POZNAN UNIVERSITY OF ECONOMICS AND BUSINESS

FINANCIAL ENGINEERING PROJECT
Stage 4

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Content

I. **Key features of EE for IRS**
II. **Calibration to market Data**
III. **Simulation of expected exposure profiles for IRS**
IV. **Valuation of IRS**

I. Key features of Expected Exposure on IRS

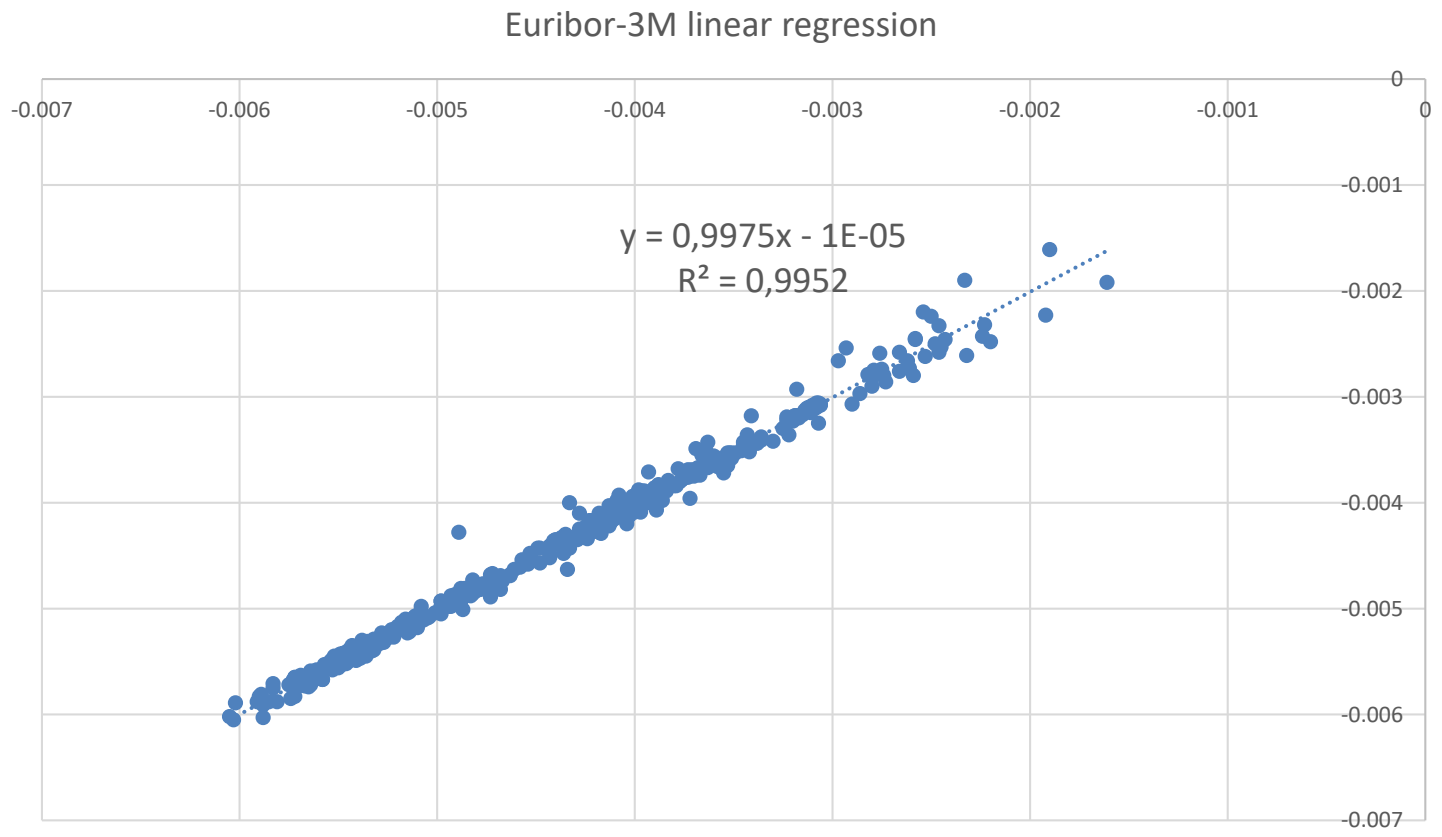
- This is the expected average credit exposure, the distribution of exposures at any particular future date before the longest maturity transaction in the netting set matures, but conditional only on positive values.
- The contract-level exposure is always positive or Zero whichever figure is greater:

$$E_i(t) = \max\{V_i(t), 0\}$$

$$EE_t = \frac{1}{N}\sum_{i=1}^{N} \max\{V_i(t), 0\}$$

II. Calibration to market data: volatility, mu & Lambda

- The least squares regression method was used to calibrate parameters f(0), lambda and sigma for Hull-White model of interest rate.



