POZNAN UNIVERSITY OF ECONOMICS AND BUSINESS

FINANCIAL ENGINEERING PROJECT
Stage III

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Content

I. **Key features of EE for FX Forward**
II. **Calibration to market data**
III. **Simulation of expected exposure profiles for FX Forward**

I. Key features of EE for FX Forward

- Expected Exposure (EE) means the average of the distribution of exposures at any particular future date before the longest maturity transaction in the netting set matures, but conditional only on positive values.
- The contract-level exposure:

$$E_i(t) = \max\{V_i(t), 0\}$$

- Expected exposure profiles are used in the calculation of various valuation adjustments, which are an integral part of derivatives pricing.
- In general, the expected exposure of FX forwards is more sensitive to calibration compared to IR swaps.

I. Key features of EE for FX Forward

- Expected Exposure (EE) of FX Forward

$$EE(t_i) = F(0, t_m) \times N(d_1) - K \times N(d_2),$$

$$d_1 = \frac{ln\frac{F}{K} + \frac{1}{2} \times \sigma^2 \times t_i}{\sigma \times \sqrt{t_i}}, d_2 = d_1 - \sigma \times \sqrt{t_i}.$$

Where:

$F(0, t_m)$: Foreign exchange rate at time t_m

K: strike price

T_m: The time to maturity

II. Calibration to market data: volatility

- EUR_PLN spot rates Historical Data on: https://stooq.pl/
- Data from 01/01/2019 to 31/12/2021 (3 years)

Volatility Formula

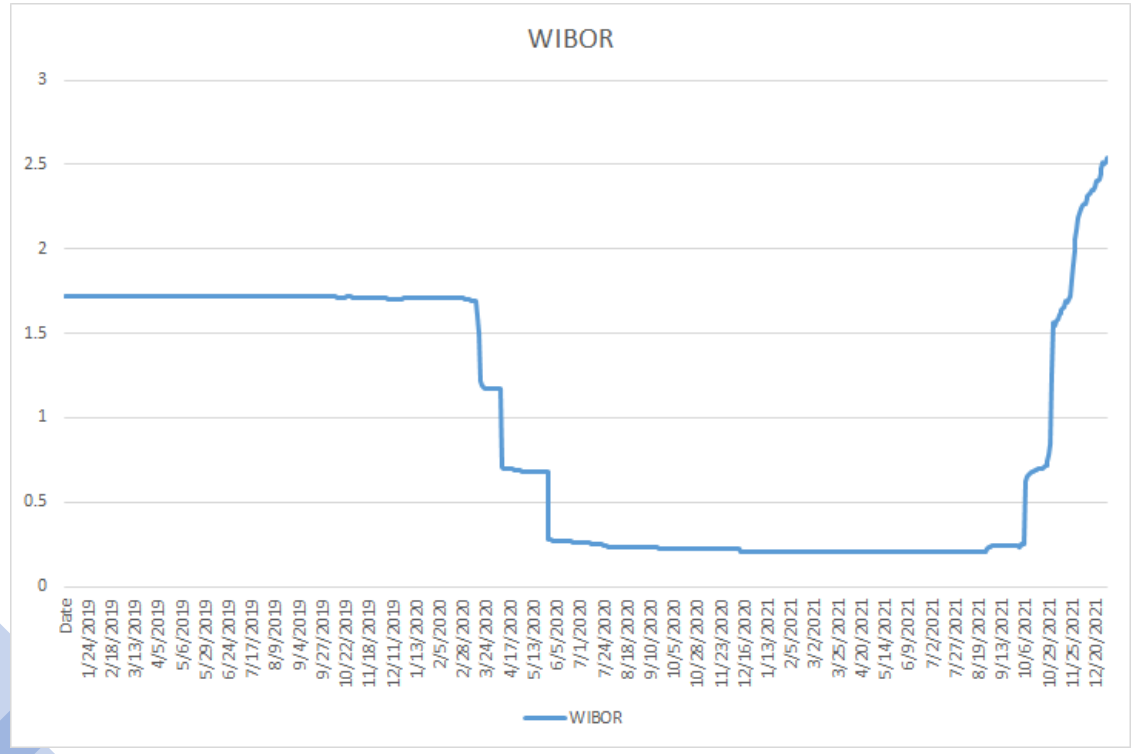
$$\text{Daily Volatility Formula} = \sqrt{\text{Variance}}$$

