Risk Management: An Introduction

FNCE 5321

Hang Bai

Objectives

- Become familiar with the range of risks facing corporations, and how to measure and manage these risks
- Become familiar with the salient features of financial asset returns
- Apply state-of-the-art risk management techniques
- Critically appraise commercially available risk management systems and contribute to the construction of tailor-made systems
- Understand the current academic and practitioner literature

Why should firms manage risk?

- Classic portfolio theory: Investors can eliminate firm-specific risk by diversifying holdings to include many different assets
- Investors should hold a combination of the risk-free asset and the market portfolio.
- Firms should not waste resources on risk management, as investors do not care about firm-specific risk.
- Modigliani-Miller: The value of a firm is independent of its risk structure.
- Firms should simply maximize expected profits regardless of the risk entailed.

Why should firms manage risk?

- **Bankruptcy:** The real costs of company reorganization or shut-down will reduce the current valuation of the firm. Risk management can increase the value of a firm by reducing the probability of default.
- **Taxes:** Risk management can help reduce taxes by reducing the volatility of earnings.

Why should firms manage risk?

- **Capital structure and the cost of capital:** a major source of corporate default is the inability to service debt. Proper risk management may allow the firm to expand more aggressively through debt financing.
- **Employee Compensation:** due to their implicit investment in firm-specific human capital, key employees often have a large and unhedged exposure to the risk of the firm they work for.

Evidence on RM practices

- In 1998 researchers at the Wharton School surveyed 2000 companies on their risk management practices including derivatives uses.
- Of the 2000 surveyed, 400 responded.
- Companies use a range of methods and have a variety of reasons for using derivatives.
- Not all risks which were managed were necessarily completely removed.
- About half of the respondents reported they use derivatives as a risk management tool.

Evidence on RM practices

- One third of derivatives users actively take positions reflecting their market views. Could increase risk rather than reduce it.
- Also standard techniques such as physical storage of goods (i.e inventory holdings), cash buffers and business diversification.
- Not everyone chooses to manage risk and risk management approaches differ across firms.
- Some firms use cash-flow volatility while others use the variation in the value of the firm as the risk management object of interest.

Evidence on RM practices

- Large firms tend to manage risk more actively than small firms, which is perhaps surprising as small firms are generally viewed to be more risky.

- However smaller firms may have limited access to derivatives markets and furthermore lack staff with risk management skills.

Does RM improve firm performance?

- The overall answer to this question appears to be YES.
- Analysis of the risk management practices in the gold mining industry found that share prices were less sensitive to gold price movements after risk management.
- Similarly, in the natural gas industry, better risk management has been found to result in less variable stock prices.
- A study also found that RM in a wide group of firms led to a reduced exposure to interest rate and exchange rate movements.

Does RM improve firm performance?

- Researchers have found that less volatile cash flow result in lower costs of capital and more investment.
- A portfolio of firms using RM outperformed a portfolio of firms that did not, when other aspects of the portfolio were controlled for.
- Similarly, a study found that firms using foreign exchange derivatives had higher market value than those who did not.
- The evidence so far paints a fairly rosy picture of the benefits of current RM practices in the corporate sector.

A brief taxonomy of risks

- **Market Risk:** the risk to a financial portfolio from movements in market prices such as equity prices, foreign exchange rates, interest rates and commodity prices.

A brief taxonomy of risks

- **Liquidity risk:** The particular risk from conducting transactions in markets with low liquidity as evidenced in low trading volume, and large bid-ask spreads.
- Under such conditions, the attempt to sell assets may push prices lower and assets may have to be sold at prices below their fundamental values or within a time frame longer than expected.
- Traditionally liquidity risk was given scant attention in RM, but the events in the fall 1998 sharply increased the attention devoted to liquidity risk.

A brief taxonomy of risks

- **Operational risk:** the risk of loss due to physical catastrophe, technical failure and human error in the operation of a firm, including fraud, failure of management and process errors.
- Operational risk-“op risk”-should be mitigated and ideally eliminated in any firm as the exposure to it offers very little return (the short-term cost savings of being careless for example).

