Air Gap Shape Optimization for Minimizing Proximity Effect Losses in an Inductor

Stéphane Gaydier[1,2,3], Itaï Zehavi[4], Théodore Cherrière[5], Peter Gangl[6]

[1] Schneider Electric, Global Technology Eybens, France
[2] Univ. Grenoble Alpes, CNRS, Grenoble INP, G2ELab, F-38000, Grenoble, France
[3] Univ. Grenoble Alpes, CNRS, Grenoble INP, LJK, 38000 Grenoble, France
[4] Electrical Engineering Department, ENS Paris-Saclay, Université Paris-Saclay, 91190 Gif-sur-Yvette, France
[5] Université Paris-Saclay, CentraleSupélec, Sorbonne Université, CNRS, GeePs, 91192 Gif-sur-Yvette, France
[6] Johann Radon Institute for Computational and Applied Mathematics (RICAM), ÖAW, 4040 Linz, Austria

Introduction

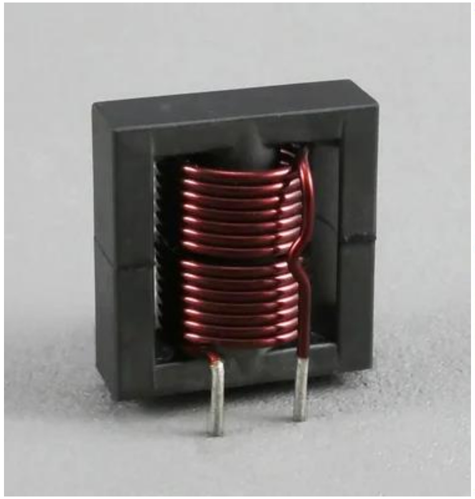
We want to optimize an **inductor** to minimize the AC losses due to the **fringing effect**.

Possible solutions :

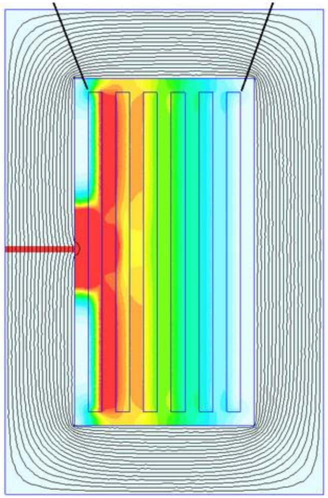
- Distributed air gaps
- Change winding distribution of the conductor [1]
- Optimize air gap profile (e.g. with heuristic methods [2])

Our proposition : We optimize the **air gap profile** using **gradient descent methods.**

- Converge faster
- More flexible parametrization



An inductor [3]



Fringing Effect [4]

[1] R.A. Jensen and C.R. Sullivan, “Optimal core dimensional ratios for minimizing winding loss in high-frequency gapped-inductor windings,”

[2] D.I. Zaikin, S. Jonasen, and S.L. Mikkelsen, “An Air-Gap Shape Optimization for Fringing Field Eddy Current Loss Reductions in Power Magnetics,”

[3] https://info.triadmagnetics.com/blog/basics-of-inductors

[4] https://www.e-magnetica.pl/doku.php/proximity_effect

Outline

Part I – Optimization Problem

Device and modeling

Device: A simplified inductor composed of a coil and a magnetic core

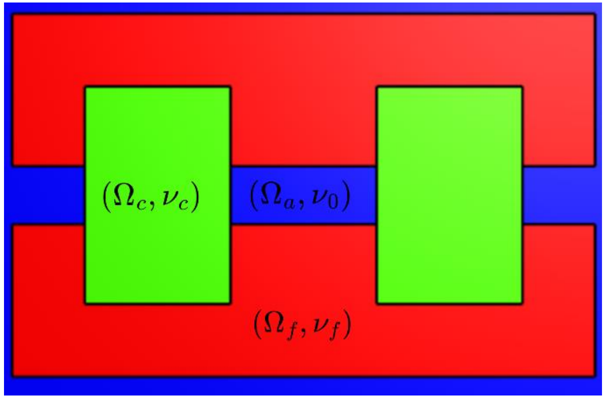
Goal: Minimizing the **AC losses** in the conductor keeping the **inductance** at $L_0 = 1$ mH.

Magneto-harmonic equation: $\boxed{-div(\underline{\nu}\nabla\underline{a}) = j}$,

where :

- $\underline{a}$ is magnetic vector potential out-of-plane component
- j is real current density,
- and $\underline{\nu}$ is the reluctivity which takes values :

Reluctivity value (ν)	Subdomain
$\nu_0 = 1/\mu_0$	Air region Ω_a
$\nu_f = \nu_0/1000$	Core region (ferrite) Ω_f
$\boxed{\nu_c = \nu_0 \exp(i\delta)}$ with $\delta = 0.1\ rad$ to model proximity effects at $\mathrm{f} = 50\ \mathrm{k}Hz$	Conductor region (copper) Ω_c



Reference design

Objective and Constraint

The retained formulas are:

AC Losses:

$$P(\underline{a}) = l_z \pi f \int_{\Omega_c} Im\left(\underline{\nu_c}\right) \left|\nabla \underline{a}\right|^2 dx$$

Inductance:

$$L(\underline{a}) = \frac{l_z}{I^2} \int_{\Omega} Re(\underline{\nu}) \left|\nabla \underline{a}\right|^2 dx$$

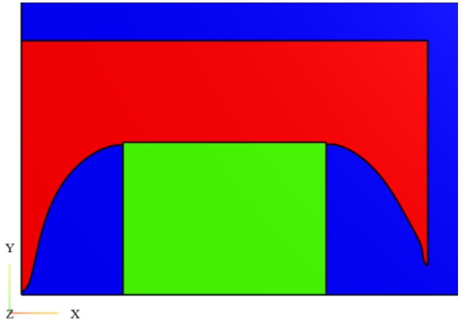
with $l_z = 1\ cm$ (thickness) and $I = 2\ A$ (current, 200 turns).

Parametrization

Let h be a **parametrization** of the air gap profile.

The following are dependent on h:

- the partition of the domain $\Omega = \Omega_{a,h} \cup \Omega_{f,h} \cup \Omega_{c,h}$,
- the reluctivity $\underline{\nu_h}$,
- the magnetic potential $\underline{a_h}$ solution of (E_h)



Parametrized air gap

$$\text{Find } \underline{a_h} \in H_0^1(\Omega) \text{ such that } \forall \underline{v} \in H_0^1(\Omega), \int_\Omega \underline{\nu_h} \nabla \underline{a_h} \cdot \nabla \underline{v}\, dx = \int_{\Omega_{c,h}} j \underline{v}\, dx \qquad (E_h)$$

Then, the objective and constraint rewrites into :

AC Losses:

$$P(h, \underline{a_h}) = l_z \pi f \int_{\Omega_{c,h}} Im\left(\underline{\nu_c}\right) \left|\nabla \underline{a_h}\right|^2 dx$$

Inductance:

$$L\left(h, \underline{a_h}\right) = \frac{l_z}{I^2} \int_\Omega Re\left(\underline{\nu_h}\right) \left|\nabla \underline{a_h}\right|^2 dx$$

Constrained Minimization Problem

Let h be a **parametrization** of the air gap profile.

