1. Use switching-algebra theorems to simplify each of the following logic functions:

a) F = AC´+ AB+ C(A+B)

Expand: $C(A+B)$

$$F = AC' + AB + AC + BC$$

$$F = AC' + AC + AB + BC$$

Single value theorem T5: $x + x' = 1$

$$F = A(C' + C) + AB + BC$$

$$F = A + AB + BC$$

Absorption law theorem: $X + XY = X$

$$\underline{\underline{F = A + BC}}$$

b) F = A´B + B´C´ + AB + B´C

Groupe the terms:

$$F = (A'B + AB) + (B'C' + B'C)$$

$$F = B(A' + A) + B'(C' + C)$$

Single value theorem: $x + x' = 1$

$$F = B + B'$$

$$\underline{\underline{F = 1}}$$

2. Prove the identity of each of the following logic equations, using algebraic manipulation:

a. AB + BC´D´+ A´BC+ C´D= B + C´D

Sort the terms:

$$AB + A'BC + BC'D' + C'D = B + C'D$$

Factorise:

$$B(A + A'C) + BC'D' + C'D = B + C'D$$

Absorption theorem for three variables: $X + XY' = X + Y$

$$B(A + C) + BC'D' + C'D = B + C'D$$

Distribute: B

$$BA + BC + BC'D' + C'D = B + C'D$$

Factorise: B

$$BA + B(C + C'D') + C'D = B + C'D$$

Simplify: $X + X'Y' = X + Y'$

$$BA + B(C + D') + C'D = B + C'D$$

Expand: B

$$BA + BC + BD' + C'D = B + C'D$$

Absorption theorem: A

$$B + BC + BD' + C'D = B + C'D$$

Absorption of $B + BC = B$

$$B + BD' + C'D = B + C'D$$

Absorption of $B + BD' = B$

$$\underline{\underline{B + C'D = B + C'D}}$$

b. $Y + X'Z + XY' = X + Y + Z$

Rearrange:

$$Y + XY' + X'Z = X + Y + Z$$

Absorption: $Y + XY' = Y + X$

$$Y + X + X'Z = X + Y + Z$$

Absorption: $X + X'Z = X + Z$

$$\underline{\underline{Y + X + Z = X + Y + Z}}$$

3. List the truth table for each of the following logic functions

AND	**OR**
$\mathbf{0 \cdot 0 = 0}$	$\mathbf{1 + 1 = 1}$
$\mathbf{1 \cdot 1 = 1}$	$\mathbf{0 + 0 = 0}$
$\mathbf{0 \cdot 1 = 0}$	$\mathbf{0 + 1 = 1}$

a. $F = X'Y + XY' + Y'Z$ (List XYZ F)

Row	**X**	**Y**	**Z**	**F**
0	0	0	0	**0**
1	0	0	1	**1**
2	0	1	0	**1**
3	0	1	1	**1**
4	1	0	0	**1**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**0**

b. $F = W'X + W(Y' + Z + X)$ (List WXYZ F)

Row	W	X	Y	Z	F
0	0	0	0	0	**0**
1	0	0	0	1	**0**
2	0	0	1	0	**0**
3	0	0	1	1	**0**
4	0	1	0	0	**1**
5	0	1	0	1	**1**
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	**1**
9	1	0	0	1	**1**
10	1	0	1	0	**0**
11	1	0	1	1	**1**
12	1	1	0	0	**1**
13	1	1	0	1	**1**
14	1	1	1	0	**0**
15	1	1	1	1	**1**

To do this faster, check what $W'X$ is equal to, if 1 then the answer will be 1 if 0 then it depends what $W(Y' + Z + X)$ gives.

c. $F = (WZ)'(X' + Y')'$ (List WXYZ F)

Row	W	X	Y	Z	F
0	0	0	0	0	**0**
1	0	0	0	1	**0**
2	0	0	1	0	**0**
3	0	0	1	1	**0**
4	0	1	0	0	**0**
5	0	1	0	1	**0**
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	**0**
9	1	0	0	1	**0**
10	1	0	1	0	**0**
11	1	0	1	1	**0**
12	1	1	0	0	**0**
13	1	1	0	1	**0**
14	1	1	1	0	**1**
15	1	1	1	1	**0**

d. $F = (((A + B')' + C')' + D)'$ (List ABCD F)