II. Calibration to market data: volatility, mu & Lambda

$$NPV(t_i) = NPV_{fix}(t_i) - NPV_{float}(t_i).$$

$$\hat{\sigma}^2 = \sigma^2 \frac{1 - e^{-2\lambda\delta}}{2\lambda}$$

Mean mu: f(0)	-0.40000%
Sigma	0.11%
Lambda	0.63
Time horizon = 36 periods	

For Euribor calculations

$$\lambda = -\frac{\ln a}{\delta}$$

$$\mu = \frac{b}{1-a}$$

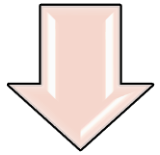
$$\sigma = sd(\epsilon)\sqrt{\frac{-2\ln a}{\delta(1-a^2)}}$$

Calibration to market data

	t	DF	f((0,t)	DF(-)	DF(+)	f(0,t) - numerical	lambda	sigma	mu(t)
0	0,0000	1,0000	-0,40%	1,0000	1,0000	-0,40%	0,63	0,11%	-0,4000%
1	0,0833	1,0003	-0,40%	1,0003	1,0003	-0,40%	0,63	0,11%	-0,4000%
2	0,1667	1,0007	-0,40%	1,0007	1,0007	-0,40%	0,63	0,11%	-0,4000%
3	0,2500	1,0010	-0,40%	1,0010	1,0010	-0,40%	0,63	0,11%	-0,4000%
4	0,3333	1,0013	-0,40%	1,0013	1,0013	-0,40%	0,63	0,11%	-0,3999%
5	0,4167	1,0017	-0,40%	1,0017	1,0017	-0,40%	0,63	0,11%	-0,3999%
6	0,5000	1,0020	-0,40%	1,0020	1,0020	-0,40%	0,63	0,11%	-0,3999%
7	0,5833	1,0023	-0,40%	1,0023	1,0023	-0,40%	0,63	0,11%	-0,3999%
8	0,6667	1,0027	-0,40%	1,0027	1,0027	-0,40%	0,63	0,11%	-0,3999%
9	0,7500	1,0030	-0,40%	1,0030	1,0030	-0,40%	0,63	0,11%	-0,3999%
10	0,8333	1,0033	-0,40%	1,0033	1,0033	-0,40%	0,63	0,11%	-0,3999%
11	0,9167	1,0037	-0,40%	1,0037	1,0037	-0,40%	0,63	0,11%	-0,3999%
12	1,0000	1,0040	-0,40%	1,0040	1,0040	-0,40%	0,63	0,11%	-0,3999%
13	1,0833	1,0043	-0,40%	1,0043	1,0043	-0,40%	0,63	0,11%	-0,3999%
14	1,1667	1,0047	-0,40%	1,0047	1,0047	-0,40%	0,63	0,11%	-0,3999%
15	1,2500	1,0050	-0,40%	1,0050	1,0050	-0,40%	0,63	0,11%	-0,3999%
16	1,3333	1,0053	-0,40%	1,0053	1,0053	-0,40%	0,63	0,11%	-0,3999%
17	1,4167	1,0057	-0,40%	1,0057	1,0057	-0,40%	0,63	0,11%	-0,3999%
18	1,5000	1,0060	-0,40%	1,0060	1,0060	-0,40%	0,63	0,11%	-0,3999%
19	1,5833	1,0064	-0,40%	1,0064	1,0064	-0,40%	0,63	0,11%	-0,3999%
20	1,6667	1,0067	-0,40%	1,0067	1,0067	-0,40%	0,63	0,11%	-0,3999%
21	1,7500	1,0070	-0,40%	1,0070	1,0070	-0,40%	0,63	0,11%	-0,3999%
22	1,8333	1,0074	-0,40%	1,0074	1,0074	-0,40%	0,63	0,11%	-0,3999%
23	1,9167	1,0077	-0,40%	1,0077	1,0077	-0,40%	0,63	0,11%	-0,3999%
24	2,0000	1,0080	-0,40%	1,0080	1,0080	-0,40%	0,63	0,11%	-0,3999%
25	2,0833	1,0084	-0,40%	1,0084	1,0084	-0,40%	0,63	0,11%	-0,3999%
26	2,1667	1,0087	-0,40%	1,0087	1,0087	-0,40%	0,63	0,11%	-0,3999%
27	2,2500	1,0090	-0,40%	1,0090	1,0090	-0,40%	0,63	0,11%	-0,3999%
28	2,3333	1,0094	-0,40%	1,0094	1,0094	-0,40%	0,63	0,11%	-0,3999%
29	2,4167	1,0097	-0,40%	1,0097	1,0097	-0,40%	0,63	0,11%	-0,3999%
30	2,5000	1,0101	-0,40%	1,0100	1,0101	-0,40%	0,63	0,11%	-0,3999%
31	2,5833	1,0104	-0,40%	1,0104	1,0104	-0,40%	0,63	0,11%	-0,3999%
32	2,6667	1,0107	-0,40%	1,0107	1,0107	-0,40%	0,63	0,11%	-0,3999%
33	2,7500	1,0111	-0,40%	1,0111	1,0111	-0,40%	0,63	0,11%	-0,3999%
34	2,8333	1,0114	-0,40%	1,0114	1,0114	-0,40%	0,63	0,11%	-0,3999%
35	2,9167	1,0117	-0,40%	1,0117	1,0117	-0,40%	0,63	0,11%	-0,3999%
36	3,0000	1,0121	-0,40%	1,0121	1,0121	-0,40%	0,63	0,11%	-0,3999%

III. Simulation of expected exposure profiles for the IRS

Simulate 10 000 paths of the IRS for each time step



Calculate IRS Value for each time step, each path

Calculate EE for each time step, each path.

