Lecture 2

Risk Return Optimization

Portfolio Management Procedure

1. Define the security universe
2. Model security risk and performance
3. Forecast returns
4. Define the portfolio's objective
5. Define portfolio's constraints
6. Simulate the candidate portfolios
7. Optimize among the portfolios
8. Assess the constructed portfolio's performance

Nature of Risk

Is Portfolio Risk a linear function of security risk?

Return Notation

Notation	Description	Formula
r^i	Return rate of asset i	
r^f	Risk-free return rate	
$\tilde{r}^i$	Excess return rate of asset i	$r^i - r^f$

Two investments: bonds and stocks

Consider the following portfolio example

	Return	**Allocation weight**
Bonds	r^b	w
Stocks	r^s	$1-w$

Return Statistics notation

Mean	**Variance**	**Correlation**
μ / E	σ^2	ρ

Коваріація та кореляція

Математичні поняття **коваріації** (англ. *covariance*) та **кореляції** (англ. *correlation*) у теорії ймовірностей та статистиці дуже схожі. Обидва описують ступінь, до якого дві випадкові величини або набори випадкових величин схильні відхилятися від своїх математичних сподівань подібним чином.

Якщо X та Y — дві випадкові величини з середніми значеннями (математичними сподіваннями) μ_X та μ_Y і стандартними відхиленнями σ_X та σ_Y відповідно, то їх коваріація та кореляція такі:

Коваріація:

$$\operatorname{cov}_{XY} = \sigma_{XY} = \mathbb{E}\left[(X - \mu_X)(Y - \mu_Y)\right]$$

$$\operatorname{corr}_{XY} = \rho_{XY} = \mathbb{E}\left[(X - \mu_X)(Y - \mu_Y)\right] / (\sigma_X \sigma_Y),$$

Отже,

$$\rho_{XY} = \sigma_{XY} / (\sigma_X \sigma_Y)$$

Portfolio Return statistics

Investment portfolio return r^p has mean and variance of

$$\mu^p = w\,\mu^b + (1 - w)\mu^s$$

$$\sigma_p^2 = w^2\,\sigma_b^2 + (1 - w)^2\sigma_s^2 + 2w(1 - w)\rho\sigma_s\sigma_b$$

Perfect correlation

Suppose that $\rho = 1$

- ➢ Then the volatility (standard deviation) of the portfolio is proportional to the asset allocation weights:

  $$\sigma_p = w\,\sigma_b + (1 - w)\sigma_s$$

- ➢ Thus, both mean and volatility are linear in the allocations.

Imperfect correlation

Suppose that $\rho < 1$

- The volatility function is convex:

  $\sigma_p < w\,\sigma_b + (1 - w)\sigma_s$

- Yet the mean return is still linear in the portfolio allocation:

  $\mu^p = w\,\mu^b + (1 - w)\mu^s$

A perfect hedge

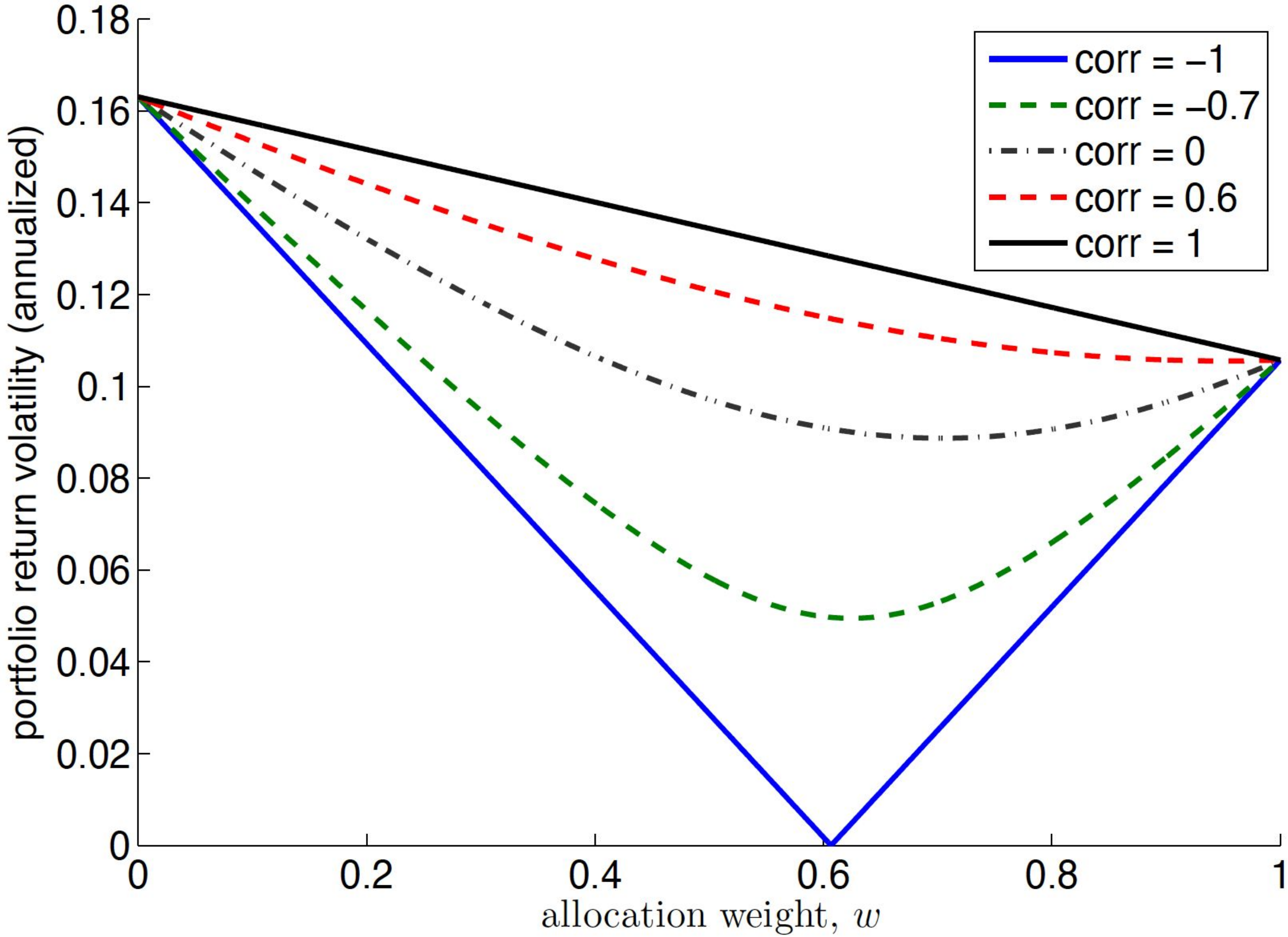
For $\rho = -1$

- ➢ The portfolio variance can be as small as desired, by choosing the appropriate allocation, *w*

- ➢ In fact, $\sigma_p = 0$ *if*

$$w = \frac{\sigma_s}{\sigma_s + \sigma_b}$$

- ➢ Thus, a riskless portfolio can be formed from the two risky assets.