We aim to solve the following minimization problem to determine the optimal h:

$$\boxed{\min_{h} \; P\left(h, \underline{a_h}\right) \text{ subject to } \begin{cases} L\left(h, \underline{a_h}\right) = 1\, mH \\ \mathrm{a_h} \text{ solution of } (E_h) \end{cases}}$$

Challenges and Solutions

Challenges :	Proposed solutions [1]:
Find good parametrizations h of the air gap.	Control point vs Adaptative Mesh deformation.
Differentiate P and L with respect to h…	**Shape derivatives**…
…and to the PDE constraint !	…obtained by the **adjoint method**.
Find descent directions (How to find d such that $\frac{dP}{dh}(h, a_h)d < 0$?) that preserve the conductor domain.	**Hilbertian extension-regularization.**

We are using a handwritten **augmented Lagrangian** algorithm to solve this constrained optimization problem.

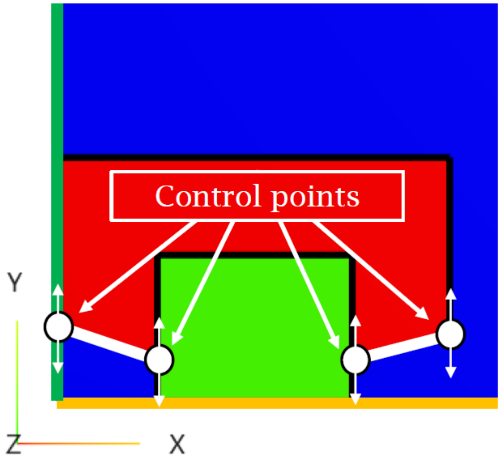
[1] G. Allaire, C. Dapogny, and F. Jouve, “Shape and topology optimization,” in Handbook of Numerical Analysis

Part II - Methodology

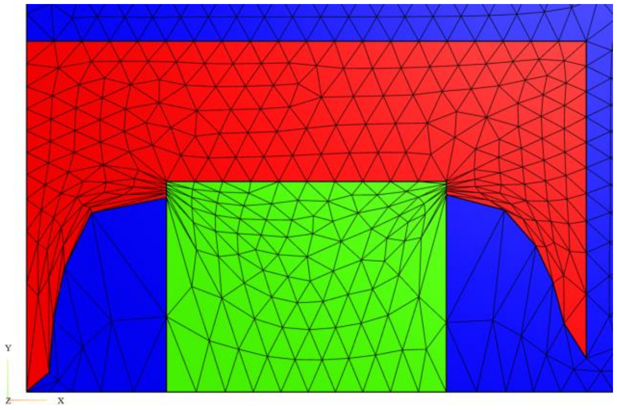
Two Choices for the Parametrization h

$$\boldsymbol{h} = (\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4)$$

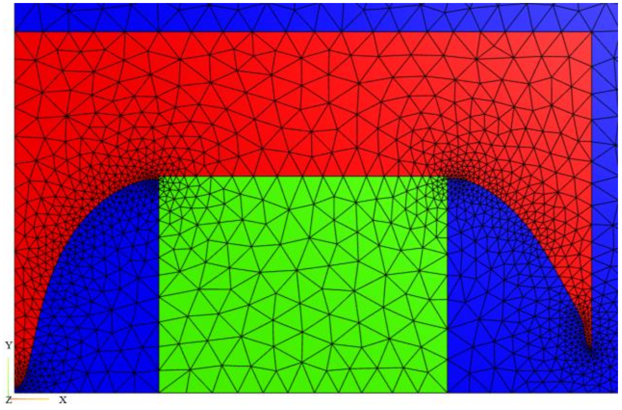
$$\boldsymbol{h} = (\boldsymbol{\Omega}_a, \boldsymbol{\Omega}_f, \boldsymbol{\Omega}_c)$$



Parametric Shape Optimization using
Control points



Geometric Shape Optimization by
mesh deformation



Geometric Shape Optimization
by **adaptative** mesh deformation
using MMG [1]

[1] https://www.mmgtools.org/

Shape derivatives

Shape derivative $\approx$ derivative of a functional w.r.t. the deformation of a domain

Considering:

- A bounded Lipschitz domain Ω
- “Small and regular enough” **deformation fields** ϕ
- A real-value function $J(\Omega)$ of the domain

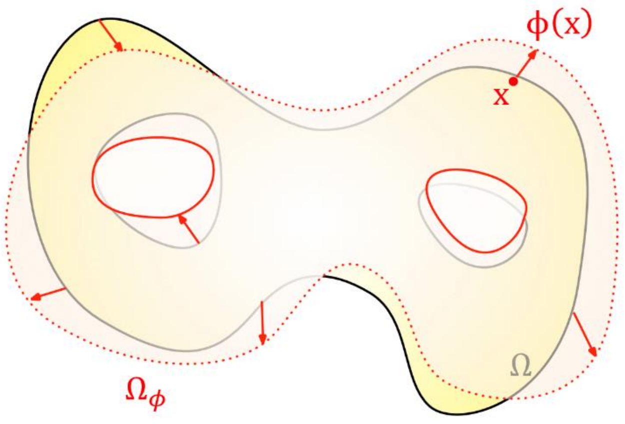
We define the **deformed domain** with respect to ϕ to be $\Omega_\phi = (Id + \phi)(\Omega)$

The **shape derivative** of $J(\Omega)$ is the derivative of $\phi \mapsto J(\Omega_\phi)$ at $\phi = 0$:

$$J(\Omega_\phi) = J(\Omega) + J'(\Omega)(\phi) + o(\phi)$$

We are searching a ϕ that is a **descent direction**:

$$\boxed{J'(\Omega)(\phi) < 0}$$



Deformed domain

Two approaches:

In our “control points” approach, ϕ is defined on the control points

In our “adaptative mesh deformation” approach, ϕ is defined on the mesh nodes.

Example of a Shape Derivative

For example, if we take a function $J(\Omega) = \int_\Omega f\,dx$,

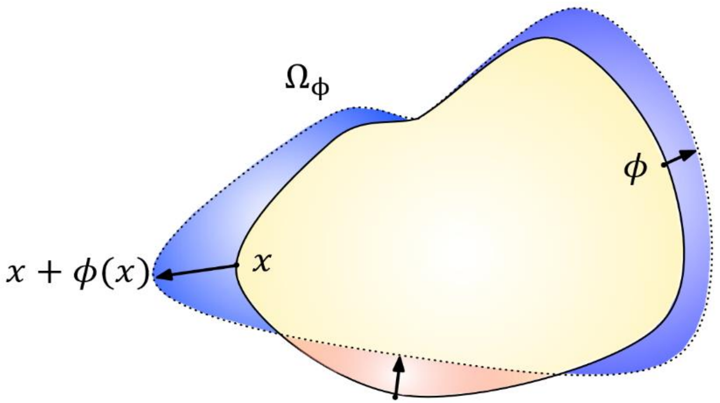
its shape derivative is:

$$J'(\Omega)(\phi) = \int_{\Omega} div(f\phi)dx = \int_{\partial\Omega} f\phi \cdot n\, ds$$

This is the **Reynolds transports theorem**!

Finding a descent direction is obvious! For example:

$$\phi = -fn$$



Minimization of the integral.
The deformation $\phi = -fn$ makes Ω
shrink in the (red) region where $f(x) > 0$,
and expend in the (blue) region where $f(x) < 0$

Shape Derivatives of Losses & Inductance

Recalling the formulas for the AC losses and the inductance :

- Losses : $P(\underline{a}) = l_z \pi f \int_{\Omega_c} Im\left(\underline{\nu_c}\right) \left|\nabla \underline{a}\right|^2 dx$
- Inductance : $L(\underline{a}) = \frac{l_z}{I^2} \int_{\Omega} Re(\underline{\nu}) \left|\nabla \underline{a}\right|^2 dx$

We introduce $\underline{\lambda_P}$ and $\underline{\lambda_L}$ the **adjoint states** given by the **adjoint equations** :

- $-div\left(\underline{\nu}^* \nabla \underline{\lambda_P}\right) = -\frac{dP}{d\underline{a}}(\underline{a})$
- $-div\left(\underline{\nu}^* \nabla \underline{\lambda_L}\right) = -\frac{dL}{d\underline{a}}(\underline{a})$

where the right-hand members are complex Fréchet-derivatives of real functions.

