Data 605 Assignment 1

Steph Roark

2/3/2019

1. Problem set 1

You can think of vectors representing many dimensions of related information. For instance, Netflix might store all the ratings a user gives to movies in a vector. This is clearly a vector of very large dimensions (in the millions) and very sparse as the user might have rated only a few movies. Similarly, Amazon might store the items purchased by a user in a vector, with each slot or dimension representing a unique product and the value of the slot, the number of such items the user bought. One task that is frequently done in these settings is to find similarities between users. And, we can use dot-product between vectors to do just that. As you know, the dot-product is proportional to the length of two vectors and to the angle between them. In fact, the dot-product between two vectors, normalized by their lengths is called as the cosine distance and is frequently used in recommendation engines.

(1) Calculate the dot product u.v where $\mathbf{u} = [0.5; 0.5]$ and $\mathbf{v} = [3; -4]$

```
u <- c(0.5, 0.5)
v <- c(3, -4)
dotuv <- u %*% v
```

The dot product of u.v is -0.5.

(2) What are the lengths of u and v? Please note that the mathematical notion of the

length of a vector is not the same as a computer science definition.

The length of u is 0.7071068.

The length of v is 5.

(3) What is the linear combination: $3\mathbf{u} - 2\mathbf{v}$?

3*[0.5,0.5]* - *2*[3, -4] = -4.5, 9.5.

(4) What is the angle between u and v

```
angle <- acos( sum(u*v) / ( sqrt(sum(u * u)) * sqrt(sum(v * v)) ) )
```

The angle theta between vectors u and v is 1.713 radians.

2. Problem set 2

Set up a system of equations with 3 variables and 3 constraints and solve for x. Please write a function in R that will take two variables (matrix A & constraint vector b) and solve using elimination. Your function should produce the right answer for the system of equations for any 3-variable, 3-equation system.

(You don't have to worry about degenerate cases and can safely assume that the function will only be tested with a system of equations that has a solution. Please note that you do have to worry about zero pivots, though. Please note that you should not use the built-in function solve to solve this system or use matrix inverses.)

The approach that you should employ is to construct an Upper Triangular Matrix and then back-substitute to get the solution. Alternatively, you can augment the matrix A with vector b and jointly apply the Gauss Jordan elimination procedure.

Once we have the co-efficient matrix in Upper Triangular Form form, we can systematically substitute one variable at a time and get the final solution. We can build a simple iterative procedure to systematically convert the coefficient matrix A into the Upper Triangular Form. We will call this procedure Pivot & Multiply.

Steps for procedure:

(1) Start with row 1 of the co-efficient matrix
(2) Pivot: The first non-zero element in the row being evaluated
(3) Multiplier: The element being eliminated divided by the Pivot
(4) Subtract Multiplier times row n from row n+1
(5) Advance to the next row and repeat

Gaussian Elimination Function with Backsubstitution

```
gauss_elim_w_backsub <- function(A,b){
    #create an augmented matrix
    aug <- cbind(A,b)
    # pivot if 1,1 is 0
    if( aug[1,1] == 0 ) {
        tmp = aug[3,]
        aug[3,] = aug[1,]
        aug[1,] = tmp
    }
    #solve matrix for reduced row echelon form using row operations
    aug[2,] = aug[2,] - aug[1,] * aug[2,1] / aug[1,1]
    aug[3,] = aug[3,] - aug[1,] * aug[3,1] / aug[1,1]
    #pivot if center is 0
    if( aug[2,2] == 0 ) {
        tmp = aug[3,]
        aug[3,] = aug[2,]
        aug[2,] = tmp
        }
    aug[3,] = aug[3,] - aug[2,] * aug[3,2] / aug[2,2]
    #back substitute to solve
    x3 <- aug[3,4]/aug[3,3]
    x2 <- (aug[2,4] - aug[2,3]*x3)/aug[2,2]
    x1 <- (aug[1,4] - aug[1,2]*x2 - aug[1,3]*x3)/aug[1,1]
    #print solution matrix
    matrix(c(x1,x2,x3), nrow=3)
}

#matrix A is the coefficients of the linear equations
A <- matrix( c(3,3,4,1,1,1,2,1,3), nrow=3, byrow = TRUE)
A
```

```
##      [,1] [,2] [,3]
## [1,]    3    3    4
## [2,]    1    1    1
## [3,]    2    1    3
```

```r
#matrix b of the augmented matrix
b <- matrix( c(20,6,13), nrow=3)
#call function on matrix A & b
b
```

```
##      [,1]
## [1,]   20
## [2,]    6
## [3,]   13
```

```r
gauss_elim_w_backsub(A,b)
```

```
##      [,1]
## [1,]    3
## [2,]    1
## [3,]    2
```

Test Case

Please test it with the system below and it should produce a solution x $= [-1.55, -0.32, 0.95]$

```r
A_test_matrix <- matrix(c(1,1,3,2,-1,5,-1,-2,4), nrow = 3, byrow = TRUE)
A_test_matrix
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]    2   -1    5
## [3,]   -1   -2    4
```

```r
b_test_matrix <- matrix(c(1,2,6), nrow = 3)
b_test_matrix
```

```
##      [,1]
## [1,]    1
## [2,]    2
## [3,]    6
```

```r
round(gauss_elim_w_backsub(A_test_matrix,b_test_matrix),2)
```

```
##       [,1]
## [1,] -1.55
## [2,] -0.32
## [3,]  0.95
```