Diversification of investment portfolio between two risky assets.

Allocation among *n* assets

Consider the following portfolio allocation problem:

- ➢ *n* risky securities,
- ➢ return volatility (std.dev.) denoted σ_i
- ➢ return covariance between security i and j denoted by $\sigma_{i,j}$
- ➢ w^idenotes the fraction of the portfolio allocated to asset *i*, which sum up to 1.

Then

$$\sigma_p^2 = \sum_{j=1}^{n} \sum_{i=1}^{n} w^i w^j \sigma_{i,j}$$

Variance of the equally weighted portfolio

Consider an equally-weighted portfolio, with $w^i = 1/n$ for each asset. Then

$$\sigma_p^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sigma_i^2 + \frac{1}{n^2}\sum_{j\neq i}\sum_{i=1}^{n}\sigma_{i,j}$$

In the example with stocks and bonds

$$\sigma_p^2 = \frac{1}{4}\sigma_b^2 + \frac{1}{4}\sigma_s^2 + \frac{1}{2}\sigma_{b,s}, \qquad \sigma_{b,s} = \rho\sigma_b\sigma_s$$

Portfolio variance as average covariances

Use the following notation for averaging the variances and covariances across the *n* assets:

$$\text{avg}\left[\sigma_i^2\right] = \frac{1}{n}\sum_{i=1}^{n}\sigma_i^2$$

$$\text{avg}\left[\sigma_{i,j}\right] = \frac{1}{n(n-1)}\sum_{j\neq i}^{n}\sum_{i=1}^{n}\sigma_{i,j}$$

So, the portfolio variance can be written as

$$\sigma_p^2 = \frac{1}{n}\text{avg}\left[\sigma_i^2\right] + \frac{n-1}{n}\text{avg}\left[\sigma_{i,j}\right]$$

Portfolio irrelevance of individual security variance

As number of securities in portfolio, *n*, gets large,

$$\lim_{n \to \infty} \sigma_p^2 = \operatorname{avg}\left[\sigma_{i,j}\right]$$

- ➢ Individual security variance is unimportant!
- ➢ Overall portfolio variance is average of individual securities covariance

Diversified portfolio

Obtained this result using equally-weighted portfolio, $w^i = 1/n$

- Don't need equal weighting, just that

$$\lim_{n \to \infty} w^i = 0$$

- That is, as *n* gets large the portfolio must have trivial exposure to security i .
- This is the sense in which portfolio must be diversified for individual variances to become unimportant.

Portfolio variance decomposition

We have discussed the equally-weighted portfolio variance:

$$\sigma_p^2 = \frac{1}{n}\text{avg}\left[\sigma_i^2\right] + \frac{n-1}{n}\text{avg}\left[\sigma_{i,j}\right]$$

Variance has a term which can be diversified to zero, and another term that remains.

Suppose that asset returns have

- identical volatilities, $\sigma_i = \sigma$
- identical correlations, $\rho_{i,j} = \rho$

Systematic Risk

$$\sigma_p^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2$$

$$\lim_{n\to\infty} \sigma_p^2 \to \rho\sigma^2$$

- A fraction ρ of the variance is systematic
- No amount of diversification can get portfolio variance lower:

$$\sigma_p^2 \geq \rho\sigma^2$$

Idiosyncratic risk

$$\sigma_p^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2$$

- ➢ Idiosyncratic risk refers to the diversifiable part of σ_p^2
- ➢ An equally-weighted portfolio has idiosyncratic risk equal to $\frac{1}{n}\sigma^2$.

Correlation and diversified portfolios

$$\sigma_p^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2$$

- ➢ For $\rho = 1$, there is no possible diversification, regardless of *n*.

- ➢ For $\rho = 0$, there is no systematic risk, only variance is remaining idiosyncratic:

$$\sigma_p^2 = \frac{1}{n}\sigma^2$$

And as *n* gets large the portfolio is riskless,

$$\lim_{n\to\infty} \sigma_p^2 = 0$$

Riskless portfolios

- ➢ Above, we found that a riskless portfolio could be created if $\rho = -1$
- ➢ Here, we found that a riskless portfolio can be created if $\rho = 0$

Question:

How did the assumptions behind these conclusions differ?

Answer

- In the case of just two underlying assets, complete diversification is achieved with $\rho = -1$
- In the case of many assets, complete diversification is achieved when all assets are uncorrelated, and the number of assets in the portfolio goes to infinity.

