Integration by Parts–

$$\boxed{\int u dv = uv - \int v du}$$

Choose u and dv wisely:

Select $u =$**IPET** in the decreasing order:

- **I** = inverse func. (ex. $\arctan(x), \ln(x)$) (one IBP at a time)
- **P** = polynomial (ex. $5x^3$)
- **E** = exponential func. (ex. a^x, e^{3x})
- **T** = trigonometric func. (ex. $\cos(2x), \sin(x)$) (E & T interchangeable)

INTEGRATION BY PARTS SUMMARY

1. For integrals of the form $\int x^n f(x)\ dx$ where $f(x) \neq$ inverse function.

   <u>**ex.**</u> $\int x^n a^{bx}\ dx, \int x^n \sin(ax)\ dx, \int x^n \cos(ax)\ dx,$

   Let $u = x^n$ and differentiate u all the way till $du = 0$.

2. For integrals of the form $\int x^n f(x)\ dx$ where $f(x)$ =inverse function.

   <u>**ex.**</u> $\int x^n \ln x\ dx, \int x^n \arcsin(ax)\ dx, \int x^n \arctan(ax)\ dx,$

   Let $u = f(x)$ and differentiate u just **once** and evaluate the 'left over' integral.

3. For integrals of the form $\int f(x) \cdot g(x)\ dx$ where $f(x) = a^x$ and $g(x) = \sin(bx)$ or $\cos(bx)$

   <u>**ex.**</u> $\int e^{2x} \sin 3x\ dx, \int 2^{3x} \cos x\ dx,$

   Let $u = a^{bx}$ and differentiate u **twice** and 'wrap around' the 'left over' integral.

Now You Try It (NYTI): Evaluate the following integrals:

1. $\displaystyle\int x5^x dx \quad (\frac{x5^x}{\ln 5} - \frac{5^x}{(\ln 5)^2} + C),$ 2. $\displaystyle\int x^5 \ln x dx \quad (\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C)$

3. $\displaystyle\int x \ln(1+x) dx \quad (\frac{x^2}{2} \ln(1+x) - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln(1+x)\right) + C),$

4. $\displaystyle\int x \arctan x dx \quad (\frac{x^2}{2} \arctan x - \frac{1}{2}(x - \arctan x) + C)$

5. $\displaystyle\int \sin x \cos x \, e^{\cos^2 x} dx \quad (-\frac{1}{2} e^{\cos^2 x} + C)$

You should be able to get #1 and #2 quickly using the tabular method.
For #5, what is a good choice for the $u-$sub?
For #3 and #4, you will need to rewrite the improper fraction to be proper.

Proper Fraction $\dfrac{p(x)}{q(x)}$ where deg $(p(x) < \deg(q(x), p(x), q(x)$ are polynomials.
(ie. deg(numerator) is strictly less than the deg(denominator)).

For #3 : $\dfrac{x^2}{x+1} = x - 1 + \dfrac{1}{x+1}$
For #4 : $\dfrac{x^2}{x^2+1} = 1 - \dfrac{1}{x^2+1}$