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Mathematical Contest In Modeling®

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What Affects the Decomposer and How?

Summary

In recent years, more and more scientific researches have focused on the factors affecting the decomposition rate of fungi. Our team simulate the decomposition of woody fibers on a given piece of land, discuss the growth rate and moisture tolerance of hyphae and establish the de-composition rate model. On this basis, we consider the influence of different species, then get the interaction model.

For Task1, we set up a mathematical model, which describes the decomposition of wood fiber by fungi in the presence of various fungi, taking the different growth rates, moisture tolerance and the interaction among different fungi into account. We use the linear regression model to analyze the data, and use the least square method to fit the trend, and get the functional expressions of the expansion rate influencing factor and the moisture tolerance in-fluencing factor, and their corresponding times in the decomposition rate model. After that, we consider the temperature factor and the normal distribution of water niche (representing the influence of humidity), multiply these four factors and introduce the time factor, and obtain the decomposition rate model representing the properties of a single fungus. After that, we modify the logistic model of single population and the Lotka-Volterra competition model of multi-population, and solve it by using the differential equation model. By combining the modified Lotka-Volterra competition model with the decomposition rate model, we obtain the interaction model .

For Task2, we take the competition factor and the inherent growth rate in the model as control variables, and make a population growth trend graph. We analyze the short-term and long-term trends of various possible situations: in the short-term, species are mainly inhibited by other species; The long-term trend is mainly limited by environmental resources, and the main results are competition (that is, one party is extinct) and coexistence (until the environmental resources are exhausted).

For Task3, we analyze the trend diagram of individual number changes of different species combinations under different environmental humidity, and find that the advantages and disadvantages of a single species are determined by its moisture tolerance and competitive ranking, while the relationship between species combinations may be competition or coexistence. In order to consider the environmental differences in the same area, we also explore the influence of temperature differences between winter and summer in the same area.

For Task4, we compare the growth trends of two kinds of fungi and three kinds of fungi in the most suitable humidity environment (temperate zone), and on this basis, observe their differences under different environmental changes. We find that when the environmental change is not so severe, the fungal activity is more affected by the environment and mutual inhibition; However, when the environment changes drastically, the combination with more species will survive better.

In conclusion, we set up a very comprehensive model, and discuss the in-fluencing factors of fungal decomposition rate, including mycelial growth rate, environmental humidity, interaction of different kinds of fungi and so on. After model analysis and evaluation, our model has very good sensitivity and ro-bustness. In addition, this model has strong universality, which can be applied to not only fungi, but also other biological fields, and has broad application prospects.

Keywords: fungal traits; wood decomposition; competitive exclusion Principle;functional biogeography

Contents

1 Introduction

1.1 Problem Background and Restatement

The decomposition of plant materials and lignocellulose is an important part of the de-composition of compounds in the carbon cycle, and fungi are the main factors of decompos-ing lignocellulose. It is found that the expansion rate of hyphae is the growth rate of fungi, and the faster the growth rate of fungi, the faster the de-composition rate of wood; Fungi that can adapt to more humidity conditions decompose wood more slowly; If the filament density is high, the decom-position speed of wood will be slow, that is, there is competition among different flora. Our team simulated the decomposition of woody fibers on a given piece of land, and simulated the existence of various types of fungi that decompose woody fibers in the same area. In addition, we also dis-cussed the impact of environmental factors and biodiversity, and analyzed and evaluated the model.

In view of the given background information and conditions, we need to solve the fol-lowing problems:

- A mathematical model was established to describe the decomposi-tion of ground litter and ligno-cellulose by fungi in the presence of various fungi, taking into account the different growth rates and moisture tolerance of fungi and the interaction among different fungi.
- Provide model analysis to describe the interaction between different types of fungi, including the analysis of short-term and long-term trends.
- Predict the relative advantages and disadvantages of each species and the combination of species that may persist. And forecast the possible conditions of different environments, including arid, semi-arid, temperate zone, arbor and tropical rain forest. The model is sensitive to rapid fluctuations in the environment, and determines the overall impact of changing atmospheric trends to assess the impact of changes in local weather patterns.
- Describe the impact of fungal community diversity on decomposition efficiency, and predict the importance and role of biodiversity when the local environment changes to varying degrees.

1.2 Our Approach and Model Overview

First of all, we establish the decomposition rate model L, and then get the interaction model I from the decomposition rate model. Decomposition rate model is composed of growth rate model and moisture tolerance model in a certain way. Growth rate model E includes expansion rate influence factor v and temperature factor Te. We obtained the mycelial growth rate data under certain temperature and humidity conditions, which was taken as the intrinsic growth rate v, and the expansion rate influencing factor V was a fitting function of the intrinsic growth rate v. We define the Temperature factor Te to show the effect of temperature on growth rate, and Te is 1 when t takes a time unit. The moisture tolerance model H includes humidity influence factor Mf and ecological water level normal distribution function N. The linear function of moisture tolerance M (difference of each isolate's competitive ranking and their humidity niche width, both scaled to [0,1]) [2] [3] is taken as humidity influencing factor, and the influence of environmental humidity on fungal activity is expressed by normal distribution. After combining the above two models in a reasonable way, the de-composition rate model is obtained by introducing time factor. We modified the logistic model of single population and the Lotka-Volterra competition model of multiple populations, and combined them with the decomposition rate model to get the de-composition rate model considering interaction.

Secondly, we analyzed the interaction and competition among different fungi, including short-term and long-term trends.

After that, we forecast the relative advantages and disadvantages of each species and the combination of species that may persist. We also consider the climate impacts of different environments, such as humidity differences under different climate conditions and temperature differences under the same climate type in the same area.

Finally, we discussed the influence of fungal community diversity on decomposition ef-ficiency, and predicted the importance and role of biodiversity of fungal community diver-sity when the local environment changed in different degrees.

To sum up, the whole model process is shown in Figure 1 .

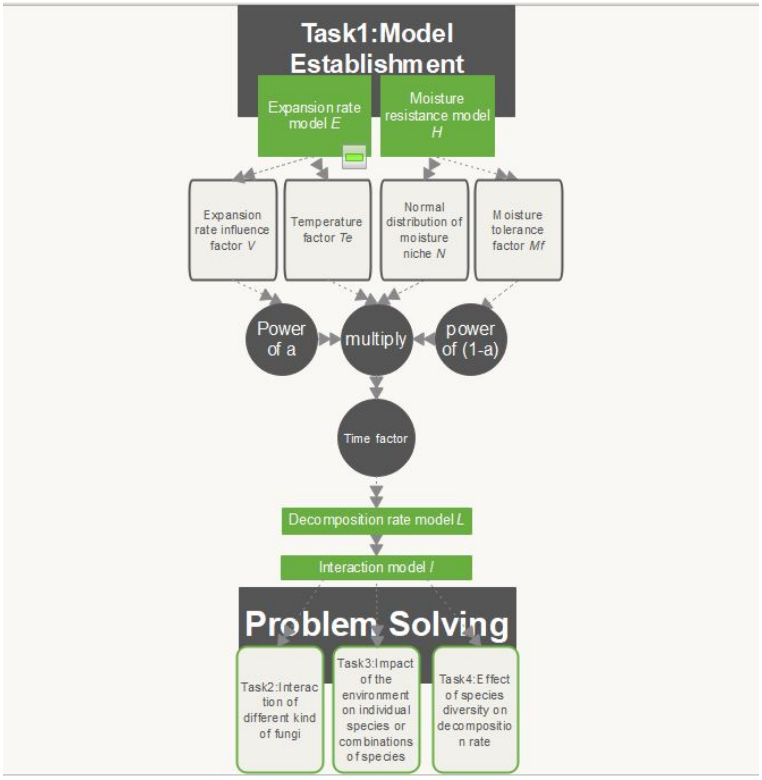

Figure 1: The whole model flow, a is calculated and fitted by data, a=0.95

1.3 Literature Review

The decomposition of compounds is a part of carbon cycle, which enables carbon to be renewed and used in other forms,The decomposition of plant materials and wood fibers is the key component of this process. Fungi are the main factor of decomposing lignocellulose, which is a field worthy of study,In recent years, the research on the decomposition rate characteristics of fungi is becoming more and more prosperous. We will summarize some documents and methods that have appeared and have great reference value for our research field.