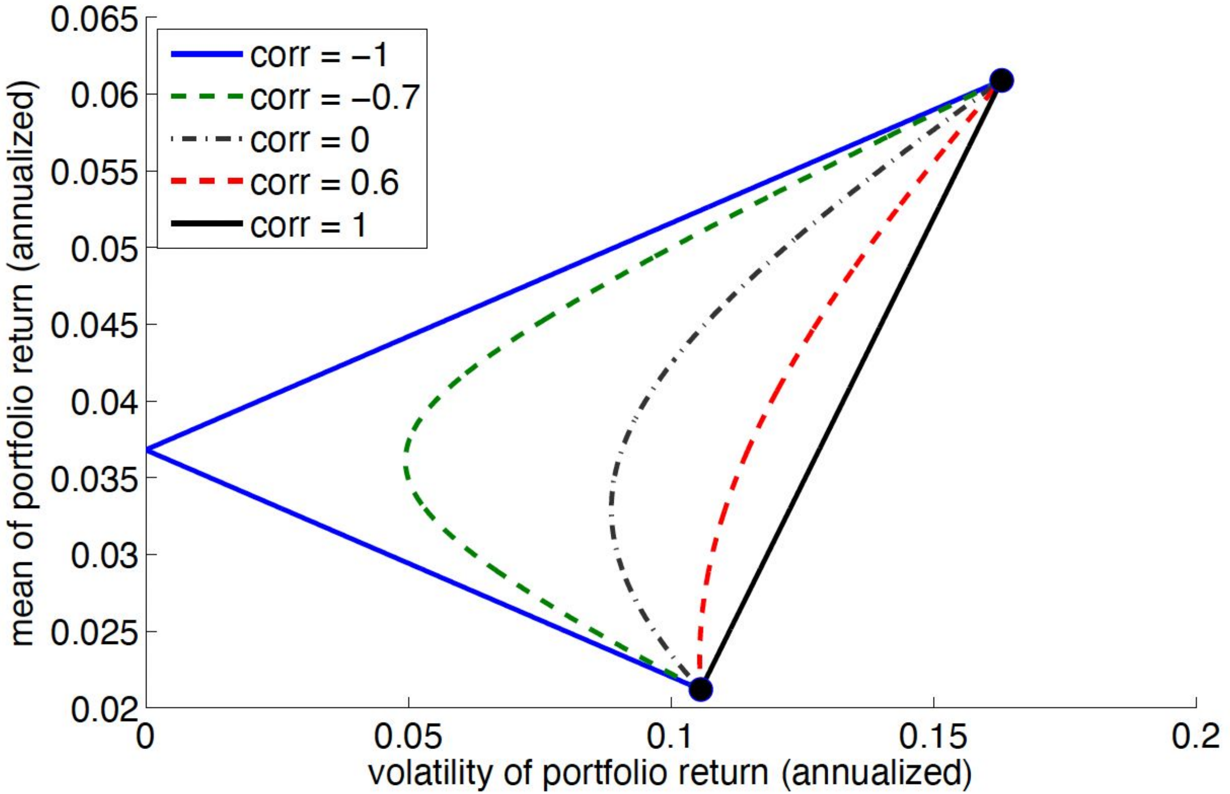
Mean-variance comparison

We want to compare risk and return...

- ➢ Use mean return to score the portfolio's benefits.
- ➢ Use variance (or volatility) of return to score the portfolio's risk.

Consider the case of two assets:

Mean-volatility space of diversification between two assets



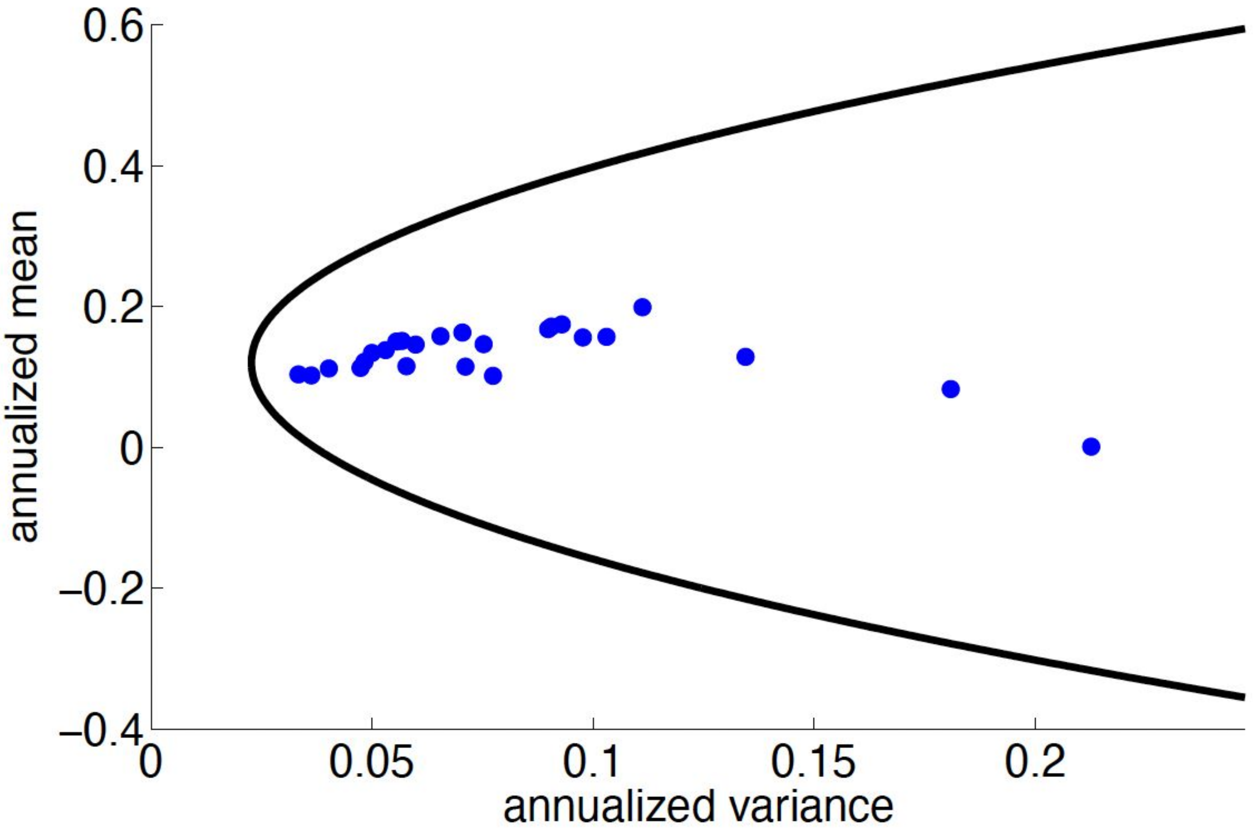
Diversification between n assets

With *n* securities, there is further potential for diversification.

