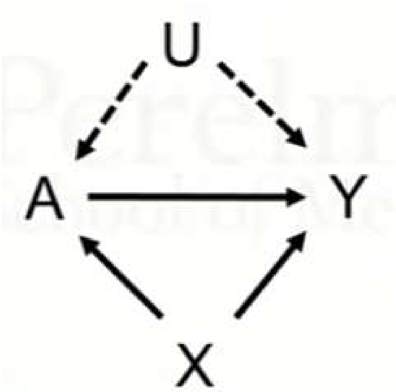
Module 5

Thursday, October 31, 2024 10:36 AM

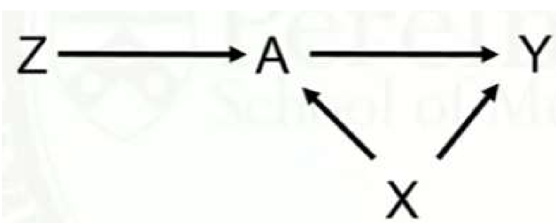
Introduction to Instrumental Variables

Suppose there are unmeasured / unobserved variables, U, that affect the treatment A and outcome Y; then there is unmeasured confounding:

- The ignorability assumption is violated
- The estimates of causal effects are biased
- We cannot control for confounders and average over the distribution if we don't observe them all



Instrumental Variables (IV) is an alternative causal inference method that does not rely on the ignorability assumption; it affects treatment A, but does not directly affect the outcome Y. In the DAG below, Z is an instrumental variable and can be thought of as encouragement, where higher values of Z more aggressively motivate people to receive treatment.



As an example:

- A: smoking during pregnancy (yes/no)
- Y: birthweight
- X: parity, mother's age, weight, etc.

There could be unmeasured confounders and it isn't ethical to randomly assign smoking to pregnant women. So we approach this problem with an Encouragement Design:

- Z: randomize to either receive encouragement to stop smoking (Z=1) or receive usual care (Z=0)

An intention-to-treat analysis would focus on the causal effect of encouragement: $E(Y^{Z=1}) - E(Y^{Z=0})$
The thing being randomized in this example is encouragement, not the smoking itself; therefore we aren't contrasting potential outcomes with different treatments, but the encouragement of treatment.

Randomized Trials with Non-Compliance

Consider the following randomized trial setup:

- Z: randomization to treatment (1 if randomized to treatment, 0 otherwise)
- A: treatment received (1 if received, 0 otherwise)
- Y: outcome

Typically, not everyone assigned treatment will actually receive the treatment; this is called non-compliance.

Non-compliance makes a randomized trial like an observational study in that there could be confounding based on treatment received. It might be reasonable to assume that treatment assignment does not directly affect Y. Each subject has two potential values of treatment:

- $A^{Z=1} = A^1$: the value of treatment if randomized to Z=1
- $A^{Z=0} = A^0$: the value of treatment if randomized to Z=0

We can think of the average causal effect of treatment assignment on treatment received as $E(A^1 - A^0)$. This is the proportion treated if everyone had been assigned to receive treatment minus the proportion treated if no one had been assigned to receive treatment. If there is perfect compliance, this expected value would be equal to 1.

This can be estimated from observed data since, by randomization and consistency:

$E(A^1) = E(A|Z = 1)$
$E(A^0) = E(A|Z = 0)$

Compliance Classes

Never-Takers: people who do not take treatment, regardless of randomization or what they are assigned; encouragement does not work. We would not learn anything about the effect of treatment from this subpopulation, as there is no variation in treatment received.

Compliers: people who take treatment when encouraged to, but not otherwise. In this group, treatment received is randomized.

Defiers: people who do the opposite of what they are encouraged to do. In this group, treatment received is randomized, but in the opposite way.

Always-Takers: people who always take treatment, regardless of randomization or what they are assigned. We would not learn anything about the effect of treatment from this population because there is no variation in treatment received.

A motivation for using IV methods is concern about possible unmeasured confounding, because if there is unmeasured confounding, we cannot marginalize over all confounders using methods like matching and IPTW. IV methods do not focus on the average causal effect for the population, but the local average treatment effect (the target of inference):

$$E(Y^{Z=1}|A^0 = 0 \cap A^1 = 1) - E(Y^{Z=0}|A^0 = 0 \cap A^1 = 1)$$
$$= E(Y^{Z=1} - Y^{Z=0}|compliers)$$
$$= E(Y^{A=1} - Y^{A=0}|compliers)$$

This is known as Complier Average Causal Effect (CACE). This is causal because it contrasts counterfactuals in a common population. There is no inference about defiers, always-takers, or never-takers.

Assumptions about IVs:

1. It is associated with the treatment
2. It affects the outcome only through its effect on treatment (exclusion restriction)
3. Monotonicity (there are no defiers)

Practice Quiz

1. An instrumental variable is one that:
   a. Affects treatment and only affects the outcome through its effect on the treatment
2. The potential treatment values of an always taker are:
   a. $A^0 = 1, A^1 = 1$
3. The monotonicity assumption implies there are no:
   a. Defiers

Causal Effect Identification and Estimation

Recall that when identifying Causal Effects, the goal is to estimate $E(Y^{A=1} - Y^{A=0}|compliers)$

Starting with the Intention to Treat (ITT) effect: $E(Y^{Z=1}) - E(Y^{Z=0}) = E(Y|Z = 1) - E(Y|Z = 0)$