We compute the **shape derivatives** using the **adjoint method** and obtain :

- $P'(\Omega)(\phi) = Re\left(l_z \pi f \int_{\Omega_c} Im\left(\underline{\nu_c}\right) A_\phi \nabla \underline{a} \cdot \left(\nabla \underline{a}\right)^* dx + \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot \left(\nabla \underline{\lambda_P}\right)^* dx - \int_{\Omega_c} j div\, \phi \underline{\lambda_P}^* dx\right)$
- $L'(\Omega)(\phi) = Re\left(\frac{l_z}{I^2} \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot \left(\nabla \underline{a}\right)^* dx + \int_{\Omega} \underline{\nu} A_\phi \nabla \underline{a} \cdot \left(\nabla \underline{\lambda_L}\right)^* dx - \int_{\Omega_c} j\, div\phi\, \underline{\lambda_L^*} dx\right)$

with the real 2×2 matrix $A_\phi = div\phi I_2 - \partial\phi - \partial\phi^\top$

Those derivative formulas don't directly reveal the descent directions; we need another step.

Hilbertian Extension-Regularization

The "Hilbertian extension-regularization" technique consists in solving a Lax-Milgram-type equation of the form:

$$\text{Find } \phi \in V \text{ such that } \forall \hat{\phi} \in V,\ \ \langle \phi, \hat{\phi} \rangle_V = J'(\Omega)(\hat{\phi})$$

where :

- V is any subspace of $\mathrm{H}^1(\Omega)$ (with wisely defined Dirichlet conditions),
- $\langle \cdot, \cdot \rangle_V$ is any inner product of V.

Then $-\phi$ is a descent direction : $J'(\Omega)(-\phi) = -\|\phi\|_V^2$.

This procedure yields a **regularized descent direction**

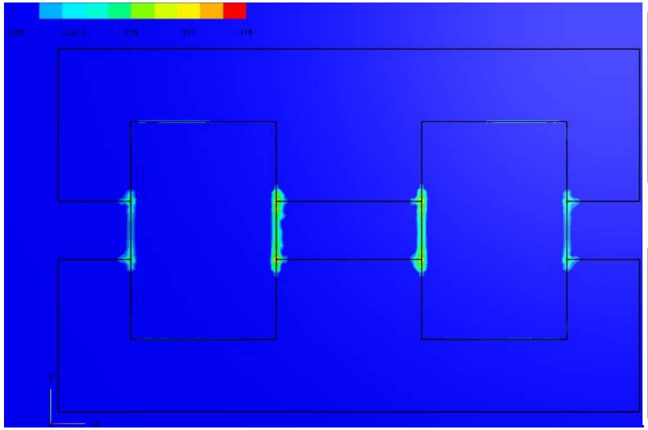
with a **structure** that can preserve the conductor's boundary.

Unregularized the Deformation Field

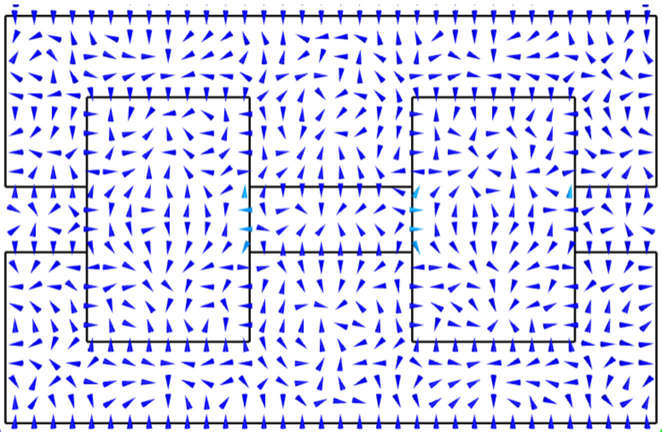
With a deformation field ϕ coming from:

$Find\ \phi \in L^2(\Omega)\ s.t. \forall \hat{\phi} \in L^2(\Omega),$

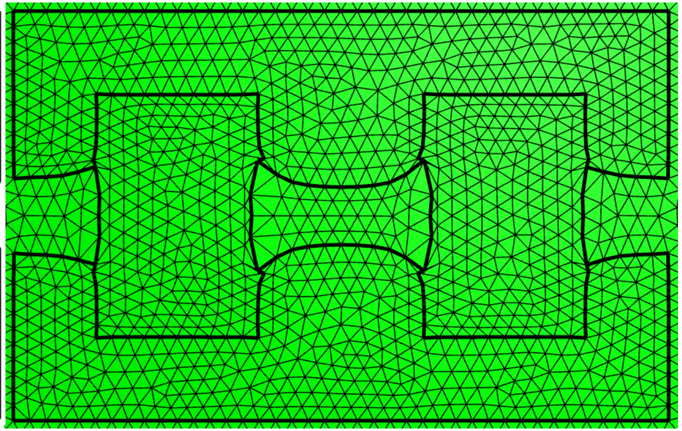
$$\langle \phi, \hat{\phi} \rangle_{L^2(\Omega)} = J'(\Omega)(\hat{\phi})$$



Norm of $-\phi$



Vector field $-\phi$



Deformed mesh

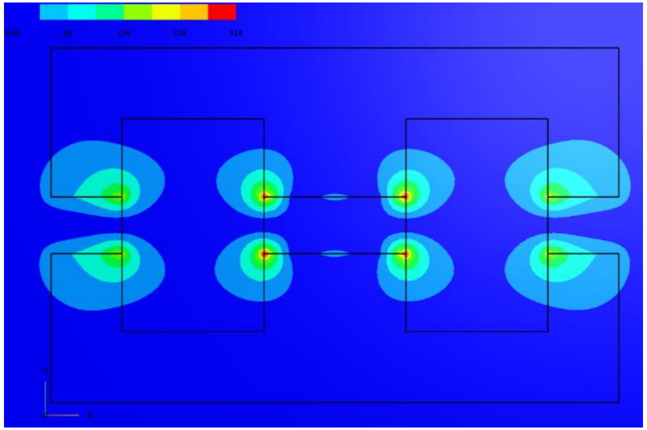
Regularized Deformation Field

With a deformation field ϕ coming from:

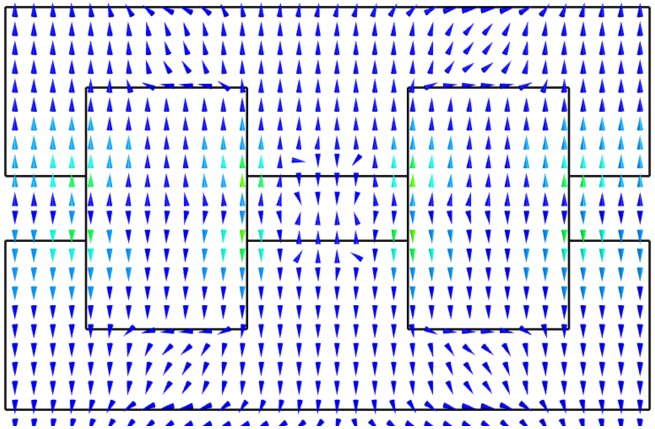
$Find\ \phi \in V\ s.t. \forall \hat{\phi} \in V,$

$$\langle \phi, \hat{\phi} \rangle_{L^2(\Omega)} + \langle \nabla\phi, \nabla\hat{\phi} \rangle_{L^2(\Omega)} = J'(\Omega)(\hat{\phi})$$