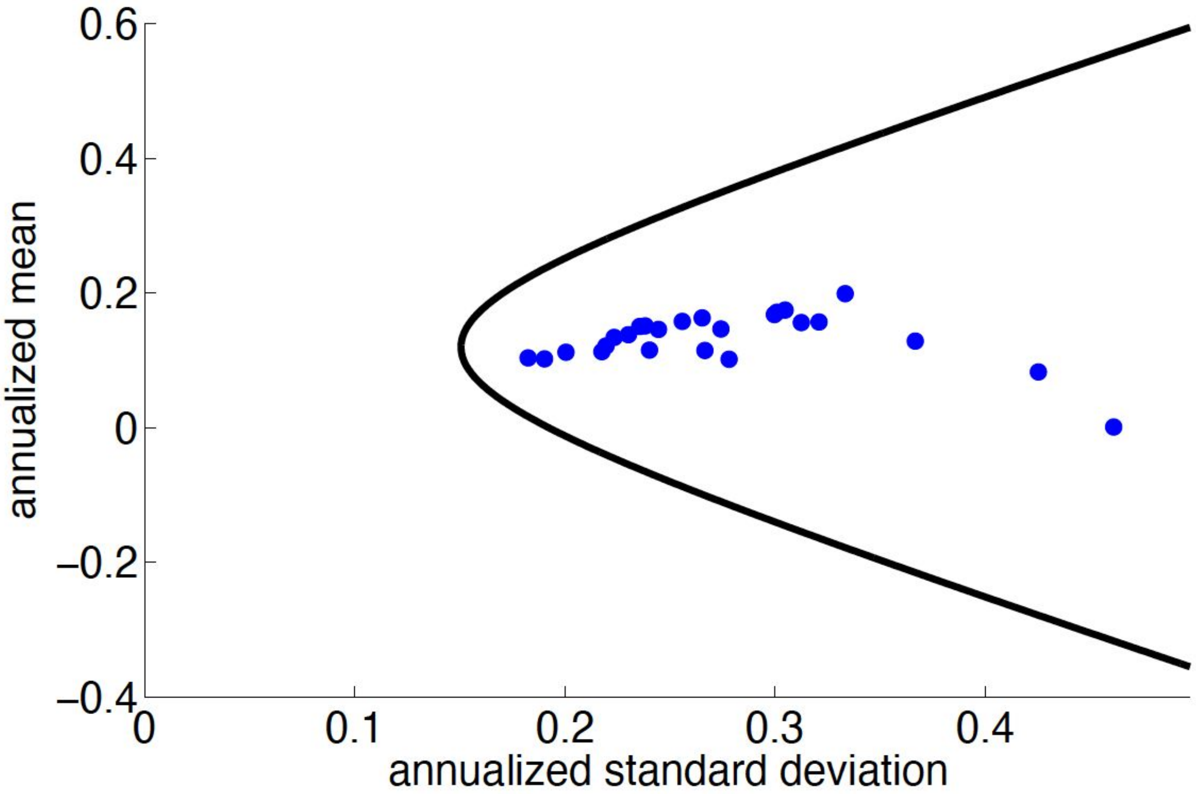
- ➢ The set of all possible portfolios formed from this basis of assets forms a convex set in mean-variance space.
- ➢ The boundary of this set is known as the mean-variance frontier, and it forms a parabola.
- ➢ The boundary of the set in mean-volatility space forms a hyperbola.

MV frontier is used to refer to both the mean-variance and mean-volatility frontiers.

Mean - Variance frontier formed by 25 US equity portfolios



Mean - Volatility frontier formed by 25 US equity portfolios



Efficient portfolio

The top segment of the MV frontier is the set of efficient MV portfolios.

- These portfolios maximize mean return given the return variance.
- Contrast this with the lower segment of the MV frontier, the inefficient MV portfolios.
- The inefficient MV portfolios minimize mean return given the return variance.

Importance of MV analysis

MV analysis is the most widely used tool in portfolio allocation.

- ➢ The model gives a tractable way to balance risk and return.
- ➢ There is a connection between MV analysis and beta-factor models.

Notation

Suppose there are *n* risky assets.

- ***r*** is an *n x 1* random vector. Each element is the return on one of the assets.
- Let μ denote the *n x 1* vector of mean returns. Let $\sum$ denote the *n x n* covariance matrix of returns.

$$\mu = \mathbb{E}[r]$$

$$\Sigma = \mathbb{E}\left[(r-\mu)(r-\mu)'\right]$$

Portfolios

- An investor chooses a **portfolio**, defined as a $n \times 1$ vector of allocation weights, ω.

- These allocation weights must sum to unity:

$$\omega' \mathbf{1} = 1$$

where $\mathbf{1}$ denotes a $n \times 1$ vector of ones.

Return Moments

The portfolio return on some portfolio, $\boldsymbol{\omega}^p$, is

$$r^p = (\boldsymbol{\omega}^p)' \boldsymbol{r}.$$

The portfolio return moments are

$$\mu^p =: \mathbb{E}[r^p] = (\boldsymbol{\omega}^p)' \boldsymbol{\mu}$$

$$\sigma_p^2 =: \text{var}(r^p) = (\boldsymbol{\omega}^p)' \Sigma \boldsymbol{\omega}^p$$