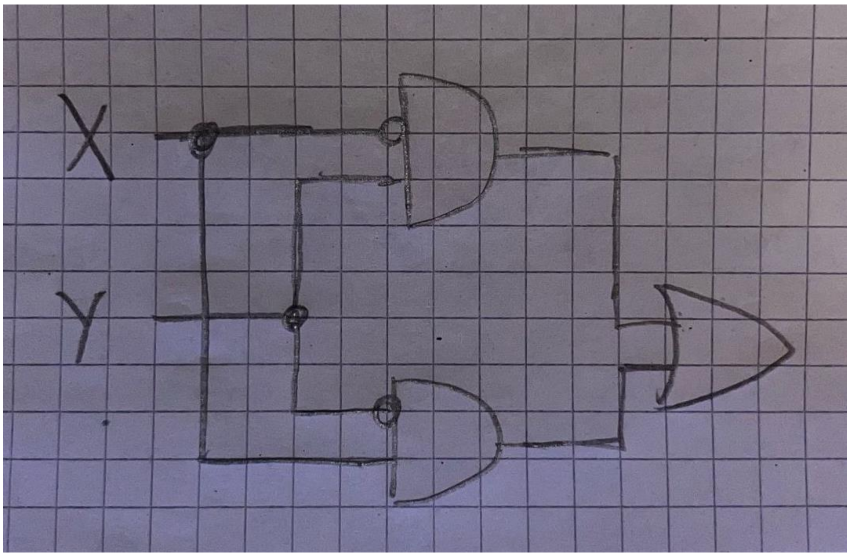
Row	A	B	C	D	F	
0	0	0	0	0	**0**	1
1	0	0	0	1	**1**	0
2	0	0	1	0	**1**	0
3	0	0	1	1	**1**	0
4	0	1	0	0	**0**	1
5	0	1	0	1	**1**	0
6	0	1	1	0	**0**	1
7	0	1	1	1	**1**	0
8	1	0	0	0	**0**	1

9	1	0	0	1	**1**	0
10	1	0	1	0	**1**	0
11	1	0	1	1	**1**	0
12	1	1	0	0	**0**	1
13	1	1	0	1	**1**	0
14	1	1	1	0	**1**	0
15	1	1	1	1	**1**	0

4. **An Exclusive OR (XOR) gate is a 2-input gate whose output is 1 if and only if exactly one of the inputs is 1. Write a truth table, sum-of-products expression, and corresponding AND-OR circuit for the Exclusive OR function.**

$$\boldsymbol{F = XY' + X'Y}$$

Row	**X**	**Y**	**F**
0	0	0	**0**
1	0	1	**1**
2	1	0	**1**
3	1	1	**0**



5. **List the truth table of the following functions, and express for each function the canonical sum and canonical product.**

a. $F = (XY + Z)(Y + X'Z)$ (List XYZ F)

Row	**X**	**Y**	**Z**	**F**
0	0	0	0	**0**
1	0	0	1	**1**
2	0	1	0	**0**
3	0	1	1	**1**
4	1	0	0	**0**
5	1	0	1	**0**
6	1	1	0	**1**
7	1	1	1	**1**

<u>Canonical sum</u>

$$F = \sum X, Y, Z(1,3,6,7)$$

$$F = (X' \cdot Y' \cdot Z) + (X' \cdot Y \cdot Z) + (X \cdot Y \cdot Z') + (X \cdot Y \cdot Z)$$

<u>Canonical product</u>

$$F = \prod X, Y, Z\ (0,2,4,5)$$

$$F = (X + Y + Z)(X + Y' + Z)(X' + Y + Z)(X' + Y + Z')$$

b. $F = X'Z + WX'Y + WYZ' + W'Y$ (List WXYZ F)

To do this faster, if one of the AND operation equal 1, then the result will become 1 too.

