1. Easy Mode

Problem 1.1. *What is the smallest r such that three disks of radius r can completely cover up a unit disk?*

Problem 1.2. *If* $\frac{\log_b 9}{\log_c 9} = 2023$, *compute* $\frac{b}{c} \mod 17$.

Problem 1.3. *Let x and y be two-digit integers such that y is obtained by reversing the digit of x. Suppose that $x^2 - y^2 = m^2$ where m is some integer, find $x + y$.*

Problem 1.4. *Calculate the exact value of the angle expressed in the form with* arctan *between 2 $C - H$ bonds in CH_4 (methane) and give out the process of calculation.*

Problem 1.5. *Given that $N = (20^{23} + 2^{915})^2 - (20^{23} - 2^{915})^2$, how many consecutive "0" are there at the end of N? (For example, 2480900 has 2 "0"s at the end.)*

Problem 1.6. *Positive integers a and b satisfy the condition*

$$\log_2(\log_{2^a}(\log_{2^b}(2^{1000}))) = 0.$$

Find the sum of all possible values of $a + b$.

2. Intermediate Mode

Problem 2.1. *In rectangle $ABCD$, $AB = 12$ and $BC = 10$. Points E and F lie inside rectangle $ABCD$ so that $BE = 9$, $DF = 8$, $\overline{BE} \parallel \overline{DF}$, $\overline{EF} \parallel \overline{AB}$, and line BE intersects segment $\overline{AD}$. What is the length of EF?*

Problem 2.2. *Triangle ABC has a right angle at C, and D is the foot of the altitude from C to AB. Points L, M and N are the midpoints of segments AD, DC and CA, respectively. If $CL = 9$ and $BM = 15$, compute BN^2.*

Problem 2.3. *Alice is thinking of a positive real number x, and Bob is thinking of a positive real number y. Given that $x^{\sqrt{y}} = 27$ and $(\sqrt{x})^y = 9$, compute xy.*

Problem 2.4. *Find the smallest integer n such that*

$$\sqrt{n + 99} - \sqrt{n} < 1$$

Problem 2.5. *Which number among the set* $\{0, 1, 2, 3, 4\}$ *stands for the imaginary number* i *mod 5? [* $\bmod n$ *stands for modulo arithmetic, which gives out the residue when divided by* n*. For example,* $9 \equiv 1 \pmod 4$ *means that 9's residue is 1 when divided by 4.]*

Problem 2.6. *Points* $A(6, 13)$ *and* $B(12, 11)$ *lie on circle* ω *in the plane. Suppose that the tangent lines to* ω *at* A *and* B *intersect at a point on the* x*-axis. What is the area of* ω*?*

3. Difficult Mode

Problem 3.1. *Let* (a, b, c) *be the real solution of the system of equations* $x^3 - xyz = 2$, $y^3 - xyz = 6$, $z^3 - xyz = 20$*. The greatest possible value of* $a^3 + b^3 + c^3$ *can be written in the form* $\frac{m}{n}$*, where* m *and* n *are relatively prime positive integers. Find* $m + n$*.*

Problem 3.2. *How many digits are in the base two representation of* $10!$ *(factorial)?*

Problem 3.3. *If* $a_1 = 1, a_2 = 0$*, and* $2a_{n+1} = 2a_n + a_{n+2}$ *for all* $n \geq 1$*, compute* $log_2(-a_{2024})$

Problem 3.4. *Given that* $\log_{10} \sin x + \log_{10} \cos x = -1$ *and that* $\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10} n - 1)$*, find* n*.*

Problem 3.5. *What is*

$$\sum_{n=1}^{2023} \frac{\left\lfloor \sqrt{n} + \sqrt{n+1} \right\rfloor}{\left\lfloor \sqrt{4n+2} \right\rfloor}$$

and prove it.

Problem 3.6. *If a DP1 student chooses 5 HL (high level) courses out of 6 and needs to take 6 classes each week for each HL course, what is the probability that he has 2 2-period classes for different HL courses in a day?*