A brief taxonomy of risks

- **Credit risk:** the risk that a counter-party may become less likely to fulfill its obligations in part or in full on the agreed upon date.
- Thus credit risk consists not only of the risk that a counterparty completely defaults on its obligations, but also that it only pays in part and/or after the agreed upon date.
- The nature of commercial banks has traditionally been to take on large amounts of credit risk through their loan portfolios.

A brief taxonomy of risks

- Today, banks spend much effort to carefully manage their credit risk exposure.

- Nonbank financials as well as nonfinancial corporations might instead want to completely eliminate credit risk as it is not a part of their core business.

A brief taxonomy of risks

- **Business risk:** the risk that changes in variables of a business plan will destroy that plan's viability, including quantifiable risks such as business cycle and demand equation risk, and non-quantifiable risks such as changes in competitive behavior or technology.
- Business risk is sometimes simply defined as the types of risks which are integral part of the core business of the firm and which should therefore simply be taken on.

Asset returns definitions

- The daily simple rate of return from the closing prices of the asset:

$$r_{t+1} = (S_{t+1} - S_t) / S_t = S_{t+1}/S_t - 1$$

- The daily continuously compounded or log return on an asset is

$$R_{t+1} = \ln(S_{t+1}) - \ln(S_t)$$

Asset returns definitions

- The two returns are fairly similar

$$R_{t+1} = \ln(S_{t+1}) - \ln(S_t) = \ln(S_{t+1}/S_t) = \ln(1 + r_{t+1}) \approx r_{t+1}$$

- The approximation holds because $\ln(x) \approx x-1$ when x is close to 1

- Let N_i be the number of units held in asset i and let $V_{PF;t}$ be the value of the portfolio on day t so that

$$V_{PF,t} = \sum_{i=1}^{n} N_i S_{i,t}$$

Asset returns definitions

- Then the portfolio rate of return is

$$r_{PF,t+1} \equiv \frac{V_{PF,t+1} - V_{PF,t}}{V_{PF,t}} = \frac{\sum_{i=1}^{n} N_i S_{i,t+1} - \sum_{i=1}^{n} N_i S_{i,t}}{\sum_{i=1}^{n} N_i S_{i,t}} = \sum_{i=1}^{n} w_i r_{i,t+1}$$

- where $w_i = N_i S_{i,t} / V_{PF,t}$ is the portfolio weight in asset i

- Most assets have a lower bound of zero on the price
- Log returns are more convenient for preserving this lower bound in the risk model because an arbitrarily large negative log return tomorrow will still imply a positive price at the end of tomorrow.

Asset returns definitions

- Tomorrow's price when using log returns is

$$S_{t+1} = \exp(R_{t+1})S_t$$

- where exp(•) denotes the exponential function
- If instead we use the rate of return definition then tomorrow's closing price is

$$S_{t+1} = (1+r_{t+1})S_t$$

- Here S_{t+1} could go negative unless the assumed distribution of tomorrow's return, r_{t+1}, is bounded below by −1

Asset returns definitions

- With log return definition, we can easily calculate the compounded return at the K−day horizon as the sum of the daily returns:

$$R_{t+1:t+K} = \ln(S_{t+K}) - \ln(S_t) = \sum_{k=1}^{K} \ln(S_{t+k}) - \ln(S_{t+k-1}) = \sum_{k=1}^{K} R_{t+k}$$

Stylized facts of asset returns

- We can consider the following list of so-called stylized facts which apply to most stochastic returns.

- We will use daily returns on the S&P500 from 01/01/2001 to 12/31/2010 to illustrate each of the features.