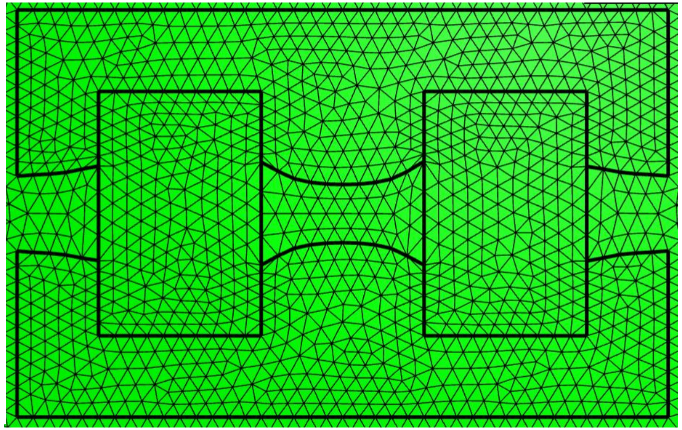
with $V = \{\phi \in H^1(\Omega) |\ \phi.\overrightarrow{e_x} = 0\ on\ \Gamma_{D,x}\ , \phi.\overrightarrow{e_y} = 0\ on\ \Gamma_{D,y}\}$



Norm of $-\phi$

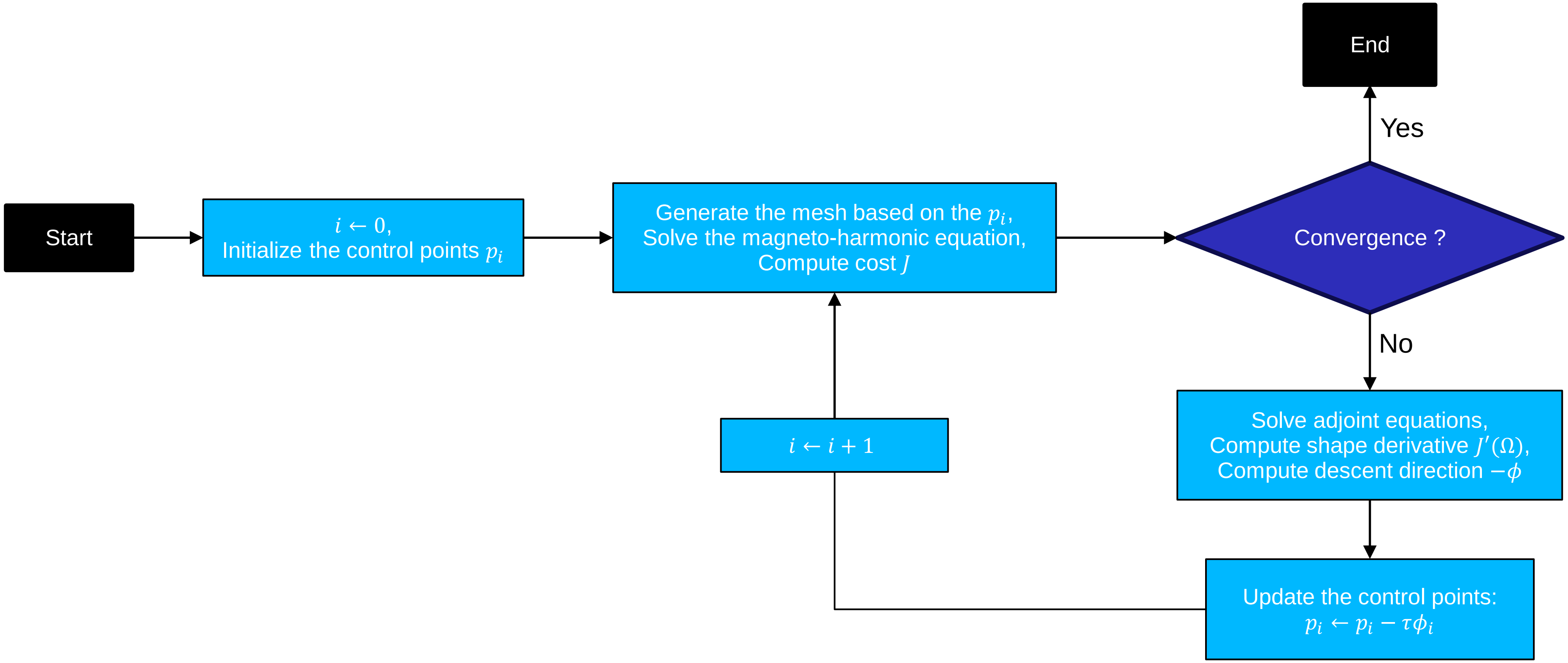


Vector field $-\phi$

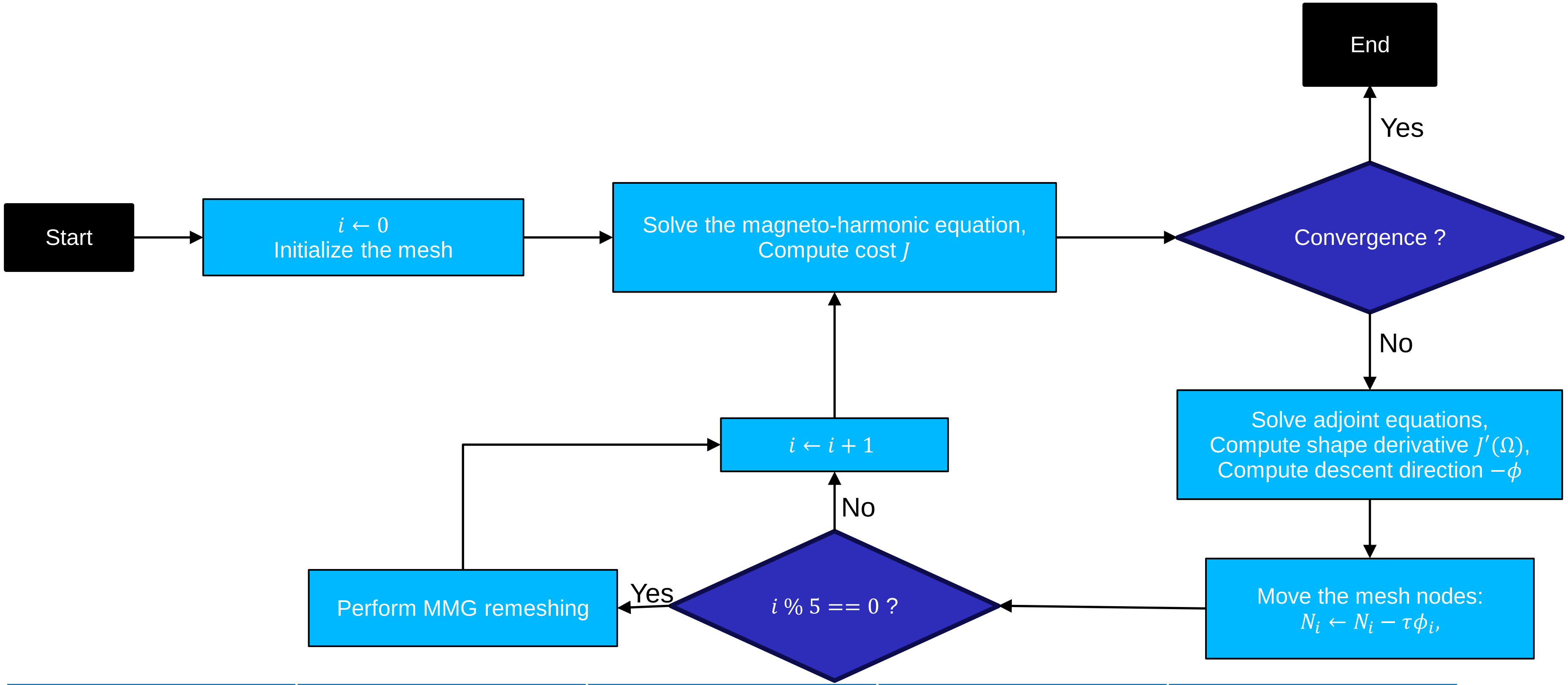


Deformed mesh

Control Points Flowchart

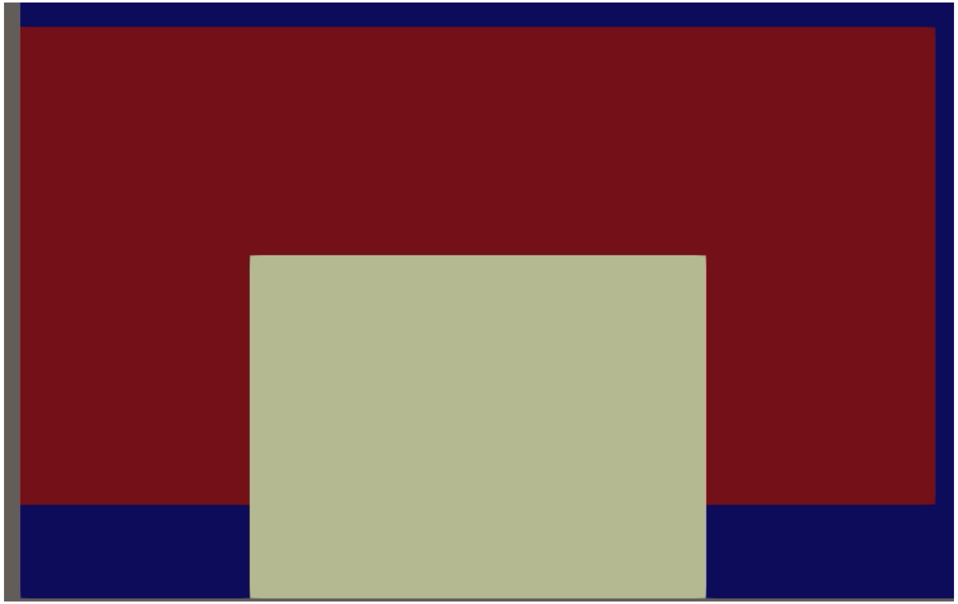


