### University of Bergen Department of Informatics

## Title of your master thesis

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# UNIVERSITETET I BERGEN Det matematisk-naturvitenskapelige fakultet

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#### Abstract

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No eros nemore impedit his, per at salutandi eloquentiam, ea semper euismod meliore sea. Mutat scaevola cotidieque cu mel. Eum an convenire tractatos, ei duo nulla molestie, quis hendrerit et vix. In aliquam intellegam philosophia sea. At quo bonorum adipisci. Eros labitur deleniti ius in, sonet congue ius at, pro suas meis habeo no.

#### Acknowledgements

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## Introduction

TODO: talk about Shortest Path, how cool of a problem it is, and how we have solved it in a bunch of different ways already.

TODO: then talk about Shortest Odd Path, how strange it is in comparison, and that we care because acutally useful problems are easier with such a subprocedure.

## **Preliminaries**

### 2.1 Graphs

In algorithms, we often use graphs as an abstract structure to represent the fundamental problem behind an algorithms problem without distractions. For example, when you want to find the fastest route to walk to the study hall, or if you want the cheapest combination of flights to take you to Kuala Lumphur, then both questions are really the same problem. If we remove all the details that are unnecessary to solve the problem, like the names of the airports and whether we are walking or flying, then we end up with a graph.

**Definition 2.1.1** (Graph). A graph G := (V, E, from, to) is given by

- V, a collection of vertices
- E, a collection of edges
- $from: E \to V$ , a mapping from each edge to its source vertex
- $to: E \to V$ , a mapping from each edge to its target vertex

**Definition 2.1.2** (Weighted graph). A weighted graph G := (V, E, from, to, w) is a graph, where  $w : E \to \mathbb{R}$  is the weight of each edge. If a graph is not weighted, we often treat it like it is weighted with all edges having unit weight, a weight of 1. An algorithm intended for weighted graphs will therefore often work on unweighted graph as well.

**Definition 2.1.3** (Directed and undirected graphs). Let G be a graph. G is said to be an undirected graph if each edge has an opposite:  $\forall e \in E \exists e' \in E : reverse(e) == e'$ . If G is not undirected, we say that G is a directed graph. Edges in directed graphs are often drawn as arrows, while edges in undirected graphs can be drawn using just a line.

TODO figures for graphs

**Definition 2.1.4** (Neighbourhood). Let G be a graph, and let  $u \in V$  be a vertex in the graph. The *neighbourhood* of u, commonly denoted as either N(u) or G[u], is defined as the vertices in G that are reachable from u using just a single edge:  $N(u) := \{to(e) \mid e \in E, from(e) == u\}$ .

**Definition 2.1.5** (Walk). A walk  $P := [p_1, p_2, ..., p_k]$  in a graph G is a sequence of edges where each edge ends where the next one starts:  $\forall i \in \{1, 2, ..., k-1\} : to(p_i) = from(p_{i+1})$ . If  $s := from(p_1)$  and  $t := to(p_k)$ , we say that P is an s-t-walk in G.

**Definition 2.1.6** (Path). A path  $P := [p_1, p_2, ..., p_k]$  in a graph G is a walk with the extra requirement that each vertex is used more than once:  $\forall i, j \in \{1, 2, ..., k\} : j \neq i + 1 \rightarrow to(p_i) \neq from(p_j)$ . If  $s := from(p_1)$  and  $t := to(p_k)$ , we say that P is path from s to t, or an s-t-path in G. Note that in some literature, a walk is referred to as a path, and a path is referred to as a simple path. In this thesis, when we refer to paths they are always simple, meaning that they never repeat any vertices. If any vertices are repeated, we will refer to it as a walk.