$$\text{cov}(r^p, \boldsymbol{r}) = \Sigma \boldsymbol{\omega}^p$$

MV Portfolio

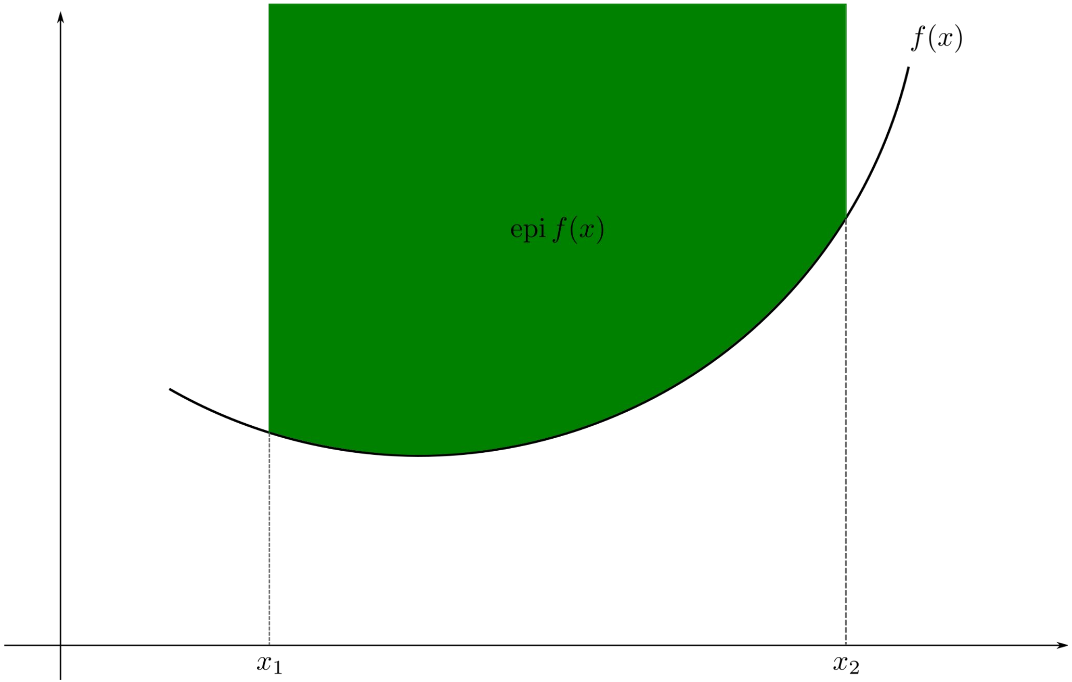
A Mean-Variance (MV) portfolio is a vector, $\boldsymbol{\omega}^*$, which solves the following optimization for some number μ^p:

$$\begin{aligned} \min_{\boldsymbol{\omega}} \quad & \boldsymbol{\omega}'\Sigma\boldsymbol{\omega} \\ \text{s.t.} \quad & \boldsymbol{\omega}'\boldsymbol{\mu} = \mu^p \\ & \boldsymbol{\omega}'\mathbf{1} = 1 \end{aligned}$$

- Note that the objective function is convex in w, given that Σ is positive definite.
- The constraint set is also convex.

Convex Function

Symmetric matrix A with real entries is **positive-definite** if the real number mAm^T is positive for every nonzero real column vector m.



MV Solution

Thus, a portfolio $\boldsymbol{\omega}^*$ is MV iff exists $\delta \in (-\infty, \infty)$ such that

$$\boxed{\begin{aligned} \boldsymbol{\omega}^* =& \delta \boldsymbol{\omega}^{\mathrm{t}} + (1-\delta)\boldsymbol{\omega}^{\mathrm{v}} \\ \boldsymbol{\omega}^{\mathrm{t}} \equiv \underbrace{\left(\frac{1}{\mathbf{1}'\Sigma^{-1}\boldsymbol{\mu}}\right)}_{\text{scaling}} \Sigma^{-1}\boldsymbol{\mu}, &\qquad \boldsymbol{\omega}^{\mathrm{v}} \equiv \underbrace{\left(\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)}_{\text{scaling}} \Sigma^{-1}\mathbf{1} \end{aligned}}$$

$\boldsymbol{\omega}^{\mathrm{t}}$ and $\boldsymbol{\omega}^{\mathrm{v}}$ are themselves MV portfolios $(\delta = 0, 1)$

Solving the MV problem

Solving with Lagrangian multipliers, (γ_1 and γ_2,) gives the unconstrained optimization:

$$\mathcal{L} = \frac{1}{2}\boldsymbol{\omega}'\Sigma\boldsymbol{\omega} - \gamma_1\left(\boldsymbol{\omega}'\boldsymbol{\mu} - \mu^p\right) - \gamma_2\left(\boldsymbol{\omega}'\mathbf{1} - 1\right)$$

The first derivative equations are (in matrix notation,)

$$\frac{\partial \mathcal{L}}{\partial \boldsymbol{\omega}'} = \Sigma\boldsymbol{\omega} - \gamma_1\boldsymbol{\mu} - \gamma_2\mathbf{1}$$

Get the first-order conditions of optimization by setting equal to zero and solve for $\boldsymbol{\omega}^*$:

$$\boldsymbol{\omega}^* = \Sigma^{-1}\begin{bmatrix}\boldsymbol{\mu} & \mathbf{1}\end{bmatrix}\begin{bmatrix}\gamma_1 \\ \gamma_2\end{bmatrix}$$

Solving the MV problem: portfolios and w^t and w^v

Rewrite this as

$$\boldsymbol{\omega}^* = \gamma_1 \Sigma^{-1} \boldsymbol{\mu} + \gamma_2 \Sigma^{-1} \mathbf{1}$$

which can be rewritten as the sum of two portfolios:

$$\boldsymbol{\omega}^* = \gamma_1 \left(\mathbf{1}' \Sigma^{-1} \boldsymbol{\mu}\right) \boldsymbol{\omega}^{\mathrm{t}} + \gamma_2 \left(\mathbf{1}' \Sigma^{-1} \mathbf{1}\right) \boldsymbol{\omega}^{\mathrm{v}}$$

where

$$\boldsymbol{\omega}^{\mathrm{t}} \equiv \frac{1}{\mathbf{1}' \Sigma^{-1} \boldsymbol{\mu}} \Sigma^{-1} \boldsymbol{\mu}, \qquad \boldsymbol{\omega}^{\mathrm{v}} \equiv \frac{1}{\mathbf{1}' \Sigma^{-1} \mathbf{1}} \Sigma^{-1} \mathbf{1}$$

Solving the MV problem: eliminate γ_2

Note that $\boldsymbol{\omega}^{\text{t}}$ and $\boldsymbol{\omega}^{\text{v}}$ are proper portfolios:

$$(\boldsymbol{\omega}^{\text{t}})' \mathbf{1} = 1, \qquad (\boldsymbol{\omega}^{\text{v}})' \mathbf{1} = 1$$