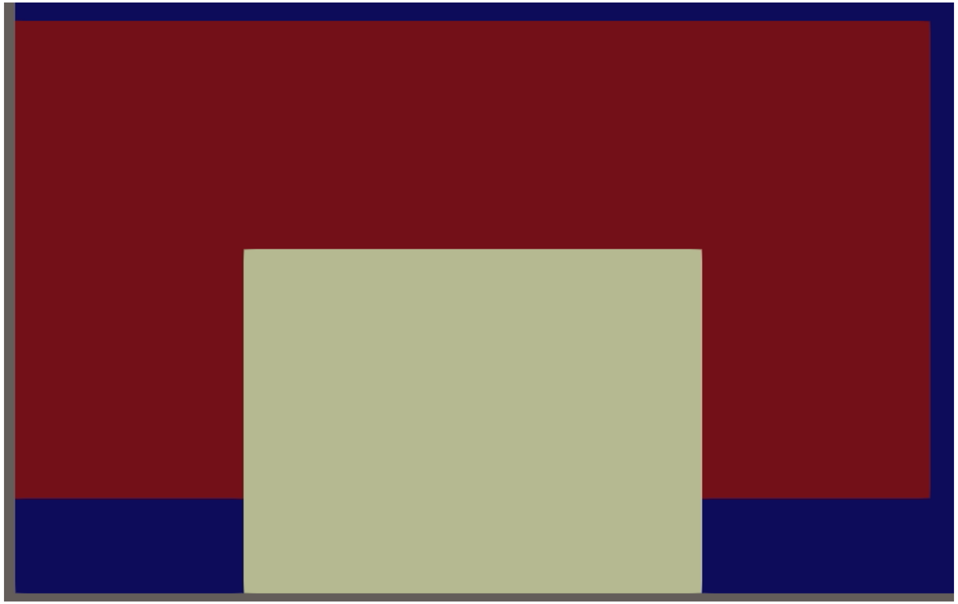
Adaptative Mesh Deformation Flowchart

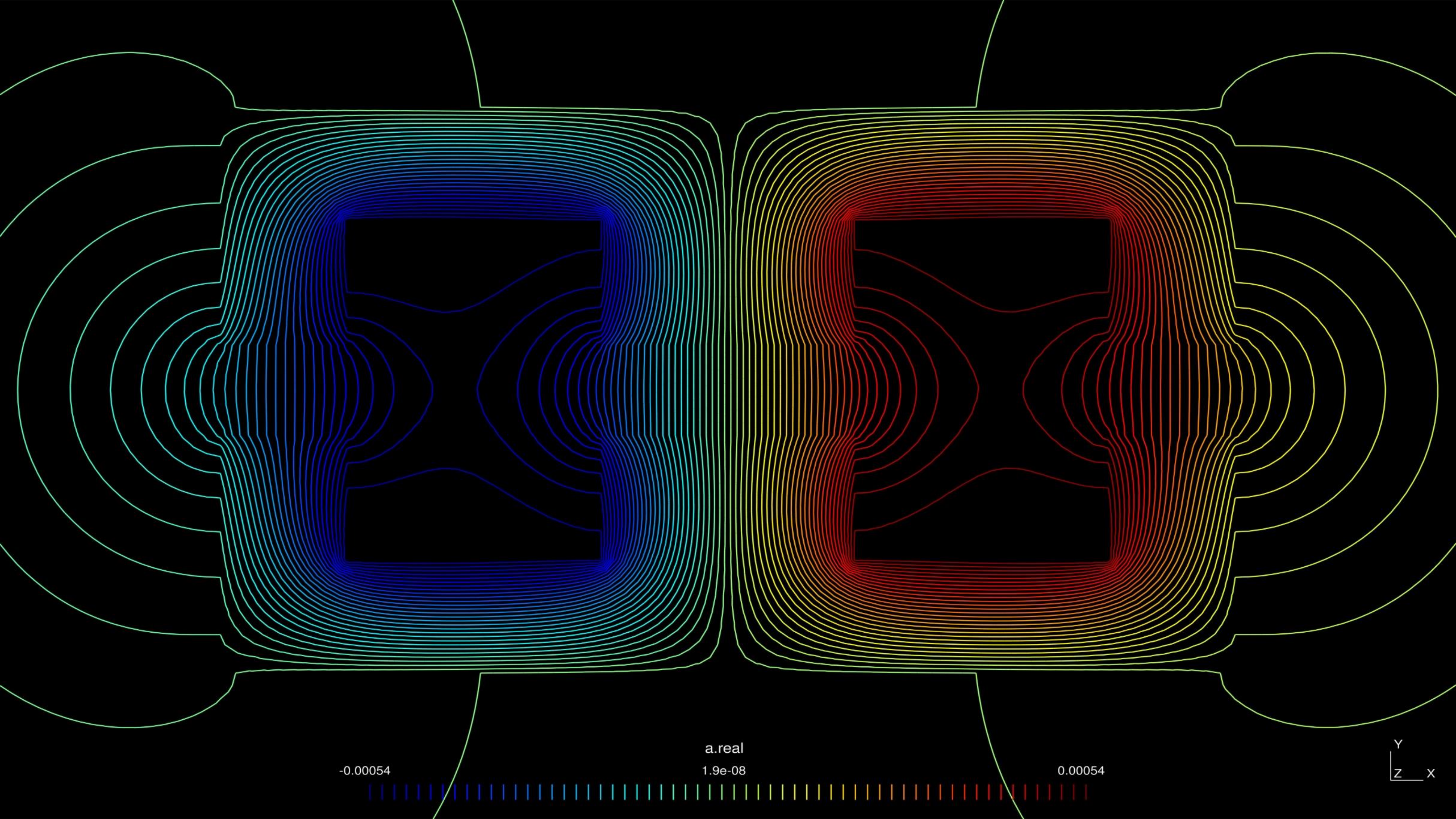


Part III – Optimization Results

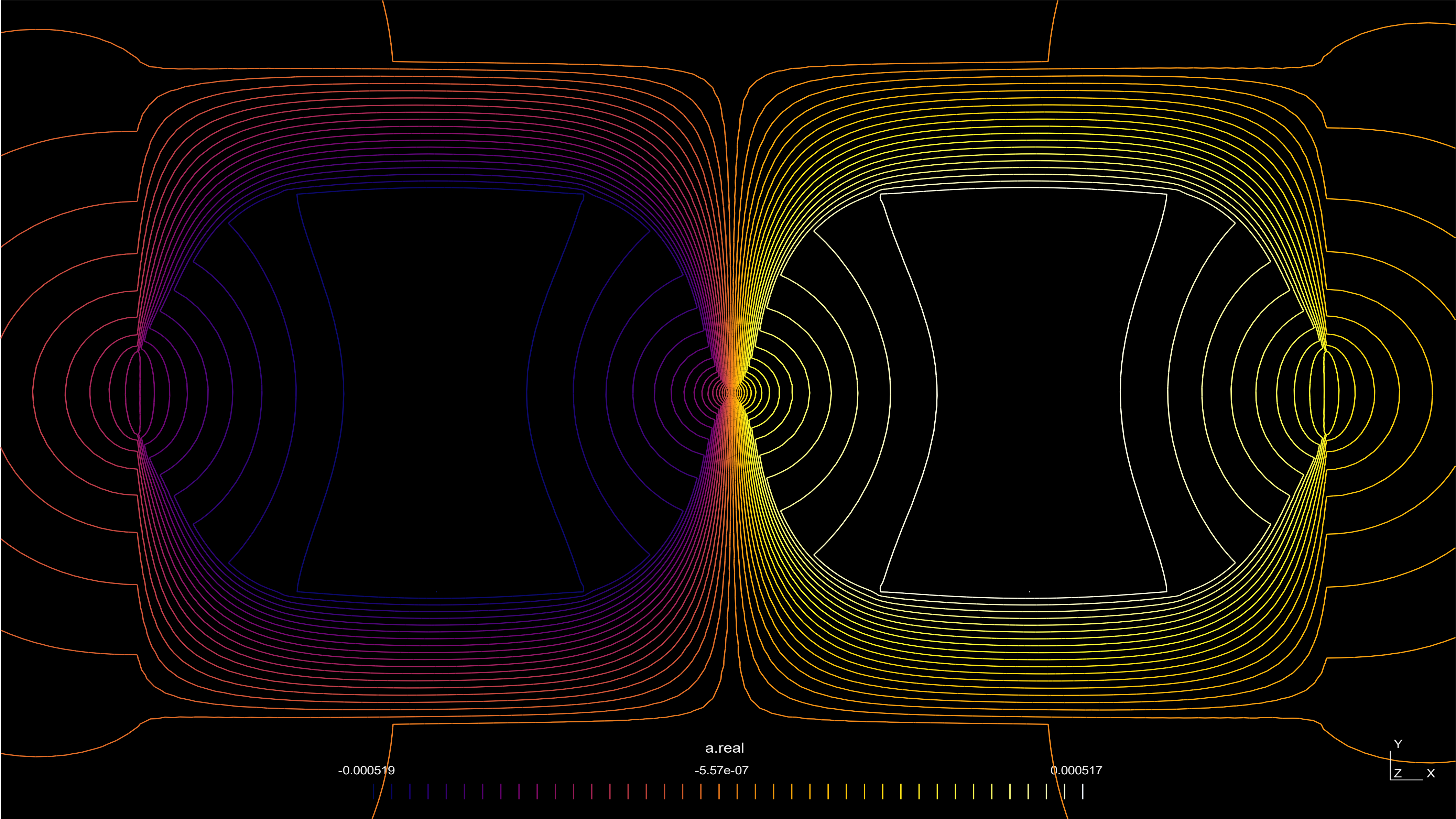
Results


Control points optimization results


Adaptative mesh deformation results



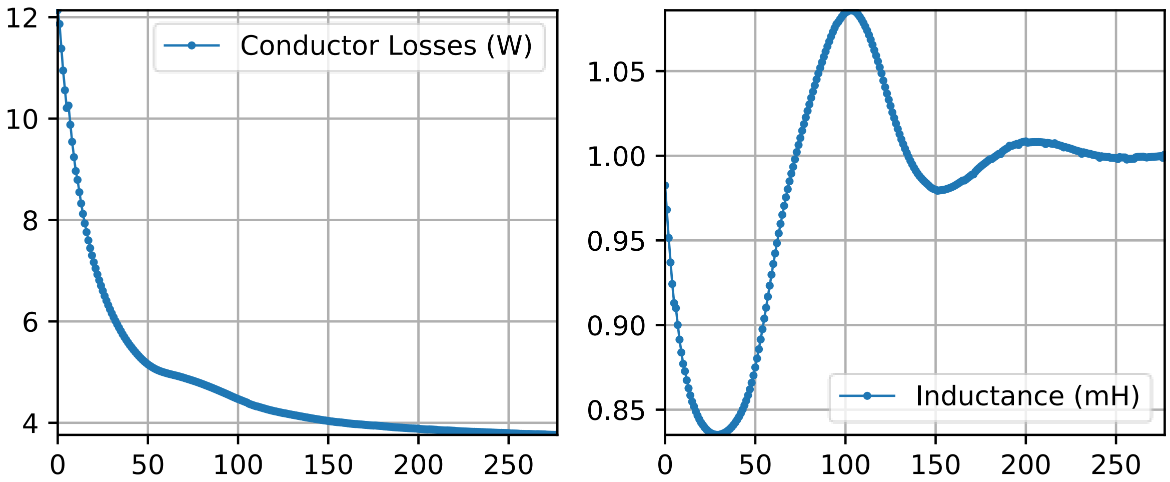
a.real
-0.00054
1.9e-08
0.00054
Y
Z
X


a.real
-0.000519
-5.57e-07
0.000517
Y
Z
X

Results – Performance and Convergence

Optimization Type	Iterations	P (W)	L (mH)
Reference	-	13.16	1.00
Control points	92	6.00	1.00
Adaptative mesh deformation	277	3.76	1.00

Optimization results



Adaptative mesh deformation convergence curves

Conclusion

We used **gradient-based** optimizations of the air gap profile of an inductor to minimize AC losses.

