"Teleportation in Astro Haunted Galaxies"

You have a teleporter that can take you from galaxy i to galaxy j. Cost to teleport is given by c(i,j), which can be arbitrary. Some galaxies are "astro-haunted" – this is specified by a(i) which can be 0 or 1 (1 means that that galaxy is "astro-haunted"). Give a polynomial time algorithm that minimizes the cost of going from galaxy 1 to galaxy n, such that you pass through no more than k astro-haunted galaxies. (You can assume that galaxies 1 and n are not astro-haunted.)

Problem Statement

all-pair shortest path problem:

Number the vertices from 1 to n. *Let* $d[i, j]^k$ *be the shortest path from* i *to* j *using only vertices from* 1, 2, ..., k *as possible intermediate vertices.*

With no intermediate vertices, any path consists of at most one edge, so $d[i, j]^0 = w[i, j]$.
In general, adding a new vertex k + 1 helps iff a path goes through it, so

$d[i,j]^k = w[i,j] \text{ if } k=0$

$= \min(d[i, j]^{k-1}, d[i, k]^{k-1} + d[k, j]^{k-1}) \text{ if } k \geq 1$

The following algorithm improves it:

$d^0 = w$
for k = 1 to n
for i = 1 to n
for j = 1 to n
$d[i, j]^k = \min(d[i, j]^{k-1}, d[i, k]^{k-1} + d[k, j]^{k-1})$

This obviously runs in $\Theta(n^3)$ time, which is asymptotically no better than n calls to Dijkstra's algorithm.

Theoretical Analysis

Step 1:
define Dis(i,j,m)= Shortest path from galaxy i to galaxy j using a maximum of m astrohaunted galaxies

Step 2:
By using Recurrence relation Find the minimum distance between galaxy i to galaxy j
Dis(i,j,m) = min { Dis(i,z,m-1) + Dis(z,j,0) for all z}

Recursive Algorithm:
For m = 1 to k
For i = 1 to n
 For j = 1 to n
  Calculate Dis(i,j,m) = min { Dis(i,z,m-1) + Dis(z,j,0) for all z}

Step 3 :
check all pairs Shortest path
Dis(i,j,0) By using All Pairs Shortest Path

Dis(i,j, k) is the base case. Dis(i,j,k) represents the distance from i to j, using a set of nodes {1..k} as possible intermediate nodes, and such that no Astro haunted intermediate nodes are allowed.

Dis(i,j,0) = c(i,j). c(i,j)=cost of Teleportation
Dis(i,j,k) = min{Dis(i,j,k-1) , Dis(i,k,k-1) + Dis(k,j,k-1)} if k is not Astro haunted galaxy
min{Dis(i,j,k-1)} if k is Astro haunted galaxy
Then, Dis(i,j,n) is the base case, which we can use for Dis(i,j,0)

Step 4 :
Find the minimum cost of Teleportation
Dis(i,j,0) = min {c(i,j); Eliminating all Astro Haunted galaxies
Each calculation of D(i,j,m) takes O(n) time.

Time Complexity
O (n^3) from the all pairs shortest path problem Each calculation of D (i, j, k) takes O (n) time. Further, there are kn^2 entries in the dynamic programming table. Therefore, the total time complexity of recursive is O (kn^3)

Thus,
Time Complexity of base case: O(n^3)
Time Complexity of recursive portion: O(k * n^3)
Total time complexity: O(k * n^3)

