

D03 NumPy 陣列運算及數學 Universal Functions (ufunc)



簡報閱讀



範例與作業



問題討論



學習心得(完成)

重要知識點

NumPy 陣列運算 - 四則運算

NumPy 陣列運算 - 次方
np.power()

NumPy 陣列運算 - 平方
根 np.sqrt()

NumPy 陣列運算 - 歐拉
數 (Euler's number) 及...



重要知識點



NumPy 陣列運算 - 取近

重要知識點

- NumPy 提供許多數學及統計的函式，這些函式的用法都很相似，針對陣列進行 element-wise 的操作，並回傳陣列做為輸出，稱為 Universal Functions (ufunc)。

NumPy 陣列運算 - 四則運算

陣列的加減乘除四則運算，可以使用運算子 (+, -, *, /) 或是呼叫函式來進行，語法及對照如下表：

運算子	函式	
$a + b$	<code>np.add(a, b)</code>	加法
$a - b$	<code>np.subtract(a, b)</code>	減法
$a * b$	<code>np.multiply(a, b)</code>	乘法
a / b	<code>np.divide(a, b)</code>	除法
$a \% b$	<code>np.mod(a, b)</code>	求餘數

以上運算子的部分，使用與 Python 相同的運算子。

element-wise 運算。規則如下：

- 兩個陣列形狀完全相同
- 比較兩個陣列的維度，如果維度的形狀相同的話，可以進行廣播
- 比較兩個陣列的維度，其中一個維度為 1 的話，可以進行廣播

NumPy 陣列運算 – 次方 np.power()

次方的運算，跟四則運算一樣，也要遵循上述的規則，才能成功進行運算。語法如下表：

運算子	函式	a 的 b 次方
<code>a ** b</code>	<code>np.power(a, b)</code>	a^b

NumPy 陣列運算 – 平方根 np.sqrt()

基本語法: `np.sqrt(a)`，對陣列進行 element-wise 的平方根。

範例：

```
[18]: np.sqrt(4)
[18]: 2.0
[19]: np.sqrt(a)
[19]: array([1.          , 1.41421356, 1.73205081, 2.          , 2.23606798])
```

1 `np.sqrt(4)`

NumPy 陣列運算 – 歐拉數 (Euler's number) 及指數函式 `np.exp()`

NumPy 提供歐拉常數 e (`np.e`)，以及指數函式 `np.exp()`，表示 e^x 。

範例：

```
[20]: np.e
[20]: 2.718281828459045
[21]: np.exp(1)
[21]: 2.718281828459045
[22]: np.exp(np.arange(5))
[22]: array([ 1.         ,  2.71828183,  7.3890561 , 20.08553692, 54.59815003])
```

NumPy 陣列運算 – 對數函式

`log` 函式如下表：

函式

`np.log(x)` 底數為 e

`np.log2(x)` 底數為 2

`np.log10(x)` 底數為 10

`np.log1p(x)` 底數為 e ，計算 $\log(1+x)$

若要使用其他底數，可以用下列的方法 (以底數 3 為例)。

```
[24]: np.log(9)/np.log(3)
```

```
[24]: 2.0
```

```
1 np.log(9)/np.log(3)
```

若是 $\log(\text{負數})$ 則會產生 nan 常數，NaN / NAN 為 nan (not a number) 的別名。

```
np.log([-1, 1, 2])
c:\python\python36_64\lib\site-packages\ipykernel_launcher.py:1: RuntimeWarning: invalid value encountered in log
    """Entry point for launching an IPython kernel.
array([      nan,  0.        ,  0.69314718])
```

NumPy 陣列運算 – 取近似值

取近似值的函式及說明如下表：

函式	常用語法	說明
round(), around()	ndarray.round(decimals=0) numpy.round(a, decimals=0) numpy.around(a, decimals=0)	在 Rounding 的方法部分，與 Python 同樣採用 IEEE 754 規範，四捨、五取最近偶數、六入，而非我們一般講的四捨五入。 round 與 around 用法及結果相同
rint()	numpy.rint(a)	Round至最近的整數
trunc()	numpy.trunc(a)	無條件捨去小數點
floor()	numpy.floor(a)	向下取整數
ceil()	numpy.ceil(a)	向上取整數
fix()	numpy.fix(a)	向0的方向取整數

NumPy 陣列運算 – 取絕對值： np.abs(), np.absolute(), np.fabs()

- np.abs() 是 np.absolute() 的簡寫，兩者完全相同；np.fabs() 的差異在於無法處理複數 (Complex)。

NumPy 陣列運算 – 點積 (dot product)