Calculate average EE for each time step

Simulation of expected exposure profiles for IRS

Simulate 10 000 paths on a 3 year horizon

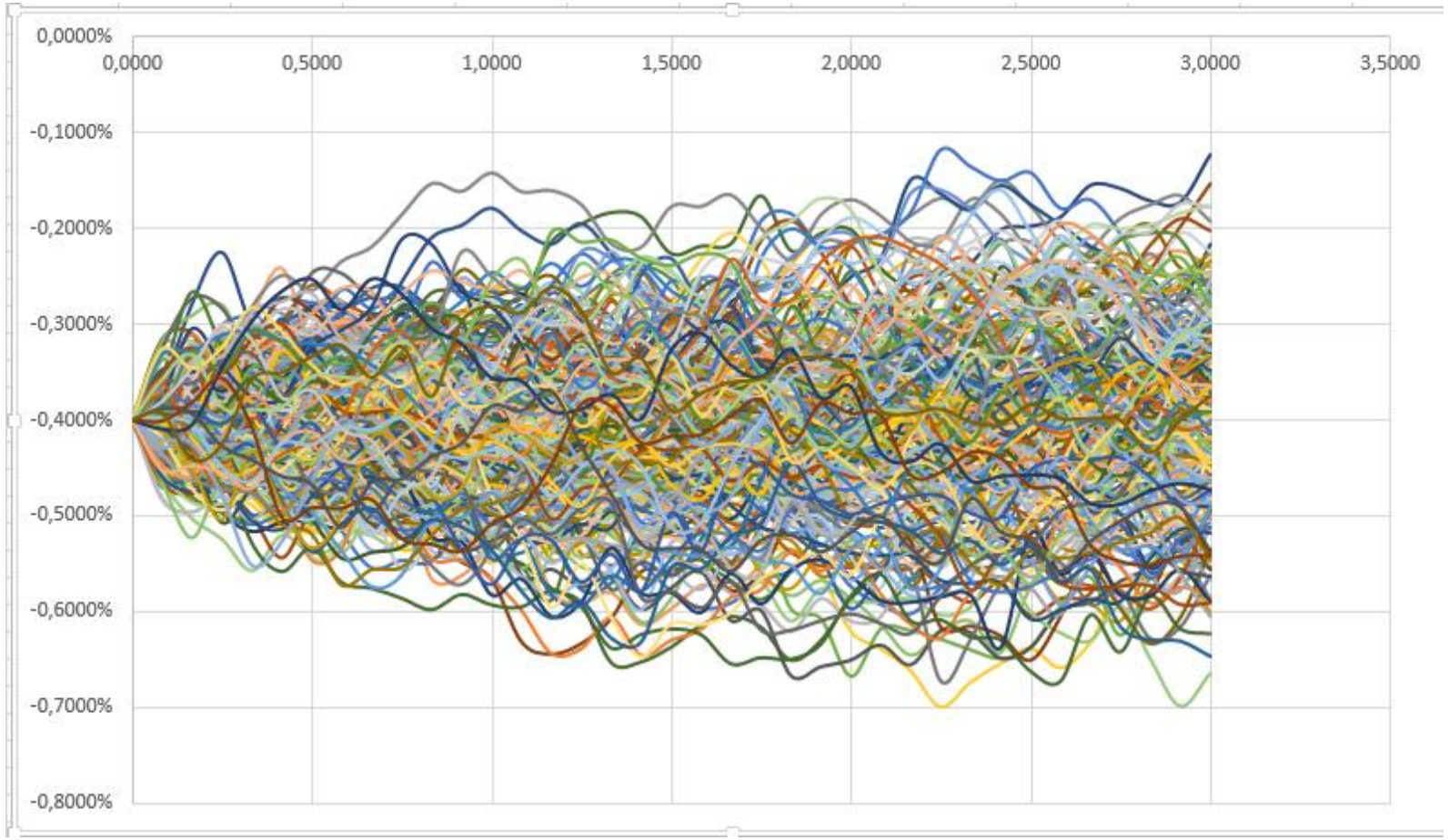
$$dr(t) = \lambda \times (\mu(t) - r(t)) \times dt + \sigma \times dW(t),$$

