

# Exploring IMUs in Inertial Navigation Systems

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## 1 Introduction

Inertial Navigation Systems (INS) play an important role in missiles and aircrafts. Sensors commonly used for the purpose include IMUs and GNSSes. Either of these cannot be used with great precision on their own. The GNSS suffers from large measurement noise. Although an IMU has quite small measurement noise, it cannot measure position directly and additionally has a slowly varying bias. Thus it will drift indefinitely over time and provide quite unreliable measures. The sensors' weaknesses do however vanish when fused together through e.g. a Kalman filter. Having an unlinear process model, the Error State Kalman Filter (ESKF) as described by Sola [1] is of great use. He proposes the use of IMU measurements as input to the process model, and GNSS measurements used in the measurement models. Then only for each GNSS measurement the whole update cycle is run. The GNSS measurements will make it possible to estimate the biases as well as prevent drift by making position directly observable.

In this report, two different datasets with GNSS and IMU measurements will be used for developing an ESKF. The first one contains simulated data, and thus also contains ground truth. This makes it possible to give a measure on how good the filter's estimate is through e.g. the NEES. However, the noises used when generating the dataset is unknown and thus need to be estimated. The second dataset contains measurements from a real UAV flying around. Since this dataset does not contain the ground truth, other methods for analyzing its performance have to be used. On the other hand, the uncertainties for both the IMU and the GNSS receiver are known.

## 2 Simulated data

Starting off with no known values for the measurement noise covariances and bias covariances, an initial guess based on common sense provides a good starting point. According to IEEE[2], modern GPS receivers have an accuracy of between 15 and 30cm in each direction. Therefore,  $R_{GNSS} = (0.40m)^2 I_3$  was used as a starting point just to give a little headroom. For the bias and noise covariances, we started off by using the values as stated in the datasheet for the IMU used in the real dataset - the STIM300 [3]. The bias  $p$ -values were left at  $p_{acc} = p_{gyro} = 10^{-16}$ , effectively making the bias noises pure Wiener processes.

[1]Juan Sola, "Quaternion kinematics for the error-state Kalman filter"

[2]Moore, Samuel K. "Superaccurate GPS Chips Coming to Smartphones in 2018", <https://spectrum.ieee.org/tech-talk/semiconductors/design/superaccurate-gps-chips-coming-to-smartphones-in-2018>

[3]Sensoror STIM300 datasheet, <https://www.sensoror.com/media/1132/ts1524r9-datasheet-stim300.pdf>

$$\sigma_{\omega} = 0.15^{\circ}/\sqrt{h} \approx 4.36 \cdot 10^{-5} \frac{rad}{\sqrt{s}} \quad (1)$$

$$\sigma_a = 0.06 \frac{m/s}{\sqrt{h}} = 0.001 \frac{m/s}{\sqrt{s}} \quad (2)$$

$$\sigma_{\omega b} = 0.5^{\circ}/h \approx 2.4 \cdot 10^{-6} \frac{rad}{s} \quad (3)$$

$$\sigma_{ab} = 0.05g \approx 4.91 \cdot 10^{-4} m/s^2 \quad (4)$$

These values did however not provide satisfactory results; the gyroscope bias would not converge, and the NEESes for attitude, accelerometer bias and gyro bias were too inconsistent, with respectively 55.3%, 4.9%, and 42.7% inside 95% confidence intervals. Ideally, we want them to be above 80%, preferably around 95%. By inspecting the graphs, the NEESes were for long periods of time outside on the upper part of the confidence intervals. Such a phenomenon is a good indication that the corresponding noise covariances for the IMU are too low; when a GNSS measurement comes in, the state has drifted quite a lot such that it needs to be significantly corrected by the GNSS measurements, and the state estimate differs significantly from the GNSS measurements. However, since the noise covariances for the IMU is too low, the filter will instead choose to trust the IMU measurements. As a result, the attitude is wrong for long periods, and the biases cannot be correctly estimated, thereof non-converging gyro bias.

In order to improve on this, we start off by increasing  $\sigma_{\omega}$  by a magnitude of 1 to  $\sigma_{\omega} = 2.4 \cdot 10^{-4} rad/\sqrt{s}$ . This increases attitude NEES CI to 86.1%, and the gyro bias to 74.5%. The only NEES CIs below 80% is now the total NEES and the biases.