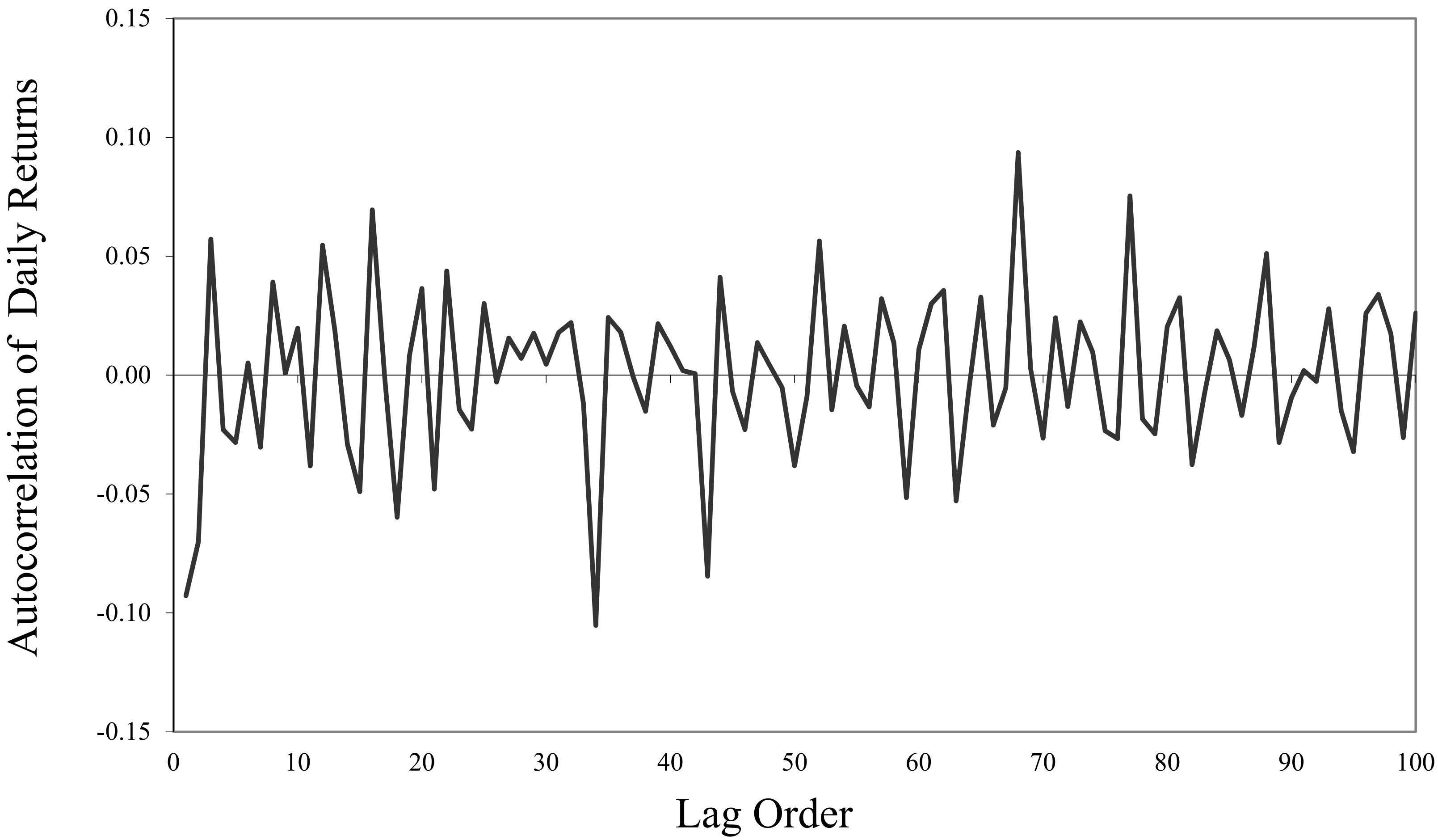
Stylized fact 1

- Daily returns have very little autocorrelation. We can write

$$Corr\left(R_{t+1}, R_{t+1-\tau}\right) \approx 0, \;\; \text{for } \tau = 1, 2, 3, \ldots, 100$$

- Returns are almost impossible to predict from their own past.