- 進行點積運算須注意形狀必須注意形狀 (shape)。若是兩個向量的點積，兩個向量的元素數目也須相同，或其中一個數目為 1 (廣播)。
- 若是兩個多維陣列 (矩陣) 的點積，則中間兩個大小要相同才能進行點積，例如：
(2,3)·(3,4)→ 成為 (2,4)
- 如果形狀不符合則無法進行點積。
- 點積運算示意圖：

$$\begin{aligned}
 A &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \\
 B &= \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix} \\
 A \cdot B &= \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} & a_{11} * b_{12} + a_{12} * b_{22} + a_{13} * b_{32} & a_{11} * b_{13} + a_{12} * b_{23} + a_{13} * b_{33} & a_{11} * b_{14} + a_{12} * b_{24} + a_{13} * b_{34} \\ a_{21} * b_{11} + a_{22} * b_{21} + a_{23} * b_{31} & a_{21} * b_{12} + a_{22} * b_{22} + a_{23} * b_{32} & a_{21} * b_{13} + a_{22} * b_{23} + a_{23} * b_{33} & a_{21} * b_{14} + a_{22} * b_{24} + a_{23} * b_{34} \end{bmatrix}
 \end{aligned}$$

延伸閱讀^[1]

ufunc的清單及文件

網站：[Universal functions \(ufunc\)](#)

- Universal function or ufunc
- Broadcasting
- Output type determination
- Use of internal buffers
- Error handling
- Casting Rules
- Overriding Ufunc behavior
- ufunc
 - Optional keyword arguments
 - Attributes
 - Methods
- Available ufuncs
 - Math operations
 - Trigonometric functions
 - Bit-twiddling functions
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 - Floating functions

Previous topic

Constants

Next topic

numpy.setbufsize

Quick search

A universal function (or `ufunc` for short) is a function that operates on `ndarrays` in an element-by-element fashion, supporting [array broadcasting](#), type casting, and several other standard features. That is, a `ufunc` is a “[vectorized](#)” wrapper for a function that takes a fixed number of specific inputs and produces a fixed number of specific outputs.

In NumPy, universal functions are instances of the `numpy.ufunc` class. Many of the built-in functions are implemented in compiled C code. The basic `ufuncs` operate on scalars, but there is also a generalized kind for which the basic elements are sub-arrays (vectors, matrices, etc.), and broadcasting is done over other dimensions. One can also produce custom `ufunc` instances using the `frompyfunc` factory function.

Broadcasting

Each universal function takes array inputs and produces array outputs by performing the core function element-wise on the inputs (where an element is generally a scalar, but can be a vector or higher-order sub-array for generalized `ufuncs`). Standard broadcasting rules are applied so that inputs not sharing exactly the same shapes can still be usefully operated on. Broadcasting can be understood by four rules:

- All input arrays with `ndim` smaller than the input array of largest `ndim`, have 1's prepended to their shapes.
- The size in each dimension of the output shape is the maximum of all the input sizes in that dimension.
- An input can be used in the calculation if its size in a particular dimension either matches the output size in that dimension, or has value exactly 1.
- If an input has a dimension size of 1 in its shape, the first data entry in that dimension will be used for all calculations along that dimension. In other words, the stepping machinery of the `ufunc` will simply not step along that dimension (the `stride` will be 0 for that dimension).

Broadcasting is used throughout NumPy to decide how to handle disparately shaped arrays; for example, all arithmetic operations (`+`, `-`, `*`, `/`, ...) between `ndarrays` broadcast the arrays before operation.

A set of arrays is called “broadcastable” to the same shape if the above rules produce a valid result, i.e., one of the following is true:

- The arrays all have exactly the same shape.
- The arrays all have the same number of dimensions and the length of each dimensions is either a common length or 1.
- The arrays that have too few dimensions can have their shapes prepended with a dimension of length 1 to satisfy property 2.

Example:

If `a.shape` is (5,1), `b.shape` is (1,6), `c.shape` is (6,) and `d.shape` is () so that `d` is a scalar, then `a`, `b`, `c`, and `d` are all broadcastable to dimension (5,6); and

- `a` acts like a (5,6) array where `a[i,0]` is broadcast to the other columns,
- `b` acts like a (5,6) array where `b[0,i]` is broadcast to the other rows,
- `c` acts like a (1,6) array and therefore like a (5,6) array where `c[i,1]` is broadcast to every row, and finally,
- `d` acts like a (5,6) array where the single value is repeated.

[下一步：閱讀範例與完成作業](#)