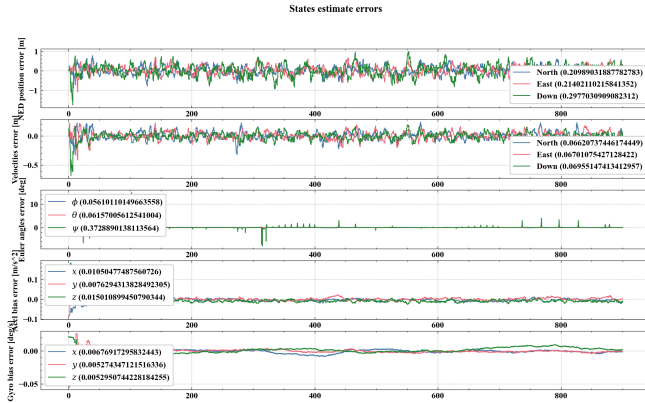
By looking at the RMSE plot of the accelerometer bias, we can get a feeling of what is actually going on; it seems that the error is sort of a random walk, which tells us that  $\sigma_{ab}$  is too low, since it fails to model the random walk in the accelerometer bias process good enough. Setting it to  $\sigma_{ab} = 3.0 \cdot 10^{-3} m/s^2$  leaves us 96.4% inside accelerometer bias CI.

We are not quite finished with the tuning yet, as the gyro bias NEES CI is at 77.4%. This can be corrected by the same arguments used for accelerometer bias - by increasing the random walk noise to  $\sigma_{\omega b} = 4.0 \cdot 10^{-6} rad/s$ .

With the parameters as described, we get a quite well performing ESKF; only the total NEES and the attitude NEES is not within the previously anticipated 95% region. This does however have a natural explanation. For the total NEES, this has to do with the fact that it contains correlations which have not been accounted for in the calculation, which makes it lower than what it actually is. Therefore, 85.2% should be considered acceptable.

It is however more complicated for the attitude NEES. By studying the error plots in Figure 1 together with state estimates in Figure 4, we might get an explanation of why; when the plane is not doing

many maneuvers in e.g. the time frame 700 to 800 seconds, its heading gets peaks in the error which are far larger than the errors of pitch and roll. This can be seen in Figure 1 by looking at the Euler angle  $\psi$ . This has to do with observability. While pitch and roll is directly observable through the gravitational force, the heading is only observable when the vessel is doing a maneuver. The roll and pitch are directly observable since the gravity acts on the accelerometer. However, we can only estimate the heading to a rough extent through calculating the velocity using the GNSS measurements when the target is moving straight ahead. When performing maneuvers, the heading is directly observable through the accelerometer. This leads to an error in the bias estimates steadily increasing as shown from 700 to 800 seconds in the NEES for the attitude. Since this error is integrated when propagated through the filter, it would perform worse when the heading is not directly observable over a longer period of time. And of course, it would not be possible to estimate the heading when the aircraft stands still since GNSS measurements then no longer could be used in the estimation. This is of course a problem for aircrafts and missiles, which might not change course that often. In those cases, including a magnetometer which can observe heading directly might improve the performance further. This would also have helped in the simulated dataset.