**Definition 2.1.7** (The cost of a path (/walk)). Let  $P := [p_1, p_2, ..., p_k]$  be a path (/walk) in a weighted graph G. The cost of P is defined as the sum of its edges:  $\sum_{i=1}^k w(p_i)$ . In a collection of paths (/walks), we say that the shortest path (/walk) is the cheapest one, the one with the lowest cost. Likewise for the longest and most expensive path (/walk). Note that in some literature, shortest may instead mean fewest edges, and the term length could mean both the number of edges or the cost of a path. For that ambiguous reason, we will from now on avoid the word length, and shortest will always mean cheapest.

Now we are ready to define the underlying problem of the example we started with. Both problems can be represented by an abstract graph, where we want to find the shortest path from one vertex to another. From a computational perspective, it does not matter whether the edges are roads or flights, or whether the vertices are crossroads or airports. Vertices and edges can therefore represent whatever we want them to.

SHORTEST PATH

**Input:** A graph G, two vertices  $s, t \in V$ 

**Output:** the shortest s-t-path in G

This thesis will focus on a curious variant of the Shortest Path problem, called Shortest Odd Path:

SHORTEST ODD PATH

**Input:** A graph G, two vertices  $s, t \in V$ 

Output: the shortest s-t-path in G that uses an odd number of edges

### 2.2 Planarity

TODO little introduction here, why do we care

**Definition 2.2.1** (Planar graph). Let G be a graph. We say that G is a *planar graph* if it is possible to draw the graph on the plane such that none of the edges intersect each other. Such a drawing is referred to as an *embedding* of G.

TODO figures of planar and non-planar graphs.

**Definition 2.2.2** (Duality of planar graphs). Let G be a planar graph, and choose any embedding of G in the plane.

TODO contine the definition. WTF is a face? What is the dual graph?

# Shortest Odd Walk

Before we start on the main topic of this thesis, we want to discuss a closely related problem:

SHORTEST ODD WALK

**Input:** A weighted graph G, two vertices  $s, t \in V$ 

**Output:** the shortest s-t-walk in G that uses an odd number of edges

The difference is simple: a walk may use the same vertices multiple times, whereas a path can not. A naïve attempt at solving Shortest Odd Path will often accidentally use the same vertices multiple times, and then be an odd walk instead. Therefore, we want to present an algorithm to solve Shortest Odd Walk first, and explain why it does not solve Shortest Odd Path.

#### 3.1 Intuition

Our algorithm will take inspiration from Dijkstra's algorithm for Shortest Path, and assume that all the edges have non-negative weights. Remember, in Dijkstra's algorithm we have an array to keep the tentative best distance to each vertex. In this algorithm, we will keep two such arrays, one for the best distance using an odd walk, and one for the best distance using an even walk. Each vertex can be scanned at most twice: once when we have found the definitive best odd walk and want to find potential improvements to the even walks of its neighbours, and similar when we find the best odd walk.

### 3.2 Psuedocode

Code Listing 3.1: Shortest Odd Walk

```
shortest_odd_walk(graph, s, t) {
 2
3
        for u in 0..n {
             even_dist[u]
                              = \infty
             odd_dist[u] = \infty
even_done[u] = false;
\begin{array}{c} 4\\5\\6\\7\\8\\9\end{array}
             odd_done[u] = false;
        even_dist[s] = 0
10
        queue = priority_queue([(0, true, s)]);
11
        while queue is not empty {
12
             (dist_u, even, u) = queue.pop()
if even {
13
14
                  if even_done[u] continue;
15
                   even_done[u] = true;
16
17
                  for edge in graph[u] {
18
                        v = to(edge);
19
                        dist_v = dist_u + weight(edge);
20
                        if dist_v < odd_dist[v] {
    odd_dist[v] = dist_v;</pre>
21
22
23
                             queue.push((dist_v, false, v));
                        }
\overline{24}
                  }
25
             }
26
             else {
27
                   if odd_done[u] continue;
28
                  odd_done[u] = true;
29
30
                  for edge in graph[u] {
\frac{31}{32}
                        v = to(edge);
                        dist_v = dist_u + weight(edge);
33
                        if dist_v < even_dist[v] {</pre>
34
                             even_dist[v] = dist_v;
35
                             queue.push((dist_v, true, v));
36
                        }
37
                  }
38
             }
39
                 odd_dist[t] < \infty {
40
                  return odd_dist[y];
41
42
        }
43
        return None;
44
  }
```

In the psuedocode we show how to find the best odd walk from the source vertex to the target vertex. If we instead want to find the best odd or even walks to all vertices, we can simply remove the if-clause around the target, and return the arrays instead.