D. S. Maynard et al.,. In the article diversity begets diversity in com-petition for space [3] published in 2017, it is proved that species richness can be used as a self-reinforcing buffer of competitive exclusion, and a method to measure the impact of competition is proposed, which has been applied in the following articles and also used for reference in our paper.

D. S. Maynard et al.,. In the article Consistent trade-offs in funga[1] published in 2019, a trait-based method was proposed to quantify the niche of fungi and explore the relationship between functional trait expression, climate and phylogeny. This paper reveals the basic balance between abiotic stress tolerance and competitiveness, that is, fungi have a wide range of heat and water niches, showing low displacement ability, and the balance between dominance and tolerance is partly related to the envi-ronmental conditions of fungi collection.

Nicky Lustenhouwer, Daniel S. Maynard, Mark A. Bradford, Daniel L. Lindner, Brad Oberle, Amy E. Zanne, and Thomas W. Crowther put forward in the article [2] published in 2020 that slow-growing strains can survive and grow better in environmental changes such as humidity and temper-ature, while fast-growing strains are often less robust to the same changes. They identified the characteristics of fungal decomposition rate, and no-ticed the relationship between some characteristics [4].

2 Assumptions and Justifications

We make the following basic assumptions to simplify the problem, each of which is properly justified.

- **assumption1:** For the purpose of this modeling exercise, we focus on the results for the middle stage and assume it is consistent for other stages of decomposition.

  justification1: As required , the fungi studied in the literature provided are most related to the decay of wood materials in the middle stage of its decay cycle, and the results of other decay stages may be different. We focus on the decomposition process in the intermediate stage, ignoring the deviation caused by other processes.

- **assumption2:** We assume that the influence of expansion rate and humidity act on decomposition rate in-dependently, and these two variables are not correlated with each other.

  justification2: In this paper, we focus on two influencing factors: mycelial expansion rate and humidi-ty,When analyzing these two factors separately, we ignore the influence of the other factor.

- **assumption3:** We assume that the decomposition rate is not affected by the flora at the same expansion rate, that is, we can fit the general trend of the influence of expansion rate on decomposition rate through different decomposition rates caused by different expansion rates.

  justification3: It can be concluded from the data source [2] that at a certain temperature, the expansion rate is mainly determined by the species of flora. Therefore, we ignore the extra influence and fit the change trend like the picture given in the paper (see fig 2) to explore the relationship between the decomposition rate and the expansion rate.

- **assumption4:** We assume that the temperature environment of fungus research involved in this paper is 22℃.

  justification4: In this paper, we mainly explore the influence of growth rate and humidity,The data given in Reference [2] involve three temperature environments, and the change trend of decomposition rate is roughly the same at these three temperatures,Therefore, we take 22℃, the envi-ronment with the most sensitive change of experimental data, as our exploration environ-ment.

- **assumption5:** We assume that there is a positive correlation between the number of fungi n and its de-composition rate l.

  justification5: When other factors are not considered, the inhibition of competition among populations is reflected in the number of individuals in the population,We measure the activity of fungi by the number of individuals.

- **assumption6:** We assume that our given land is a litter and lignocellulose with a mass of 1 regardless of size and shape.

  justification6: In order to simulate and control variables, we ignore size and shape. The data given in the reference is decomposition rate, so we assume that the initial mass is 1, and the decomposi-tion rate is equivalent to the concept of lost mass.

- **assumption7:** In this paper, apart from discussing species combination and biodiversity, the interaction of only two species is considered for the time being.

  justification7: In the model we use, the interaction of different fungi is reflected in the growth rate model, and we use the logistic growth model. In this model, the increase of influence types will only lead to the decrease of influence factors, but will not affect the overall change trend. There-fore, in order to facilitate simulation and understanding, when we focus on other influencing factors, we first assume that only two kinds of fungi interact with each other, and then con-sider the interaction of multiple species when considering the interaction of multiple species or exploring biodiversity.

- **assumption8:** Our time factor t takes 122 days as an unit.

  justification8: The experimental data we obtained [2] records the average data within 122 days,We take 122 days as a time unit, discuss the short-term trend with one time unit (122 days), and dis-cuss the long-term trend with infinite time units (positive infinite days). In addition to dis-cussing both long-term and short-term trends when different fungi interact with each other, other discussions in this paper consider short-term effects, that is, one time unit (122 days),Since the initial mass is 1, the three concepts of decomposition rate, decomposition rate and mass loss are equivalent when discussing short-term effects.

3 Notations

The key notations used are listened in Table 1, and the description of some symbols will be given in the following paper. We use uppercase letters to represent the dependent variable (function or model) and lowercase letters as independent variables.

4 Task1: Model Establishment

4.1 Growth rate model

4.1.1 Data Description

When studying the growth rate model, we used the experimental data given in paper-tables 4 [2]. The experimental data is to measure the av-erage mycelial growth rate of different fungal species in 122 days at dif-ferent temperatures,We take the experimental data at 22℃ and get a set of data with fungal species as independent variable and growth rate as de-pendent variable. In the following research, we will take this data as the Hyphal Extension Rate v in the growth rate model, and which is a parameter determined by fungi species.

4.1.2 The Establishment of growth rate model

Growth rate model E describes the influence of fungus growth and expansion on its activity (i.e., decomposition rate), including its own expansion rate and temperature. Therefore, we introduce expansion rate factor and temperature factor to describe these two effects.

Expansion rate factor v refers to the influence of expansion rate of different species on decomposition rate, and its independent vari-able is intrinsic growth rate. In order to make our model more consistent with the real experimental results (as shown in Figure 2), we assume the form of expansion rate factor as V(v)=k$_1 v^c + b_1$.

According to the trend in fig,2, c is about 0.5 times, and its accurate value can be calculated by fitting. We used the point estimation method and the least square method, and analyzed the mathematical expectation, variance and correlation co-efficient,Finally, we fitted Figure 3, and calculated k$_1 = 6.386, b_1 = 3.970, andc = 0.5636$.

Table 1: Product Index Information

Symbols	Descrption	percent
L	Decomposition rate	percent
v_L	Decomposition rate	-
$I(L)$	Interaction model	percent
$L(E, H)$	Decomposition rate model	-
$E(V, Te)$	Growth rate model	-
$V(v)$	Expansion rate influence factor	-
$Te(T)$	Temperature factor	-
$H(M_f, N)$	Moisture resistance model	-
$Mf(M)$	Tolerance factor, linear function of tolerance	-
$M = r - w$	Moisture trade-off(Moisture tolerance)	-
$N(x_1, x_2, h)$	Normal distribution of water niche	-
w	Moisture niche width(x1-x2)	MPa
x_1	Moisture niche maximum	MPa
x_2	Moisture niche minimum	MPa
$Ti(t)$	Time factor	-
v	Hyphal Extension Rate	mm/day
h	Ambient humidity	percent
T	Ambient temperature	°C
t	Time	122day
n	The number of species in a certain group	-
r	Competitive ranking	-
$K(t)$	Environmental capacity	-
M_d =L	The decomposed mass	percent*1

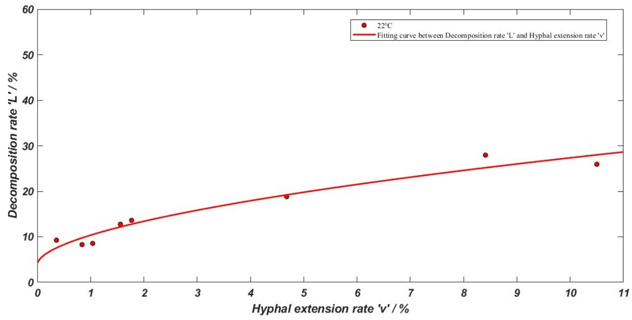

Figure 3: By fitting the curve of decomposition rate L versus intrinsic growth rate v, $k_1 = 6.386, b_1 = 3.970, c = 0.5636$

So we get the specific form of expansion rate factor as follows V(v)=6.386 $v^{0.5636} + 3.970$

Temperature factor Te takes e as the base and k0T as the in-dex, which indicates that when the environmental temperature devi-ates from the suitable living temperature, the fungal activity de-creases in the form of exponential function.

The growth rate model is obtained by combining the expansion rate factor and the temperature factor in a certain way.

4.2 Moisture resistance model

4.2.1 Data Description

When studying the moisture resistance model, we used the experi-mental data given in references (25)[1]. The experimental data gives the maximum and minimum values of the suitable ecological water level of various fungi, and gives the competitive ranking obtained by using Elo ranking system[5],These data all take fungi species as independent vari-ables.

