```

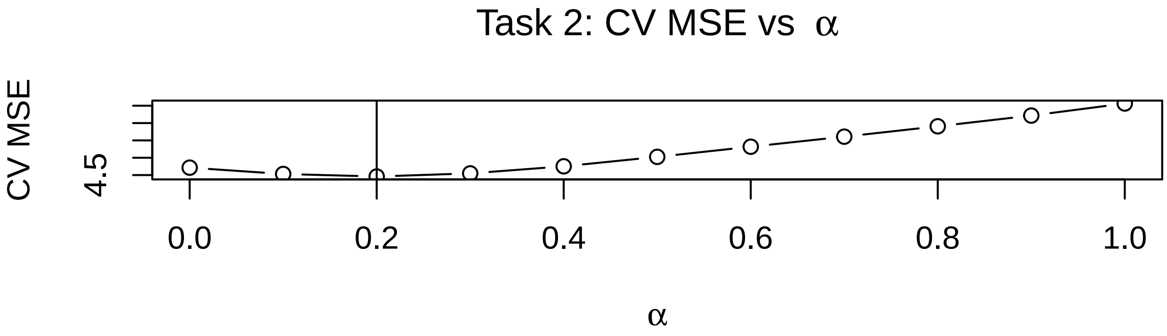
The first step of pretraining is to fit a multi-response Gaussian model and extract the offset and support.

```
mtask.fit = cv.glmnet(x, y, family = "mgaussian",
                      keep = TRUE,
                      foldid = foldid,
                      type.measure = "mse")

offset = mtask.fit$fit.preval[, , mtask.fit$lambda == mtask.fit$lambda.1se]

coefs = coef(mtask.fit, s = "lambda.1se")
coefs = cbind(coefs$y1, coefs$y2)
support = which((rowSums(coefs) != 0)[-1])
```

And now, we'll fit a separate model for each response. We will loop over values of α $(0, 0.1, \ldots, 1)$; for each α, we fit a model for each response using the offset and support defined above and modified by α. We also record the minimum CV mean squared error for each model – this is how we will perform model selection.

```
cv.error = c(NULL, NULL)
alphalist = seq(0, 1, length.out = 11)

for(alpha in alphalist){
  pf = rep(1/alpha, p)
  pf[support] = 1

  y1_fit = cv.glmnet(x, y[, 1],
                     foldid = foldid,
                     offset = (1 - alpha) * offset[, 1],
                     penalty.factor = pf,
                     family = "gaussian",
                     type.measure = "mse")

  y2_fit = cv.glmnet(x, y[, 2],
                     foldid = foldid,
                     offset = (1 - alpha) * offset[, 2],
                     penalty.factor = pf,
                     family = "gaussian",
                     type.measure = "mse")

  cv.error = rbind(cv.error, c(min(y1_fit$cvm), min(y2_fit$cvm)))
}

par(mfrow = c(2, 1))
plot(alphalist, cv.error[, 1], type = "b",
     xlab = expression(alpha), ylab = "CV MSE",
     main = bquote("Task 1: CV MSE vs " ~ alpha))
abline(v = alphalist[which.min(cv.error[, 1])])

plot(alphalist, cv.error[, 2], type = "b",
     xlab = expression(alpha), ylab = "CV MSE",
     main = bquote("Task 2: CV MSE vs " ~ alpha))
abline(v = alphalist[which.min(cv.error[, 2])])
```





The optimal values of α for the two responses are pretty close, and we could choose to use one α for both responses (say, the α that minimizes the average CV for both class). Here, we will choose to use two separate values of α. We train our final models:

```
best.alpha.1 = alphalist[which.min(cv.error[, 1])]
best.alpha.2 = alphalist[which.min(cv.error[, 2])]

pf = rep(1/best.alpha.1, p)
pf[support] = 1
y1_fit = cv.glmnet(x, y[, 1], foldid = foldid,
                   offset = (1 - best.alpha.1) * offset[, 1],
                   penalty.factor = pf,
                   family = "gaussian",
                   type.measure = "mse")

pf = rep(1/best.alpha.2, p)
pf[support] = 1
y2_fit = cv.glmnet(x, y[, 2], foldid = foldid,
                    offset = (1 - best.alpha.2) * offset[, 2],
                    penalty.factor = pf,
                    family = "gaussian",
                    type.measure = "mse")
```

There are two natural baselines: one is the performance of the multi-response model used in the first step of pretraining, and the other is a separate model for each response:

```
y1_fit_no_pretrain = cv.glmnet(x, y[, 1], foldid = foldid,
                                family = "gaussian", type.measure = "mse")

y2_fit_no_pretrain = cv.glmnet(x, y[, 2], foldid = foldid,
                                family = "gaussian", type.measure = "mse")
```

Compare performance for task 1:

```
test_offset = predict(mtask.fit, xtest, s = "lambda.1se")[, , 1]

test_mtask_pred = predict(mtask.fit, xtest, s = "lambda.min")[, , 1]

cat("Response 1 MSE overall model:",
    round(assess.glmnet(test_mtask_pred[, 1], newy = ytest[, 1])$mse, 2))
#> Response 1 MSE overall model: 4.25

cat("Response 1 MSE with pretraining:",
    round(assess.glmnet(y1_fit, xtest, newy = ytest[, 1],
                        newoffset = (1 - best.alpha.1) * test_offset[, 1])$mse,
          2))
#> Response 1 MSE with pretraining: 4.26

cat("Response 1 MSE individual model:",
    round(assess.glmnet(y1_fit_no_pretrain, xtest, newy = ytest[, 1])$mse, 2))
#> Response 1 MSE individual model: 4.48
```

And performance for task 2:

```
cat("Response 2 MSE overall model:",
    round(assess.glmnet(test_mtask_pred[, 2], newy = ytest[, 2])$mse, 2))
#> Response 2 MSE overall model: 5.21

cat("Model 2 MSE with pretraining:",
    round(assess.glmnet(y2_fit, xtest, newy = ytest[, 2],
                        newoffset = (1 - best.alpha.2) * test_offset[, 2])$mse,
          2))
#> Model 2 MSE with pretraining: 4.98

cat("Model 2 MSE individual model:",
    round(assess.glmnet(y2_fit_no_pretrain, xtest, newy = ytest[, 2])$mse, 2))
#> Model 2 MSE individual model: 5.8
```

We find that pretraining improves performance for response 2, and has performance close to that of the overall model for response 1.