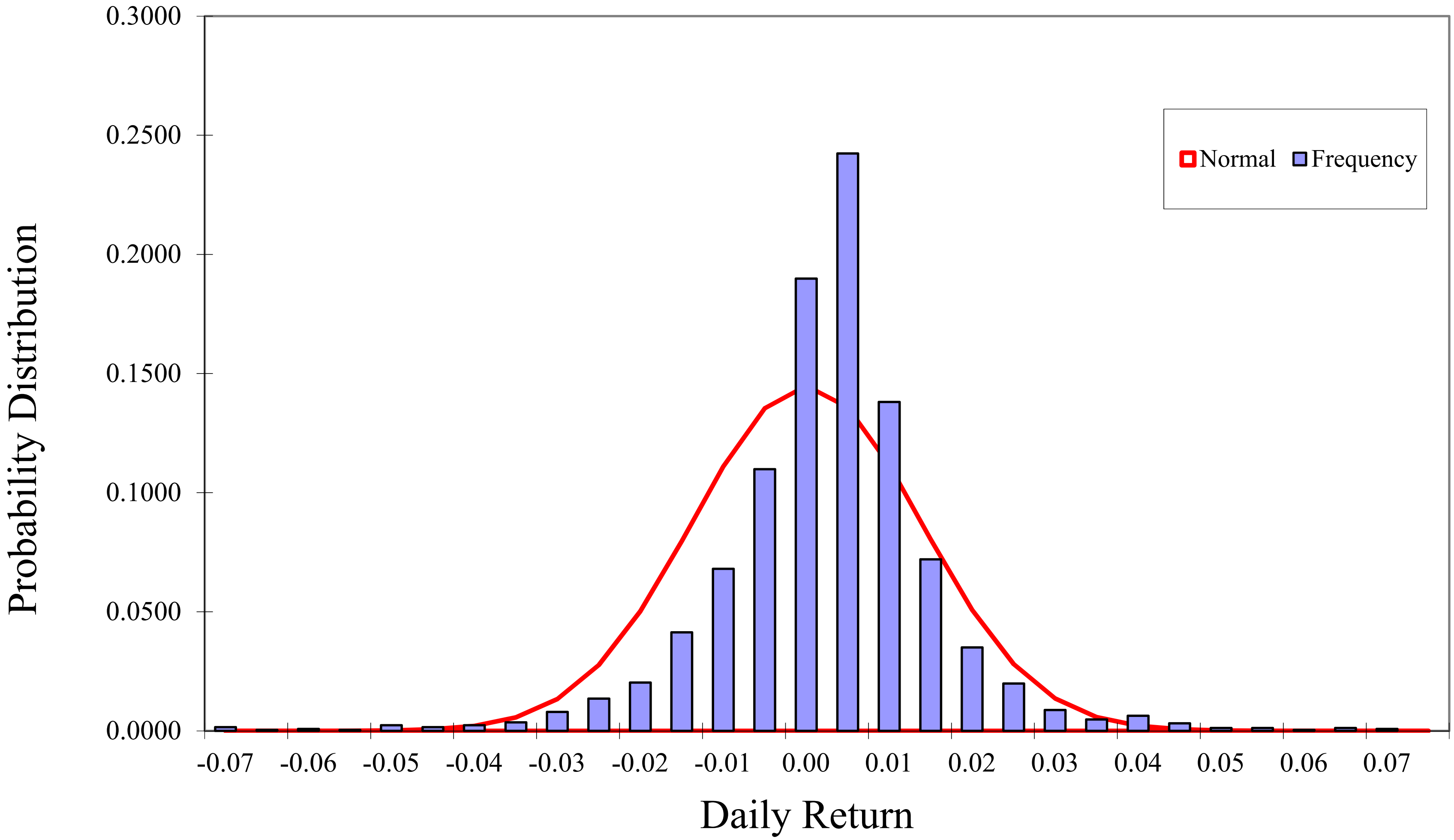
Figure 1.1
Autocorrelation of Daily S&P 500 Returns
Jan 1, 2001 - Dec 31, 2010



Stylized fact 2

- The unconditional distribution of daily returns have fatter tail than the normal distribution.
- Fig.1.2 shows a histogram of the daily S&P500 return data with the normal distribution imposed.
- Notice how the histogram has longer and fatter tails, in particular in the left side, and how it is more peaked around zero than the normal distribution.
- Fatter tails mean a higher probability of large losses than the normal distribution would suggest.

