Homework 2 – part a Posted 10 Feb 2019 // Due 1535 18 Feb 2019

1. Suppose that one letter is to be selected at random from the 42 letters in the sentence, "The shortest distance between two points is a taxi." If Y denotes the number of letters in the word in which the selected letter appears, what is the value of E(Y)?

2. Suppose that $X$and Y have a continuous joint distribution for which the joint ppf is:

   $f(x, y) = 12y^2$ for $0 \leq y \leq x \leq 1$

   Find the value of $E(XY)$.

3. Suppose that three random variables X_1, X_2, X_3 form a random sample from the uniform distribution on the interval $[0, 1]$. Find $E[(X_1 - 2X_2 + X_3)^2]$.

4. X has pdf

   $f(x) = e^{-x}, \quad x > 0$

   $Y = e^{\frac{3X}{4}}$

   Find $E(Y)$

5. X is the outcome of rolling a fair die.

   $Y = g(X) = 2X^2 + 1$

   Find $E(Y)$

6. X has pdf

   $f(x) = 2(1 - x), \quad 0 < x < 1$

   $Y = (2X + 1)$

   Find $E(Y^2)$.

7. Remember the binomial theorem: $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k \quad for n \in \mathbb{Z}^+$

   Show that $E[(ax + b)^n] = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(X^{n-i})$

8. The proportion of defective parts in a large shipment is p. A random sample of n parts is selected from the shipment. Let X denote the number of defective parts in the sample, and Y denote the number of good parts in the sample. Find $E(X - Y)$.

   If the sample size is 20 and p is 5%, what is $E(X - Y)$? Write out your answer as a complete sentence that expresses the meaning of your result.