Row	W	X	Y	Z	F
0	0	0	0	0	**0**
1	0	0	0	1	**1**
2	0	0	1	0	**1**
3	0	0	1	1	**1**
4	0	1	0	0	**0**
5	0	1	0	1	**0**
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	**0**
9	1	0	0	1	**1**
10	1	0	1	0	**0**
11	1	0	1	1	**1**
12	1	1	0	0	**0**
13	1	1	0	1	**0**
14	1	1	1	0	**1**
15	1	1	1	1	**0**

<u>Canonical sum</u>

$$F = \sum X, Y, Z(1,2,3,6,7,9,11,14)$$

$$\begin{aligned} F = {} & (W' \cdot X' \cdot Y' \cdot Z) + (W' \cdot X' \cdot Y \cdot Z') + (W' \cdot X' \cdot Y \cdot Z) \\ & + (W' \cdot X \cdot Y \cdot Z') + (W' \cdot X \cdot Y \cdot Z) \\ & + (W \cdot X' \cdot Y' \cdot Z) + (W \cdot X' \cdot Y \cdot Z) \\ & + (W \cdot X \cdot Y \cdot Z') \end{aligned}$$

<u>Canonical product</u>

$$F = \prod X, Y, Z\ (0,4,5,8,10,12,13,15)$$

$$\begin{aligned} F = {} & (W + X + Y + Z)(W + X' + Y + Z)(W + X' + Y \\ & + Z')(W' + X + Y + Z)(W' + X + Y' \\ & + Z)(W' + X' + Y + Z)(W' + X' + Y \\ & + Z')(W' + X' + Y' + Z') \end{aligned}$$

6. **Write the canonical sum and canonical product for each of the following logic functions.**
   a. $F = \sum A, B, C(1,2,5,7)$

From the given canonical sum, it is to be seen be made this truth table.

Row	A	B	C	F
0	0	0	0	**0**
1	0	0	1	**1**
2	0	1	0	**1**
3	0	1	1	**0**
4	1	0	0	**0**
5	1	0	1	**1**
6	1	1	0	**0**
7	1	1	1	**1**

$$F = (A' \cdot B' \cdot C) + (A' \cdot B \cdot C') + (A \cdot B' \cdot C) + (A \cdot B \cdot C)$$

   b. $F = \prod W, X, Y\ (0,2,4,5,7)$

From the given canonical product, it is to be seen be made this truth table

Row	A	B	C	F
0	0	0	0	**0**
1	0	0	1	**1**
2	0	1	0	**0**
3	0	1	1	**1**
4	1	0	0	**0**
5	1	0	1	**0**
6	1	1	0	**1**
7	1	1	1	**0**

$$F = (A + B + C)(A + B' + C)(A' + B + C)(A' + B + C')(A' + B' + C')$$

7. **Convert each of the following expressions into canonical sum and canonical product. Hint: compute the truth tables.**

From the result of the truth table, it can be read of what combinations are the mintemrs and maxterms.

a. $F = (A'B + C)(B + C'D)$

Row	A	B	C	D	F
0	0	0	0	0	**0**
1	0	0	0	1	**0**
2	0	0	1	0	**0**
3	0	0	1	1	**0**
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	**1**
7	0	1	1	1	**1**
8	1	0	0	0	**0**
9	1	0	0	1	**0**
10	1	0	1	0	**0**
11	1	0	1	1	**0**
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	**1**
15	1	1	1	1	**1**

$$F = \sum A, B, C(4,5,6,7,14,15)$$

$$F = A'BC'D' + A'BC'D + A'BCD' + A'BCD + ABCD' + ABCD$$

$$F = \prod A, B, C(0,1,2,3,8,9,10,11,12,13)$$

$$F = (A + B + C + D) \cdot (A + B + C + D') \cdot (A + B + C' + D) \cdot (A + B + C' + D') \cdot (A' + B + C + D) \cdot (A' + B + C + D') \cdot (A' + B + C' + D) \cdot (A' + B + C' + D') \cdot (A' + B' + C + D) \cdot (A' + B' + C + D')$$

b. $F = X' + X(X + Y')(Y + Z)$

Row	X	Y	Z	F
0	0	0	0	**1**
1	0	0	1	**1**
2	0	1	0	**1**
3	0	1	1	**1**
4	1	0	0	**0**
5	1	0	1	**1**
6	1	1	0	**1**
7	1	1	1	**1**