In the tolerance factor, we get the ecological water level width by subtracting the minimum value from the maximum value of our ecological water level,After scaling the competition ranking and ecological water level width to [0,1], we get the ecological tolerance,The tolerance factor is a linear function of ecological tolerance.

In the normal distribution function of ecological water level, we use the maximum and minimum values of suitable ecological water level to measure whether the environmental humidity is suitable for fungi to survive.

4.2.2 The Establishment of moisture resistance model

The moisture tolerance model H describes the effect of environ-mental humidity on fungal activity (i.e., decomposition rate). Therefore, we introduce tolerance factor and ecological water level normal distribution to describe these two effects, and multiply these two factors to get the moisture tolerance model we need.

Tolerance factor Mf refers to the ability of a species to bear changes in environmental humidity, which is based on E. In order to make our model more consistent with the real experimental results (as shown in Figure 4), we set the index as a linear function of tolerance (Mf(r,w)=k2M+b2). R and w here have been scaled to [0,1], and M=r-w . We use linear regression fitting, and finally we get K2 = 0.7625 and B2 = 2.593, as shown in Figure 5 .

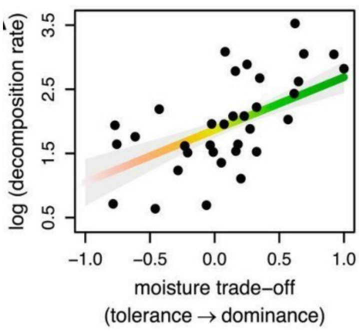

Figure 4: The relationship between the moisture tolerance of various fungi and the re-sulting wood decomposition rate (percent mass loss over 122 days, log transformed).

We tested the linear regression model and made a residual diagram, as shown in Figure 6 . It can be known from the residual graph that there is no singular data, so the linear re-gression fitting model is effective.

So we get the specific expression of tolerance factor as follows

M f(r, w)=0.7625 M+2.593=0.7625(r-w)+2.593

The normal distribution n of ecological water level indicates the influence of actual environmental humidity on the activity of fungi. Taking the center of ecological water level width as the optimum humidity, the peak value of normal distribution function is 1 . The farther away from the optimum

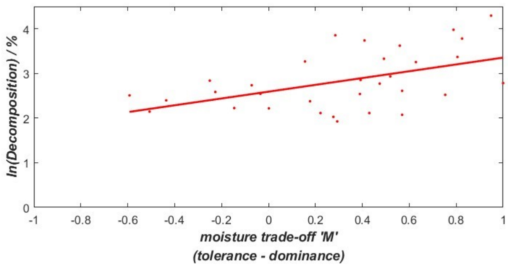

Figure 5: k2 = 0.7625 and b2 = 2.593 are obtained by fitting the curve of decomposition rate L versus tolerance M.

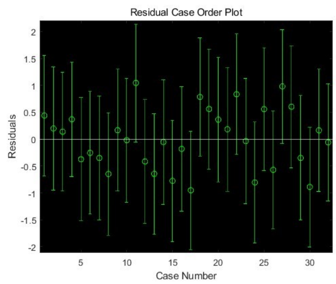

Figure 6: Residual diagram

humidity, the smaller the value of normal distribution function, that is, humidity is not suitable for the survival of fungi, and the activity of fungi decreases. The specific form is shown in the formula.

$$\mathrm{N}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left(\frac{(h-\mu)^2}{2\sigma^2}\right)} \div \frac{1}{\sqrt{2\pi}\sigma} = e^{\left(\frac{(h-\mu)^2}{2\sigma^2}\right)}$$

In which the mathematical expectation is taking the center value of ecological water level breadth.

That is ,the mathematical expectation= the normal distribution peak value is located at the center of ecological water level breadth. The standard deviation is value to($\mathrm{x}_1 - x_2$)

The normal distribution function divided by the coefficient indicates that the peak value of the normal distribution function of ecological water level is 1 .

The moisture tolerance model is obtained by combining tolerance factor with normal distribution of ecological water level.

4.3 Decomposition rate model

Decomposition rate model L is composed of growth rate model E and moisture re-sistance model H. The expansion rate factor V in growth rate model E and tolerance M in moisture toler-ance model H are related to decomposition rate model L , so we think that the influence of expansion rate factor V and tolerance M on decomposition rate is expressed as the product of a power of expansion rate factor V and (1-a) power of moisture tolerance. We get the spe-cific value of A through trend fitting, and the trend is given by the literature in the topic, shown as Figure 2 and Figure 4 . We get the fitting curve by taking multiple values of A, and finally come to the conclusion that when a=0.95, the decomposition rate model is closest to the real trend.

Then, the influence of Temperature factor te and normal distribution of water niche n on decomposition rate model l was considered, and the time factor ti (t) = q * e ((t/122)-1)+p was introduced. Ti(t)=0 when the time factor satisfies t=0; When t=122, Ti(t)=1,The calcu-lated q = 1.582 and q = 0.5800 .

So we get the expression of decomposition rate model, as shown in the formula .

[L=V$^{a} * M^(1-a) * N * Te * Ti(t)$]

L=V$^{a} \cdot M^{1-a} \cdot e^{-\frac{(h-\mu)^2}{2\sigma^2}} \cdot e^{k_0 T} \cdot \left(q e^{\frac{t}{122}-1} + p\right)$

We can explain the meaning of time factor and temperature factor through Figure 7 . In our paper,

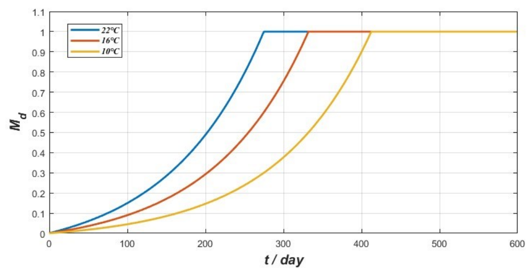

Figure 7: The loss mass Md (i.e., decomposition rate L) increases exponentially with time and de-creases with the decrease of temperature

except for the long-term trend, t is taken as a time unit, that is, t=122, so we can consider the time factor Ti(t)=1 .

In order to better reflect the influence of various factors on the decomposition rate, we first analyze the model without considering the influence of interaction. We visualize the model, and take the intrinsic growth rate as the X axis and the moisture tolerance as the Y axis, and make a three-dimensional curve of the decomposition rate model, as shown in Fig-ure 8 . It can be seen from fig,8 that the decomposition

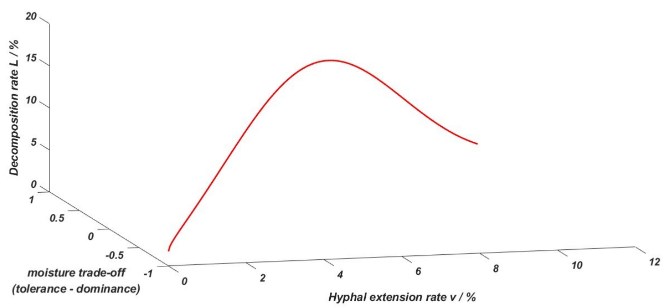

Figure 8: Three-dimensional curve of decomposition rate model

rate model shows both the growth rate of fungi and the environmental humidity without considering the mutual influence of dif-ferent populations, so the model is suitable for various fungi with different growth rates and moisture tolerance.

4.4 Interaction model

On the basis of decomposition rate model, considering the interaction of different fungi, we obtain the interaction model .

4.4.1 logistic growth model of single population

If the quality of decomposition products in the considered land is constant, that is, the outside world cannot supplement the ground litter and wood fiber to the considered land, then the environmental capacity K can be set as a constant.

We use mathematical differential equation model, biological continuous single popula-tion model and logistic growth model.

$$\frac{dn(t)}{dt} = vn(t)[1 - \frac{n(t)}{K}]$$

$$n(t) = \frac{n_0 e^{vt}}{1 + n_0(e^{vt} - 1)/K} = \frac{Kn_0}{n_0 - (n_0 - K)e^{-vt}}$$

With N(t) as the number of bacteria, let K=10000, and properly select N_0

And r (let $N_0 = 10, r = 0.5$).

