Introduction to Time Series Analysis

PRACTICAL TIME SERIES ANALYSIS

THISTLETON AND SADIGOV

Objectives

- Define a time series
- Get familiar with 'astsa' package

Definition

- Time series is a data set collected through time.

Correlation

- Sampling adjacent points in time introduce a correlation.

Areas

- Economics and financial time series
- Physical time series
- Marketing time series
- Demographic time series
- Population time series
- Etc.

“astsa” package

- Package by Robert H. Shumway and David S. Stoffer
- Contains data sets and scripts to accompany “Time Series Analysis and Its Applications: With R Examples”
- https://cran.r-project.org/web/packages/astsa/astsa.pdf

What We’ve Learned

- Definition of a time series (we will re-define it in a slightly different way)
- The package titled ‘astsa’

Some Time Plots

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Objectives

- See some examples of time series.
- Produce meaningful time plots.

Some time series from ‘astsa’

- jj
- flu
- globtemp
- globtempl
- star

Johnson and Johnson Quarterly Earnings (jj)

- US company Johnson and Johnson
- Quarterly earnings
- 84 quarters
- 1st quarter of 1960 to 4th quarter of 1980

Pneumonia and influenza deaths in the U.S. (flu)

- Monthly pneumonia and influenza deaths per 10,000 people
- 11 years
- From 1968 to 1978

Land-ocean temperature deviations (globtemp)

- Global mean land-ocean temperature deviations
- Deviations from base period 1951-1980 average
- Measured in degrees centigrade
- For the years 1880-2015.
- http://data.giss.nasa.gov/gistemp/graphs/

Land (only) temperature deviations (globtempl)

- Global mean [land only] temperature deviations
- Deviations from base period 1951-1980 average
- Measured in degrees centigrade
- For the years 1880-2015.
- http://data.giss.nasa.gov/gistemp/graphs/

Variable Star (star)

- The magnitude of a star taken at midnight
- For 600 consecutive days
- The data are from "The Calculus of Observations, a Treatise on Numerical Mathematics", by E.T. Whittaker and G. Robinson

What We've Learned

- Time series exist in variety of areas
- How to produce meaningful time plots

Stationarity

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Objectives

- Get some intuition for (weak) stationary time series

No systematic change in mean

i.e., No trend

No systematic change in variation

No periodic fluctuations

The properties of one section of a data are much like the properties of the other sections of the data

For an non-stationary time series,
we will do some transformations to
get stationary time series

What We've Learned

In a (weak) stationary time series, there is no

- systematic change in mean (no trend)
- systematic change in variance
- periodic variations

Autocovariance function

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Objectives

- recall random variables and covariance of two random variables
- characterize time series as a realization of a stochastic process
- define autocovariance function

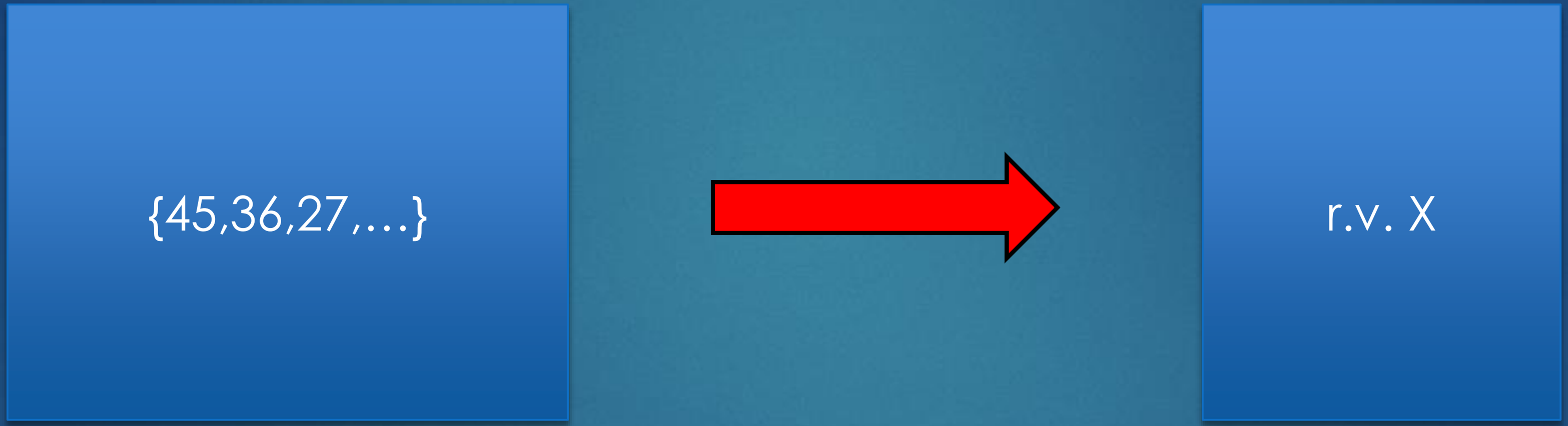
Random variables

- Random variable is defined

$$X : S \to \mathbb{R}$$

where S is the sample space of the experiment.