$$F = \sum X, Y, Z(0,1,2,3,5,6,7)$$

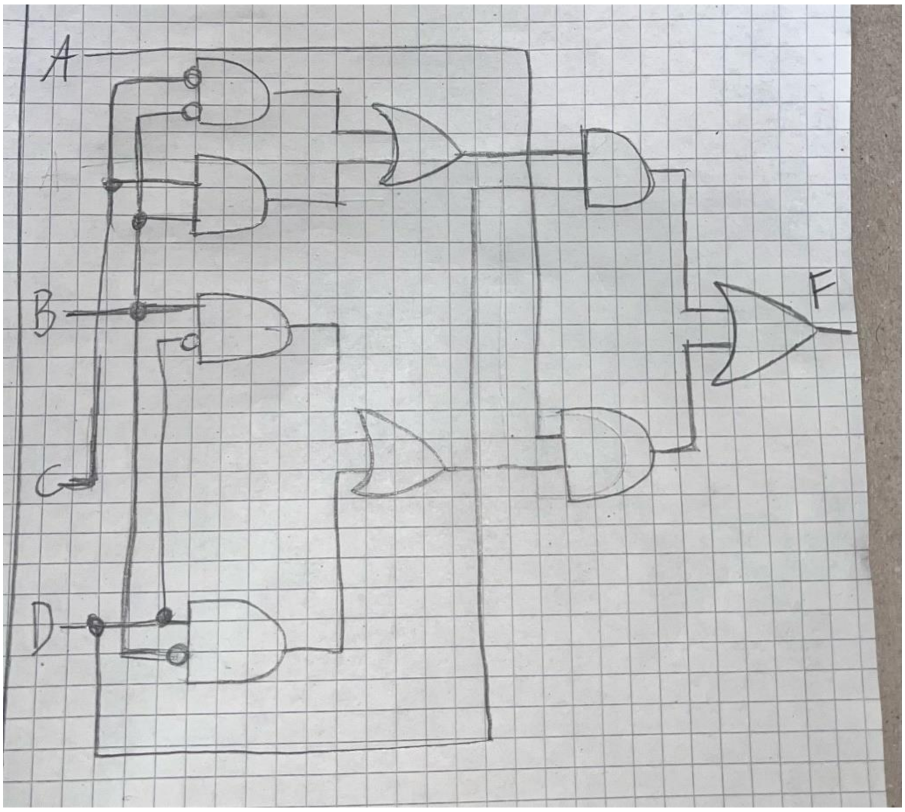
$$F = (A' \cdot B' \cdot C') + (A' \cdot B' \cdot C) + (A' \cdot B \cdot C') + (A' \cdot B \cdot C) + (A \cdot B' \cdot C) + (A \cdot B \cdot C') + (A \cdot B \cdot C)$$

