Multitask learning or coaching (Gaussian responses)

```
#> Loading required package: ptLasso
#> Loading required package: ggplot2
#> Loading required package: glmnet
#> Loading required package: Matrix
#> Loaded glmnet 4.1-8
#> Loading required package: gridExtra
```

Multitask learning consists of data X with two or more responses $y_1, \ldots, y_j$. We usually assume that there is shared signal across the responses, and that performance can be improved by jointly fitting models for the responses.

Here, we suppose that we wish to predict multiple **Gaussian responses**. (If the goal is to predict multiple responses of a different type, see the section "Multi-response data with mixed response types".)

Pretraining is a natural choice for multitask learning – it allows us to pass information between models for the different responses. The overview for our approach is to:

1. fit a multi-response Gaussian model,
2. extract the support (shared across responses) and offsets (one for each response), and
3. fit a model for each response, using the shared support and appropriate offset.

Importantly, in Step 1, we use regularization so that the multi-response Gaussian model is forced to choose the same support for all responses $y_1, \ldots, y_j$. This encourages learning *across* all responses in the first stage; in the second stage, then, we find features that are specific to each individual response y_k.

This is all done with the function `ptLassoMult` (for multi-response ptLasso).

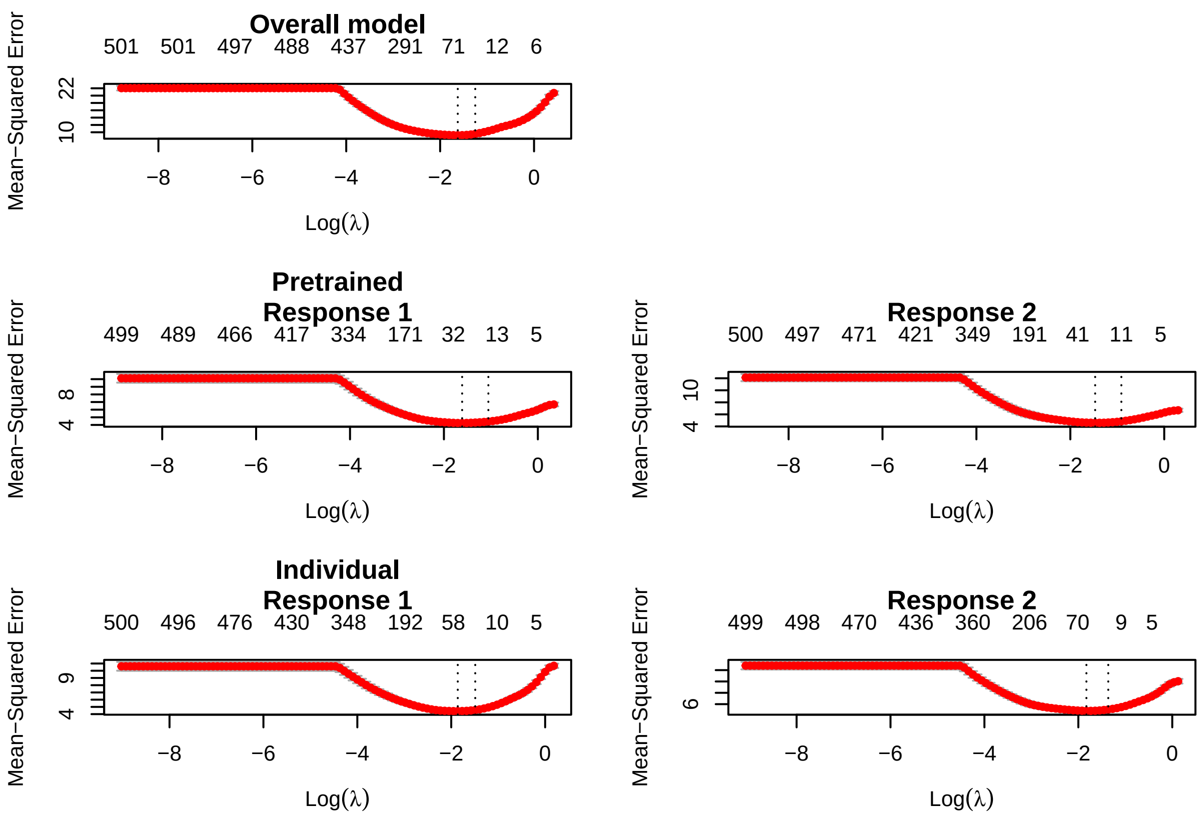
We will illustrate this with simulated data with two Gaussian responses; the two responses share the first 5 features, and they each have 5 features of their own. The two responses are quite related, with Pearson correlation around 0.5.

```r
set.seed(1234)

n = 1000; ntrain = 500;
p = 500
sigma = 2

x = matrix(rnorm(n*p), n, p)
beta1 = c(rep(1, 5), rep(0.5, 5), rep(0, p - 10))
beta2 = c(rep(1, 5), rep(0, 5), rep(0.5, 5), rep(0, p - 15))

mu = cbind(x %*% beta1, x %*% beta2)
y  = cbind(mu[, 1] + sigma * rnorm(n),
           mu[, 2] + sigma * rnorm(n))
cat("SNR for the two tasks:", round(diag(var(mu)/var(y-mu)), 2))
#> SNR for the two tasks: 1.6 1.44

xtest = x[-(1:ntrain), ]
ytest = y[-(1:ntrain), ]

x = x[1:ntrain, ]
```

```r
y = y[1:ntrain, ]

cat("Correlation between two tasks:", cor(y[, 1], y[, 2]))
#> Correlation between two tasks: 0.5218575
```