$$\text{Annual Volatility Formula} = \sqrt{252} \times \sqrt{\text{Variance}}$$

Daily volatility	0.00334642
Annualized volatility	0.053122775

II. Calibration to market data: mu

- Wibor-3M and Euribor-3M Historical Data on: https://stooq.pl/
- Data from 01/01/2019 to 31/12/2021 (3 years), excl. 09.04.2020-04.11.2021



Due to Covid-19 outbreak in 2020-2021, in accordance with relief measures the National Bank of Poland decreased key reference rate on April 9, 2020 to support the economy. This measure was in force until November 4, 2021. We excluded this time interval from calculations as it distorted long-term differential between Wibor-3M and Euribor-3M

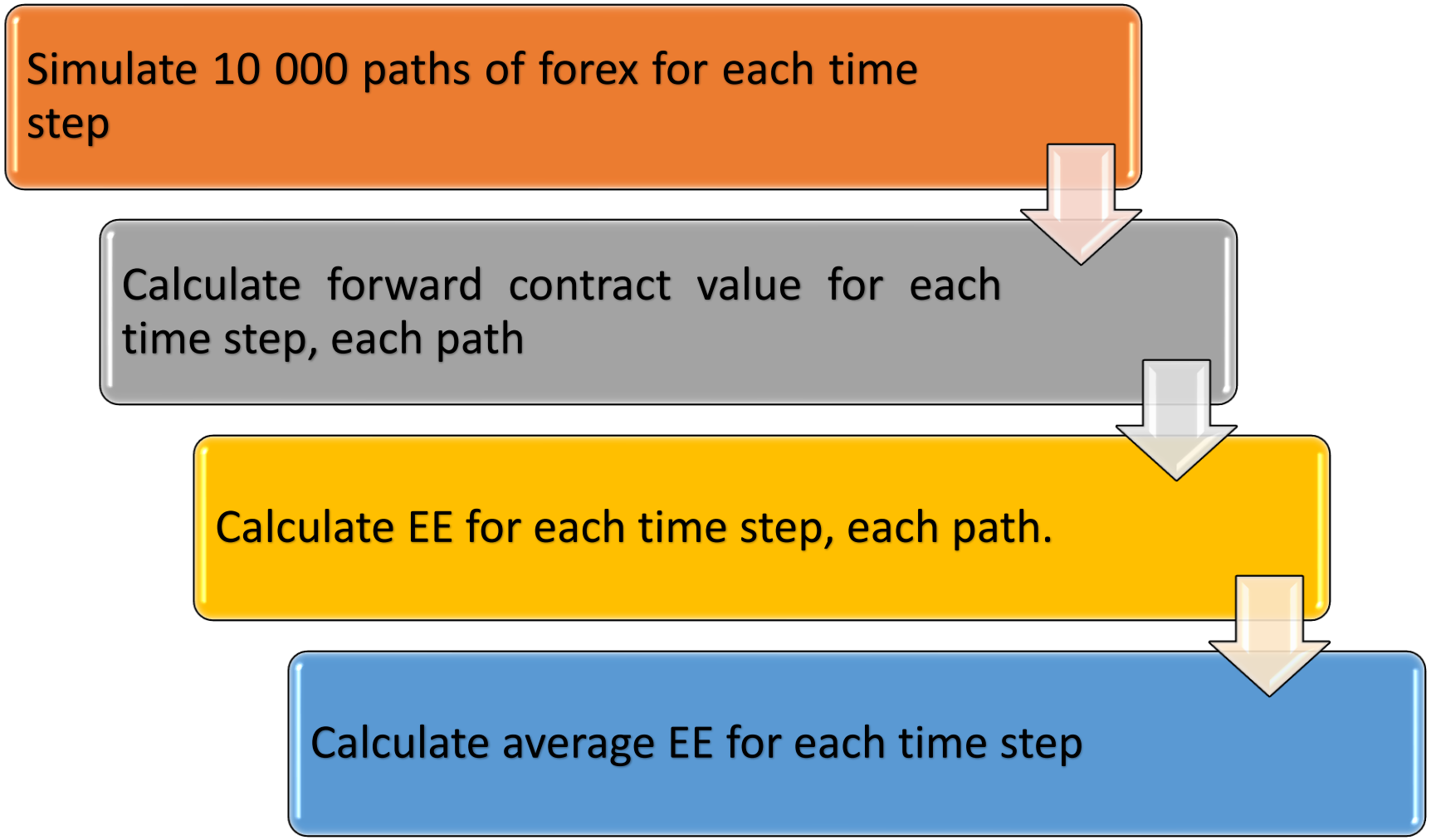
Excluding covid window (09.04.2020-04.11.2021)	
Average WIBOR	1.73%
Average EURBOR	-0.39%
Average mu	2.12%

