FNCE 5321: Finanical Risk Modeling II
Spring 2017
Homework 1

Instructions: You can work in groups of up to 5 students. Please hand in only one copy per group and indicate clearly the members of your group on the first page of your submitted homework.

Problems

1. Which of the following is not a stylized fact on asset returns?

   A) Daily returns have very little autocorrelation.

   B) Unconditonal ditribution of daily returns have fatter left tails than the normal distribution.

   C) Variance of returns have very little autocorrelation.

   D) At short hozirons such as daily, the volatility of returns completely dominates the mean of returns.

2. Which of the following is not an example of a stationary time series?

   A) daily returns on the S&P 500 index

   B) closing prices on the S&P 500 index

   C) price-earning ratio on the S&P 500 index

   D) daily return volatility on the S&P 500 index

3. Which of the following statements is not correct in describing an AR(1) time series?

$$X_t = \phi_0 + \phi_1 X_{t-1} + \epsilon_t, \qquad |\phi_1| < 1$$

   A) The autocorrelation of X_t at the first lag is ϕ_1.

   B) The partial autocorrelation of X_t at the first lag is ϕ_1.

   C) The partial autocorrelation of X_t at the second lag is ϕ_1^2.

   D) X_t is stationary.

4. Which of the following is not a measure of risk?

   A) autocorrelation
   B) VaR
   C) expected shortfall
   D) volatility

5. Suppose the return of your portfolio is normally distributed with a mean of 0 and a standard deviation of 2%. What is the 1% VaR of your portfolio? (Hint: The one-percentile point of a standard normal distribution is -2.326).

   A) -4.65%
   B) 4.65%
   C) -2.33%
   D) 2.33%

6. What is the difference between expected shortfall and VaR? What is the theoretical advantage of expected shortfall over VaR?

7. An autoregressive time series model has the following estimated autocorrelations at its first, second, and third lags: 0.8, 0.5, 0.4. Is an AR(1) model appropriate to describe this time series? Explain why.

   For the questions 8-14, use the data in HW1Data.csv. The data contain daily closing prices on a stock market index for the period July 2nd 1962 to December 30th 2016.

8. Remove the prices that are simply repeats of the previous day's price because they indicate a missing observation due to a holiday. Calcuate daily log returns as $R_{t+1} = \ln(S_{t+1}) - \ln(S_t)$ where S_{t+1} is the closing price on day $t + 1$, S_t is the closing price on day t, and $\ln(\cdot)$ is the natural logarithm. Plot the closing prices and returns over time.

9. Calculate and report the mean and standard deviation of daily log returns.

10. Calculate the first through 100th lag autocorrelations of daily log returns. Plot the autocorrelations against the lag order.

11. Calculate the first through 100th lag autocorrelations of squared daily log returns. Plot the autocorrelations against the lag order. Do squared daily log returns have more or less autocorrelation compared with daily log returns?

12. Suppose you own a portfolio which is 100% invested in this market index. Calculate the 1-day, 1% VaRs on each day using Historical Simulation with a 250-day moving estimation window. Plot the VaRs.

13. Repeat the above exercise to calculate the 1-day, 5% VaRs on each day. Are the 5% VaRs smaller or larger than the 1% VaRs? Explain why with intuition.

14. Formally test whether the daily closing prices and the daily log returns are stationary using augmented Dickey-Fuller tests (Hint: Use the adf.test command in R. Install the 'tseries' package first if you have not done so).

15. Suppoese X_t follows the following AR(1) model: $X_t = 0.9X_{t-1} + \epsilon_t$, where $\sigma_\epsilon = 0.1$. Simulate an episode of X_t with 1 million observations (Hint: Use the R command arima.sim). Calculate the first through 10th lag autocorrelations of the simulated series. Compare the estimated autocorrelations with the theoretical counterparts (Hint: the theoretical autocorrelations for AR(1) is $\rho_\tau = \phi_1^\tau$, where τ is the lag.)

16. Suppose $Z_t = X_t + Y_t$, where Y_t follows an AR(1) model: $Y_t = 0.5Y_{t-1} + \epsilon_t$, where $\sigma_\epsilon = 0.1$. Is an AR(1) model appropriate to describe Z_t?