Exercise 5.3.2

Saurabh Steixner-Kumar
(social): in - ◯ - 𝕐

Contents

1 Question

Exercise 5.3.2 Hearing Loss Assessment Trial. Grant (1974) reported data from a screening of a pre-school population intended to assess the adequacy of a school nurse assessment of hearing loss in relation to expert assessment. Of those children assessed as having hearing loss by the expert, 20 were correctly identified by the nurse and 7 were missed. Of those assessed as not having hearing loss by the expert, 417 were correctly diagnosed by the nurse but 103 were incorrectly diagnosed as having hearing loss. These data correspond to a = 20, b = 7, c = 103, d = 417. Once again, examine the posterior distributions of the interesting variables, and reach a scientific conclusion. Once again, what would your two- or three-sentence "take home message" conclusion be to your customers?

2 Comments/Solution

Compared to the expert, the nurse displays a bias to classify children as having hearing loss. In addition, the nurse misses 7 out of 27 children with hearing loss. The nurse is doing a poor job, and this is reflected in the 95% credible interval (ETI) for kappa of (approximately, up to sampling) (.12, .28).

The model used to calculate the required values and the plots is scripted below. Copy/pasting the given code will generate the same result on your own machine.

3 Code

3.1 libraries

The libraries required for the script and the plots.

```r
# clears workspace
rm(list=ls())
#load libraries
library(rstan)
library(bayestestR)
library(tidyr)
library(ggplot2)
library(patchwork)
```

3.2 Data

The data required for this particular stan model.

```r
# data   initialization
# Hearing Loss
y <- c(20, 7, 103, 417)
# to be passed on to Stan
stan_data <- list(y=y)
```

3.3 Stan code

Stan code, that can be written in R as such or in a separate new file with stan extension.

```stan
write("// Stan code here in this section

// Kappa Coefficient of Agreement
data {
  int<lower=0> y[4];
}
parameters {
  // Underlying Rates
  // Rate Objective Method Decides 'one'
  real<lower=0,upper=1> alpha;
  // Rate Surrogate Method Decides 'one' When Objective Method Decides 'one'
  real<lower=0,upper=1> beta;
  // Rate Surrogate Method Decides 'zero' When Objective Method Decides 'zero'
  real<lower=0,upper=1> gamma;
}
transformed parameters {
  simplex[4] pi;
  real xi;
  real psi;
  real kappa;
  // Probabilities For Each Count
  pi[1] = alpha * beta;
  pi[2] = alpha * (1 - beta);
  pi[3] = (1 - alpha) * (1 - gamma);
  pi[4] = (1 - alpha) * gamma;

  // Derived Measures
  // Rate Surrogate Method Agrees With the Objective Method
  xi = alpha * beta + (1 - alpha) * gamma ;
  // Rate of Chance Agreement
  psi = (pi[1] + pi[2]) * (pi[1] + pi[3]) + (pi[2] + pi[4]) * (pi[3] + pi[4]);
```

```
  // Chance-Corrected Agreement
  kappa = (xi - psi) / (1 - psi);
}
model {
  alpha ~ beta(1, 1);  // could be removed
  beta ~ beta(1, 1);   // could be removed
  gamma ~ beta(1, 1);  // could be removed
  // Count Data
  y ~ multinomial(pi);
} // ",

"5_3_2.stan")
```

3.4 code in R to run stan

Running stan through R (with the required input parameters).

```r
myinits <- list(
  list(alpha=.5, beta=.5, gamma=.5),  # chain 1 starting value
  list(alpha=.5, beta=.5, gamma=.5))  # chain 2 starting value

# parameters to be monitored:
parameters <- c("kappa", "xi", "psi", "alpha", "beta", "gamma", "pi")

# The following command calls Stan with specific options.
# For a detailed description type "?stan".
mod_fit <- stan(file="5_3_2.stan",
                data=stan_data,
                init=myinits,  # If not specified, gives random inits
                pars=parameters,
                iter=2000,
                chains=2,
                thin=1,
                warmup=100,  # Stands for burn-in; Default = iter/2
                seed=123  # Setting seed; Default is random seed
)
```