Average	t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
-0,4000%	0,0000	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%	-0,4000%
-0,4005%	0,0833	-0,4261%	-0,3584%	-0,3208%	-0,4069%	-0,3731%	-0,3982%	-0,4212%	-0,4135%	-0,3684%	-0,3843%	-0,3608%	-0,3983%	-0,4111%	-0,4391%	-0,4204%	-0,4041%
-0,4008%	0,1667	-0,4268%	-0,3553%	-0,3331%	-0,3868%	-0,3614%	-0,4035%	-0,4237%	-0,4474%	-0,3943%	-0,3244%	-0,3855%	-0,3743%	-0,3742%	-0,4433%	-0,3776%	-0,4480%
-0,4006%	0,2500	-0,3876%	-0,3537%	-0,3282%	-0,4036%	-0,3305%	-0,3387%	-0,4195%	-0,4297%	-0,3876%	-0,3280%	-0,3642%	-0,3323%	-0,3091%	-0,4551%	-0,3677%	-0,4710%
-0,4014%	0,3333	-0,3997%	-0,4030%	-0,3743%	-0,3506%	-0,3235%	-0,3476%	-0,3819%	-0,4101%	-0,3784%	-0,3171%	-0,3834%	-0,3731%	-0,3449%	-0,4342%	-0,3939%	-0,4899%
-0,4009%	0,4167	-0,3862%	-0,3984%	-0,3799%	-0,3242%	-0,3523%	-0,3274%	-0,3793%	-0,4439%	-0,3787%	-0,3025%	-0,3601%	-0,3797%	-0,3898%	-0,4582%	-0,4122%	-0,5149%
-0,4007%	0,5000	-0,3599%	-0,3441%	-0,4139%	-0,3299%	-0,3643%	-0,3358%	-0,4099%	-0,3725%	-0,4167%	-0,3200%	-0,3057%	-0,3184%	-0,4158%	-0,4546%	-0,3899%	-0,4827%
-0,4001%	0,5833	-0,3888%	-0,4080%	-0,4085%	-0,3537%	-0,3684%	-0,3333%	-0,4244%	-0,4452%	-0,4400%	-0,3105%	-0,2920%	-0,3017%	-0,4347%	-0,5102%	-0,4034%	-0,4832%
-0,4002%	0,6667	-0,3756%	-0,3584%	-0,3997%	-0,3683%	-0,3944%	-0,3967%	-0,4288%	-0,4642%	-0,4612%	-0,3577%	-0,2632%	-0,2880%	-0,4009%	-0,5125%	-0,4037%	-0,5188%
-0,4005%	0,7500	-0,3200%	-0,3925%	-0,3858%	-0,3668%	-0,3875%	-0,4394%	-0,3917%	-0,4113%	-0,4591%	-0,3141%	-0,2623%	-0,3033%	-0,3726%	-0,4784%	-0,4245%	-0,4873%
-0,4004%	0,8333	-0,3302%	-0,3638%	-0,4127%	-0,3230%	-0,3812%	-0,4060%	-0,3700%	-0,4600%	-0,4856%	-0,3141%	-0,2856%	-0,2675%	-0,3599%	-0,5657%	-0,4379%	-0,5438%
-0,4005%	0,9167	-0,3157%	-0,4049%	-0,3847%	-0,2873%	-0,4179%	-0,4016%	-0,3508%	-0,4720%	-0,4923%	-0,2863%	-0,3117%	-0,2664%	-0,3219%	-0,5624%	-0,5087%	-0,5203%
-0,4001%	1,0000	-0,3230%	-0,4383%	-0,3764%	-0,2656%	-0,3615%	-0,3615%	-0,3331%	-0,4856%	-0,4826%	-0,3089%	-0,3355%	-0,2723%	-0,3411%	-0,5321%	-0,4958%	-0,5504%
-0,3997%	1,0833	-0,3073%	-0,4282%	-0,3850%	-0,2602%	-0,3436%	-0,3699%	-0,3073%	-0,4281%	-0,4788%	-0,3292%	-0,2805%	-0,2727%	-0,3189%	-0,5089%	-0,5057%	-0,5213%
-0,3995%	1,1667	-0,3213%	-0,4380%	-0,3684%	-0,3007%	-0,3682%	-0,3541%	-0,2958%	-0,4192%	-0,5207%	-0,3027%	-0,3511%	-0,2143%	-0,2748%	-0,5781%	-0,4703%	-0,5381%
-0,3992%	1,2500	-0,3448%	-0,4665%	-0,3419%	-0,2815%	-0,3929%	-0,3219%	-0,2726%	-0,4059%	-0,5263%	-0,3088%	-0,3874%	-0,1957%	-0,3132%	-0,5913%	-0,4963%	-0,5150%
-0,3985%	1,3333	-0,3619%	-0,4883%	-0,3614%	-0,2580%	-0,3714%	-0,3098%	-0,2934%	-0,3989%	-0,5511%	-0,2738%	-0,4237%	-0,1837%	-0,3127%	-0,5802%	-0,5161%	-0,5403%
-0,3990%	1,4167	-0,4165%	-0,5196%	-0,3697%	-0,2761%	-0,4159%	-0,2994%	-0,3335%	-0,4247%	-0,5247%	-0,3260%	-0,4618%	-0,1881%	-0,3294%	-0,6135%	-0,4868%	-0,5399%
-0,3992%	1,5000	-0,4380%	-0,5418%	-0,3438%	-0,2801%	-0,4722%	-0,2829%	-0,3014%	-0,4172%	-0,5142%	-0,2890%	-0,4896%	-0,2266%	-0,3067%	-0,5971%	-0,4344%	-0,5084%