The expected value of Y among people assigned treatment is a weighted average of the expected value of Y given Z=1 in the three subpopulations (not including defiers as monotonicity says they do not exist). Let NT = never-takers, AT = always-takers, C=compliers, then:

$$E(Y|Z = 1) = E(Y|Z = 1 \cap AT)P(AT) + E(Y|Z = 1 \cap NT)P(NT) + E(Y|Z = 1 \cap C)P(C)$$

Note that among always-takers and never-takers, encouragement (Z) doesn't do anything. Therefore:

$E(Y|Z = 1 \cap AT) = E(Y|AT)$
$E(Y|Z = 1 \cap NT) = E(Y|NT)$

Also note that by randomization:

$P(AT|Z) = P(AT)$

Thus we can simplify the expected value of Y given Z:

$$E(Y|Z=1) = E(Y|AT)P(AT) + E(Y|NT)P(NT) + \ E(Y|Z=1 \cap C)P(C)$$

And the equation looks similar for the subpopulation of Z=0:

$$E(Y|Z=0) = E(Y|AT)P(AT) + E(Y|NT)P(NT) + \ E(Y|Z=0 \cap C)P(C)$$

Therefore the Intention to Treat effect can be simplified:

$$E(Y|Z=1) - E(Y|Z=0)$$
$$= E(Y|AT)P(AT) + E(Y|NT)P(NT) + \ E(Y|Z=1 \cap C)P(C) - E(Y|AT)P(AT) + E(Y|NT)P(NT) + \ E(Y|Z=0 \cap C)P(C)$$
$$= \mathrm{E}(Y|Z=1 \cap C)P(C) - \ E(Y|Z=0 \cap C)P(C)$$

This implies:

$$\frac{E(Y|Z=1) - E(Y|Z=0)}{P(C)} = \mathrm{E}(Y|Z=1 \cap C) - \mathrm{E}(Y|Z=0 \cap C)$$

Then this can be rewritten as:

$$\frac{E(Y|Z=1) - E(Y|Z=0)}{P(C)} = \mathrm{E}(Y^{a=1}|C) - \mathrm{E}(Y^{a=0}|C) = CACE$$

Now we have that $P(C) = E(A|Z=1) - E(A|Z=0)$

- Since the difference above is (proportion who are AT or C) - (proportion who are AT)

Then the Complier Average Causal Effect formula can further be simplified as the causal effect of treatment assignment on the outcome (ITT) over the causal effect of treatment assignment on the treatment received:

$$CACE = \frac{E(Y|Z=1) - E(Y|Z=0)}{E(A|Z=1) - E(A|Z=0)}$$

Note:

- In cases of perfect compliance, CACE = ITT
- The denominator will always be between 0 and 1, so CACE will be at least as large as ITT
  - The ITT is an underestimate of CACE since some people assigned to treatment do not take it

IVs in Observational Studies

IVs can be used in observational studies and they can be thought of as randomizers in natural experiments. If Z is binary, it is encouragement yes/no. If Z is continuous, it is a "dose" of encouragement. The key challenge is to think of a variable that affects treatment but not the outcome (exclusion restriction). The validity of this restriction often comes down to domain knowledge.

Two Stage Least Squares

Two stage least squares is a method for estimating a causal effect in the IV setting. In ordinary least squares, there is an assumption that the error term is independent from the predictor variables, which is not true in in the IV setting.

- Stage 1: A new variable is created using the IV
  - Regress treatment received A on the instrumental variable Z
- Stage 2: the model-estimated values from stage one are used in place of the actual values of the problematic predictors to compute an OLS model for the response of interest.
  - Regress the outcome Y on the fitted value from stage 1

Weak Instruments

The strength of an IV is how well it predicts treatment received.

- A Strong Instrument is highly predictive; encouragement greatly increases probability of treatment
- A Weak Instrument is weakly predictive; encouragement barely increases probability of treatment

The strength can be measured as an estimate of the proportion of compliers, using the observed proportion of treated subjects for Z=1 and Z=0: $E(A|Z=1) - E(A|Z=0)$

Weak instruments cause issues, such as very large variance estimates, which means the estimate of causal effects is unstable.

Practice Quiz

1. The second stage of two stage least squares is:
   a. Regression of the outcome on the predicted value of treatment

2. A strong instrument is one that:
    a. Is highly predictive of treatment

IV Quiz

1. The Intention to Treat effect is:
    a. The causal effect of treatment assignment on the outcome
2. The potential treatment values of a complier are:
    a. $A^1 = 1, A^0 = 0$
3. Treatment assignment has no impact on treatment received for:
    a. Always takers and never takers
4. In instrumental variable analysis, what is meant by a local treatment effect?
    a. Treatment effect among compliers only
5. The monotonicity assumption is necessary for estimation of:
    a. Complier Average Causal Effect
6. If we do not make the monotonicity assumption, somebody who was observed to have Z=1 and A=0 is:
    a. Either a never taker or defier
7. If we do make the monotonicity assumption, somebody who was observed to have Z=1 and A=0 is:
    a. Definitely a never taker
8. The assumption that the instrumental variable Z only affects the outcome through its effect on treatment is called:
    a. The Exclusion Restriction
9. The first stage of two stage least squares is:
    a. Regression of treatment received on the instrumental variable
10. The strength of an instrumental variable can be measured by:
    a. The association of an instrumental variable and the treatment
11. A weak instrument will tend to lead to estimates of causal effects that have:
    a. Large standard errors
12. Instrumental variable methods do not require:
    a. The Ignorability Assumption