We leverage classical techniques such as:

- Control points, mesh deformations, remeshing
- Shape derivates
- Adjoint method
- Hilbertian extension-regularization

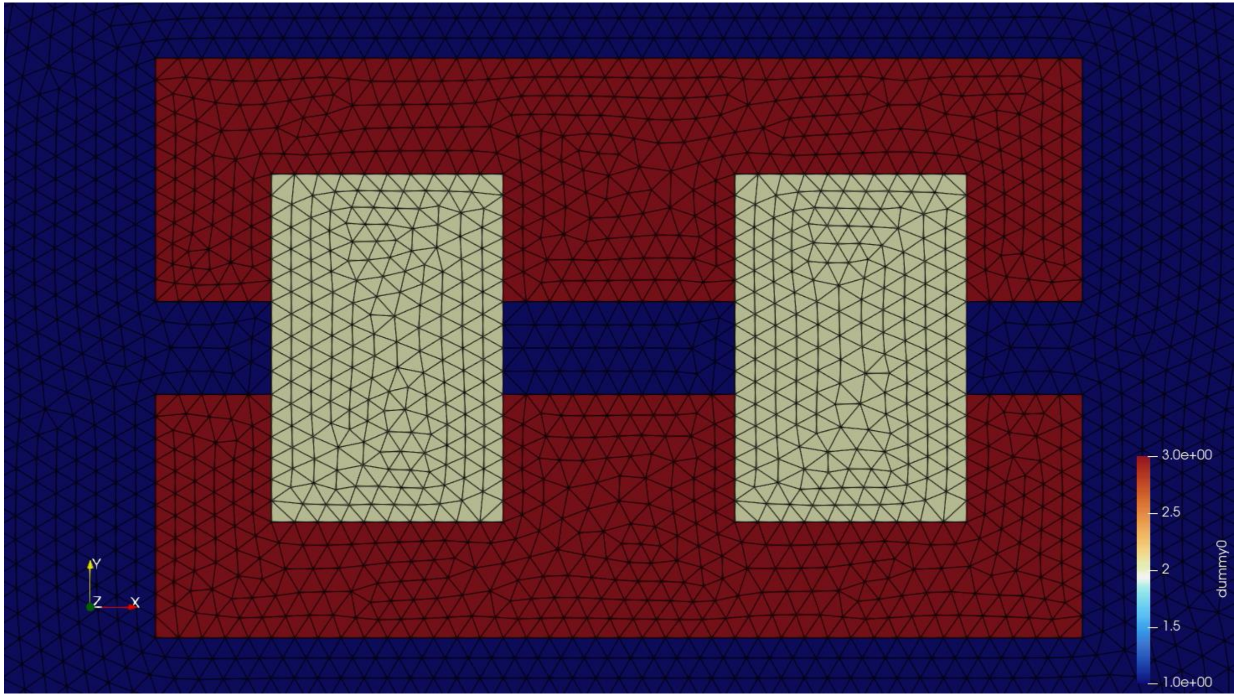
Perspective:

- Include magnetic saturation in the optimization
- Find better topology of the magnetic core
- Include iron losses

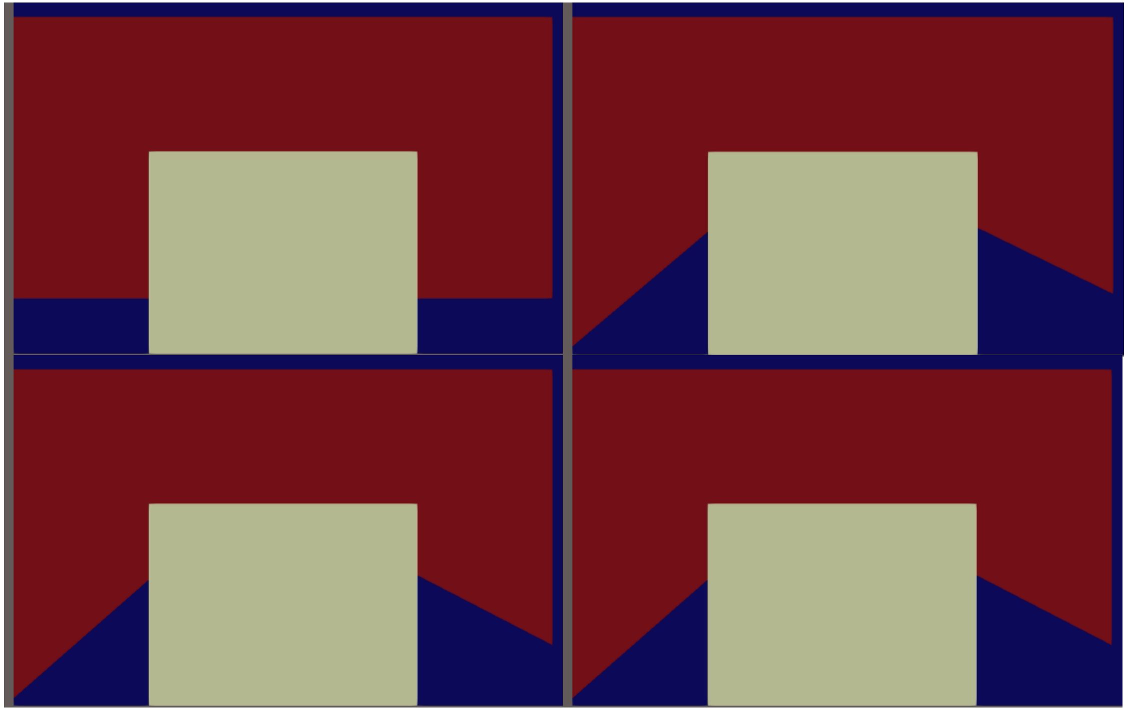
Thank You!