Figure 1.2
Histogram of Daily S&P 500 Returns and the Normal Distribution
Jan 1, 2001 - Dec 31, 2010



Stylized fact 3

- The stock market exhibits occasional, very large drops but not equally large up-moves.
- Consequently the return distribution is asymmetric or negatively skewed. This is clear from Figure 1.2 as well.
- Other markets such as that for foreign exchange tend to show less evidence of skewness.

Stylized fact 4

- The standard deviation of returns completely dominates the mean of returns at short horizons such as daily.
- It is typically not possible to statistically reject a zero mean return.
- Our S&P 500 data have a daily mean of 0.0056% and a daily standard deviation of 1.3771%.

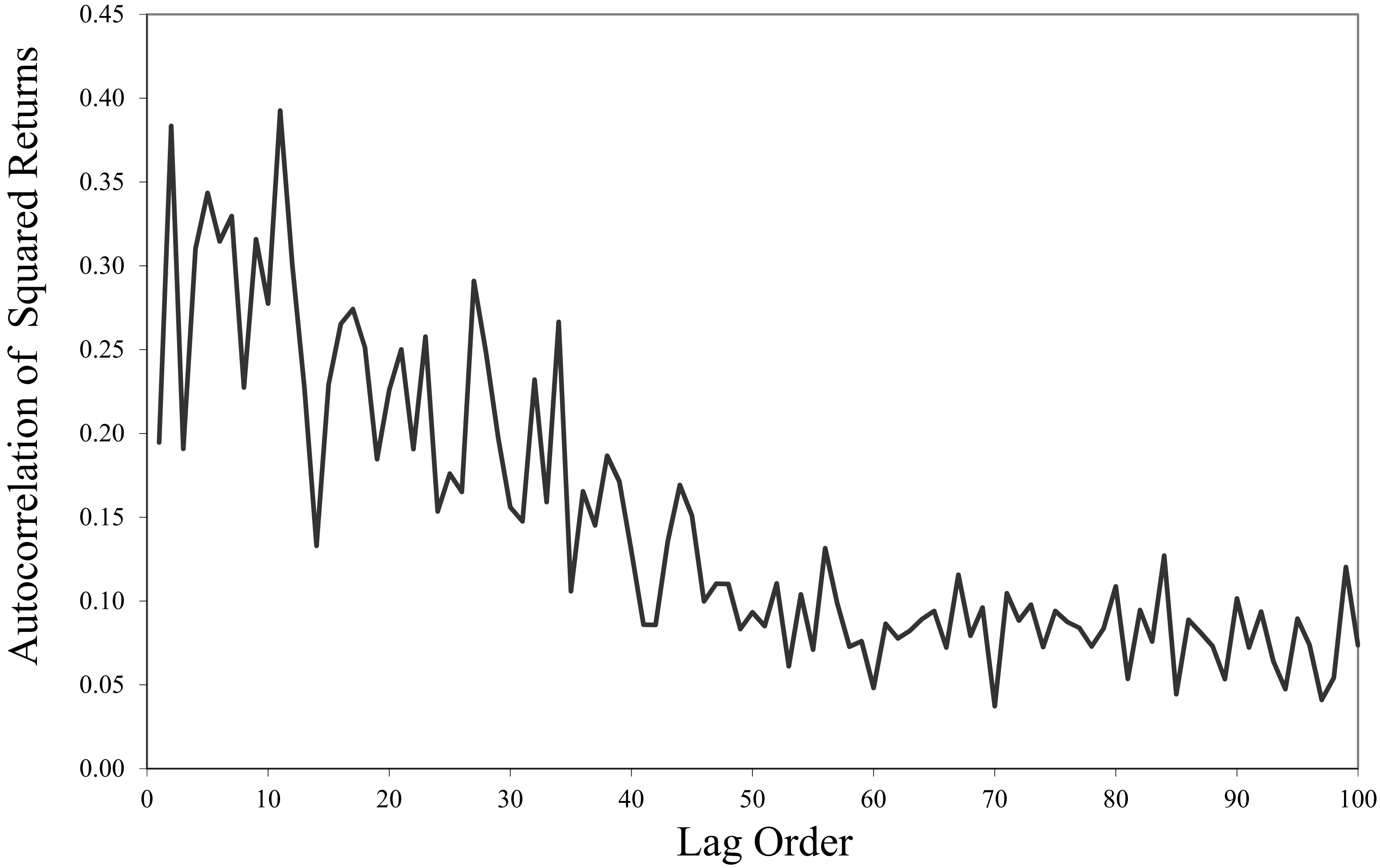
Stylized fact 5

- Variance measured for example by squared returns, displays positive correlation with its own past.
- This is most evident at short horizons such as daily or weekly.
- Fig 1.3 shows the autocorrelation in squared returns for the S&P500 data, that is

$$Corr\left(R_{t+1}^2, R_{t+1-\tau}^2\right) > 0, \quad \text{for small } \tau$$

- Models which can capture this variance dependence will be presented in Chapters 4 & 5

Figure 1.3
Autocorrelation of Squared Daily S&P 500 Returns
Jan 1, 2010 - Dec 31, 2010



Stylized fact 6

- Equity and equity indices display negative correlation between variance and returns.
- This often termed the leveraged effect, arising from the fact that a drop in stock price will increase the leverage of the firm as long as debt stays constant.
- This increase in leverage might explain the increase variance associated with the price drop. We will model the leverage effect in Chapters 4 and 5.

Stylized fact 7

- Correlation between assets appears to be time varying.
- Importantly, the correlation between assets appear to increase in highly volatile down-markets and extremely so during market crashes.
- We will model this important phenomenon in Chapter 7

Stylized fact 8

