FNCE 5321: Finanical Risk Modeling II
Spring 2017
Homework 3

Instructions: You can work in groups of up to 5 students. Please hand in only one copy per group and indicate clearly the members of your group on the first page of your submitted homework.

Important: Include the codes with your solution.

Problems

1. Your employer offers you retirement accounts at Vanguard, which has a variety of funds you can choose from. You are thinking about allocating your retirement money among a US equity index fund (symbol **VIIIX**) and an internatial equity index fund (symbol **VGTSX**). The first thing you would like to know is the extent of diversification your portfolio would potentially glean. In other words, you want to have a sense of the dynamic correlation of the two funds over time. For more details on the two equity funds, refer to https://personal.vanguard.com/us/funds/snapshot?FundId=0854&FundIntExt=INT and https://personal.vanguard.com/us/funds/snapshot?FundId=0113&FundIntExt=INT.

   (a) Download daily closing prices of the two indexes using the **quantmod** package in R for the period from January 1st 1998 to April 7th 2017. Calculate their daily stock returns respectively. What is the unconditional correlation of the two indexes?

   (b) Estimate their dynamic correlations using the RiskMetrics model (i.e. exponentially smoothed correlation), setting the parameter $\lambda = 0.94$. Make a plot of the estimated correlations. (Hint: First use a Garch(1,1) model to standardize returns).

   (c) Estimate their dynamic correlations using a Garch(1,1) type of model, setting the paramters $\alpha = 0.05$ and $\beta = 0.9$. Make a plot of the estimated correlations. (Hint: First use a Garch(1,1) model to standardize returns).

   (d) The correlation among equity indexes is typically interpreted as the extent of integration among equity markets. Based on your estimations in Part (b) and (c), does the rest of the world become more integrated with the U.S. over time in the equity market, or the other way around?

2. Instead of estimating the term structure of VaR or expected shortfalls, this exercise tries to estimate the term structure of volatility using Monte Carlo simulations. All the steps keep the same, except that in the last step we estimate the volatility of simulated returns at each horizon.

   We will be using standard normal shocks throughout this exercise. The number of Monte Carlo simulations is denoted by MC. Use a MC of at least 50000 (you can use more simulations as long as your computer allows). The term structure horizon is denoted by T. Set $T = 500$.

   Suppose we have already estimated the one-period ahead volatility σ_{t+1} with historical data. The goal is to estimate the term structure of volatility $\hat{\sigma}_{t+1:t+k}$ for $k = 1, \cdots, T$ using Monte Carlo simulations. $\hat{\sigma}_{t+1:t+k}$ is the estimated volatility of returns over the next k periods, i.e. $R_{t+1} + \cdots + R_{t+k}$.

   (a) Using Monte Carlo simulations, estimate the term structure of volatility with the RiskMetrics model. That said, you update future volatility using the RiskMetrics model. Start the simulation by setting $\sigma_{t+1} = 0.01$. Plot $\frac{\hat{\sigma}_{t+1:t+k}}{\sigma_{t+1}\sqrt{k}}$ as a function of k.

   (b) What is the theoretical value of $\frac{\hat{\sigma}_{t+1:t+k}}{\sigma_{t+1}\sqrt{k}}$ defined in Part (a)? Is the plot in Part (a) close to the theoretical value?

   (c) Using Mone Carlo simualations, estimate the term structure of volatility with the Garch(1,1) model. That said, you update future volatility using the Garch(1,1) model. Supposed you have estimated $\omega = 1e - 6, \alpha = 0.05, \beta = 0.9$ based on historical data. Start the simulation by setting $\sigma_{t+1} = \sigma \equiv \sqrt{\frac{\omega}{1-\alpha-\beta}}$. Plot $\frac{\hat{\sigma}_{t+1:t+k}}{\sigma_{t+1}\sqrt{k}}$ as a function of k.

   (d) For Part (c), does the value of σ_{t+1} affect the pattern of $\frac{\hat{\sigma}_{t+1:t+k}}{\sigma_{t+1}\sqrt{k}}$? You can answer this question by repeating Part (c) but perturbing the values of σ_{t+1}, for example, resetting $\sigma_{t+1} = 0.5\sigma$ and $\sigma_{t+1} = 3\sigma$.

   (e) What is the theoretical value of $\hat{\sigma}_{t+1:t+k}$ estimated in Part (c)? Check how close the Monte Carlo estimations in Part (c) are to the theoretical counterparts. An easy way to check it is to divide the estimated $\hat{\sigma}_{t+1:t+k}$ by its theoretical counterpart, make a plot, and compare with the horizon line which crosses the y-axis at (0, 1).

3. For the student t distribution, the lower the degree of freedom, the fatter the tails. This exercise helps you visualize the fact.

   (a) Using the **rt** command in R, simulate 10000 observations from a student t distribution with the degree of freedom equal to 5. Do the same thing for t distributions with degrees of freedom equal to 20 and 100 respectively. You end up with three t-distributed samples with different degrees of freedom. Calculate the means and standard deviations of the three samples.

   (b) To compare the extent of deviation from normality, it is better to first standardize all three samples to have a mean of zero and a standard deviation of one. Use the **scale** function in R to do it. Make QQ plots of the three standardized samples and arrange them in one plot (Hint: Use **par(mfrow=c(1,3))** before you start plotting). Do you find the pattern you expect, that is, the lower the degree of freedom, the fatter the tails?