-0,3993%	1,5833	-0,4293%	-0,5071%	-0,3402%	-0,2892%	-0,4544%	-0,2502%	-0,3061%	-0,3796%	-0,4659%	-0,3171%	-0,4830%	-0,2188%	-0,3411%	-0,5727%	-0,4194%	-0,5186%
-0,3989%	1,6667	-0,4736%	-0,4711%	-0,3299%	-0,2625%	-0,4749%	-0,2934%	-0,2765%	-0,4325%	-0,4644%	-0,2977%	-0,4561%	-0,2162%	-0,3672%	-0,5741%	-0,5035%	-0,4676%
-0,3987%	1,7500	-0,4441%	-0,4751%	-0,2946%	-0,2598%	-0,4638%	-0,3249%	-0,2624%	-0,4240%	-0,4973%	-0,2526%	-0,4009%	-0,1662%	-0,3241%	-0,5483%	-0,4978%	-0,5304%
-0,3988%	1,8333	-0,4133%	-0,4439%	-0,3322%	-0,2364%	-0,4688%	-0,3012%	-0,2563%	-0,4885%	-0,4418%	-0,2635%	-0,4207%	-0,2235%	-0,3242%	-0,4755%	-0,5179%	-0,5358%
-0,3986%	1,9167	-0,3805%	-0,4753%	-0,3318%	-0,2046%	-0,4342%	-0,3298%	-0,3512%	-0,4834%	-0,4103%	-0,2900%	-0,4982%	-0,2055%	-0,3081%	-0,4799%	-0,5187%	-0,5623%
-0,3982%	2,0000	-0,3594%	-0,4404%	-0,3158%	-0,2012%	-0,4444%	-0,3271%	-0,3522%	-0,5094%	-0,3994%	-0,2179%	-0,4992%	-0,2016%	-0,3697%	-0,4151%	-0,5237%	-0,6211%
-0,3986%	2,0833	-0,3373%	-0,3919%	-0,2938%	-0,2457%	-0,4847%	-0,2993%	-0,3192%	-0,4938%	-0,4518%	-0,2174%	-0,5117%	-0,2418%	-0,4387%	-0,4282%	-0,5529%	-0,6394%
-0,3983%	2,1667	-0,3497%	-0,3971%	-0,2950%	-0,2336%	-0,4751%	-0,3088%	-0,2673%	-0,5419%	-0,4665%	-0,2289%	-0,4992%	-0,2678%	-0,4247%	-0,3852%	-0,5312%	-0,6627%
-0,3979%	2,2500	-0,3582%	-0,3920%	-0,3170%	-0,2595%	-0,4528%	-0,2643%	-0,2511%	-0,5729%	-0,4571%	-0,2546%	-0,4590%	-0,2977%	-0,4473%	-0,3516%	-0,5506%	-0,7000%
-0,3978%	2,3333	-0,3400%	-0,4223%	-0,3005%	-0,2729%	-0,4067%	-0,2400%	-0,2368%	-0,5898%	-0,4225%	-0,2780%	-0,4556%	-0,3655%	-0,4119%	-0,3333%	-0,5270%	-0,6742%
-0,3976%	2,4167	-0,3161%	-0,4199%	-0,3652%	-0,2685%	-0,4409%	-0,2612%	-0,2020%	-0,5696%	-0,4281%	-0,2618%	-0,4710%	-0,4108%	-0,4218%	-0,3198%	-0,5442%	-0,6550%
-0,3978%	2,5000	-0,3238%	-0,4677%	-0,3775%	-0,2917%	-0,4363%	-0,2676%	-0,1982%	-0,6039%	-0,4626%	-0,2623%	-0,4493%	-0,4300%	-0,4221%	-0,2639%	-0,5490%	-0,6389%
-0,3977%	2,5833	-0,2736%	-0,4584%	-0,3886%	-0,2646%	-0,4060%	-0,3370%	-0,1887%	-0,5862%	-0,4969%	-0,2525%	-0,4419%	-0,4169%	-0,3871%	-0,2869%	-0,5441%	-0,6592%
-0,3978%	2,6667	-0,3422%	-0,3932%	-0,3843%	-0,3249%	-0,4146%	-0,3209%	-0,1557%	-0,5804%	-0,4709%	-0,2341%	-0,4255%	-0,3494%	-0,4209%	-0,2228%	-0,5092%	-0,6326%
-0,3978%	2,7500	-0,3423%	-0,4147%	-0,4003%	-0,2911%	-0,4455%	-0,3018%	-0,1590%	-0,5148%	-0,4880%	-0,2527%	-0,4112%	-0,3600%	-0,4252%	-0,2531%	-0,4532%	-0,5751%
-0,3976%	2,8333	-0,3354%	-0,4360%	-0,3992%	-0,2889%	-0,4495%	-0,2849%	-0,1720%	-0,5141%	-0,4489%	-0,2681%	-0,4543%	-0,3547%	-0,3978%	-0,2311%	-0,4107%	-0,5661%
-0,3978%	2,9167	-0,3659%	-0,4406%	-0,4399%	-0,3194%	-0,3922%	-0,2756%	-0,1734%	-0,4873%	-0,4373%	-0,2600%	-0,4284%	-0,3826%	-0,4487%	-0,2232%	-0,3796%	-0,5573%
-0,3978%	3,0000	-0,4166%	-0,4864%	-0,4484%	-0,4043%	-0,3862%	-0,2709%	-0,1240%	-0,4339%	-0,4877%	-0,2825%	-0,4735%	-0,3569%	-0,4356%	-0,2405%	-0,3933%	-0,5399%