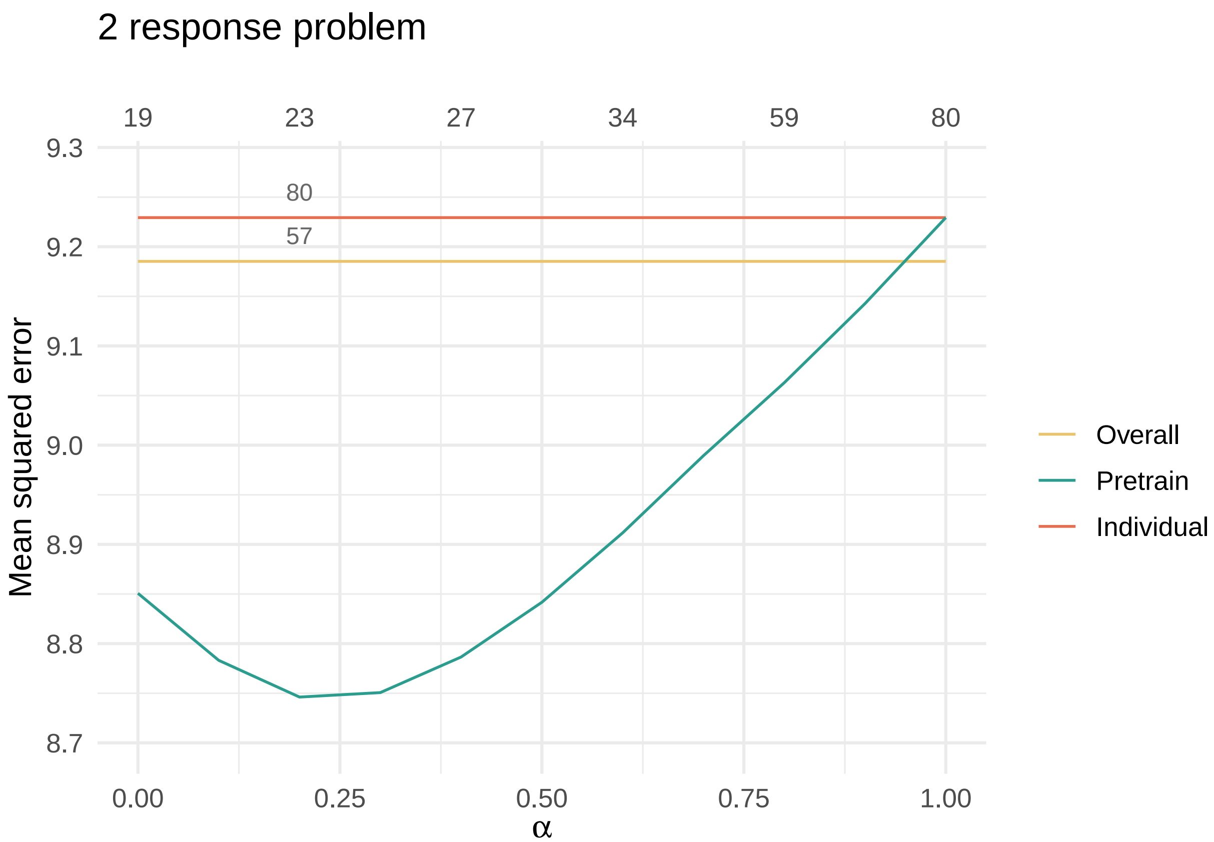
Now, we are ready to call `ptLassoMult` with our covariates `x` and response matrix `y`. (The syntax is nearly identical to that of `ptLasso`, and as in `ptLasso`, the default value of $\alpha = 0.5$.) A call to `plot` shows the CV curves over the lasso parameter λ for each model.

```r
fit = ptLassoMult(x, y)
plot(fit)
```



To choose the parameter α, we can use `cv.ptLassoMult`. The syntax is as in `cv.ptLasso`; and just like `cv.ptLasso`, we can view the CV curve for pretraining together with the overall model (multi-response Gaussian model) and the individual model (a separate Gaussian model for each response) using `plot`.

```r
fit = cv.ptLassoMult(x, y)
plot(fit)
```



The `predict` function works the same as in `ptLasso`; if `ytest` is supplied, it will print the mean squared error as well as the support size for the pretrained, overall and individual models, using the single α that minimizes the the average CV MSE across both responses.

```
preds = predict(fit, xtest, ytest=ytest)
preds
#>
#> Call:
#> predict.cv.ptLasso(object = fit, xtest = xtest, ytest = ytest)
#>
#>
#>
#> alpha =  0.2
#>
#> Performance (Mean squared error):
#>
#>            allGroups  mean response_1 response_2
#> Overall        9.394 4.697      4.227      5.168
#> Pretrain       8.907 4.453      4.186      4.721
#> Individual     9.465 4.733      4.243      5.222
#>
#> Support size:
#>
#> Overall    57
#> Pretrain   23 (19 common + 4 individual)
#> Individual 80
```

However, as in `predict` with `ptLasso`, we can choose to use the value of α that minimizes the CV MSE for *each* response.

```
preds = predict(fit, xtest, ytest=ytest, alphatype = "varying")
preds
#>
#> Call:
#> predict.cv.ptLasso(object = fit, xtest = xtest, ytest = ytest,
#>     alphatype = "varying")
#>
#>
#> alpha:
#> group_1 group_2
#>     0.3     0.2
#>
#>
#> Performance (Mean squared error):
#>            allGroups  mean response_1 response_2
#> Overall        9.394 4.697      4.227      5.168
#> Pretrain       8.877 4.438      4.156      4.721
#> Individual     9.465 4.733      4.243      5.222
#>
#>
#> Support size:
#>
#> Overall    57
#> Pretrain   23 (19 common + 4 individual)
#> Individual 80
```