II. Calibration to market data

Excluding covid window (09.04.2020-04.11.2021)	
Average WIBOR	1.73%
Average EURBOR	-0.39%
Average mu	2.12%

Calibration							
Step	T	DF CC	DF CC	Fwd	lnF(T i+1)-lnF(t i)	Mu	Sigma
0	0.0000	1.0000	1.0000	4.5892			5.31%
1	0.0833	1.0003	0.9986	4.5973	0.1765%	2.12%	5.31%
2	0.1667	1.0006	0.9971	4.6054	0.1765%	2.12%	5.31%
3	0.2500	1.0010	0.9957	4.6136	0.1765%	2.12%	5.31%
4	0.3333	1.0013	0.9942	4.6217	0.1765%	2.12%	5.31%
5	0.4167	1.0016	0.9928	4.6299	0.1765%	2.12%	5.31%
6	0.5000	1.0019	0.9914	4.6381	0.1765%	2.12%	5.31%
7	0.5833	1.0023	0.9900	4.6463	0.1765%	2.12%	5.31%
8	0.6667	1.0026	0.9885	4.6545	0.1765%	2.12%	5.31%
9	0.7500	1.0029	0.9871	4.6627	0.1765%	2.12%	5.31%
10	0.8333	1.0032	0.9857	4.6709	0.1765%	2.12%	5.31%
11	0.9167	1.0036	0.9843	4.6792	0.1765%	2.12%	5.31%
12	1.0000	1.0039	0.9828	4.6875	0.1765%	2.12%	5.31%
13	1.0833	1.0042	0.9814	4.6957	0.1765%	2.12%	5.31%
14	1.1667	1.0045	0.9800	4.7040	0.1765%	2.12%	5.31%
15	1.2500	1.0049	0.9786	4.7124	0.1765%	2.12%	5.31%
16	1.3333	1.0052	0.9772	4.7207	0.1765%	2.12%	5.31%
17	1.4167	1.0055	0.9758	4.7290	0.1765%	2.12%	5.31%
18	1.5000	1.0058	0.9744	4.7374	0.1765%	2.12%	5.31%
19	1.5833	1.0061	0.9730	4.7458	0.1765%	2.12%	5.31%
20	1.6667	1.0065	0.9716	4.7541	0.1765%	2.12%	5.31%
21	1.7500	1.0068	0.9702	4.7625	0.1765%	2.12%	5.31%
22	1.8333	1.0071	0.9688	4.7710	0.1765%	2.12%	5.31%
23	1.9167	1.0074	0.9674	4.7794	0.1765%	2.12%	5.31%
24	2.0000	1.0078	0.9660	4.7878	0.1765%	2.12%	5.31%
25	2.0833	1.0081	0.9646	4.7963	0.1765%	2.12%	5.31%
26	2.1667	1.0084	0.9632	4.8048	0.1765%	2.12%	5.31%
27	2.2500	1.0088	0.9618	4.8133	0.1765%	2.12%	5.31%
28	2.3333	1.0091	0.9604	4.8218	0.1765%	2.12%	5.31%
29	2.4167	1.0094	0.9590	4.8303	0.1765%	2.12%	5.31%
30	2.5000	1.0097	0.9576	4.8388	0.1765%	2.12%	5.31%
31	2.5833	1.0101	0.9563	4.8474	0.1765%	2.12%	5.31%
32	2.6667	1.0104	0.9549	4.8559	0.1765%	2.12%	5.31%
33	2.7500	1.0107	0.9535	4.8645	0.1765%	2.12%	5.31%
34	2.8333	1.0110	0.9521	4.8731	0.1765%	2.12%	5.31%
35	2.9167	1.0114	0.9508	4.8817	0.1765%	2.12%	5.31%
36	3.0000	1.0117	0.9494	4.8903	0.1765%	2.12%	5.31%