Experimental Analysis

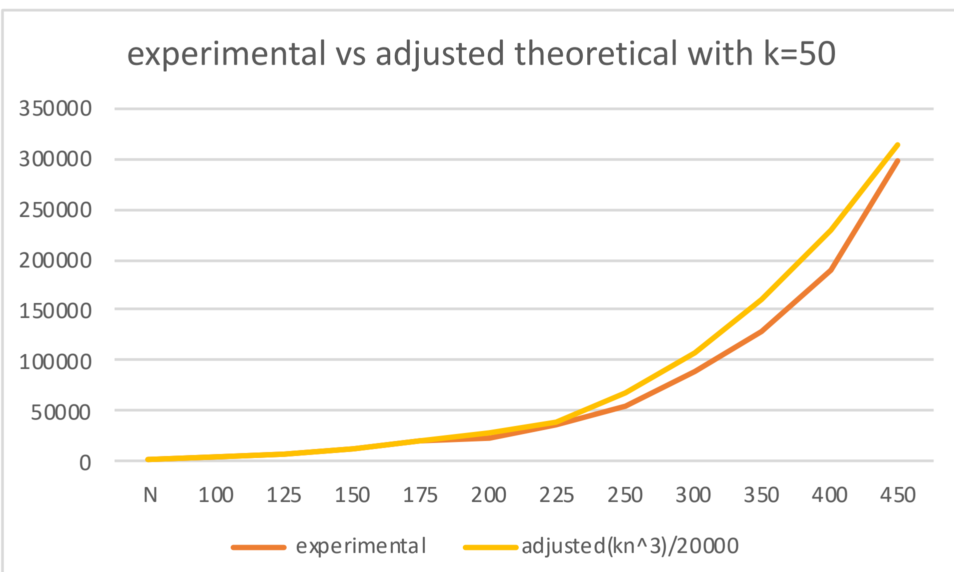
Output Numerical Data

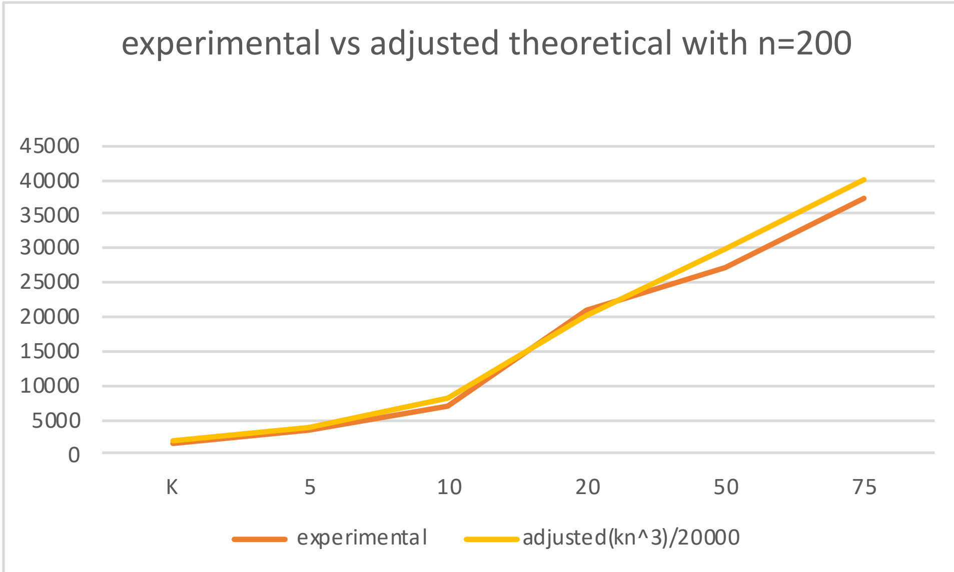
K	N	experimental	theoretical(kn^3)	adjusted(kn^3)/20000
50	100	2667	50000000	2500
50	125	4959	97656250	4882.8125
50	150	7428	168750000	8437.5
50	175	12145	267968750	13398.4375
50	200	20904	400000000	20000

50	225	24349	569531250	28476.5625
50	250	36872	781250000	39062.5
50	300	55952	1350000000	67500
50	350	90260	2143750000	107187.5
50	400	129353	3200000000	160000
50	450	189195	4556250000	227812.5
50	500	296579	6250000000	312500

N	K	experimental	theoretical(kn^3)	adjusted(kn^3)/20000
200	5	1620	40000000	2000
200	10	3533	80000000	4000
200	20	6907	160000000	8000
200	50	20904	400000000	20000
200	75	27034	600000000	30000
200	100	37382	800000000	40000

Graph





Conclusions

The curve shows that experimental time complexity line and adjusted theoretical line are similar no matter when k is fixed or n is fixed, so the analysis may be correct.