Simulation of expected exposure profiles for IRS

Simulate 10 000 paths for each time step

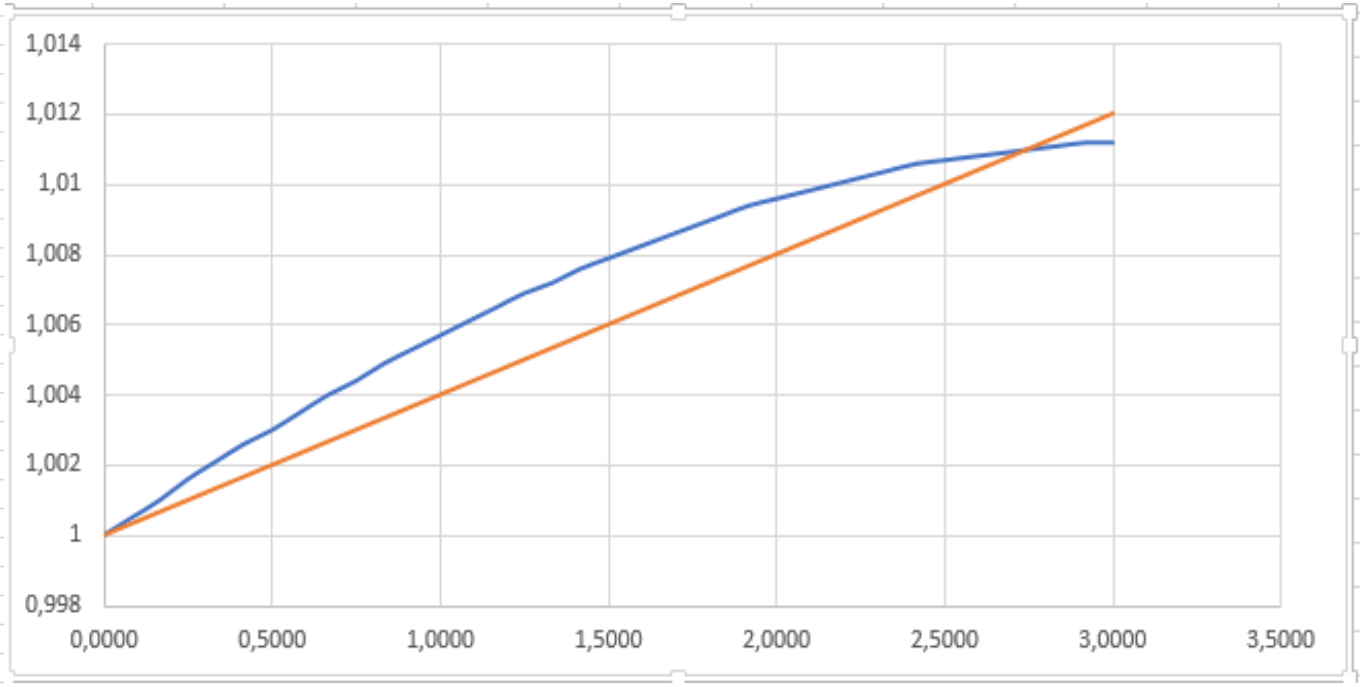
Calculate IRS value for each time step, each path