The relationship between the number of bacteria and time is an S-shaped curve, as shown in Figure 9

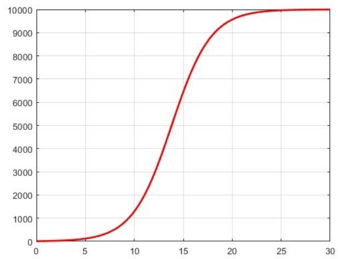

Figure 9: The relationship between the number of bacteria and time is S-shaped curve

So far, we have got the logistic model without time factor.

As the mass of decomposition product is 1, the mass of decomposition product de-creases with time, so the environmental capacity decreases, so the constant k must be re-vised to K(t). We refer to the non-autonomous single-species model in the continuous sin-gle-species model in biology to modify it, and get the logistic model whose parameters change with time.

$$\frac{dn(t)}{dt} = v(t)n(t)[1 - \frac{n(t)}{K(t)}]$$

K(t) is the environmental capacity, which changes with time and is set as

K(t)=$10000[1 - \mathrm{M_d}(\mathrm{v}, \mathrm{M}, \mathrm{t})]$

n(t) is the number of bacteria, and n(0) and v are properly selected (let $n_0 = 10, v = 0.5$).

The relationship between quantity and time is S-shaped, as shown in Figure 10 .

4.4.2 Multi-population Lotka-Volterra Competition Model

There are two populations, A/B, and when they live alone in a natural environment, the evolution of quantity follows logistic law. n1(t),n2(t) are the number of two species, v1,v2are the fixed growth rate , and K1,K2 are the maximum capacity .So, for population A, there is

dn1(t)/dt=$v_1 n_1 \left(1 - \frac{n_1}{K_1}\right)$

Where the factor $\left(1 - \frac{n_1}{K_1}\right)$

reflects the retarding effect of A on its own growth due to its consumption of limited resources,

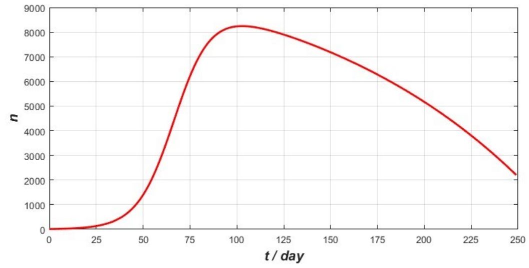

Figure 10: The curve of the relationship between quantity and time is S-shaped

(1-n$_1/K_1$)

can be interpreted as the amount of food supplied to A consumed per unit amount of A (the total amount of food is set as 1).

When two populations live in the same natural environment, the influence of the con-sumption of the same limited resource by B on the growth of A can be reasonably subtracted from another term in the factor$\left(1-\frac{n_1}{K_1}\right)$,

which is proportional to the quantity of population B (relative to K2), and the equation of the growth of population A can be obtained as

dn1(t)/dt=v$_1 n_1\left(1-\frac{n_1}{K_1}-\sigma_1\frac{n_2}{K_2}\right)$

We consider competitive factor 1 is σ_1

Here competitive factor 1 means that the amount of food consumed per unit quantity B (relative to K2) to support A is competitive factor 1 times the amount of food consumed per unit quantity A (relative to C) to support A.

Similarly, the existence of A also affects the growth of B, and the equation of popula-tion B should be

dn2(t)/dt=v$_2 n_2\left(1-\sigma_2\frac{n_1}{K_1}-\frac{n_2}{K_2}\right)$

A corresponding explanation can be given for σ_2

Next, we make some modifications to the parameters. First, we cor-respond the competition factor 1 and 2 to the decomposition rate L(v, M, 122)=L(v, M) $\cdot 1 = L(v,M) corresponding to 122 days. That is:$

1: $\sigma_1 = L_1\left(v_1, M_1, 122\right) : L_1\left(v_2, M_2, 122\right)$

$\sigma_2 : 1 = L_2\left(v_1, M_1, 122\right) : L_2\left(v_2, M_2, 122\right)$

Because the food of the two colonies is ground litter and wood fiber, it can be considered that the blocking effect is the same. That is:

1: $\sigma_1 = \sigma_2 : 1 = L\left(v_1, M_1, 122\right) : L\left(v_2, M_2, 122\right)$

Secondly, K is modified. K(t) is the environmental capacity, which changes with time, and is set as

K(t)=10000$[1-M_d(v,M,t)]$

where M$_d(v,M,t)=L(v,M,t)=\left[L\left(v_1,M_1\right)\cdot\frac{n_1}{n_1+n_2}+L\left(v_2,M_2\right)\cdot\frac{n_2}{n_1+n_2}\right]\cdot\left(ke^{\frac{t}{122}-1}-\mathrm{b}\right)$

Figure 11 means that B is weaker than A in the competition for resources supporting A, and competitive factor2>1 means that A is stronger than B in the competition for resources supporting B,

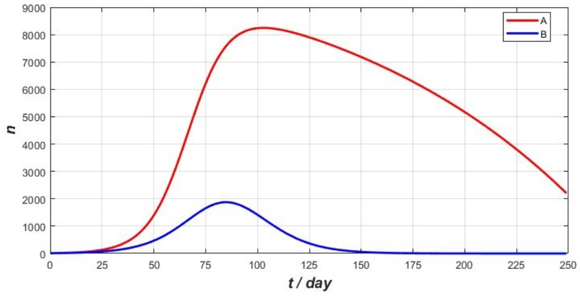

Figure 11: $\sigma_1 < 1, \sigma_2 > 1$

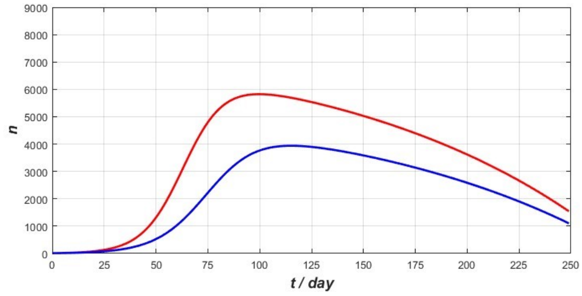

Figure 12: $\sigma_1 < 1, \sigma_2 < 1$

so population B will eventually die out, and population A will decrease to 0 after approaching the maximum capacity it can reach.When competitive factor1<1, competitive factor2<1 , the situation is just the opposite.

In Figure 12, because B is weaker in competitive re-sources and A is weaker in competitive resources, it can reach an unstable state of coexistence between two parties.And an unstable state can then be reached in which both coexist.

5 Task2: Interaction of Different Fungi

We discuss short-term trends in one time unit (122 days) and long-term trends in infinite time units (i.e., when T tends to infinity). Here, we only consider the interaction between two fungi, and assume that the initial numbers of the two species are equal (n01=n02=10),The following label "1" indicates species A, and the following label "2" indicates species B,The environmental capacity is N1=N2=10000 .

After a lot of data processing and the interaction of fungi species, we can divide the results into competition and coexistence.

On the one hand, 1 Competition: species B is extinct, and species A continues to survive until resources are exhausted

According to our interaction model, the competition factor σ_1, σ_2.

When the intrinsic growth rate is equal, that is, v$_1 = v_2 = 0.1$,

the number of indi-viduals in the two populations is shown in fig,13 .