From data to a model



Discrete vs. Continuous r.v.

X= {20, 37, 57, …}

VS.

Y in (10, 60)

- 20 is a realization of r.v. X
- 30.29 is a realization of a r.v. Y

Covariance

- X, Y are two random variables.
- Measures the linear dependence between two random variables

$$CoV(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = Cov(Y,X)$$

Stochastic Processes

Collection of a random variables

$$X_1, X_2, X_3, \ldots$$

$$X_t \sim distribution\ (\mu, \sigma^2)$$

Time series as a realization of a stochastic process

$$X_1, X_2, X_3, \ldots$$

$$30, 29, 57, \ldots$$

Autocovariance function

$$\gamma(s,t) = Cov(X_s, X_t) = E[(X\_s - \mu_s)(X_t - \mu_t)]$$

$$\gamma(t,t) = E[(X_t - \mu_t)^2] = Var(X_t) = \sigma_t^2$$

Autocovariance function cont.

$$\gamma_k = \gamma(t, t+k) \approx c_k$$

What We've Learned

- the definition of a stochastic processes
- how to characterize time series as realization of a stochastic process
- how to define autocovariance function of a time series

Autocovariance coefficients

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Objectives

- Recall the covariance coefficient for a bivariate data set
- Define autocovariance coefficients for a time series
- Estimate autocovariance coefficients of a time series at different lags

Covariance

- X, Y are two random variables.
- Measures the <u>linear</u> dependence between two random variables

$$Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$$

Estimation of the covariance

- We have a paired dataset

$$(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$$

- Estimation of covariance (cov() in R)

$$s_{xy} = \frac{\sum_{t=1}^{N}(x_t - \bar{x})(y_t - \bar{y})}{N-1}$$

Autocovariance coefficients

- Autocovariance coefficients at different lags $\gamma_k = Cov(X_t, X_{t+k})$
- c_k is an estimation of γ_k.
- We assume (weak) stationarity

Estimation

$$c_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{N}$$

where

$$\bar{x} = \frac{\sum_{t=1}^{N} x_t}{N}$$

Routine in R

- acf() routine (next video lecture)
- acf(time_series, type='covariance')

Purely random process

- Time series with no special pattern
- We use rnorm() routine

What We've Learned

- Definition of autocovariance coefficients at different lags
- Estimate autocovariance coefficients of a time series using acf() routine

The autocorrelation function

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Objectives

- Define the autocorrelation function
- Obtain corellograms using acf() routine
- Estimate autocorrelation coefficients at different lags using acf() routine

The autocorrelation function (ACF)

- We assume weak stationarity
- The autocorrelation coefficient between X_t and X_{t+k} is defined to be

$$-1 \leq \rho_k = \frac{\gamma_k}{\gamma_0} \leq 1$$

- Estimation of autocorrelation coefficient at lag k

$$r_k = \frac{c_k}{c_0}$$

Another way to write r_k

$$r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2}$$

acf() routine

- We have already used it for autocovariance coefficients
- It plots autocorrelation coefficients at different lags: Correlogram
- It always starts at 1 since $r_0 = \frac{c_0}{c_0} = 1$

What We've Learned

- Definition of the autocorrelation function (ACF)
- How to produce correlograms using acf() routine
- How to estimate the autocorrelation coefficients at different lags using acf() routine.

Random Walk

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Objectives

- Get familiar with the random walk model
- Simulate a random walk in R
- Obtain the correlogram of a random walk
- See the difference operator in action

Model

Location at previous step (or price of the stock yesterday)

Location at time t (or a price of a stock today)

$$X_t = X_{t-1} + Z_t$$

$$Z_t \sim \ Normal\ (\mu, \sigma^2)$$

White noise (residual)

$$X_0 = 0$$

$$X_1 = Z_1$$

$$X_2 = Z_1 + Z_2$$

$$\dots$$

$$X_t = \sum_{i=1}^{t} Z_i$$

$$E[X_t] = E\left[\sum_{i=1}^{t} Z_i\right] = \sum_{i=1}^{t} E[Z_i] = \mu t$$

$$Var[X_t] = Var\left[\sum_{i=1}^{t} Z_i\right] = \sum_{i=1}^{t} Var[Z_i] = \sigma^2 t$$

Simulation

- $X_1 = 0$
- $Z_t \sim Normal\ (0, 1)$
- $X_t = X_{t-1} + Z_t$ for $t = 2,3, \dots, 1000$
- Plot and ACF

Removing the tend

$X_t = X_{t-1} + Z_t$ → $X_t - X_{t-1} = Z_t$ → $\nabla X_t = Z_t$ → *Purely Random Process*

$$\nabla X_t = X_t - X_{t-1}$$

Difference operator

- diff() to remove the trend
- Plot and ACF differenced time series

What We've Learned

- Random Walk model
- How to simulate a random walk in R
- How to get stationary time series from a random walk using diff() operator