Simulation of expected exposure profiles for IRS

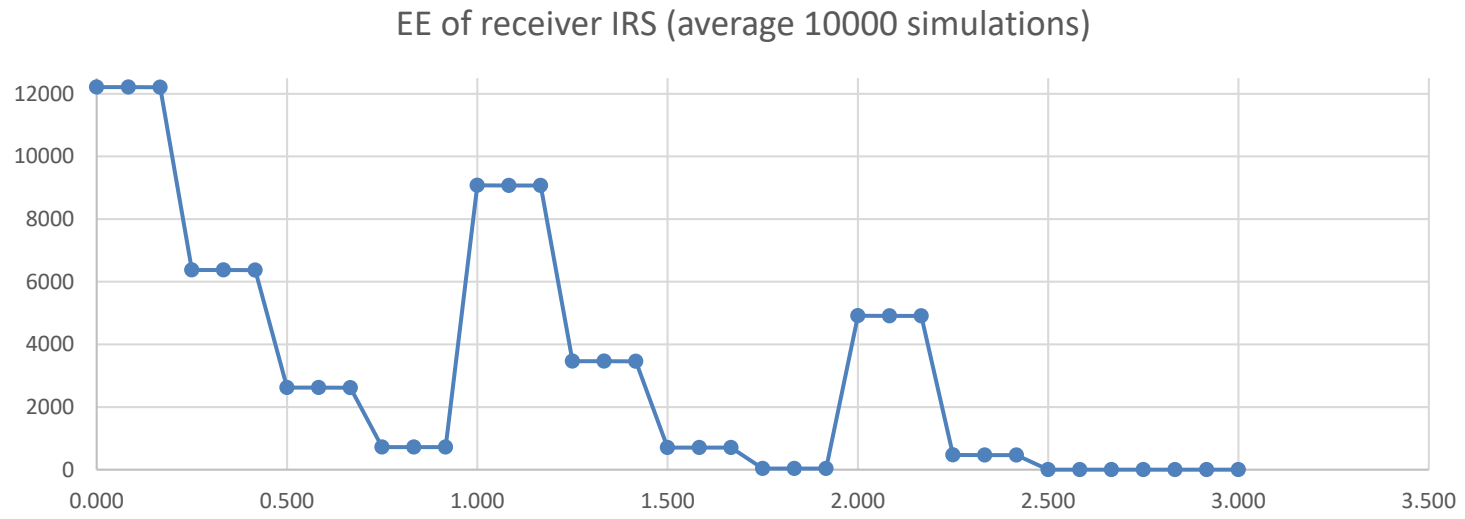
Check whether you can replicate the initial yield curve used in the drift calibration?

Month	EUR_Rat	EUR_DF	DF_simulation
0	-0,64%	1	1
1	-0,63%	1,0005	1,000334
2	-0,63%	1,001	1,000668
3	-0,62%	1,0016	1,001002
4	-0,62%	1,0021	1,001337
5	-0,61%	1,0026	1,001672
6	-0,61%	1,003	1,002006
7	-0,60%	1,0035	1,00234
8	-0,60%	1,004	1,002674
9	-0,59%	1,0044	1,003009
10	-0,58%	1,0049	1,003344
11	-0,58%	1,0053	1,003679
12	-0,57%	1,0057	1,004013
13	-0,56%	1,0061	1,004348
14	-0,56%	1,0065	1,004682
15	-0,55%	1,0069	1,005017
16	-0,54%	1,0072	1,00535
17	-0,53%	1,0076	1,005685
18	-0,53%	1,0079	1,006019
19	-0,52%	1,0082	1,006354
20	-0,51%	1,0085	1,006689
21	-0,50%	1,0088	1,007023
22	-0,49%	1,0091	1,007358
23	-0,49%	1,0094	1,007693
24	-0,48%	1,0096	1,008027
25	-0,47%	1,0098	1,008362
26	-0,46%	1,01	1,008697
27	-0,45%	1,0102	1,009031
28	-0,44%	1,0104	1,009366
29	-0,43%	1,0106	1,0097
30	-0,43%	1,0107	1,010035
31	-0,42%	1,0108	1,01037
32	-0,41%	1,0109	1,010705
33	-0,40%	1,011	1,01104
34	-0,39%	1,0111	1,011375
35	-0,38%	1,0112	1,01171
36	-0,37%	1,0112	1,012046