$$F = \prod X, Y, Z(4)$$

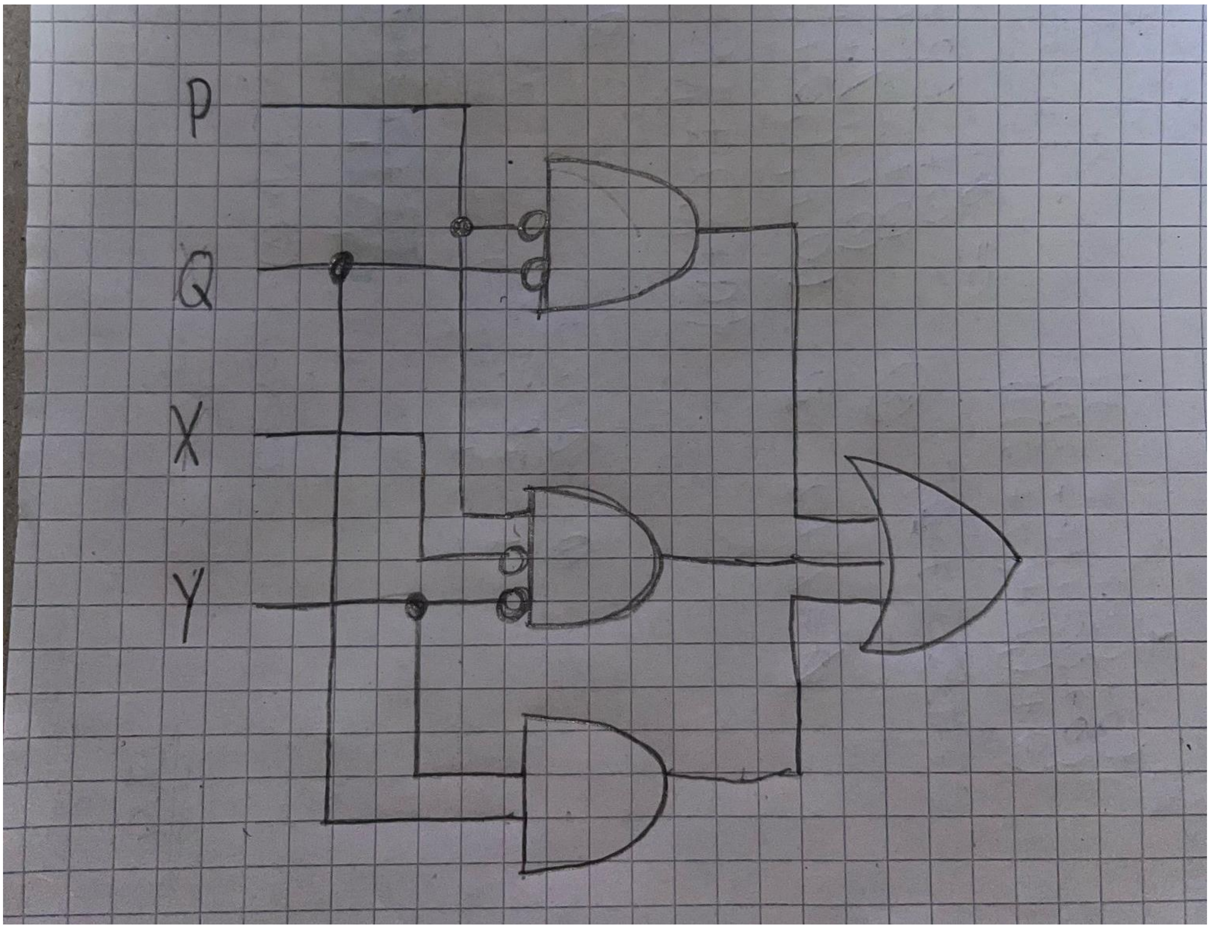
$$F = (X' + Y + Z)$$

8. **Draw the logic diagram for the following Boolean expressions. The diagram should correspond exactly to the equation. Assume that the complements of the inputs are not directly available as inputs, but rather obtained by using inverter gates.**

a. $F = A(BD' + B'D) + D(BC + B'C')$



b. $F = PX'Y' + P'Q' + YQ$



9. Calculate the DeMorgan equivalents for the following logic expressions:

a. $(X` + Y)'$

$$(X')' \cdot Y'$$

$$\underline{\underline{X \cdot Y'}}$$

b. $X'Y$

$$(X' \cdot Y)'$$

$$(X')' + Y'$$

$$\underline{\underline{X + Y'}}$$

c. $(X' + YZ')$

$$(X')' \cdot Y' + (Z')'$$

To keep the same calculation order

$$\left(\underline{X \cdot (Y' + Z)}\right)'$$

d. $X'Y' + XZ'$

$$\left(\left((X')' + (Y')'\right) \cdot \left(X' + (Z')'\right)\right)'$$

$$\left(\left(X + Y\right) \cdot \left(X' + Z\right)\right)'$$

To keep the same calculation order

$$\left(\underline{(X + Y') \cdot (X' + Z)}\right)'$$

10. Write the Boolean equation and draw the logic diagram for the circuit whose output is defined by the following truth table.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Make the canonical sums and simplify it and that can be the function for this truth table.

Canonical Sum:

$$F = \sum A, B, C(0,3,4,6)$$

$$F = (A' \cdot B' \cdot C') + (A' \cdot B \cdot C) + (A \cdot B' \cdot C') + (A \cdot B \cdot C')$$

Simplify the canonical sum:

$$F = (A' \cdot B' \cdot C') + (A' \cdot B \cdot C) + (A \cdot B' \cdot C') + (A \cdot B \cdot C')$$

$$\left((A + A') \cdot B' \cdot C'\right) + (A' \cdot B \cdot C) + (A \cdot B \cdot C')$$