III. Simulation of expected exposure profiles for FX Forward



III. Simulation of expected exposure profiles for FX Forward

Simulate 10 000 paths of forex for each time step

Calculate forward contract value for each time step, each path

$$S(t_{i+1}) = S(t_i) \times \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right) \times \Delta t + \sigma \times \left(W(t_{i+1}) - W(t_i)\right)\right].$$

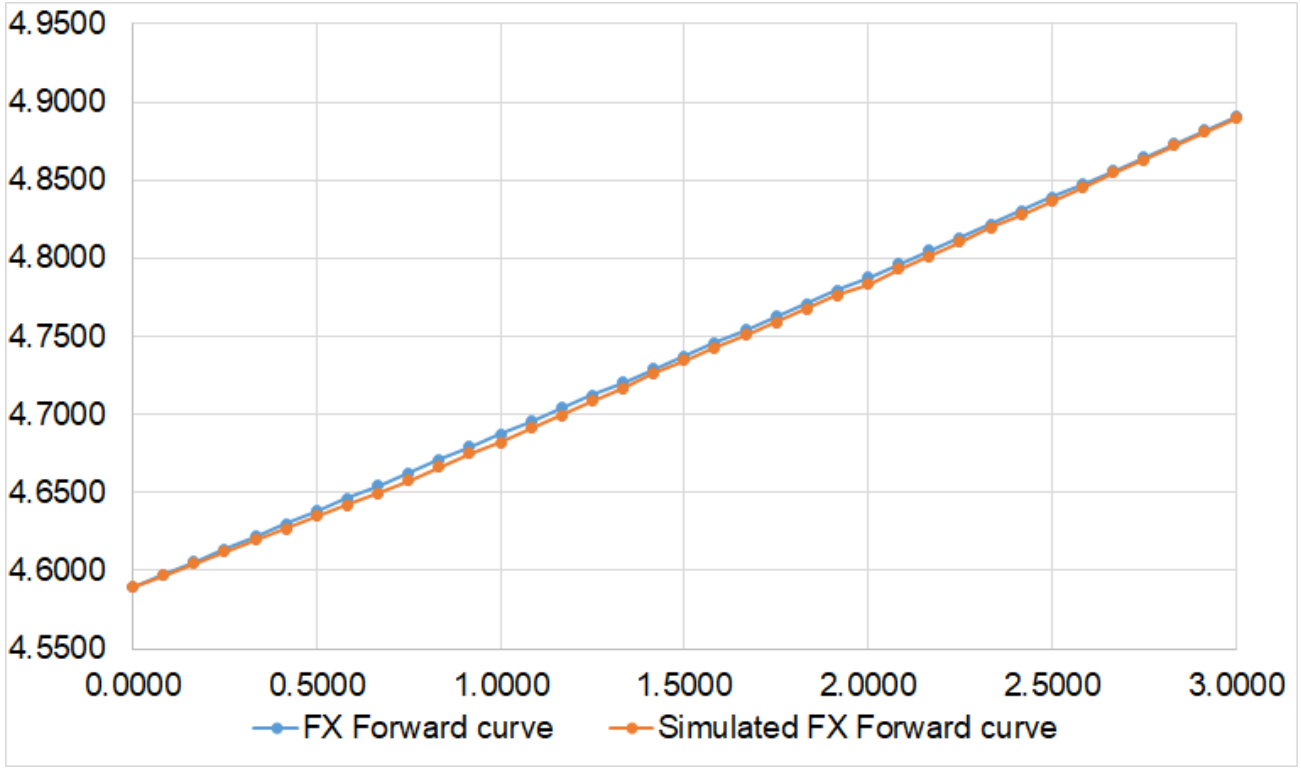
Simulation												
Average	1	2	3	4	5	6	7	8	9	10	11	12
4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892	4.5892
4.5982	4.7392	4.5967	4.5767	4.5479	4.6266	4.6356	4.5952	4.4801	4.5931	4.6574	4.6799	4.5301
4.6064	4.8269	4.6548	4.6350	4.4721	4.6498	4.6338	4.5394	4.4635	4.4739	4.8055	4.7343	4.5591
4.6146	4.7232	4.7047	4.6408	4.4933	4.6090	4.7155	4.5887	4.3800	4.5342	4.8241	4.7893	4.5761
4.6222	4.8681	4.7471	4.6368	4.5542	4.6607	4.7113	4.7230	4.4222	4.5058	4.8147	4.8863	4.5458
4.6311	4.7885	4.8666	4.7087	4.6660	4.5083	4.7746	4.7933	4.4230	4.3916	4.8718	4.9649	4.5542
4.6400	4.8123	4.8984	4.7161	4.6404	4.5337	4.7072	4.8236	4.3608	4.4065	4.8095	4.9627	4.4660
4.6492	4.9224	4.9033	4.7336	4.7018	4.5127	4.7796	4.9256	4.3825	4.5456	4.7741	4.9151	4.4532