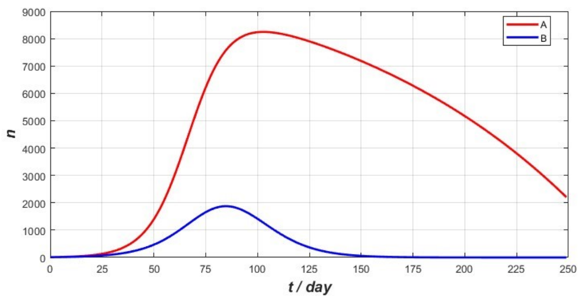

Figure 13: $\sigma_1 < 1, \sigma_2 > 1$

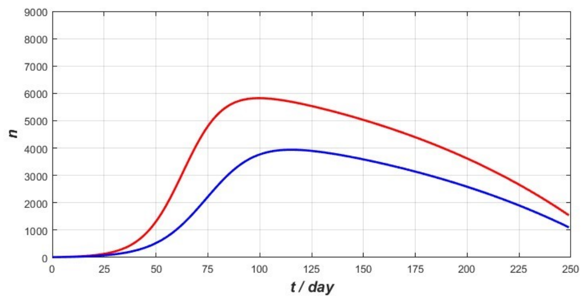

Figure 14: $\sigma_1 < 1, \sigma_2 < 1$

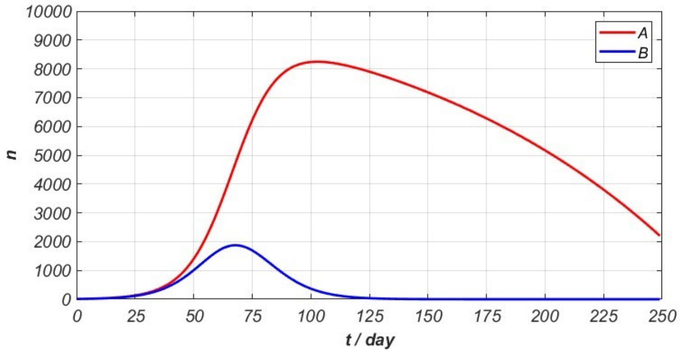

Figure 15: the inherent growth rate is equal, that is, $v_1 = v_2 = 0.1$

When the intrinsic growth rate v1=0.1 and v2=0.2, the number of individuals in the two populations is shown in fig,14 .

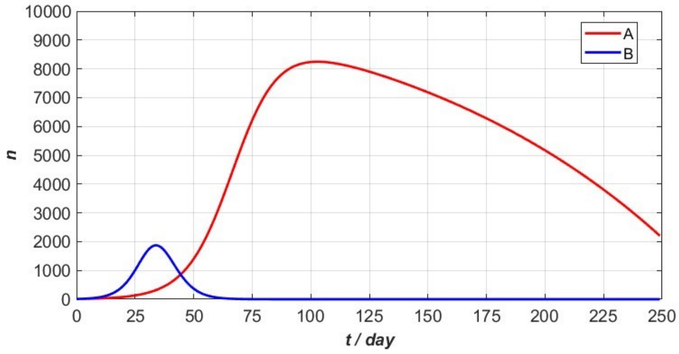

Figure 16: intrinsic growth rate $v_1 = 0.1, v_2 = 0.2$

When the intrinsic growth rate v1 = 0.2 and v2 = 0.1, the number of individuals in the two populations is shown in fig,15 .

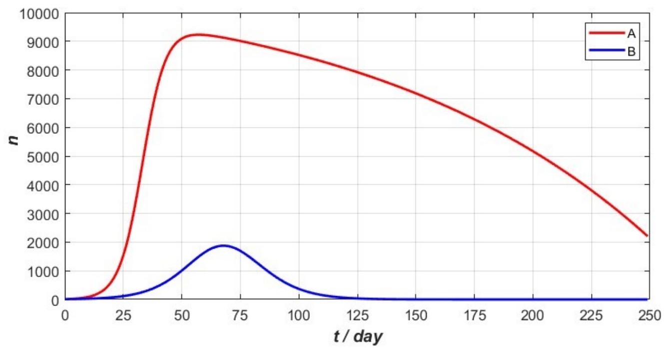

Figure 17: intrinsic growth rate v1=0.2, v2=0.1

Short-term trend: In a short time, because the initial number of species A and B is far less than the environmental capacity, the trend of individual number of A and B is increasing. When the time reaches a time unit (122 days), both species A and B have a trend of increasing at first and then decreasing. The curve of species B is concave and decreases to close to 0 in a short time, which indicates that the main reason for the decrease of species B is the inhibition of A.

Long-term trend: when t tends to infinity, species b becomes extinct due to the inhibition of species a, and then species a survives until resources are exhausted. The curve of species A is convex, which shows that the main reason for the de-crease of species A is the depletion of resources. It can be seen that the envi-ronmental capacity is limited and the number of individuals cannot grow in-definitely.

Overall trend: species b will be extinct due to the inhibition of species a in the short term, and species a will be extinct due to resource depletion after species b is extinct in the long term.

On the other hand, coexistence: A and B coexist until environmental resources are exhausted According to our interaction model, the competition factor $\sigma_1 = 0.5, \sigma_2 = 0.9$.

In this case, the inherent growth rate is equal, that is, v1=v2=0.1, and the num-ber of individuals with two populations is shown in fig,16 .

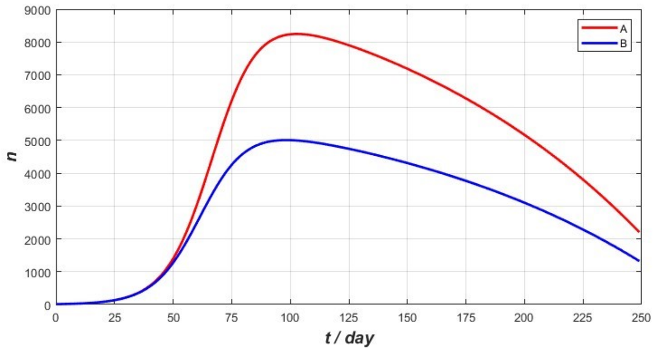

Figure 18: the inherent growth rate is equal, that is, v1=v2=0.1

Short-term trend: In a short time, because the initial number of species A and B is far less than the environmental capacity, the trend of individual number of A and B is increasing. When the time reaches a time unit (122 days), both species A and species B have a trend of increasing at first and then decreasing, and their curves are convex, which shows that the main reason for the decrease in quan-tity is the lack of environmental resources.

Long-term trend: when t tends to infinity, the number of species a and species b gradually drops to 0, and the curves are convex, which shows that the reason for the decrease of the number when they coexist is resource depletion. With lim-ited environmental capacity, the number of individuals cannot grow indefinite-ly,In this case, species A and species B coexist until the environmental re-sources are exhausted.

General trend: species A and species B coexist, and their mutual influence is less than that of the environment, and finally they die out together because of the exhaustion of environmental resources.

6 Task3: Influence of Environment on Individual Species or Spe-cies Combination

Still only consider the interaction between the two species. We think that the competition ranking [3] is affected by the positive correlation of inherent growth rate.

In our model, the change of environmental humidity will affect the water niche function N, thus affecting the decomposition rate, which is finally shown in the coefficient of our interaction model. We use the change trend of the number n of the two species under this condition to show its influence.

We use this curve to predict the relative advantages and disadvantages of individual species and their combinations.

Our model is sensitive to rapid environmental fluctuations,We use humidity changes to analyze possible situations in five climatic environments: arid, semi-arid, temperate zone, arbor and tropical rain forest, and discuss en-vironmental differences in the same area with different temperature conditions. The humidity of arid, semi-arid, temperate, arbor and tropical rain forest is increasing gradually, and temperate is the best condition for most fungi to survive. According to the different competition ranking and tolerance of the two species, we discuss them in four categories, among which we focus on analysis 1, as the other three cases involve a large number of data trend charts.

Firstly, species A has weak tolerance and strong competitiveness,B has strong tolerance and weak competitiveness

- under the current optimal conditions (temperate zone), the change trend of quanti-ty is shown in fig,17

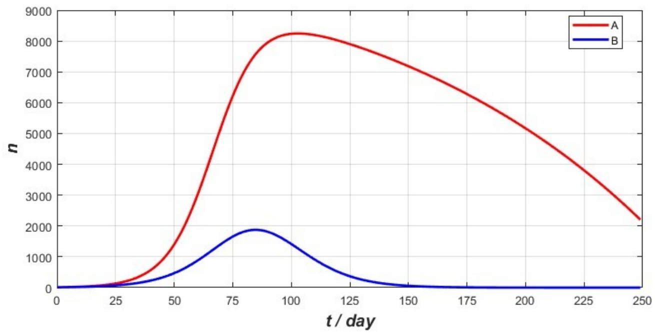

Figure 19: The quantitative change trend of temperate zone

- When the humidity changes, the humidity value is at the critical point of A's tolerance (semi-arid), the decomposition rate corresponding to A decreases, which leads to the slow growth of its number,However, because A is ranked higher than B in competition, A will not be extinct, and the final situation is that A and B coexist.

  However, the tolerance of B is stronger, that is, the water level width corresponding to B is larger, the humidity after change is still within its tolerance width, and the decomposi-tion rate corresponding to B is not affected. Because A is greatly affected by humidity, the inhibitory effect of A on B is weakened, which is not enough to make B extinct.