Complement law: $X + X' = 1$

$$\left((1) \cdot B' \cdot C'\right) + (A' \cdot B \cdot C) + (A \cdot B \cdot C')$$

Identity law: $X \cdot 1 = X$

$$(B' \cdot C') + (A' \cdot B \cdot C) + (A \cdot B \cdot C')$$

$$\left(\left((A \cdot B) + B'\right) \cdot C'\right) + (A' \cdot B \cdot C)$$

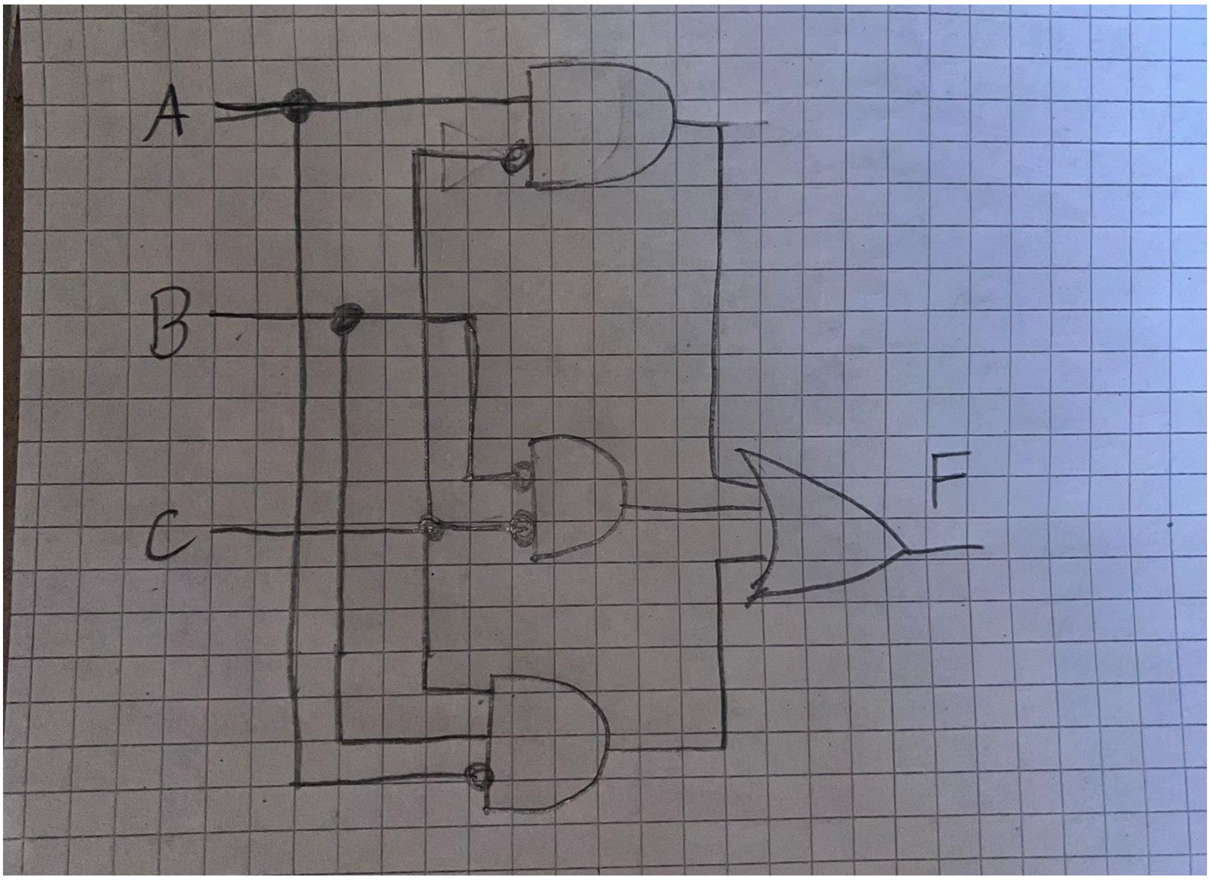
Redundancy law: $X + (X' \cdot Y) = X + Y$

$$(A + B') \cdot C' + (A' \cdot B \cdot C)$$

Expansion:

$$F = (A \cdot C') + (B' \cdot C') + (A' \cdot B \cdot C)$$

Now, it is possible to draw a logic diagram.


A
B
C
F