4 Outputs

4.1 Model summary

In order of definition.

```
## Inference for Stan model: 5_3_2.
## 2 chains, each with iter=2000; warmup=100; thin=1;
## post-warmup draws per chain=1900, total post-warmup draws=3800.
##
##          mean se_mean   sd  2.5%   25%   50%   75%  97.5% n_eff
## kappa    0.20    0.00 0.04  0.12  0.17  0.20  0.23   0.28  1770
## xi       0.80    0.00 0.02  0.76  0.79  0.80  0.81   0.83  3908
## psi      0.75    0.00 0.02  0.71  0.73  0.75  0.76   0.78  3337
## alpha    0.05    0.00 0.01  0.03  0.04  0.05  0.06   0.07  2610
## beta     0.72    0.00 0.08  0.54  0.67  0.73  0.78   0.86  1157
## gamma    0.80    0.00 0.02  0.77  0.79  0.80  0.81   0.83  4559
```

```
## pi[1]      0.04      0.00 0.01     0.02     0.03     0.04     0.04     0.05  1928
## pi[2]      0.01      0.00 0.00     0.01     0.01     0.01     0.02     0.03  1460
## pi[3]      0.19      0.00 0.02     0.16     0.18     0.19     0.20     0.22  4568
## pi[4]      0.76      0.00 0.02     0.72     0.75     0.76     0.77     0.79  3918
## lp__   -389.81      0.03 1.21 -392.88 -390.39 -389.51 -388.93 -388.43  1423
##        Rhat
## kappa    1
## xi       1
## psi      1
## alpha    1
## beta     1
## gamma    1
## pi[1]    1
## pi[2]    1
## pi[3]    1
## pi[4]    1
## lp__     1
##
## Samples were drawn using NUTS(diag_e) at Thu Nov 05 21:27:10 2020.
## For each parameter, n_eff is a crude measure of effective sample size,
## and Rhat is the potential scale reduction factor on split chains (at
## convergence, Rhat=1).
```
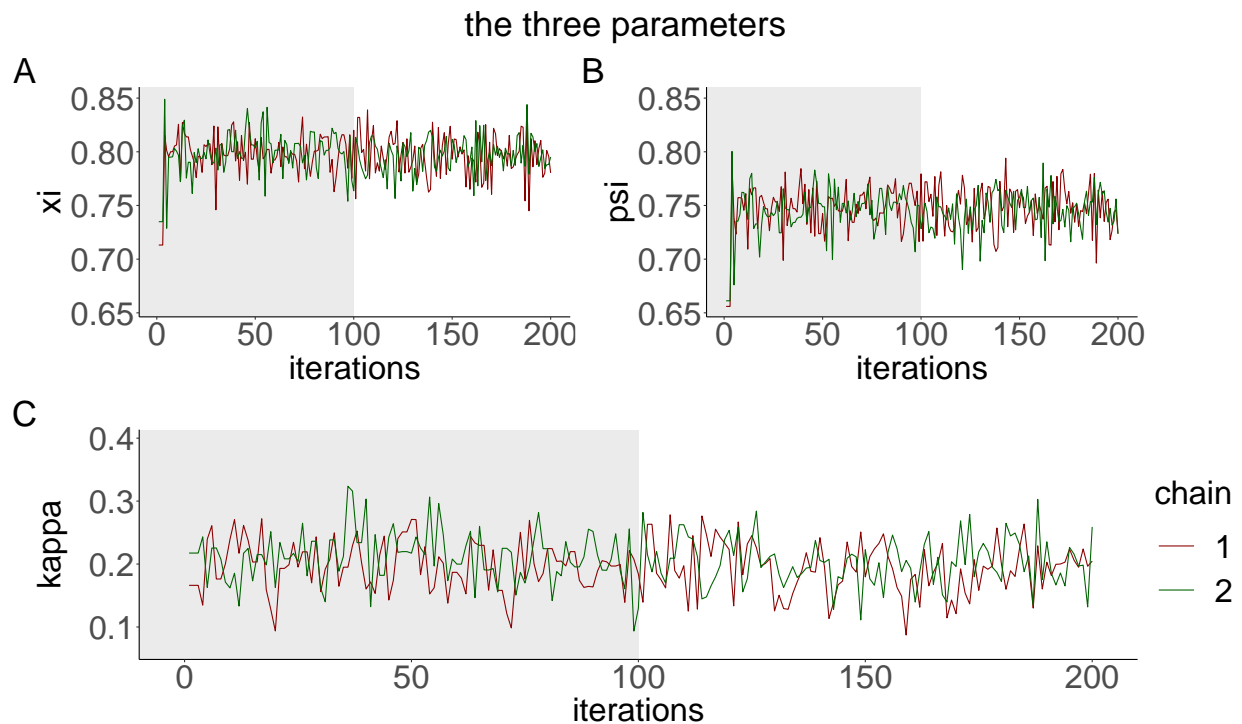
4.2   Plots

