Data 605 Assignment 3

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1. Problem set 1

(1) What is the rank of the matrix A?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 3 \\ 0 & 1 & -2 & 1 \\ 5 & 4 & -2 & -3 \end{bmatrix}$$

```
A <- matrix(c(1,2,3,4,-1,0,1,3,0,1,-2,1,5,4,-2,-3), nrow = 4)

rankMatrix(A)
```

```
## [1] 4
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 8.881784e-16
```

(2) Given an mxn matrix where m > n, what can be the maximum rank? The minimum rank, assuming that the matrix is non-zero?

The rank of a matrix for rectangular matrices can be no greater than the smaller of the row or column dimension. For invertible (square) matrices, the rank is the same as the dimension of the matrix and for rectangular matrices, the rank is the smaller of m and n, which in this case is n.

The maximum number of linearly independent vectors in a matrix is equal to the number of non-zero rows in its row echelon matrix. Therefore, to find the rank of a matrix, we simply transform the matrix to its row echelon form and count the number of non-zero rows.

The minimum rank would be the number of non-zero rows and if at least 1 row is non-zero for a non-zero matrix, then the minimum rank is 1.

(3) What is the rank of matrix B?

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 3 \\ 2 & 4 & 2 \end{bmatrix}$$

```
B <- matrix(c(1,2,3,3,6,3,2,4,2), nrow = 3)

rankMatrix(B)
```

```
## [1] 2
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
```

```
## [1] 6.661338e-16
```

```r
refmatrix(B)
```

```
##      [,1] [,2] [,3]
## [1,]    1    3    2
## [2,]    0    0    0
## [3,]    0   -6   -4
```

The number of non-zero rows is 2 and therefore the rank is 2.

2. Problem Set 2

Compute the eigenvalues and eigenvectors of the matrix A. You'll need to show your work. You'll need to write out the characteristic polynomial and show your solution.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$Av = \lambda * v$ $(\lambda * I^n - A)v = 0v$ $det(\lambda * I^n - A) = 0$

$$\lambda * I^3 - A = \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix}$$

$$I^3 - A = \begin{bmatrix} \lambda - 1 & -2 & -3 \\ 0 & \lambda - 4 & -5 \\ 0 & 0 & \lambda - 6 \end{bmatrix} * \begin{bmatrix} \lambda - 1 & -2 \\ 0 & \lambda - 4 \\ 0 & 0 \end{bmatrix}$$

$= (\lambda - 1)(\lambda - 4)(\lambda - 6) + (-2)(-5)(0) + (3)(0)(0) - (\lambda - 2)(0)(-2) - (0)(-5)(\lambda - 1) - (0)(\lambda - 4)(-3)$

$p((\lambda) = (\lambda - 1)(\lambda - 4)(\lambda - 6) = 0$

$p((\lambda) = \lambda^3 - 11\lambda^2 + 34\lambda - 24 = 0$

Factoring the polynomial: $(\lambda - 1)(\lambda - 4)(\lambda - 6) = 0$

Eigenvalues are $\lambda = 1$, $\lambda = 4$, and $\lambda = 6$.

Eigenvectors:

For $\lambda = 1$,

$$\begin{bmatrix} 0 & -2 & -3 \\ 0 & -3 & -5 \\ 0 & 0 & -5 \end{bmatrix}$$

$0V1 - 2V2 - 3V3 = 0$

$0V1 - 3V2 - 5V3 = 0$

$0V1 + 0V2 - 5V3 = 0$

V1 is not contrained by these equations and can be any value.

V3 = 0 from the third equation which gives V2 = 0.

So, we can set V1 = 1 or any value.

The eigenspace is for $\lambda = 1$:

$$\begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = 4$,

$$\begin{bmatrix} 3 & -2 & -3 \\ 0 & 0 & -5 \\ 0 & 0 & -2 \end{bmatrix}$$

$3V1 - 2V2 - 3V3 = 0$

$0V1 + 0V2 - 5V3 = 0$

$0V1 + 0V2 - 2V3 = 0$

From the first equation: $3V1 = 2V2$ $V1 = (2/3)V2$

$V3 = 0$ from the second and third equation.

If we choose a V1 of 2, the eigenspace is for $\lambda = 4$:

$$\begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

For $\lambda = 6$,

$$\begin{bmatrix} 5 & -2 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

$5V1 - 2V2 - 3V3 = 0$

$0V1 + 2V2 - 5V3 = 0$

$0V1 + 0V2 - 0V3 = 0$

From the second equation: $2V2 = 5V3$ $V2 = (5/2)V3$

From the first equation: $V1 = (2V2 + 3V3)/5$

If we choose a V3 of 10, the eigenspace is for $\lambda = 6$:

$$\begin{bmatrix} V1 \\ V2 \\ V3 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \\ 10 \end{bmatrix}$$