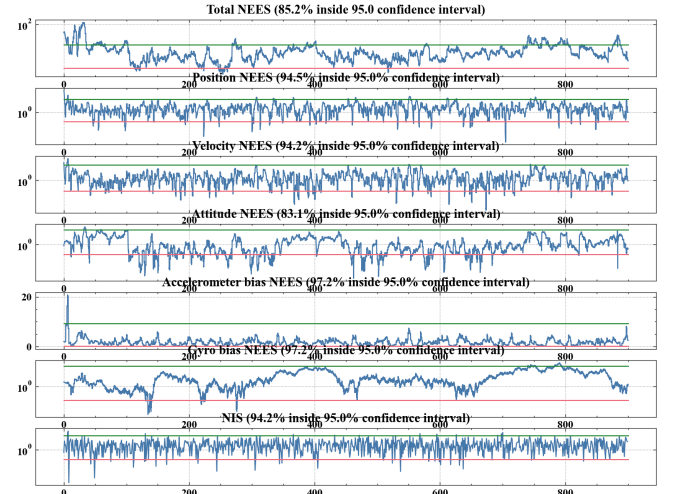


**Figure 1:** The errors for the different states in the simulated dataset - estimated vs real

As an additional step in evaluating the filter performance, we approximate the misalignment matrices  $S_a$  and  $S_g$  to the identity matrices. The result we get from this calculation is quite poor; velocity, attitude, gyro bias and accelerometer bias NEES CI are at 17.1%, 3.0%, 1.4%, and 14.2% respectively, which are far from ideal. Total NEES is at 0%, and NIS is at 51.3%. The low NEES CIs for attitude and biases should however come as no surprise; since the IMU now points in the wrong direction, its biases cannot be estimated and so we cannot remove the biases from the state estimates. Instead, the biases propagate through the filter and makes the nominal state drift away from its true state between each GNSS measurement. This is also apparent in the state errors for the biases, which shows that they vary a lot when they in fact should

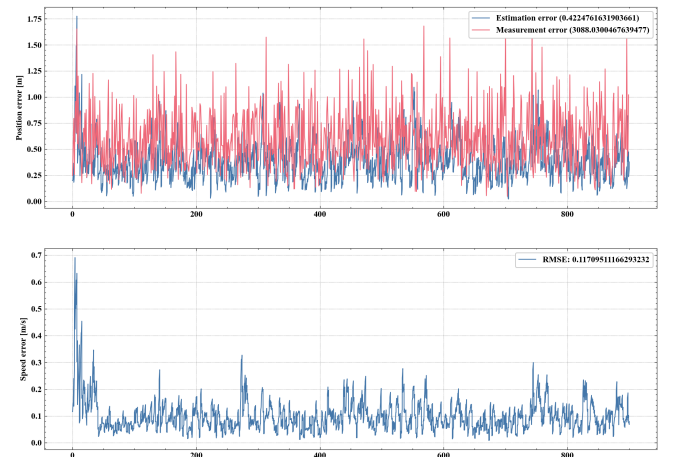
have stabilized. It should however be noted that we do not get any track loss nor large oscillations in position estimates, which tells us that the GNSS estimates still to a degree is considered.

This observation gives us an indication of how important it is to get these correction matrices right. In a real world scenario, this level of accuracy in the misalignment matrices might not be obtainable, and so the filter would perform poorly. One would however be able to observe this and tune the matrices offline after a flight until the performance is satisfactory. But this would be a difficult task since we now do not have the ground truth available. Rather, it might be better to simply allow for higher noise levels in the IMU so that any misalignments do not result in this large errors.



**Figure 2:** The NEESes for the simulated dataset with corresponding 95% confidence intervals. Final tuning parameters used.

By examining the RMSE in Figure 2, it is clear that there are no significant errors that stand out nor are the errors very high. This is true both for the speed and for the position.



**Figure 3:** The RMS errors for the position and velocity in the simulated dataset.

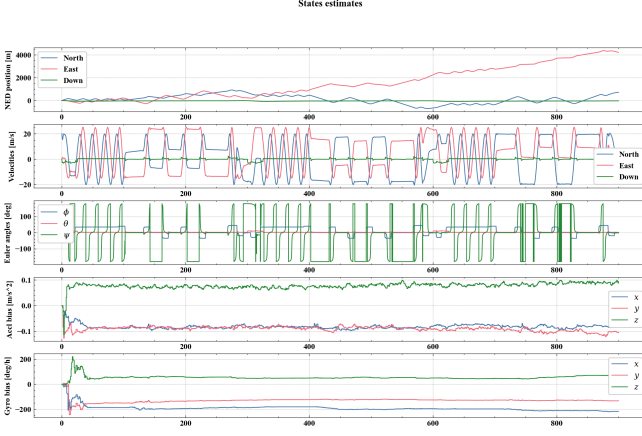


Figure 4: The state estimates for the simulated dataset.