Given that $\mathbf{1}'\boldsymbol{\omega}^* = \mathbf{1}'\boldsymbol{\omega}^{\text{t}} = \mathbf{1}'\boldsymbol{\omega}^{\text{v}} = 1$, the equation above implies

$$1 = \gamma_1 \left(\mathbf{1}'\Sigma^{-1}\boldsymbol{\mu}\right) + \gamma_2 \left(\mathbf{1}'\Sigma^{-1}\mathbf{1}\right)$$

Use this to rewrite the MV vector as

$$\boldsymbol{\omega}^* = \delta\boldsymbol{\omega}^{\text{t}} + (1 - \delta)\boldsymbol{\omega}^{\text{v}}$$

where

$$\delta \equiv \gamma_1 \left(\mathbf{1}'\Sigma^{-1}\boldsymbol{\mu}\right)$$

MV Solution

Thus, a portfolio $\boldsymbol{\omega}^*$ is MV iff exists $\delta \in (-\infty, \infty)$ such that

$$\boxed{\begin{aligned} \boldsymbol{\omega}^* &= \delta \boldsymbol{\omega}^{\mathrm{t}} + (1-\delta)\boldsymbol{\omega}^{\mathrm{v}} \\ \boldsymbol{\omega}^{\mathrm{t}} \equiv \underbrace{\left(\frac{1}{\mathbf{1}'\Sigma^{-1}\boldsymbol{\mu}}\right)}_{\text{scaling}} \Sigma^{-1}\boldsymbol{\mu}, &\qquad \boldsymbol{\omega}^{\mathrm{v}} \equiv \underbrace{\left(\frac{1}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)}_{\text{scaling}} \Sigma^{-1}\mathbf{1} \end{aligned}}$$

$\boldsymbol{\omega}^{\mathrm{t}}$ and $\boldsymbol{\omega}^{\mathrm{v}}$ are themselves MV portfolios ($\delta = 0, 1$)

GMV and zero-tangency portfolios

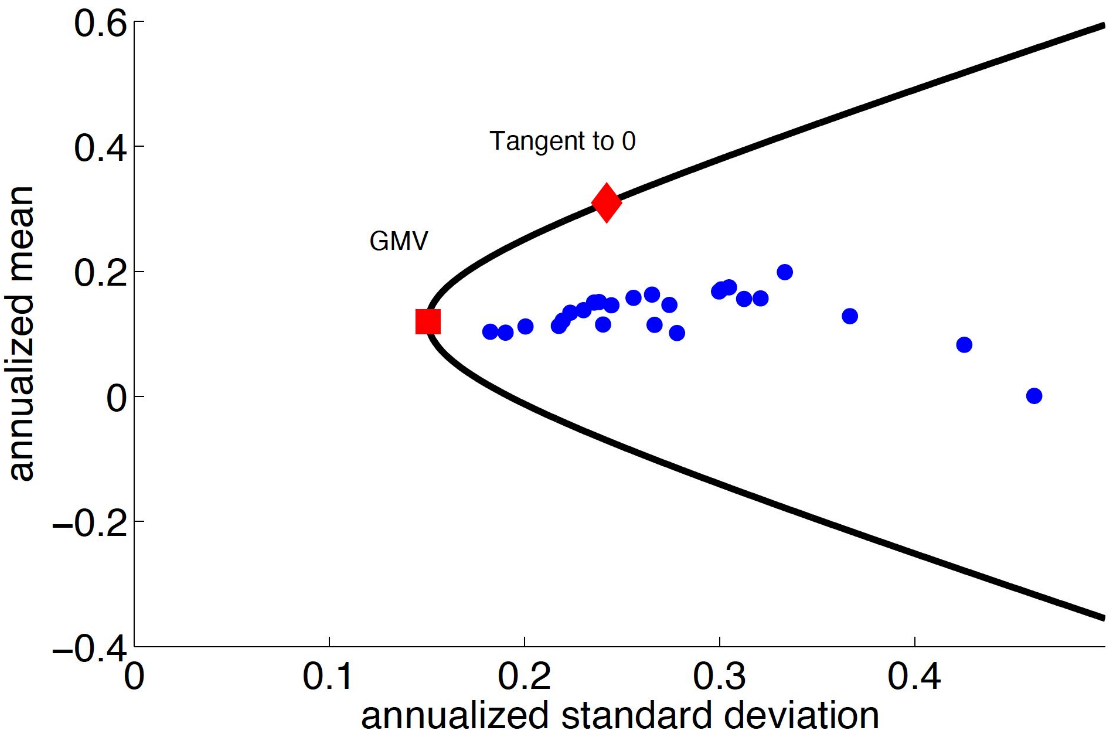
$\boldsymbol{\omega}^{\mathrm{v}}$ is the Global Minimum Variance (GMV) portfolio. It solves,

$$\min_{\boldsymbol{\omega}} \quad \boldsymbol{\omega}'\Sigma\boldsymbol{\omega}$$
$$\text{s.t.} \quad \boldsymbol{\omega}'\mathbf{1} = 1$$

- This is the same as the MV problem, but dropping the first constraint, $(\boldsymbol{\omega}'\boldsymbol{\mu} = \mu^p.)$

$\boldsymbol{\omega}^{\mathrm{t}}$ is the portfolio tangent to the mean-volatility frontier and going through the origin. (See next slide.)

Two useful MV portfolios



The Global Minimum Variance portfolio

Zero-Tangency portfolio

MV Investors

Consider MV investors, the investors for whom mean and variance of returns are sufficient statistics of the investment.

- Such investors will hold an MV portfolio, $\boldsymbol{\omega}^*$.
- Thus, these investors are holding linear combination of just two risky portfolios, $\boldsymbol{\omega}^{\mathrm{t}}$ and $\boldsymbol{\omega}^{\mathrm{v}}$.
- So if in real markets all investors were MV investors, everyone would simply invest in two funds.
- Those wanting higher mean returns would hold more in the high-return MV, $\boldsymbol{\omega}^{\mathrm{t}}$, while those wanting safer returns would hold more in the low-return MV, $\boldsymbol{\omega}^{\mathrm{v}}$.

With a riskless asset

Now consider the existence a risk-free asset with return, r^f.