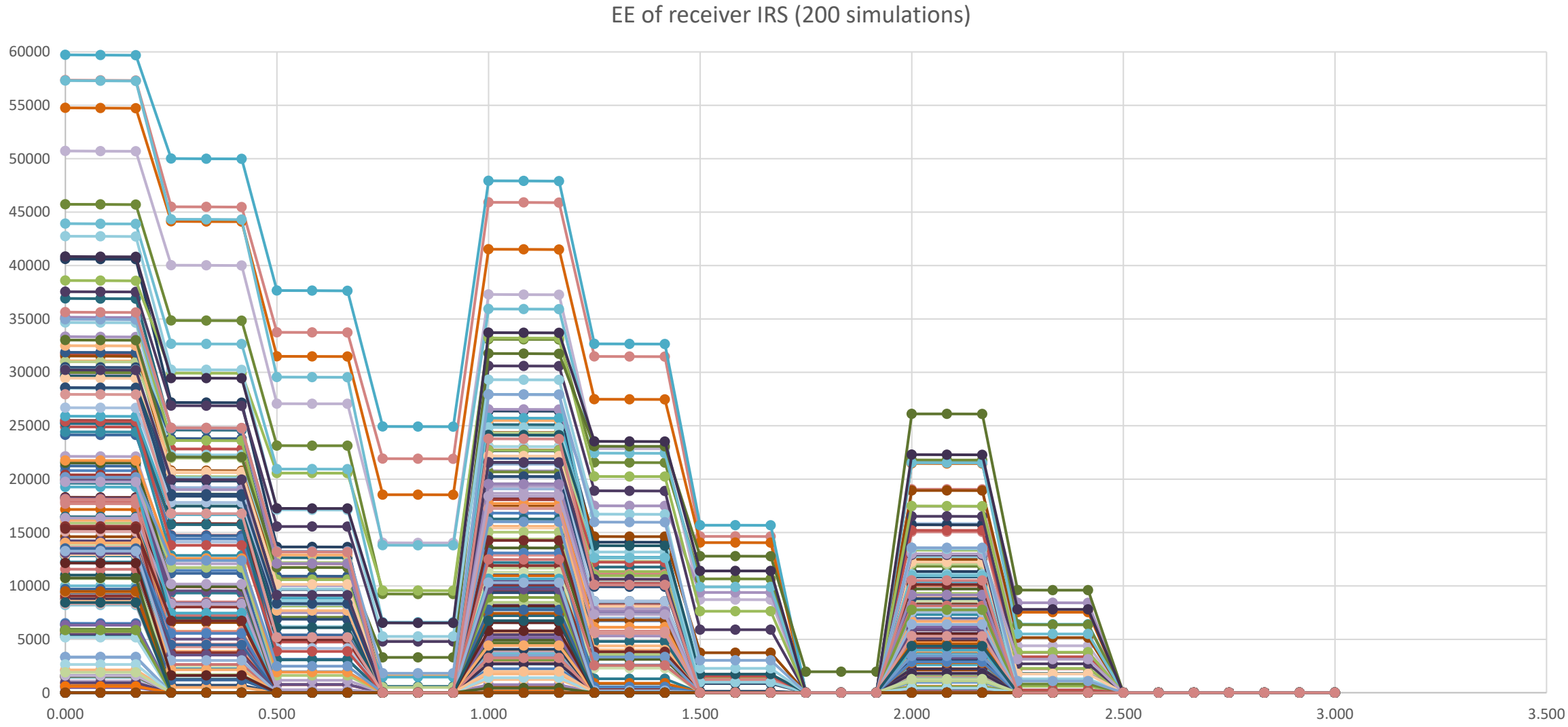


IV.IRS Valuation - NPV

Notional	**10.000.000**	**Receiver swap - we receive fixed**																				
Fixed rate	**-0,37%**																					
	0,000	**0,083**	**0,167**	**0,250**	**0,333**	**0,417**	**0,500**	**0,583**	**0,667**	**0,750**	**0,833**	**0,917**	**1,000**	**1,083**	**1,167**	**1,250**	**1,333**	**1,417**	**1,500**	**1,583**	**1,667**	**1,750**
DF		1,0003	1,0007	1,0010	1,0013	1,0017	1,0020	1,0023	1,0027	1,0030	1,0033	1,0037	1,0040	1,0043	1,0047	1,0050	1,0053	1,0057	1,0060	1,0060	1,0067	1,0070
Forward rates																						
Forward Euribor		-0,3997%	-0,3998%	-0,3996%	-0,3995%	-0,3992%	-0,3994%	-0,3997%	-0,4000%	-0,4005%	-0,4002%	-0,4008%	-0,4007%	-0,4002%	-0,4008%	-0,4008%	-0,4003%	-0,4003%	-0,4003%	-0,4007%	-0,4006%	-0,4007%
Forward Euribor quaterly				-0,399%			-0,399%			-0,400%			-0,401%			-0,401%			-0,400%			-0,400%
Floating CFs (FV)				-9986,25			-9979,28			-10007,96			-10012,54			-10014,55			-10002,16			-10012,16
Fixed CFs (FV)													-37100									
Net cash flows (FV)				9986,25			9979,28			10007,96			-27087,46			10014,55			10002,16			10012,16
Net cash flows (PV)				9996,24			9999,23			10038,01			-27196,01			10064,76			10062,37			10082,52
NPV	**8594**	**8591**	**8588**	**-1401**	**-1401**	**-1400**	**-11379**	**-11375**	**-11371**	**-21376**	**-21368**	**-21361**	**5733**	**5731**	**5729**	**-4287**	**-4286**	**-4284**	**-14285**	**-14285**	**-14275**	**-24283**
Exposure: max (NPV, 0)	8594	8591	8588	0	0	0	0	0	0	0	0	0	5733	5731	5729	0	0	0	0	0	0	0



The NPV progression with time shows negative NPVs due to differences in payment frequencies
With the monthly steps, there are no upward humps between 3M payments since the Interest rate is very low and negative


EE of receiver IRS (200 simulations)
60000
55000
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0.000
0.500
1.000
1.500
2.000
2.500
3.000
3.500

Thank you for listening!