### 3 Real dataset

For the real dataset, a UAV is tracked by an IMU and a GNSS-receiver when flying in upwards spirals, some simple planar motion with simple U-turns followed by long stretches of straight flying, and a downwards spiral before landing. The dataset does not contain any ground truth, and so its performance can only be evaluated by means of consistent NIS and the ability to follow the path created by the GNSS. A confidence interval of 95% is chosen for the NIS in this case.

The UAV uses the STIM300 IMU and a Ublox-8 GNSS-receiver. The datasheet for the IMU does fortunately list the Allan variances for the gyro and accelerometer measurement noise and bias noise. These are mentioned in (1) through (4), and are used as an initial starting point for the noises in the ESKF filter. For  $P_{acc}$  and  $P_{gyro}$  the same values as for the simulated dataset was used, that is  $10^{-16}$ .

For the GNSS-receiver, an accuracy is estimated and outputted for each GNSS-measurement. Since this is the most accurate measurement of the GNSS noise that we have available, it was used as a starting point for the  $R_{GNSS}$ , where the covariance is simply the square of this accuracy multiplied by  $I_3$ .

These values together give a very good starting point for the tuning process. Without any further adjustments, we get a relatively good 95% NIS confidence interval with 71.2% within the interval. It should however be mentioned that the recording starts before the flight. By not considering the measurements captured before the flight, we get a slightly better NIS - 81.5% is within the bounds after the biases have converged. An ideal filter would have had 95% of the NIS across time within the bounds of the confidence interval, so 81.5% is not perfect. However, it is not that far off, and so 81.5% should be considered acceptable as well. Also, it should be added that the NIS is calculated through GNSS measurements, and so it could really be the case that it is quite consistent with the ground truth instead. We did test with different parameters, but a NIS above 86% was not achievable. Since the Allan variances we initially started with is consistent across different devices because they are taken directly from

the datasheet, they are preferable to use in e.g. mass production. Therefore, these are the values we ended up with.

An ESKF shows its excellence in its estimation of the biases, and so this is also a property which needs to be examined in order to evaluate the performance of the filter. By looking at Figure 5, we see that when the heading is directly observable during the spiraling in the time span 250 to 600 seconds, the heading gyro and accelerometer biases quickly changes and converges to their true biases. In other words, the filter is able to estimate these biases. The claim is further supported by the fact that we observe small corrections to the biases in the time span 2500 to 3500 during the U-turns. Also, the biases are kept quite stable throughout the flight, which is preferable. This indicates that e.g. the heading is estimated by means of GNSS velocity when it is not directly observable by the accelerometer.

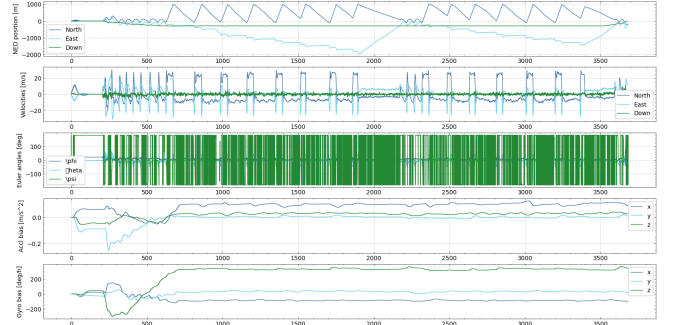


Figure 5: State estimates for the UAV

By examining the dataset it is clear that the IMU and GNSS measurements are logged as soon as the UAV is switched on and prepared for flight. This time before any flight results in some noisy predictions because the system needs to be affected by a force either gravity or acceleration in order to estimate the heading. This means that the initial errors will be higher than for the actual flight trajectory. One option is to disregard the samples where the UAV is on the ground in order to start with data samples where there are acceleration acting on the IMU as well as the gravity in order to quickly estimate the heading. However, this was not done because then the initial values and the covariances would not be set by the filter by the time the plane is flying, but rather the person initializing the program. Also the latter is a more realistic example of the use case for such a filter on an UAV. As mentioned earlier a magnetometer would be helpful in determining the heading right away.