- Suppose there are still n risky assets available, still notating the risky returns as $\boldsymbol{r}$
- Let $\boldsymbol{w}$ denote a $n \times 1$ vector of portfolio allocations to the n risky assets.
- Since the total portfolio allocations must add to one, we have

$$\text{allocation to the risk-free rate} = 1 - \boldsymbol{w}'\mathbf{1}$$

Mean excess returns

$\boldsymbol{\mu}$ denotes the vector of mean returns of risky assets, $\mathbb{E}[\boldsymbol{r}]$.

Let μ^p denote the mean return on a portfolio.

$$\mu^p = \left(1 - \boldsymbol{w}'\boldsymbol{1}\right) r^f + \boldsymbol{w}'\boldsymbol{\mu}$$

Use the following notation for excess returns:

$$\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} - \boldsymbol{1} r^f$$

Thus the mean return and mean excess return of the portfolio are

$$\begin{aligned} \mu^p &= r^f + \boldsymbol{w}'\tilde{\boldsymbol{\mu}} \\ \tilde{\mu}^p &= \boldsymbol{w}'\tilde{\boldsymbol{\mu}} \end{aligned}$$

Variance of returns

- The risk-free rate has zero variance and zero correlation with any security.
- Let Σ continue to denote the $n \times n$ covariance matrix of *risky* assets, (and is positive semi-definite.)
- The return variance of the portfolio, $\boldsymbol{w}^p$ is

$$\sigma_p^2 = \boldsymbol{w}'\Sigma\boldsymbol{w}$$

The $\tilde{MV}$ problem with a riskless asset

A Mean-Variance portfolio with risk-free asset ($\tilde{MV}$) is a vector, $\boldsymbol{w}^*$, which solves the following optimization for some mean excess return number $\tilde{\mu}^p$:

$$\min_{\boldsymbol{w}} \quad \boldsymbol{w}'\Sigma\boldsymbol{w}$$

$$\text{s.t.} \quad \boldsymbol{w}'\tilde{\boldsymbol{\mu}} = \tilde{\mu}^p$$

- In contrast to the MV problem, there is only one constraint.
- The allocation weight vector, $\boldsymbol{w}$ need not sum to one, as the remainder is invested in the risk-free rate.

Solving the $\tilde{MV}$ problem

Solving the problem is straitforward:

1. Set up the Lagrangian with just one constraint.
2. The FOC is sufficient given the convexity of the problem.
3. Finally, substitute the Lagrange multiplier using the constraint.

Refer to the solution as an $\tilde{\text{M}}\text{V}$ portfolio.

The MV˜ problem solution

$$\boldsymbol{w}^* = \tilde{\delta}\ \boldsymbol{w}^{\mathrm{t}}$$

for the portfolio

$$\boldsymbol{w}^{\mathrm{t}} = \underbrace{\left(\frac{1}{\mathbf{1}'\Sigma^{-1}\tilde{\boldsymbol{\mu}}}\right)}_{\text{scaling}} \Sigma^{-1}\tilde{\boldsymbol{\mu}}$$

and allocation

$$\tilde{\delta} = \left(\frac{\mathbf{1}'\Sigma^{-1}\tilde{\boldsymbol{\mu}}}{(\tilde{\boldsymbol{\mu}})'\,\Sigma^{-1}\tilde{\boldsymbol{\mu}}}\right)\tilde{\mu}^{p}$$

MV˜ portfolio variance formula

The return variance of an $\tilde{\text{MV}}$ portfolio is given by

$$\frac{(\tilde{\mu}^p)^2}{(\tilde{\boldsymbol{\mu}})'\Sigma^{-1}\tilde{\boldsymbol{\mu}}}$$

This implies that the return volatility (standard-deviation) is linear in the absolute value of the mean excess return:

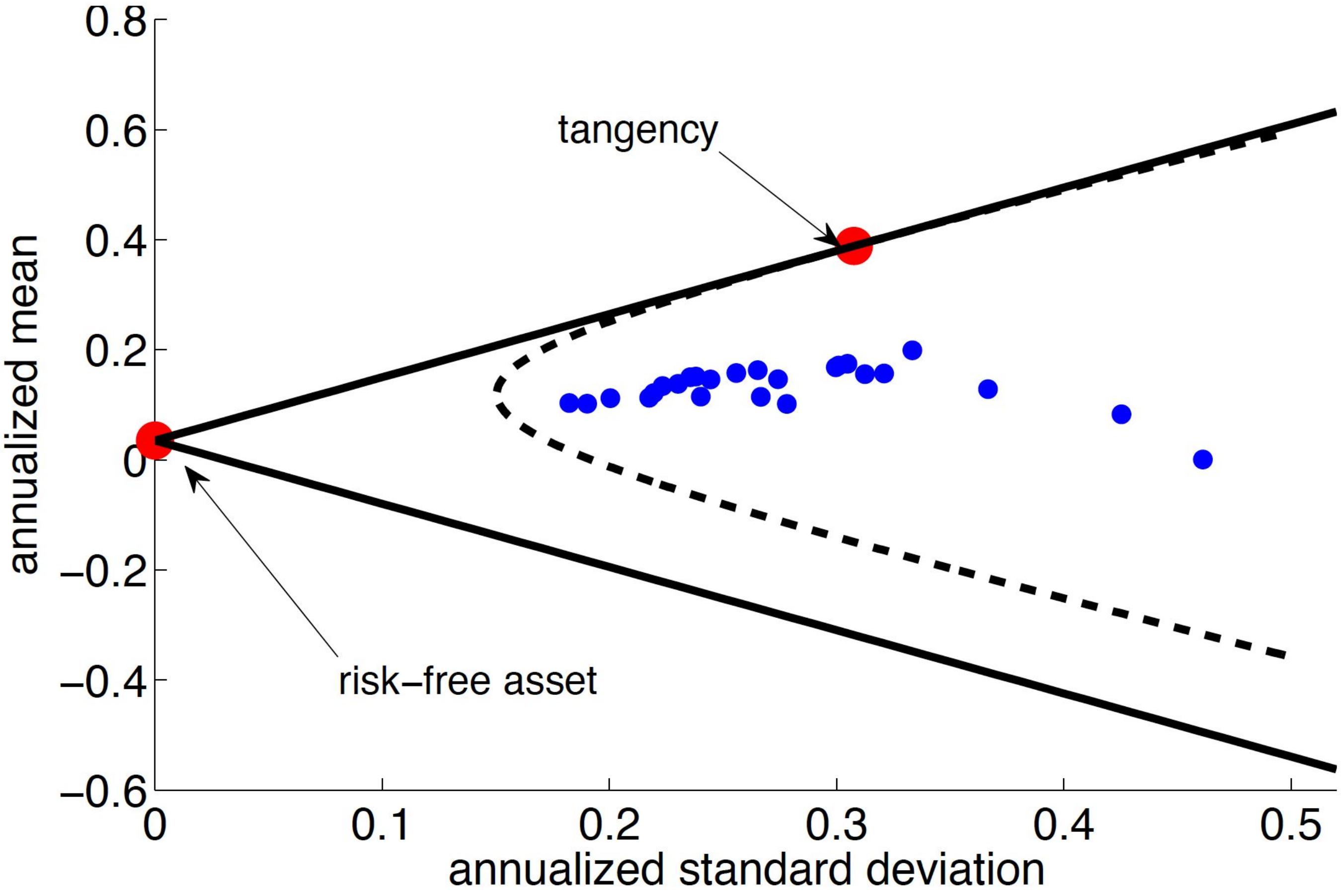
$$\frac{|\tilde{\mu}^p|}{\sqrt{(\tilde{\boldsymbol{\mu}})'\Sigma^{-1}\tilde{\boldsymbol{\mu}}}}$$