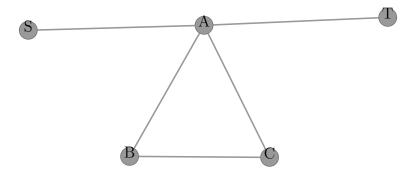


Figure 3.1: No odd s-t-path exist, but we still have many odd s-t-walks

### 3.3 Analysis

Consider 3.1. There are no odd paths from s to t, yet we have an infinite amount of odd walks. Our algorithm would first find an odd walk to a, then an even walk to b, then an odd walk to c, then an even walk to a, and lastly an odd walk to t. However, the algorithm would search a twice, once for each parity, and the resulting walk is not a path. Therefore this algorithm cannot be used to solve Shortest Odd Path.

#### 3.3.1 Correctness

TODO. Is this necessary?

### 3.3.2 Complexity

Because of our odd\_done and even\_done arrays, we can guarantee that each vertex is scanned at most twice, once for each parity. For each scan, we loop through each of the neighbours in linear time, and consider putting them in the queue. A vertex may be put into the queue many times before it is scanned, in the worst case once for each of its neighbours. That means that we put vertices in the queue at most O(m) times, for a total running time of O(m), and removing all of them takes a total of  $O(m \cdot log m)$ .

In total, the algorithm runs in  $O(m \cdot log m)$ .

TODO: er dette i det hele tatt riktig? Det føles feil. Wikipedia gir at Dijkstra kjører på  $O((n+m) \cdot log n)$ .

## 3.3.3 Benchmarking

TODO

## 3.3.4 Discussion

Shortest Odd Path

#### 4.1 Intuition

Now that we have tried out some algorithms for Shortest Odd Walk, we are finally ready to add the restriction that each vertex is used at most once, and thus solve Shortest Odd Path. The algorithm we are about to present is based on Ulrich Derigs' algorithm from 1985 [Der85], though with some improvements.

#### 4.1.1 Reduction to Shortest Alternating Path

Consider first another related problem:

SHORTEST ALTERNATING PATH

**Input:** A weighted graph G := (V, E), two vertices  $s, t \in V$ , and a set  $F \subseteq E$ 

**Output:** the shortest s-t-path in G where every other edge used is in F.

Derig observed that Shortest Odd Path can be reduced to a special case of Shortest Alternating Path, by constructing what we will refer to as a *mirror graph*.

**Definition 4.1.1** (Mirror graph). Let G = (V, E) be a graph, and  $s, t \in V$  be two vertices. We construct a new graph  $H \supset G$ , where for each vertex  $u \in V \setminus \{s, t\}$  we add a 'mirror' vertex u', and a connecting 'mirror' edge between them. The vertices in V(H) that are also in V(G) are referred to as the 'real' vertices, and the newly added vertices are referred to as the 'mirror' vertices. In addition, for any vertex  $u \in V(H) \setminus \{s, t\}$ , real or not, we define mirror(u) as u's mirror on the other side. We usually label mirror vertices with an ' at the end of the real counterpart's label. For example, if G is the graph in Figure 4.1, then Figure 4.2 would be its corresponding mirror graph H.

Our reduction from Shortest Odd Path to Shortest Alternating Path follows:

- 1. Let (G, s, t) be an instance of Shortest Odd Path.
- 2. Construct H as the mirror graph of G, and let F be the set of mirror edges in H. Now (H, s, t, F) is an instance of Shortest Alternating Path.
- 3. Let P' be the shortest alternating path of (H, s, t, F), if one exists. If none exist, then we do not have any odd s-t-paths in G either.