  Therefore, it corresponds to coexistence trend chart 18:

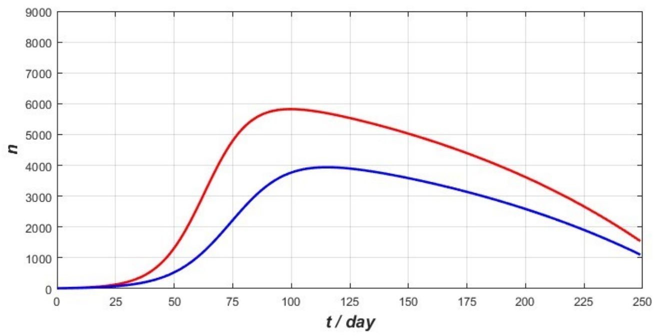

Figure 20: coexistence trend chart of semi-arid region

- When the humidity change far exceeds the critical value of A (drought), the decomposition rate of A is greatly reduced and the interaction factors are reduced; However, because the humidity change is still within the water niche width of B, although the decomposition rate of B is affected, the affected degree is far less than that of A, so the interaction factor of B is greater than that of A.

  With sufficient resources in a short time, A's competitive ranking is greater than B's, so A grows faster than B at first, but after a long time, the number of A decreases to 0, and the change trend of the number is shown in Figure 19 .

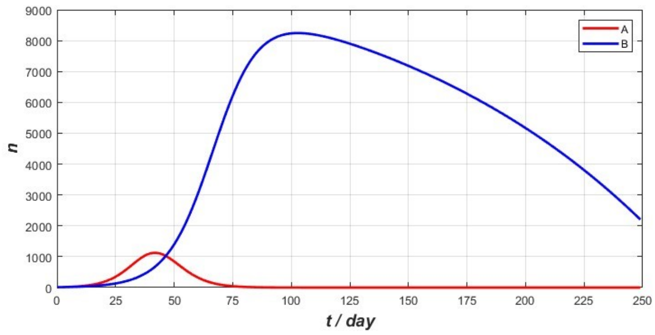

Figure 21: Trend chart of drought quantity

By the same token, when exploring the climate environment of temperate zone, arbor forest and tropical rain forest, we get the following trends:

Temperate zone: A continues to survive, B eventually dies

Arbor: A and B survive together

Rainforest: A eventually dies, B continues to survive

Secondly, A: The moisture tolerance is stronger; B: The moisture tolerance is weaker; and spe-cies A and species B have equal or similar competitive rankings.

Thirdly, A: the competition ranking is higher; B: The competition ranking is lower; and there is little difference in moisture tolerance between species A and species B.

At last, the competition ranking and moisture tolerance of species A and species B are similar.

Therefore, we can predict the relative advantages and disadvantages of individual spe-cies and their combinations.

For a single species, its tolerance determines its suitable survival degree in different environments,The better the tolerance, the larger the range of environmental humidity it can adapt to. Its competition ranking is affected by its own expansion rate,The higher the competition ranking, the greater its expansion rate, the stronger its activity and the stronger its competition with other species. In addition, the interaction with other species should also consider the impact of biodiversity, which will be discussed in detail in Task7: The impact of species diversity on decomposition rate.

For species combination, when one side can adapt to the environment while the other side can't, or when one side has too strong inhibition on the other side, it will lead to the ex-tinction of one species, while the other species will become extinct after exhausting the en-vironmental resources. Under certain conditions, the two species can coexist (as shown in Figure 13), and under such conditions, the species will not die out so quickly. If we can con-sider the input of the environment or the positive interaction between the two species, the two species may coexist in the environment for a long time. As for the combination of more than two species, we will discuss in detail in Task7: The influence of species diversity on decomposition rate.

Next, we will discuss the influence of different temperatures on fungal activity in the same area, and we will take temperate environment as an example for analysis. The temper-ate zone has distinct seasons, and the temperature difference between summer and winter is very large,The average temperature in the coldest is lower than 0℃, and the average tem-perature in the hottest month is higher than 22℃. We take the temperature environment of 22℃ and 0℃ for comparison. Assume that species A has weak tolerance and strong competitiveness; B has strong tolerance and weak competitiveness.

At 22℃, the trend of fungi quantity changing with time is shown in fig,20.

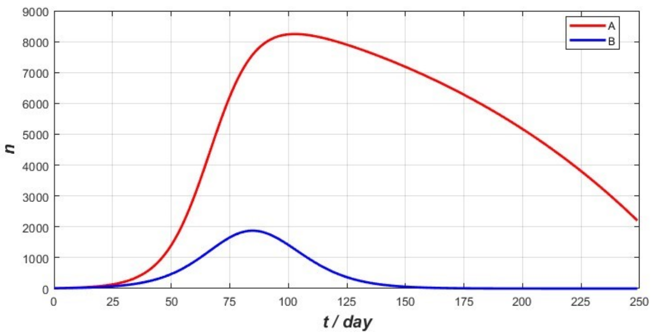

Figure 22: Quantity change trend at 22℃

At 0℃, the change trend of fungi quantity with time is shown in fig,21.

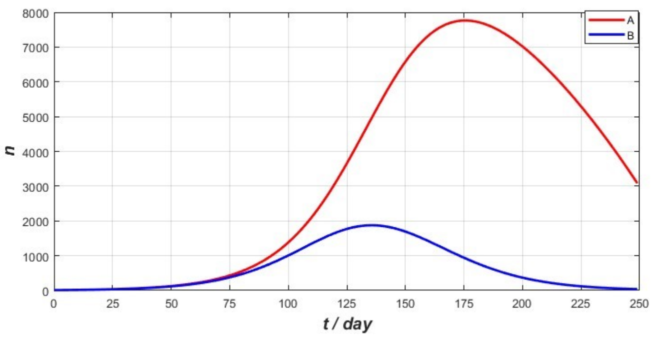

Figure 23: Change trend of quantity at 0℃

Comparing the two graphs, it can be found that the quantity changes faster and the maximum value is smaller at 22℃. Here, the influence of temperature factor is reflected, and which affects the expansion rate. The temperature factor decreases exponentially as the temperature is far away from the optimal temperature (here, 22℃), so the growth rate of fungi is lower and the number changes more slowly at 0℃.

Therefore, temperature has a certain influence on the rate of quantity change,The closer the tem-perature is to the optimum temperature (22℃), the greater the expansion rate, the faster the growth of fungus quantity and the greater the decomposition rate.

7 Task4: Effects of Species Diversity on Decomposition Rate

In order to explore the influence of biodiversity on the decomposition rate, we compare the decom-position rates under the same conditions (temperate zone) when there are fewer species (2 species) and when there are more species (3 species), and get Figure 22 and Figure 23 . In which, the tolerance is C > B >A

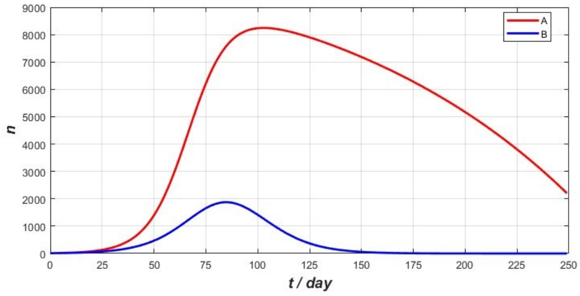

Figure 24: change trend of two species in temperate environment

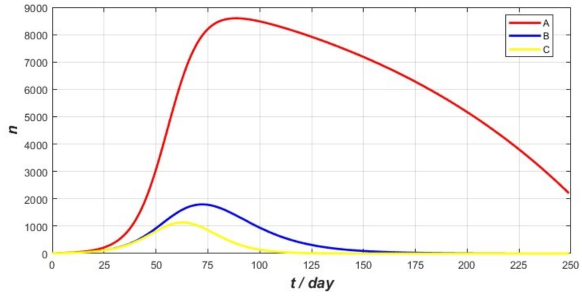

Figure 25: change trend of three species in temperate environment

We consider the decomposition rate as the fungal activity changes positively with the number of individuals, so the change trend of the number of individuals shown in the figure is the change trend of de-composition rate. The environmental capacity is limited, but under the same circumstances, the existence of one more species C does not greatly inhibit the decomposition rate of the other two species. Comparing fig,17 with fig,18, we can see that the change trend of species b is basically un-changed, but the attenuation trend of species a is slowed down. It can be seen that species diversity not only competes or inhibits other species, but is also beneficial to the balance of ecosystems, so that other species can coexist better.