4.6574	4.8097	4.8912	4.7994	4.4752	4.4225	4.6620	4.9579	4.4141	4.6633	4.7531	4.8585	4.4598
4.6658	4.8619	4.8827	4.8745	4.6313	4.4031	4.5925	5.0162	4.4877	4.7231	4.9326	4.7438	4.4020
4.6737	4.8511	4.7616	4.8931	4.5856	4.3501	4.6069	4.9099	4.4298	4.7644	4.9229	4.7515	4.5324
4.6814	4.8309	4.7217	5.0298	4.6994	4.4291	4.5086	4.8237	4.4820	4.8436	4.9664	4.7944	4.5401
4.6895	4.8030	4.7234	4.9955	4.7620	4.5511	4.4639	4.8668	4.4999	4.7719	5.0022	4.7742	4.5599
4.6976	4.7496	4.7248	4.8897	4.8038	4.5764	4.4408	4.7804	4.5740	4.7851	5.0308	4.8652	4.4576
4.7078	4.7587	4.8750	5.0069	4.8442	4.5801	4.4238	4.8830	4.6234	4.7252	4.9242	5.0609	4.4488
4.7160	4.7742	4.9135	4.8875	4.8594	4.5419	4.4163	4.9407	4.6824	4.7750	4.8332	4.9378	4.5259
4.7242	4.6984	4.9306	4.9827	4.8547	4.6066	4.3016	4.8882	4.7722	4.6809	4.7690	4.9354	4.5270
4.7332	4.6783	4.9010	4.8668	4.8754	4.6020	4.2245	4.9667	4.8453	4.8107	4.8500	5.0005	4.4367

Simulation of FX forward

III. Simulation of expected exposure profiles for FX Forward

Simulate 10 000 paths of forex for each time step

Calculate forward contract value for each time step, each path



III. Simulation of expected exposure profiles for FX Forward

Calculate EE for each time step, each path.