Tangency portfolio

The result is that any $\tilde{\text{MV}}$ portfolio is a combination of the tangency portfolio, $\boldsymbol{w}^{\text{t}}$, and a position in the riskless asset.

- The tangency portfolio, $\boldsymbol{w}^{\text{t}}$ invests 100% in risky assets, $\mathbf{1}'\boldsymbol{w}^{\text{t}} = 1$.
- $\boldsymbol{w}^{\text{t}}$ is the unique portfolio which is on the risky MV frontier as well as the $\tilde{\text{MV}}$ frontier expanded by the risk-free asset.
- $\boldsymbol{w}^{\text{t}}$ is the point on the risky MV frontier at which the tangency line goes through the risk-free rate. (See the figure below.)

Illustration of the MV˜ frontier when a riskless asset is available.
In this case, the MV˜ portfolio frontier consists of two straight lines. The curved frontier is the MV frontier when a riskless asset is unavailable.



Tangency Portfolio and the Sharpe ratio

For an arbitrary portfolio, $\boldsymbol{w}^p$,

$$\mathrm{SR}\left(\boldsymbol{w}^p\right) = \frac{\mu^p - r^f}{\sigma^p} = \frac{\tilde{\mu}^p}{\sigma^p}$$

The tangency portfolio, $\boldsymbol{w}^{\mathrm{t}}$, is the portfolio on the risky MV frontier with maximum Sharpe ratio.

$$\mathrm{SR}\left(\boldsymbol{w}^*\right) = \pm\sqrt{\left(\tilde{\boldsymbol{\mu}}\right)' \Sigma^{-1} \tilde{\boldsymbol{\mu}}}$$

The SR magnitude is constant across all $\tilde{\mathrm{MV}}$ portfolios.
(Sign depends on whether part of the efficient or inefficient frontier.)

Capital Market Line

The Capital Market Line (CML) is the efficient portion of the M̃V frontier.

- The CML shows the risk-return tradeoff available to M̃V investors.
- The slope of the CML is the maximum Sharpe ratio which can be achieved by any portfolio.
- The inefficient portion of the M̃V frontier acheives the minimum (negative) Sharpe ratio by shorting the tangency portfolio.

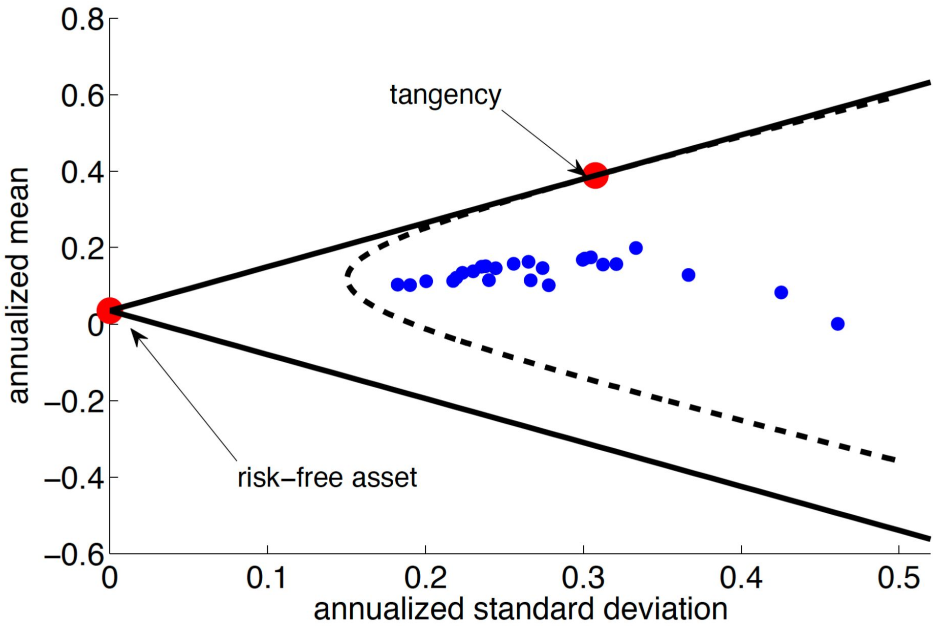


Illustration of the MV˜ frontier when a riskless asset is available.
In this case, the MV˜ portfolio frontier consists of two straight lines. The curved frontier is the MV frontier when a riskless asset is unavailable.

Two-fund separation

Two-fund separation. Every $\tilde{\text{MV}}$ portfolio is the combination of the risky portfolio with maximal Sharpe Ratio and the risk-free rate.

Thus, for an $\tilde{\text{MV}}$ investor the asset allocation decision can be broken into two parts:

1. Find the tangency portfolio of risky assets, $\boldsymbol{w}^t$.
2. Choose an allocation between the risk-free rate and the tangency portfolio.