In order to explore the importance and role of biodiversity when the environment changes in different degrees, we use the temperate envi-ronment mentioned above as a benchmark, and then simulate two different environmental changes as a comparison.

The first kind of environment: semi-arid or arbor forest, with little change in humidity,The change trend of two species is shown in Figure 24, and the change trend of three species is shown in Figure 25 .

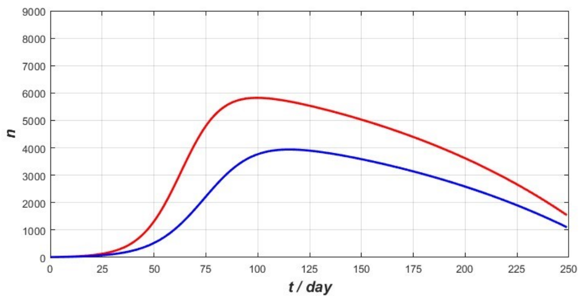

Figure 26: Change trend of two species in semi-arid or arbor forest environment

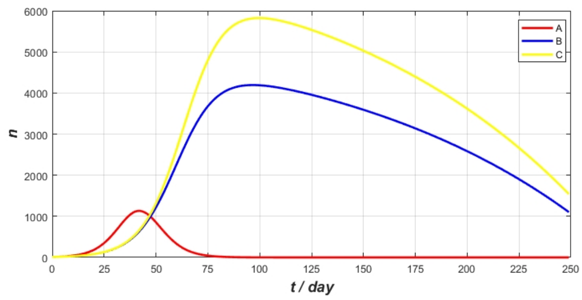

Figure 27: Change trend of three species in semi-arid or arbor forest environment

Because of the strong tolerance of species C, the activity of species C is the best in the environment deviating from the suitable humidity, which has played an inhibitory role on species A. The tolerance of species B is stronger than that of species A, and species A is suppressed, so the living environment of species B becomes better.

In the second environment: drought or tropical rain forest, the humidity changes greatly,The change trend of the number of two species is shown in Figure 26, and the change trend of the number of three species is shown in Figure 27 .

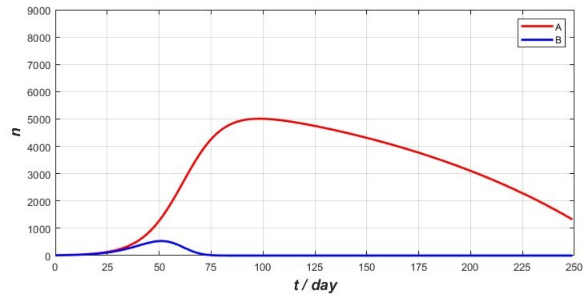

Figure 28: variation trend of two species in drought or tropical rain forest environment

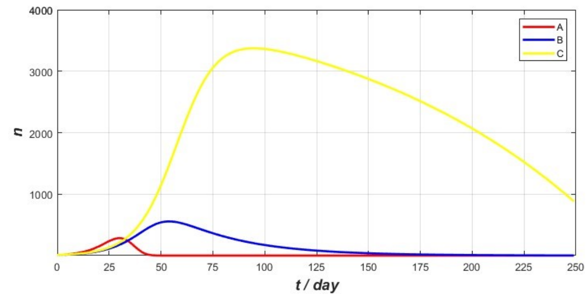

Figure 29: variation trend of three species in drought or tropical rain forest environ-ment

Similarly, in semi-arid or arbor forest environment, in arid or tropical rain forest environment, because of the strong tolerance of species C, the living condition of species C is better. Because species A is limited by its own tolerance and inhibited by species C, the living condition of species A is worse than that of two species. However, the tolerance of species B is stronger than that of species A, and because species A is suppressed, the living condition of species B is better. When the environment changes drastically, the overall survival of fungi is better when there are more species.

In a word, when the environmental changes are not so severe, the fungal activity is more affected by the environment and mutual inhibition. When the environment changes drastically, the combination with more species will survive better. From this, we can draw the conclusion that species diversity plays a very important role in the balance of ecosystem, whether the environment changes drastically or not. Especially when the environment changes drastically, systems with good species diversity tend to have better living conditions.

8 Analysis of the Model

When establishing the model, in order to simplify the operation, we made some as-sumptions and approximations, which may cause some errors to the operation results:

- We ignore the influence of size and shape, and assume that the initial mass is 1, which will be different from the actual complex situation.
- When describing the influence of environmental humidity on decomposition rate, we use normal distribution function,In fact, if more actual data can be used for fit-ting, a more accurate function form may be obtained.

When simplifying the data, we do a lot of trade-offs and averaging operations, which will lead to inaccurate experimental data. In addition, the data we use is experimental data, which has the influence of experimental error.

9 Model Evaluation and Futher Discussion

9.1 Strength

When building the model, we considered all kinds of factors, simplified it and revised it when necessary. Firstly, we consider the growth rate of fungi and environmental humidity, consider the influence of time when discussing the long-term trend, and consider the influ-ence of temperature when discussing the environmental change in the same area. Finally, based on the established decomposition rate model, we consider the interaction between different fungi. In a word, we have considered the influencing factors of the interaction model in a very comprehensive way and established a very perfect model.

Through the analysis in the text, we can conclude that the mod-el has strong sensitivity and robustness.

In addition, the model has strong universality, which can be used for different fungi and other species.

9.2 Possible Improvements

There are some errors in our model: in order to simplify the operation, we made some assumptions and approximations, which may cause some errors in the operation results; When simplifying the data, we do a lot of trade-offs and averaging operations, which will lead to inaccurate experimental data,When analyzing the model, we focus on the conditions that need to be considered at present, select other conditions as fixed values and temporarily ignore the influence of other conditions,In fact, if we can consider more environmental con-ditions, we may get more accurate model analysis.

The experimental data we use is not comprehensive enough, which will lead to the analysis trend not completely equal to reality. In addition, the data we use is experimental data, which has the influence of experimental error.

9.3 Further Discussion

If we can get more accurate data, then we can get more accurate function expression and build a more comprehensive and realistic model.

Our model has strong universality, which is not only applicable to the selected fungi in the data, but also applicable to various fungi or organisms. The model can be used to simu-late the coexistence of various organisms, including evaluating short-term and long-term trends, taking into account its own growth rate, the influence of environmental temperature and humidity, and the interaction of different species. Therefore, this model can play a very important role in biological research, ecosystem, animal husbandry economy, environmental protection and other fields.

10 Conclusion

In this paper, we discussed the growth rate and tolerance of hyphae, established the de-composition rate model, and on this basis, considered the influence of different species, and got the interaction model.

- **Result of Task 1**
  We established the interaction model according to the process shown in Figure 30 . We use the linear regression model to analyze the data, and use the least square method to fit the trend, and get the functional expressions of the expansion rate influencing factor and the tolerance influencing factor, and their times. After that, we considered the temperature factor and the normal distribution of water niche (representing the influence of humidity), multiplied these four factors and introduced the time factor, and obtained the decomposition rate model without considering the influence of interaction. After that, we modified the logistic model of single population and the Lotka-Volterra competition model of multi-population, and solved it by using the differential equation model,By combining them with the decomposition rate model, we got the decomposition rate model considering interaction.

  In the presence of various fungi, the model describes the decomposi-tion of ground litter and wood fiber by fungi, and considers the different growth rates, moisture tolerance and the interaction among different fungi.