- Even after standardizing returns by a time-varying volatility measure, they still have fatter than normal tails.
- We will refer to this as evidence of conditional non-normality.
- It will be modeled in Chapters 6 and 9.

Stylized fact 9

- As the return-horizon increases, the unconditional return distribution changes and looks increasingly like the normal distribution.
- Issues related to risk management across horizons will be discussed in Chapter 8.

A generic model of asset returns

- Based on the above of stylized facts our model of individual asset returns will take the generic form

$$R_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1}, \quad \text{with } z_{t+1} \sim \text{i.i.d. } D(0, 1)$$

- The conditional mean return is thus mu_{t+1} and the conditional variance

$$E_t[R_{t+1} - \mu_{t+1}]^2, \text{ is } \sigma^2_{t+1}.$$

- The random variable z_{t+1} is an innovation term, which we assume is identically and independently distributed (i.i.d.) as D(0,1).

A generic model of asset returns

JP Morgan's RiskMetrics model for dynamic volatility

•The volatility for tomorrow, time t+1, is computed at the end of today, time t, using the following simple updating rule:

$$\sigma_{t+1}^2 = 0.94\sigma_t^2 + 0.06R_t^2$$

- On the first day of the sample, $t = 0$, the volatility can be set to the sample variance of the historical data available.

From asset returns to portfolio returns

- The value of a portfolio with n assets at time *t* is the weighted average of the asset prices using the current holdings of each asset as weights:

$$V_{PF,t} = \sum_{i=1}^{n} N_i S_{i,t}$$

- The return on the portfolio between day *t*+1 and day *t* is then defined as $r_{PF,t+1} = V_{PF,t+1}/V_{PF,t} - 1$ when using arithmetic returns

- When using log returns return on the portfolio is:

$$R_{PF,t+1} = \ln\left(V_{PF,t+1}\right) - \ln\left(V_{PF,t}\right)$$

Introducting the VaR risk measure

- Value-at-Risk - What loss is such that it will only be exceeded $p \cdot 100\%$ of the time in the next K trading days?