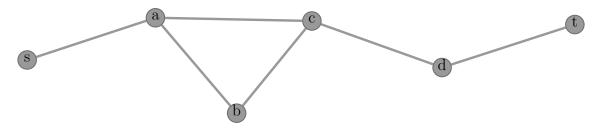


Figure 4.1: Our input graph G, for Shortest Odd Path

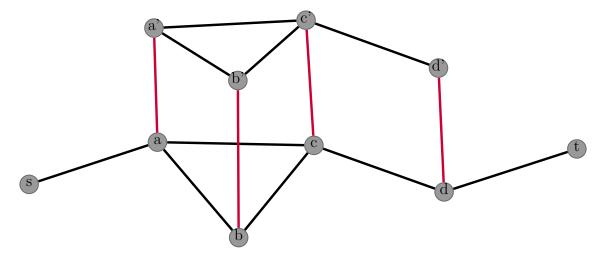


Figure 4.2: The mirror graph H of G, with mirror edges marked in red and mirror vertices labeled with an  $^{\prime}$ 

- 4. Construct P by filtering out mirror edges from P', and for each edge  $(u', v') \in E(H) \setminus (F \cup E(G))$  from the mirror side of H we replace it by the corresponding edge  $(u, v) \in E(G)$  from the real side.
- 5. Now P is the shortest odd s-t-path in G.

For example, if our input G for Shortest Odd Path is Figure 4.1, then H and F could look like Figure 4.2. One of the two possible alternating paths is P' := [(s,a),(a,a'),(a',b'),(b',b),(b,c),(c,c'),(c',d'),(d',d),(d,t)]. When we filter out mirror edges and replace edges from the mirror side with their real counterparts, we end up with P := [(s,a),(a,b),(b,c),(c,d),(d,t)], which is one of the two possible odd paths of G.

#### TODO dette kan kanskje formuleres bedre?

To see why the reduction works, simply observe that for each step we take in the graph, we have to go to the other side of the mirror. If we take another step, we get back to the same side again. It is only when we reach the target vertex t that we do not have

to go to the other side. Therefore, to reach a neighbour of t, we must have used an even number of mirror edges and an even number of non-mirror edges, and when we take the last step to reach t we have used an odd number of edges and thus found an odd path. If this alternating s-t-path in H is the shortest such path, then the corresponding path in G must also be the shortest odd s-t-path in G. The reduction works also in the weighted case, as long as each edge (u', v') on the mirror side get the same weight as their real counterpart, and all the mirror edges get the same (usually 0) weight. The interested reader may see [Der85] for more details on this reduction.

#### 4.1.2 Adapting previous work

Ball and Derigs [MOB83] have shown how to efficiently solve Shortest Alternating Path. In their algorithms, subgraphs are shrunk into psuedonodes whenever possible, to make the graph smaller. The drawback is that certain psuedonodes must later be expanded again, which is the most complicated and expensive part of their algorithms. In our case, however, we have a special case of Shortest Alternating Path. The set F is, with the exception of s and t, a perfect matching of H, and we will therefore never have to expand psuedonodes after shrinking them. The curious reader may visit [MOB83] for more on these algorithms and why our almost-perfect matching is a simpler case.

TODO siter [Edm65] her et sted

### 4.2 Psuedocode

#### Code Listing 4.1: Main

```
fn main(Graph input_graph, int s, int t) -> Option<(int, List<Edge>)> {
 2
3
        init(input_graph, s, t);
 4
        while ! control() {}
5
\begin{array}{c} 6 \\ 7 \\ 8 \end{array}
        if d_{minus}[t] == \infty \{
              // The graph is a no-instance, no odd s-t-paths exist
9
10
        Edge current_edge = pred[t];
List<Edge> path = [current_edge];
while from(current_edge) != s {
11
12
13
              current_edge = pred[mirror(from(current_edge))];
14
15
              if from(current_edge) < input_graph.n() {</pre>
16
                   path.push(current_edge);
17
             }
18
              else {
19
                   path.push(shift_edge_by(current_edge, -input_graph.n()));
20
21
        }
\overline{2}2
        return Some(d_minus[t], path);
23
```

