Group Makabaka. - Exercise 1.

3.(1). Card shows a black side: $P(\text{SideUp}=B) = \frac{2+0+1}{2+2+2} = 1/2$.

Card shows a white side: $P(\text{SideUp}=W) = \frac{2+0+1}{2+2+2} = 1/2$.

(2). $P(\text{SideUp}=B) = 1/2$. Let One side black $= 1B$.
Two side black $= 2B$

$$P(2B|1B) = \frac{P(1B|2B)\,P(2B)}{P(1B)}$$

Since $P(1B) = 1/2$ (1 side up is black).
$P(1B|2B) = 1$ (SideUp is black if two sides are black)
$P(2B) = 1/3$ (one of three cards is two side black).

$$\Rightarrow P(2B|1B) = \frac{P(1B|2B)\,P(2B)}{P(1B)} = \frac{1 \times 1/3}{1/2} = 2/3$$

(3). $$P(\text{Otherside}=B|1W) = \frac{P(1W|\text{Otherside}=B)\,P(\text{Otherside}=B)}{P(1W)}$$

Since $P(1W) = 1/2$.
$P(1W|\text{Otherside}=B) = 1/2$ (Not Sure).
$P(\text{Otherside}=B) = 2/3$.

$$\Rightarrow P(\text{Otherside}=B|1W) = \frac{1/2 \times 2/3}{1/2} = 2/3.$$

7(2). The posterior contour lines is smaller than prior contour lines. Which means the posterior estimation is more precise. This is because the prior distribution represents initial beliefs which maybe uncertain, while posterior distribution represents updated beliefs with more data.