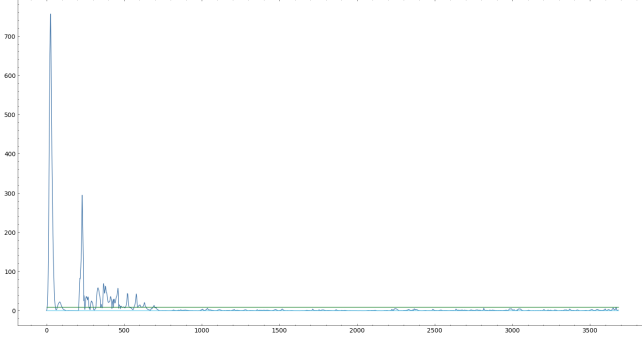
The tuning process for such a dataset is quite cumbersome as it is necessary to have a sufficient length of data to test such that the biases converge and are observable constant. Also, several flight patterns should be included in the data that is tested in order to prove consistency. The filter was hard to tune any better than with the values from the datasheet of the IMU. This is however a good indicator that the datasheet from the manufacturer is correct, which one would expect.

From looking at the plot of the flight path, it is clear that the plane follows a very smooth trajectory in the xy-plane. However, in the z-direction, the path is more squiggly. This is due to the fact that GNSS can be around 3 times more accurate in the horizontal plane than in the vertical plane. The measured GNSS accuracy from the dataset is simply a scalar which we multiply by the identity matrix. It may therefore give better results to weigh this accuracy different in the different directions in order to take advantage to this skewed accuracy. This has not been done in this report, as it was not known how this "averaged" GNSS accuracy was calculated.

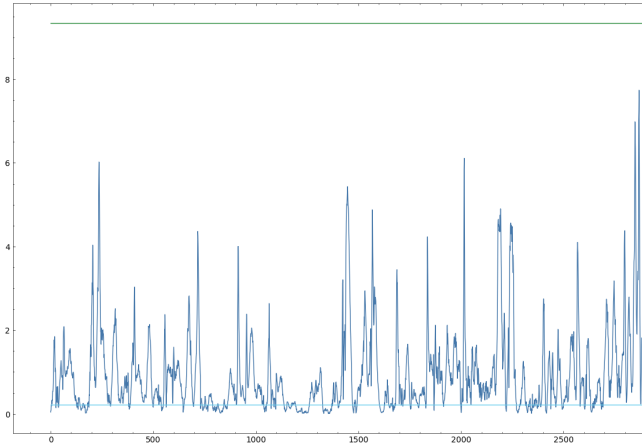
$$S_a = \begin{bmatrix} -0.05025913 & 1.00205951 & 0.04763634 \\ -1.02434848 & -0.06948449 & -0.01969145 \\ -0.03871402 & -0.04830625 & 1.0005252 \end{bmatrix} \quad (5)$$

$$S_g = \begin{bmatrix} -0.02428008 & 0.99812423 & 0.04542147 \\ -0.99982234 & -0.02422883 & -0.02551925 \\ -0.02715895 & -0.05322133 & 0.99485115 \end{bmatrix} \quad (6)$$

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$



**Figure 6:** NIS for the UAV over the whole test period. Average: 71.2%



**Figure 7:** NIS for the UAV after the biases have converged. Starting from 800s. Average: 81.5%

The mounting of the IMU is important for the results for the ESKF. It might be challenging to mount the IMU along the body axes of the UAV in a real world scenario due to balance, space constraints etc. The misalignment can be mitigated by applying a rotation matrix to the measurements before they are used in the ESKF. This was tested with swapping out the misalignment matrices  $S_a$  and  $S_g$ , defined in (5) and (6), with  $S$ , which is defined in (7). When both these matrices,  $S_a$ ,  $S_g$ , is equal to  $S$ , the biases will not converge towards the same values, and in our tests, the biases appear more constant. This is not ideal, the biases are supposed to be slowly varying. However there was negligible difference in the results over time.

## 4 Conclusion

For the simulated dataset the ESK filter works very well. It has some trouble determining the heading, but this is due to lack of observability when traveling in a straight line. The model is also very sensitive to misalignment errors.

The performance of the real dataset with the UAV is harder to quantify due to the lack of ground truth. However, the performance appears to be satisfactory and the filter is successful in keeping the track and with relative low errors. The real dataset is also not so prone to fail when using simplified misalignment matrices, although it gives worse performance.