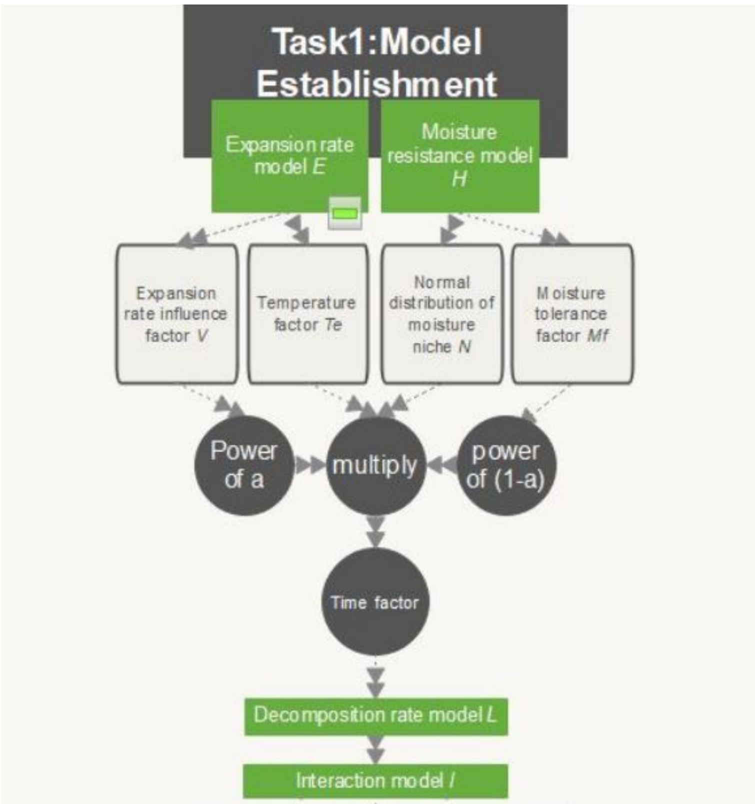

Figure 30: Model Establishment

- **Result of Task 2**
  We need to analyze the interaction between different fungi and get their short-term and long-term trends. Therefore, we take the competition factor and the inherent growth rate in the model as control variables, and make a population growth trend graph.

  We analyze the short-term and long-term trends of various possible situations: in the short term, species are mainly inhibited by other species; The long-term trend is mainly limited by environmental resources, and the main results are competition (that is, one party is extinct, as shown in Figure 13/14/15) and coexistence (until the environmental resources are exhausted, as shown in Figure 16).

- **Result of Task 3**
  We need to predict the relative advantages and disadvantages of species or combinations of species, and consider the environmental impact. Therefore, we use tolerance and competition ranking to show the difference of different species, and humidity change to show the difference of environment, and get the change trend of individual number of different species combinations under different environmental humidity.

  For a single species, its tolerance determines its suitable survival degree in different environments,The better the tolerance, the larger the range of environmental humidity it can adapt to. Its competition ranking is affected by its own expansion rate,The higher the competition ranking, the greater its expansion rate, the stronger its activity and the stronger its competition with other species. In addition, the interaction with other species should also consider the impact of biodiversity.

  For species combination, when one side can adapt to the environment while the other side can't, or when one side has too strong inhibition on the other side, it will lead to the ex-tinction of one species, while the other species will become extinct after exhausting the en-vironmental resources.

Under certain conditions, the two species can coexist (as shown in Figure 13), and under such conditions, the species will not die out so quickly. If we can con-sider the input of the environment or the positive interaction between the two species, the two species may coexist in the environment for a long time.

In order to consider the environmental difference in the same area, we also explored the influence caused by the temperature difference between winter and summer in the same area, and concluded that the influence of temperature factor will expand the rate, and the temperature factor will decrease exponentially as the temperature moves away from the optimal temperature (here, 22℃). Therefore, temperature has a certain influence on the rate of quantity change,The closer the temperature is to the optimum temperature (22℃), the greater the expansion rate, the faster the growth of fungus quantity and the greater the decomposition rate.

- **Result of Task 4**
  Species diversity not only competes or inhibits other species, but is also beneficial to the balance of ecosystems, so that other species can coexist better.

  When the environmental change is not so severe, the fungal activity is more affected by the environment and mutual inhibition. When the envi-ronment changes drastically, the combination with more species will sur-vive better. From this, we can draw the conclusion that species diversity plays a very important role in the balance of ecosystem, whether the en-vironment changes drastically or not. Especially when the environment changes drastically, systems with good species diversity tend to have better living conditions.

In conclusion, we set up a very comprehensive model, and discussed the in-fluencing factors of fungal decomposition rate, including mycelial growth rate, environmental humidity, interaction of different kinds of fungi and so on. After model analysis and evaluation, our model has very good sensitivity and ro-bustness. In addition, this model has strong universality, which can be applied to not only fungi, but also other biological fields, and has broad application prospects.

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What's New About the Decomposer?

Fungi is a kind of eukaryote, and the most common fungi in nature are mushrooms, molds and yeasts also included. More than 70,000 kinds of fungi have been discovered, which might be only a small half of all existing fungi. Once upon a time, most fungi were classified as animals or plants, but now fungi belong to their own phylum, which is different from plants, animals and bacteria. The biggest difference between fungi and other three organisms is that fungi have cell wall containing chitin as the main component, which is different from plant cell wall which is made of cellulose.

As we all know, fungi, like bacteria and microorganisms, play the role of decomposers in environmental recycling. Decomposer, also called restorer, refers to bacteria, fungi, actinomycetes and other organisms with decomposition ability in the ecosystem, including some protozoa and saprophagous animals. They can decompose complex organic matter in animal and plant residues into simple inorganic matter, and release it into the environment for producers to use again. The process conducted by decomposer also releases energy, and its function is opposite to that of producers. The role of decomposers in the ecosystem is very important,If there are no decomposers, animal and plant residues will accumulate into disasters, materials will be locked in organic matter and will no longer participate in the cycle. The material circulation function of the ecosystem will be terminated and the ecosystem will collapse. Different stages require different creatures to play the role of decomposers. [6] In short, decomposers are organisms that decompose the organic matter of dead organisms, and they decompose organisms into various inorganic substances, thus enhancing the soil fertility.

In addition, in recent years, there are new researches aiming at the role of fungal decomposers. The decomposition of compounds is a part of carbon cycle, which enables carbon to be renewed and used in other forms. The decomposition of plant materials and wood fibers is the key component of this process. Fungi are the main factor of decomposing lignocellulose, which is a field worthy of studying. In recent years, the researches on the decomposition rate characteristics of fungi are becoming more and more prosperous.

D. S. Maynard et al. in the article Diversity Begets Diversity in Competition for Space [3] published in 2017, proved that species richness can serve as a self-reinforcing buffer for competitive exclusion, and proposed a method to measure the impact of competition.

D. S. Maynard et al. in the article Consistent Trade-offs in Funga [1] published in 2019, proposed a trait-based method to quantify the niche of fungi and explore the relationship between functional trait expression, climate and phylogeny. This paper reveals the basic balance between abiotic stress tolerance and competitiveness, that is, fungi have a wide range of heat and water niches, showing low displacement ability, and the balance between dominance and tolerance is partly related to the environmental conditions of fungi collection.

Nicky Lustenhouwer, Daniel S. Maynard, Mark A. Bradford, Daniel L. Lindner, Brad Oberle, Amy E. Zanne, and Thomas W. Crowther put forward in an article [2] published in 2020 that slow-growing strains can survive and grow better in environmental changes such as humidity and temperature, while fast-growing strains are often less robust to the same changes. They identified the characteristics of fungal decomposition rate, and noticed the relationship between some characteristics [4].

Therefore, the recent researches on the decomposition rate of fungi have obtained novel conclusions, and part of these conclusions even conflict with people's inertial thinking. The research on fungal decomposition is a promising field waiting for people to explore.

EXPANSION:

In addition to the role of decomposer, in nature, the roots of some plants have symbiotic relationship with fungi, which is called mycorrhiza. Mycorrhiza is divided into ectomycorrhiza, endomycorrhiza and endomycorrhiza.

1,Ectomycorrhizae: mycelia of fungi form are on the surface of roots and wrap on the surface of young roots, sometimes invading cortical cells, but not entering cells. At this time, mycelia replace the function of root hairs and increase the absorption area of roots, such as pine.

2,Endomycorrhizae: hyphae invade into epidermal and cortical cells through cell walls, strengthening absorption function and promoting material transportation in roots, such as citrus and walnut;

3,Endophytic mycorrhiza: There are also hyphae that not only wrap on the surface of young roots, but also go deep into cells, which are called endophytic mycorrhiza, such as apples and willows.

For plants, hyphae can absorb water, inorganic salts, etc,to supply plants, and at the same time produce plant hormones and vitamin B to promote the growth of roots; for fungi, plants supply fungi with organic nutrients such as saccharides and amino acids.

There are more than 2000 higher plants that can form mycorrhiza, such as Platycladus orientalis, Populus tomentosa, Ginkgo biloba, wheat, onion and so on. Plants with mycorrhiza can't grow normally without fungi, so it is necessary to inoculate and infect the required fungi in advance to facilitate successful afforestation on wasteland. [7]