Introduction to Moving Average processes

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Objectives

- Identify Moving average processes

Intuition

X_t is a stock price of a company

Each daily announcement of the company is modeled as a noise

Effect of the daily announcements (noises Z_t) on the stock price (X_t) might last few days (say 2 days)

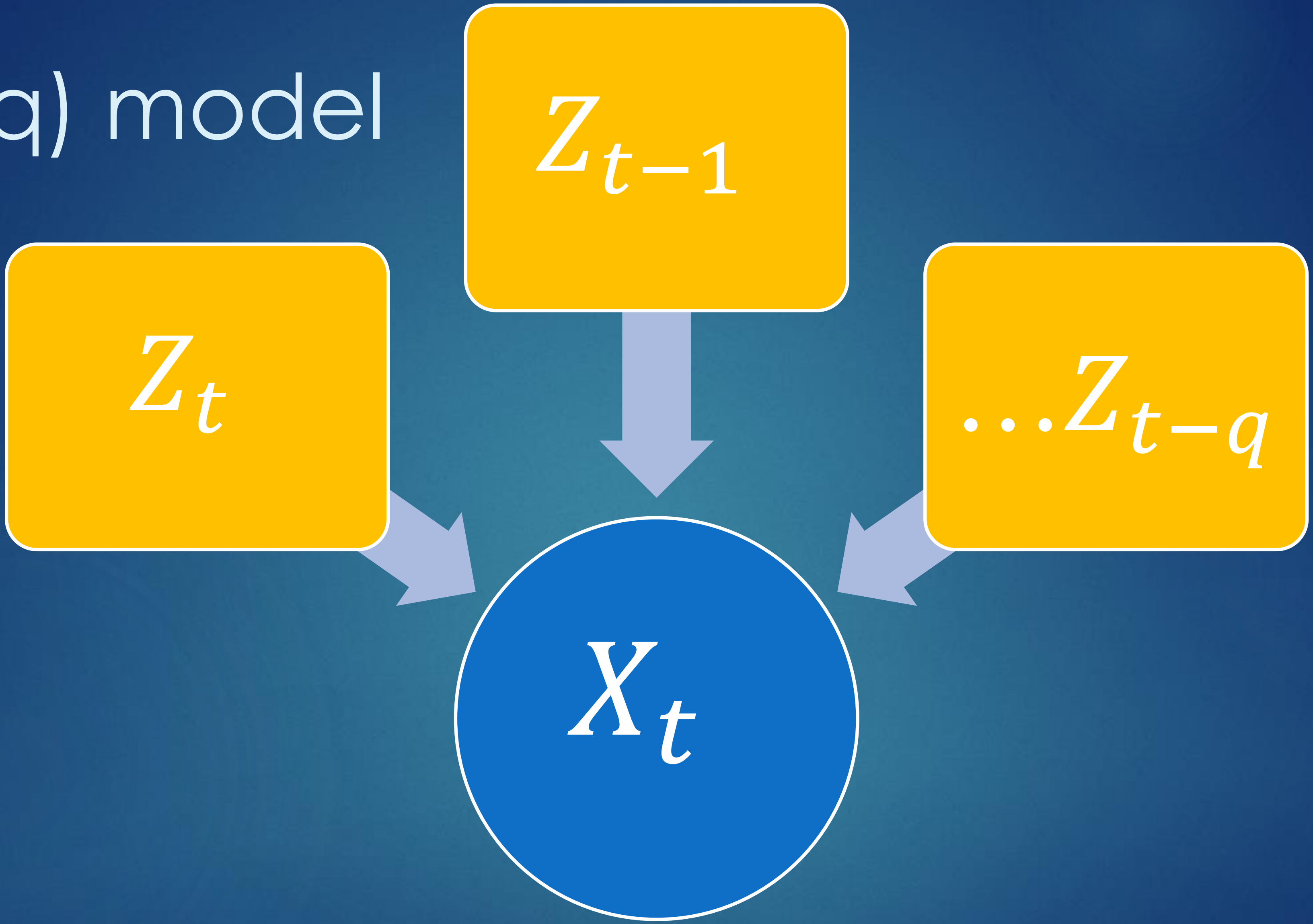
Stock price is linear combination of the noises that affects it

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$$

Moving average model of order 2

MA(2)

MA(q) model



$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + \ldots + \theta_q Z_{t-q}$$

Z_i are i.id. & $Z_i \sim Normal\ (\mu, \sigma^2)$

What We’ve Learned

- How to identify Moving average processes MA(q)

Simulating MA(2) process

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Objectives

- Simulate a moving average process
- Interpret correlogram of a Moving average process

MA(2) process

$$X_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$$

Simulation - MA(2) model

$$X_t = Z_t + 0.7\, Z_{t-1} + 0.2\, Z_{t-2}$$

$$Z_t \sim Normal\,(0, 1)$$

What We’ve Learned

- How to simulate MA processes in R
- That ACF of MA(q) cuts off at lag q