- *VaR* is often defined in dollars, denoted by $\$VaR$

- $\$VaR$ loss is implicitly defined from the probability of getting an even larger loss as in

$$\Pr(\$Loss > \$VaR) = p$$

Introducing the VaR risk measure

- Note by definition that $(1-p)100\%$ of the time, the \$*Loss* will be smaller than the *VaR*.

- Also note that for this course we will use *VaR* based on log returns defined as

$$\Pr(-R_{PF} > VaR) = p \Leftrightarrow$$
$$\Pr(R_{PF} < -VaR) = p$$

Introducing the VaR risk measure

- Now we are $(1-p)100\%$ confident that we will get a return better than $-VaR$.
- It is much easier to gauge the magnitude of VaR when it is written in return terms
- Knowing that the $\$VaR$ of a portfolio is \$500,000 does not mean much unless we know the value of the portfolio

- The two *VaR*s are related as follows:

$$\$VaR = V_{PF}(1 - \exp(-VaR))$$

Introducing the VaR risk measure

- Suppose our portfolio consists of just one security
- For example an S&P 500 index fund
- Now we can use the Risk-Metrics model to provide the *VaR* for the portfolio.
- Let VaR^{p}_{t+1} denote the p .100% *VaR* for the 1-day ahead return, and assume that returns are normally distributed with zero mean and standard deviation $\sigma_{PF,t+1}$. Then:

$$\Pr\left(R_{PF,t+1} < -VaR^{p}_{t+1}\right) = p \Leftrightarrow$$
$$\Pr\left(R_{PF,t+1}/\sigma_{PF,t+1} < -VaR^{p}_{t+1}/\sigma_{PF,t+1}\right) = p \Leftrightarrow$$
$$\Pr\left(z_{t+1} < -VaR^{p}_{t+1}/\sigma_{PF,t+1}\right) = p \Leftrightarrow$$
$$\Phi\left(-VaR^{p}_{t+1}/\sigma_{PF,t+1}\right) = p$$

Introducing the VaR risk measure

- $\Phi(z)$ calculates the probability of being below the number *z*
- $\Phi^{-1}{}_{P} = \Phi^{-1}(P)$ instead calculates the number such that *p*.100% of the probability mass is below $\Phi^{-1}{}_{P}$
- Taking $\Phi^{-1}(*)$ on both sides of the preceding equation yields the *VaR* as

$$-VaR^{p}_{t+1}/\sigma_{PF,t+1} = \Phi^{-1}(p) \Leftrightarrow$$
$$VaR^{p}_{t+1} = -\sigma_{PF,t+1}\Phi^{-1}_{p}$$

Introducing the VaR risk measure

- If we let $p = 0.01$ then we get $\Phi^{-1}_P = \Phi^{-1}_{0.01} = \approx -2.33$

- If we assume the standard deviation forecast, $\sigma_{PF,t+1}$ for tomorrow's return is 2.5% then:

$$
\begin{aligned}
VaR^p_{t+1} &= -\sigma_{PF,t+1}\Phi^{-1}_p \\
&= -0.025(-2.33) \\
&= 0.05825
\end{aligned}
$$

Introducing the VaR risk measure

- $\Phi^{-1}{}_{P}$ is always negative for $p < 0.5$

- The negative sign in front of the *VaR* formula is needed because *VaR* is defined as a positive number

- Here *VaR is interpreted* such that there is a 1% chance of *losing* more than 5.825% of the portfolio value today.