#### Code Listing 4.2: Initialization

```
fn init(Graph input_graph, int s, int t) {
 2
3
        graph = create_mirror_graph(input_graph);
4
5
        for u in 0..n {
             d_plus[u] = \infty;
6
7
             d_{minus}[u] = \infty;
             pred[u] = null;
8
             completed[u] = false;
9
10
        d_plus[s] = 0;
        completed[s] = true;
11
12
13
        for edge in graph[s] {
    pq.push(Vertex(weight(edge), to(edge)));
14
             d_minus[to(edge)] = weight(edge);
pred[to(edge)] = e;
15
16
17
        }
18
```

#### Code Listing 4.3: Control, the main loop

```
13
                       if base_of(from(edge)) == base_of(to(edge)) {
14
                            pq.pop();
15
                       }
16
                       else {
17
                            break;
18
19
                  }
20
             }
21
        }
22
23
        if pq.is_empty() {
24
             // No odd s-t-paths in G exist :(
\begin{array}{c} 25 \\ 26 \end{array}
             return true;
27
28
        match pq.pop() {
             Vertex(delta, 1) => {
29
                  if 1 == t {
30
                       // We have found a shortest odd s-t-path has been
                           \hookrightarrow found :)
31
                       return true;
32
                  }
33
                  grow(1, delta)
\frac{34}{35}
             Blossom(delta, edge) => {
36
                  blossom(e);
37
38
        }
39
        return false;
40
```

#### Code Listing 4.4: Grow

```
fn grow(int 1, int delta) {
   int k = mirror(1);
   d_plus[k] = delta;
   scan(k);
}
```

#### Code Listing 4.5: Scan

```
fn scan(int u) {
 2
3
         completed[u] = true;
         int dist_u = d_plus[u];
 4
         for edge in graph[u] {
               int v = to(edge);
int new_dist_v = dist_u + weight(edge);
 5
6
7
8
9
               if ! completed[v] {
                     if new_dist_v < d_minus[v] {
    d_minus[v] = new_dist_v;</pre>
10
                           pred[v] = edge;
11
12
                           pq.push(Vertex(new_dist_v, v));
13
                     }
14
               }
15
               else if d_plus[v] < \infty and base_of(u) != base_of(v) {
                    pq.push(Blossom(d_plus[u] + d_plus[v] + weight(edge)));
if new_dist_v < d_minus[v] {
    d_minus[v] = new_dist_v;</pre>
16
17
18
19
                           pred[v] = e;
                     }
20
\overline{21}
               }
22
         }
23
   }
```

Code Listing 4.6: Blossom

```
fn blossom(Edge edge) {
2
       (int, List < Edge > , List < Edge > ) (b, p1, p2) =

→ backtrack_blossom(edge);
3
4
5
       List<int> to_scan1 = set_blossom_values(p1);
       List<int> to_scan2 = set_blossom_values(p2);
6
7
8
9
       set_edge_bases(b, p1);
       set_edge_bases(b, p2);
10
       for u in to_scan1 {
11
           scan(u);
12
13
       for v in to_scan2 {
14
           scan(v);
15
16
```

#### Code Listing 4.7: Backtrack blossom

```
fn backtrack_blossom(Edge edge) -> (int, List<Edge>, List<Edge>){
    // TODO
}
```