Calculate average EE for each time step

$$E_i(t) = \max\{V_i(t), 0\}$$

$$NPV(t_i) = \left[S(t_i) \times \frac{DF_{CCY1}(t_i, t_M)}{DF_{CCY2}(t_i, t_M)} - \mathrm{K}\right] \times N \times DF_{CCY2}(t_i, t_m),$$

EE_simulation	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0
2,851	5,404	312	11,131	5,524	2,754	204	-9,393	-11,414	20,423	-11,181
4,041	13,418	1,863	10,645	4,039	234	15,882	-7,633	-2,881	16,224	-12,527
4,985	18,284	-7,175	19,212	2,345	2,086	4,015	-21,036	-3,784	18,023	-12,476
5,769	22,755	3,916	-3,488	2,707	12,770	-3,689	-15,895	933	21,874	-5,358
6,425	8,202	16,192	-16,733	5,885	1,790	673	-9,501	9,413	9,312	-6,337
6,994	19,530	3,909	-21,710	5,519	8,427	-11,364	-19,287	-5,779	2,430	343
7,585	12,580	2,741	-21,165	7,632	8,405	-10,234	-34,120	-7,976	3,745	-7,047
8,155	13,614	519	-15,391	4,603	24,575	2,105	-27,648	8,480	5,449	4,230
8,739	14,969	-13,690	-8,380	10,937	33,763	-6,921	-21,528	8,469	5,822	3,566
9,219	10,621	-23,329	7,047	1,552	44,569	-16,926	-30,320	4,323	18,405	14,134
9,723	493	-24,589	4,073	-6,953	39,305	-17,280	-29,249	7,133	12,369	-2,583
10,080	-3,059	-23,772	9,148	-10,996	48,217	-19,378	-24,975	1,227	17,994	641
10,508	-14,600	-26,860	-3,671	-17,878	41,840	-13,707	-33,530	566	22,501	-4,038
10,897	-4,342	-25,476	-813	-19,684	41,953	-15,193	-32,382	1,504	25,025	-1,322
11,222	-8,868	-11,900	-3,158	-24,162	33,800	-26,446	-33,992	1,496	35,716	1,407
11,726	-9,052	-15,474	-1,105	-25,507	37,660	-23,261	-43,106	-1,360	34,261	-481
12,099	-10,508	-13,703	-1,849	-25,589	30,022	-29,662	-33,598	-15,960	21,761	3,606
12,413	-17,503	-13,603	-13,071	-24,313	22,402	-34,993	-43,285	-5,959	17,875	8,669
12,764	-5,907	-3,916	-9,295	-28,410	11,094	-35,693	-45,675	-6,083	27,667	1,112
13,145	-8,962	2,460	-745	-26,049	3,369	-27,025	-44,137	1,079	22,629	7,110
13,356	-9,703	9,816	-1,906	-35,891	-6,242	-39,883	-41,916	5,843	33,068	21,458
13,707	3,556	25,561	7,551	-41,081	-401	-52,790	-34,080	-6,670	36,495	10,661

