Answers to questions in

Lab 1: Filtering operations

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Program: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

Each pair of (p, q) positions produces a sine wave in the spatial domain with a different frequency and direction. Smaller values of (p, q) result in lower frequencies (sparser fringes), while larger values of (p, q) result in higher frequencies (denser fringes)

For (125, 1), it is actually the frequency equivalent of (128 - 125, 1) = (3, 1). This means that (125, 1) produces frequencies similar to the lower frequencies of (3, 1), while (5, 1) and (3, 1) have similar frequencies, resulting in similar stripe densities.

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**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

The coordinates (p,q) represent a frequency component The Fourier basis function is of complex exponential form e^(i2π(px+qy)/N) which is essentially equivalent to a combination of sine and cosine. The generated stripes are oriented orthogonally to (p,q), the frequency magnitude determines the density of the stripes, and the direction is determined by the ratio of (p,q).

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:



SZ = 128

1/N

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

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The direction of a sine wave is determined by the direction of the vector of (p,q) and the wavelength is given by the up equation where w1 and w2 (in lecture note ) represent p and q here

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**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

just at (a time or place) q is more than half the size of the image, the frequency values ‘fold back’ to the lower frequency region. For example, for an image size N = 128, the frequency of coordinate (125, 1) is equivalent to (128 - 125, 1) = (3, 1), which is similar to the low-frequency component but in the opposite direction. This is a periodic feature of the Fourier transform.

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

Used to convert frequency coordinates (u, v) to centred coordinates (uc, vc). This moves the low-frequency components in the frequency domain to the centre of the image, making the distribution of high-frequency components and low-frequency components more intuitive and helpful for spectral analysis.

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

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**Question 8**: Why is the logarithm function applied?

Answers:

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

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**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

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**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

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**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers:

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

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**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

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**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers:

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**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

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