#### Code Listing 4.8: Set blossom values

```
fn set_blossom_values(List<Edge> path) -> List<int> {
23456789
        List < int > to_scan = [];
        for edge in path {
   int u = from(edge);
              int v = to(edge);
              int w = weight(edge);
              in_current_cycle[u] = false;
              in_current_cycle[v] = false;
10
              // We can set a d_minus if d_plus[v] + w < d_minus[u] {
11
12
13
                   d_minus[u] = d_plus[v] + w;
14
                   pred[u] = reverse(edge);
15
              }
16
17
              int m = mirror(u);
18
              // We can set a d_plus, and scan it
              if d_minus[u] < d_plus[m] {
    d_plus[m] = d_minus[u];</pre>
19
\frac{1}{20}
                   to_scan.push(m);
\frac{1}{22} 23
              }
        }
24
\overline{25}
        return to_scan;
26
   }
```

#### Code Listing 4.9: Set edge bases

```
fn set_edge_bases(int b, List<Edge> path) {
    for edge in path {
        let u = from(edge);
        let m = mirror(edge);
        set_base(u, b);
        set_base(m, b);
}
```

#### Code Listing 4.10: Basis

```
fn init(Graph input_graph, int s, int t) {
    // omitted
    Graph graph = create_mirror_graph(input_graph, s, t);
    for u in 0..graph.n() {
        basis[u] = u;
    }
    // omitted

8    fn set_base(int b, int u) {
        basis[u] = b;
    }
    fn get_base(int u) -> int {
        if u != basis[u] {
            basis[u] = get_base(basis[u]);
        }
        return basis[u];
}
```

### 4.3 Notes on implementing the psuedocode

- 4.4 Analysis
- 4.4.1 Complexity
- 4.4.2 Benchmarking methodology
- 4.4.3 Results
- 4.4.4 Discussion

**Network Diversion** 

### 5.1 Intuition

### 5.1.1 Bottleneck Paths

Before we reveal the algorithm for Network Diversion, we will first look at a curious little problem that we call Shortest Bottleneck Path.

SBP: SHORTEST BOTTLENECK PATH

**Input:** A graph G, two vertices  $s, t \in V$ , and a 'bottleneck' edge  $b \in E$ 

**Output:** the shortest s-t-path in G that goes through the bottleneck b

There is no obvious way to solve SBP. One might attempt to find the shortest paths from s to from(b) and from to(b) to t, but those two paths might overlap and reuse the same vertices, and therefore would their concatenation not necessarily be a simple path.

Instead we create a new graph H, by subdividing all edges in G except b, like seen in figure TODO. The key point to see here is that any odd s-t-path in H must necessarily go through the bottleneck, otherwise it would not be odd. We can visualize it by 'stepping through' the edges in H. If we start on our right leg, then in the beginning every time we reach a vertex that is also in G, we reach it by stepping on our left leg. That continues until we use the bottleneck edge, and from then on we step on all vertices from G using our right leg. If we require that we must end at t on our right leg, then the path must be odd, and any odd path must go through the bottleneck. Therefore we can simply run our Shortest Odd Path algorithm on H, and if such a path exists we can reverse the subdivision of the edges in the path and the result is the Shortest Bottleneck Path in G.

### 5.1.2 Extending the idea to multiple edges

If we extend the problem to have multiple bottleneck edges, and we have to go through all of them, then our idea will not work. That is good, because otherwise we would have solved the Traveling Salesman Problem in polynomial time and complexity theory as we know it would break down. TODO fact check. The problem is that we have no way of knowing whether we have used the marked edges 1, or 3, or 5, etc. times, because in all of them we hit vertices from G using our right leg. We can, however, use this idea to find paths that use a certain set of edges an odd amount of times. And as it turns out, that is exactly what we need to solve Network Diversion.

TODO: Intuition for ND

## 5.2 Psuedocode

### Code Listing 5.1: Main

- 5.3 Analysis
- 5.3.1 Complexity
- 5.3.2 Benchmarking methodology
- 5.3.3 Results
- 5.3.4 Discussion

# Conclusion

## **Bibliography**

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## Appendix A

### Generated code from Protocol buffers

Code Listing A.1: Source code of something

1 System.out.println("Hello Mars");