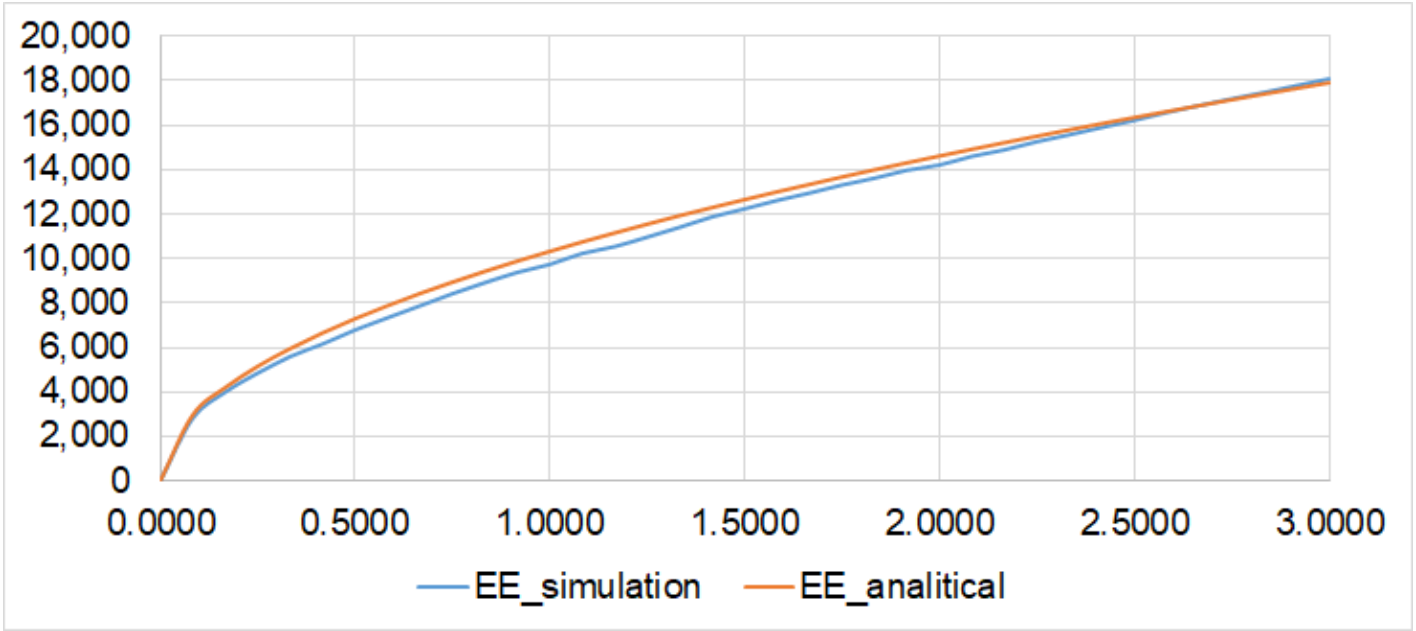
III. Simulation of expected exposure profiles for FX Forward

Check whether you can replicate the initial FX Forward used in the drift calibration?

EE_analitical	EE_simulation
0	0
2,992	2,792
4,231	3,869
5,182	4,806
5,983	5,616
6,690	6,403
7,328	6,986
7,915	7,584
8,462	8,189
8,975	8,762
9,460	9,348
9,922	9,732
10,363	10,201
10,786	10,585
11,193	10,976
11,586	11,273
11,966	11,628
12,334	12,043
12,691	12,456
13,039	12,754
13,377	13,095
13,708	13,452
14,030	13,797
14,345	14,156
14,654	14,455
14,956	14,778
15,252	15,190
15,542	15,396
15,827	15,723
16,107	15,989
16,382	16,317
16,653	16,603
16,919	16,986
17,181	17,221
17,439	17,608
17,694	17,831
17,945	18,011

$$EE(t_i) = F(0, t_m) \times N(d_1) - K \times N(d_2),$$

$$d_1 = \frac{ln\frac{F}{K} + \frac{1}{2} \times \sigma^2 \times t_i}{\sigma \times \sqrt{t_i}}, d_2 = d_1 - \sigma \times \sqrt{t_i}.$$



Thank you for listening!