4.2.1   Plot (chains)

The initial movement of the chains are shown here (including the warmup phase). The two chains begin from the initial starting points of as defined in the input parameters of the stan model.
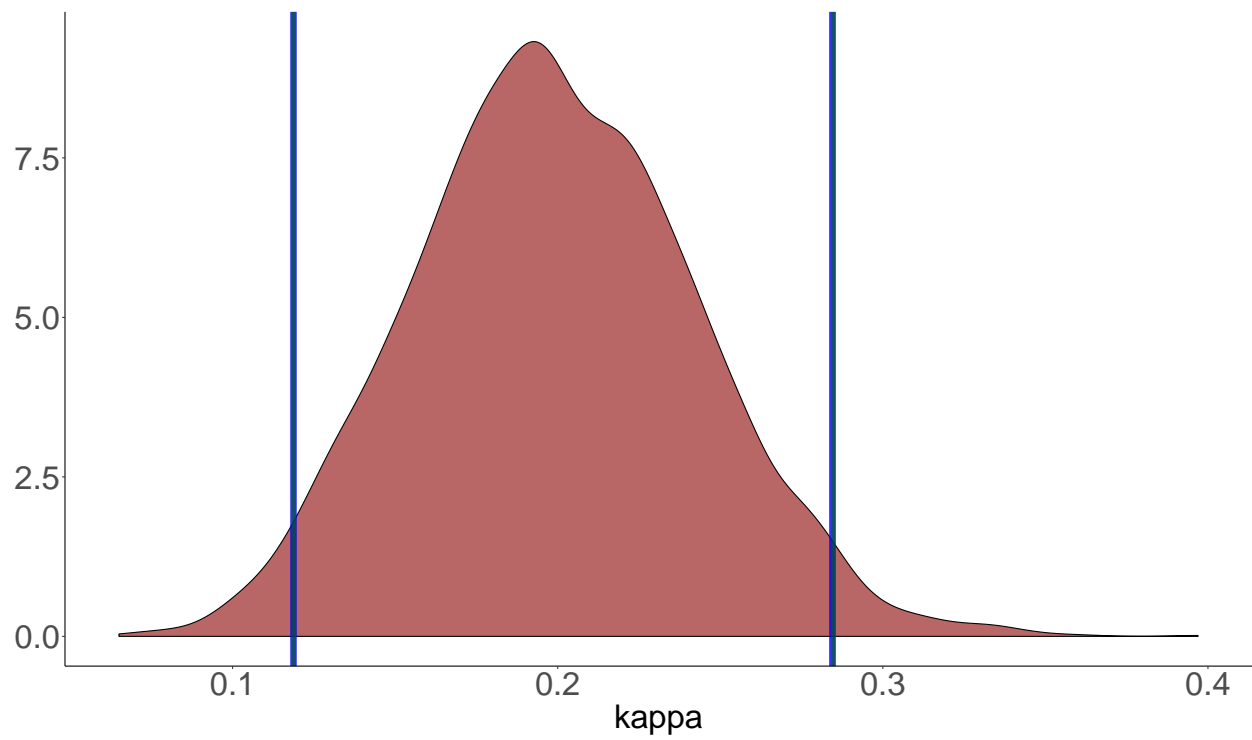
4.2.2 Plot (posterior)