Code available on zenodo

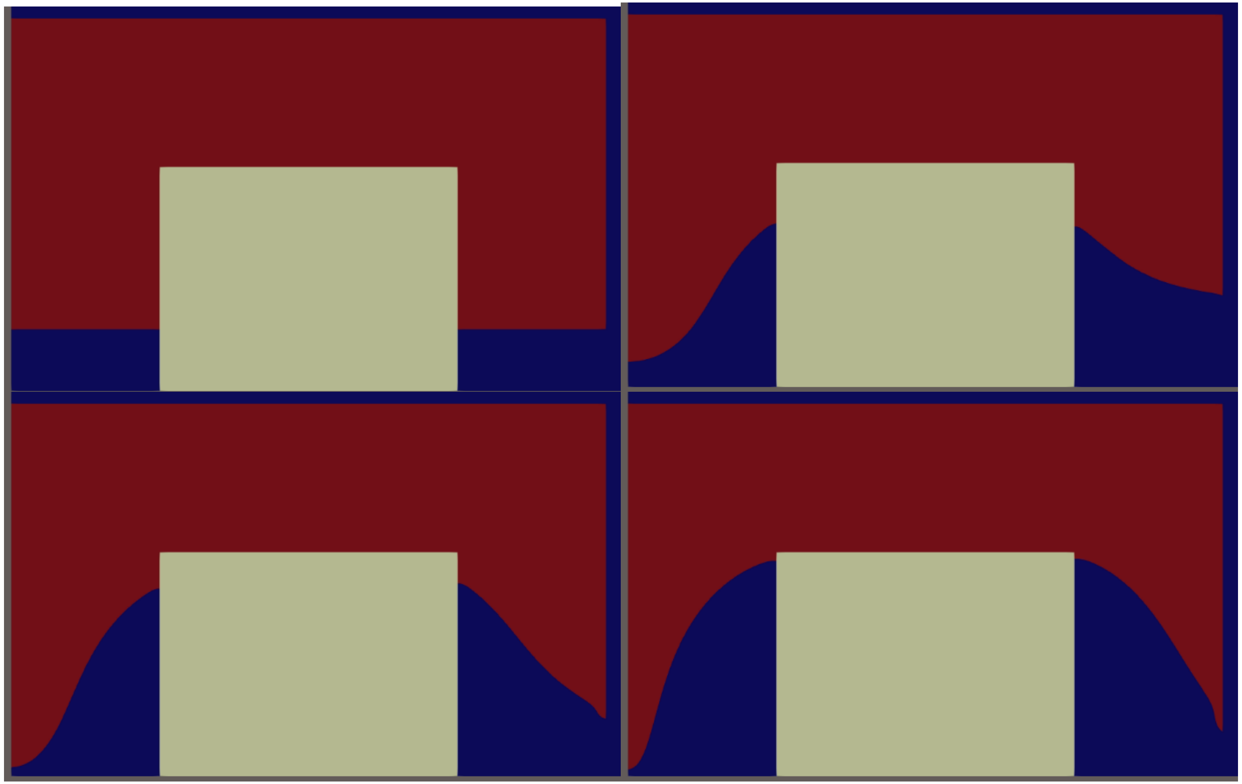
Results



Results – Control Points



Results – Adaptative Mesh Deformation



Shape derivative of $J(\Omega) = \int_\Omega f dx$

The change of variables $T = Id + \phi$ allows to transport the integral $J(\Omega_\phi)$ onto the fixed reference domain Ω:

$$J(\Omega_\phi) = \int_{\Omega_\phi} f dx = \int_\Omega |\det(I + \nabla\phi)| \, f \circ (Id + \phi) dx.$$

Considering that:

- $|\det(I + \nabla\phi)|$ is Fréchet differentiable at $\phi = 0$, expanding to $1 + div(\phi) + o(\phi)$.
- $f \circ (Id + \phi)$ is Fréchet differentiable at $\phi = 0$, expanding to $f + \nabla f \cdot \phi + o(\phi)$.

By the product rule, $\phi \mapsto F(\Omega_\phi)$ is Fréchet differentiable at 0, and its derivative is the **volume form** :

$$J'(\Omega)(\phi) = \int_\Omega (div(\phi) f + \nabla f \cdot \phi) dx = \int_\Omega div(f\phi) dx.$$

Alternatively, if Ω is a bounded and Lipschitz domain, an integration by parts gives the **surface form** :

$$J'(\Omega)(\phi) = \int_{\partial\Omega} \phi \cdot n \, ds \, .$$

<u>Structural theorem :</u> Under mild assumptions, if $\phi . n = 0$ on $\partial\Omega$ then $J'(\Omega)(\phi) = 0$.

The derivation can be difficult for elaborate functions, particularly those constrained by PDEs.

Adjoint Method of $E(p) = F(a_p)$

We want to differentiate a $E(p) = F(a_p)$ (e.g., the magnetic energy $E(p) = \int_D j a_p dx$) with respect to p, an arbitrary parametrization of the air gap profile. Here, j is the current density, and a_p is the magnetic potential.

We introduce two equations:

- The **state equation** $K(p)a_p = J$
- The **adjoint equation** : $K(p)^* \lambda_p = -\frac{dF}{da}(a_p)$ where $K(p)^*$ is the adjoint operator of $K(p)$.

Then, the sensitivities of $E(p)$ with respect to p can be expressed as :

$$\frac{dE}{dp}(p) = \left\langle \lambda_p, \frac{dK}{dp}(p) a_p \right\rangle$$

The scalar product shown above varies based on how $K(p)$, a_p, and λ_p are represented:

- if they are in <u>discretized</u> form, it's an $\mathbb{R}^n$ scalar product, reflecting a "<u>discretize then optimize</u>" approach,
- if they are in <u>variational</u> form, it's an $L^2(D)$ scalar product, reflecting an "<u>optimize then discretize</u>" approach.

Jean Céa's method offers an elegant way to obtain this result.

Adjoint Method of $E(p) = F(a_p)$

The state equation can be expressed in two ways:

- **Discretized Form:** A linear system where $K(p)$ is the stiffness matrix and J is the discretized current density vector:

$$K(p)a_p = J$$

- **Variational Form:** Where $K(p)$ is a linear operator and J is a linear form:

$$\begin{aligned} K(p):\ H^1(D) &\to H^{-1}(D) \\ a &\mapsto \left(v \mapsto \int_D \nu(p)\nabla a \cdot \nabla v dx\right) \\ J : H^1(D) &\to \mathbb{R} \\ v &\mapsto \int_D jv\, dx \end{aligned}$$

Glimpse of Jean Céa's method 1/3

To find the derivative of $E(p)$ with respect to p, we introduce a lagrangian function:

$$L(p, a, \lambda) = F(a) + \langle \lambda, K(p)a - J \rangle$$

A key observation is that $E(p) = L(p, a_p, \lambda)$ for any λ. Applying the chain rule, we get:

$$\frac{dE}{dp}(p) = \frac{\partial L}{\partial p}(p, a_p, \lambda) + \left\langle \frac{\partial L}{\partial a}(p, a_p, \lambda), \frac{da_p}{dp}(p) \right\rangle, \qquad \forall \lambda.$$

The challenge here is that $\frac{da_p}{dp}(p)$ is difficult to define rigorously and compute.

Jean Céa's proposal is to choose λ such that the second term vanishes.

This is achieved by finding a saddle point of $\boldsymbol{L(p,\cdot,\cdot)}$.

Glimpse of Jean Céa’s method 2/3

Recalling the lagrangian function we posed :

$$L(p,a,\lambda) = F(a) + \langle \lambda, K(p)a - J \rangle$$

<u>Canceling with respect to λ</u>, we obtain :

$$\left\langle \frac{\partial L}{d\lambda}(p,a,\lambda), \hat{\lambda} \right\rangle = \left\langle \hat{\lambda}, K(p)a - J \right\rangle = 0, \qquad \forall \hat{\lambda},$$

which recovers our original **state equation** $K(p)a_p = J$.

<u>Canceling with respect to a</u>, we obtain

$$\left\langle \frac{\partial L}{da}(p,a,\lambda), \hat{a} \right\rangle = \left\langle \frac{dF}{da}(a), \hat{a} \right\rangle + \left\langle \lambda, K(p)\hat{a} \right\rangle = 0, \qquad \forall \hat{a}.$$

Using the definition of the adjoint operator $K(p)^*$, this restates to

$$\left\langle \frac{dF}{da}(a), \hat{a} \right\rangle + \left\langle K(p)^* \lambda, \hat{a} \right\rangle = 0, \qquad \forall \hat{a},$$

which recovers our original **adjoint equation** $K(p)^* \lambda_p = -\frac{dF}{da}(a_p)$.

Thus, a saddle point of $\boldsymbol{L(p,\cdot,\cdot)}$ is $\boldsymbol{(a,\lambda) = (a_p, \lambda_p)}$.

Glimpse of Jean Céa’s method 3/3

Recalling the lagrangian function we posed :

$$L(p, a, \lambda) = F(a) + \langle \lambda, K(p)a - J \rangle$$

At the saddle point $(a, \lambda) = (a_p, \lambda_p)$, we have:

$$\frac{dE}{dp}(p) = \frac{\partial L}{\partial p}(p, a_p, \lambda_p) + \cancel{\left\langle \frac{\partial L}{\partial a}(p, a_p, \lambda_p), \frac{da_p}{dp}(p) \right\rangle}$$

Thus, the formula for sensitivities is :

$$\boxed{\frac{dE}{dp}(p) = \frac{\partial L}{\partial p}(p, a_p, \lambda_p) = \left\langle \lambda_p, \frac{dK}{dp}(p) a_p \right\rangle}$$

Applying this approach to shape derivatives demands special care

and yields more intricate expressions.