The plot of the rate of agreement (ξ), rate of agreement by chance (ψ) and the chance-corrected agreement rate (κ) values per chain superimposed on each other.
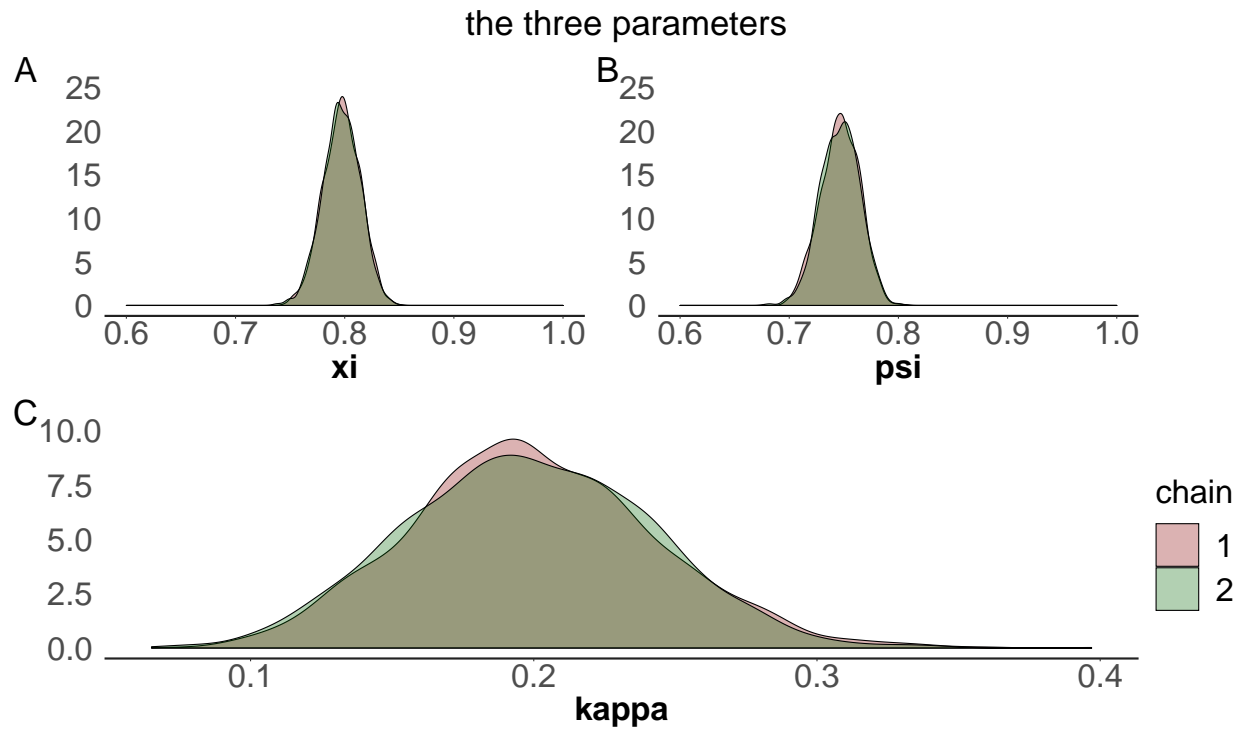
4.2.3 Plot (posterior with CI)

The plot of the combined κ value with confidence intervals. (HDI CI : blue; ETI CI : green)

4.3 Confidence interval value

The confidence interval values.

```
## # Highest Density Interval
##
##       95% HDI
##  [0.12, 0.28]
```

```
## # Equal-Tailed Interval
##
##       95% ETI
##  [0.12, 0.28]
```