Introducing the VaR risk measure

- If the value of the portfolio today is \$2 million, the \$*VaR* would simply be

$$\begin{aligned}\$VaR &= V_{PF}\left(1-\exp(-VaR)\right)\\ &= 2{,}000{,}000\left(1-\exp(-0.05825)\right)\\ &= \$113{,}172\end{aligned}$$

- For the next figure, note that we assume $K = 1$ and $p = 0.01$

Figure 1.4
Value at Risk (VaR) from the Normal Distribution
Return Probability Distribution

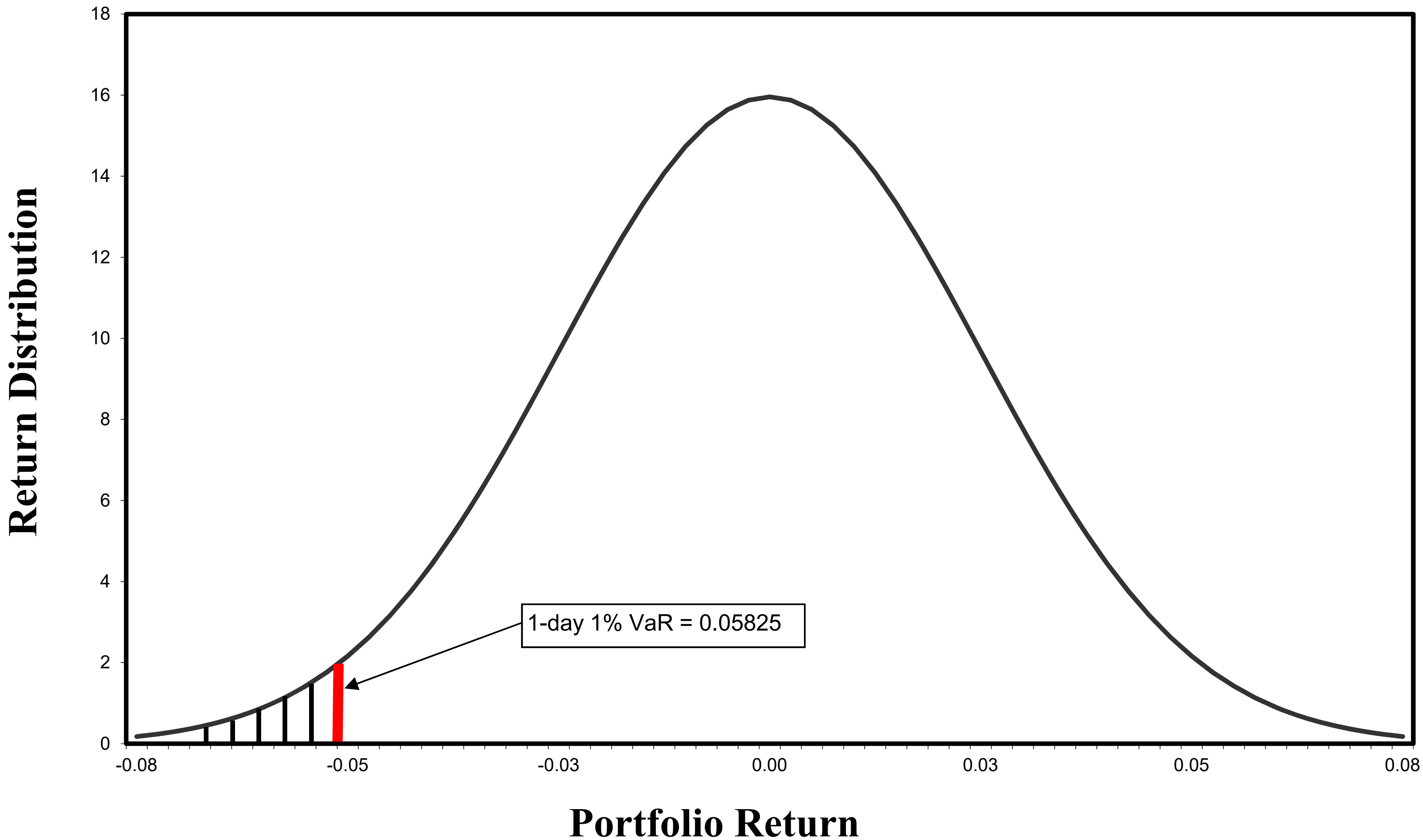
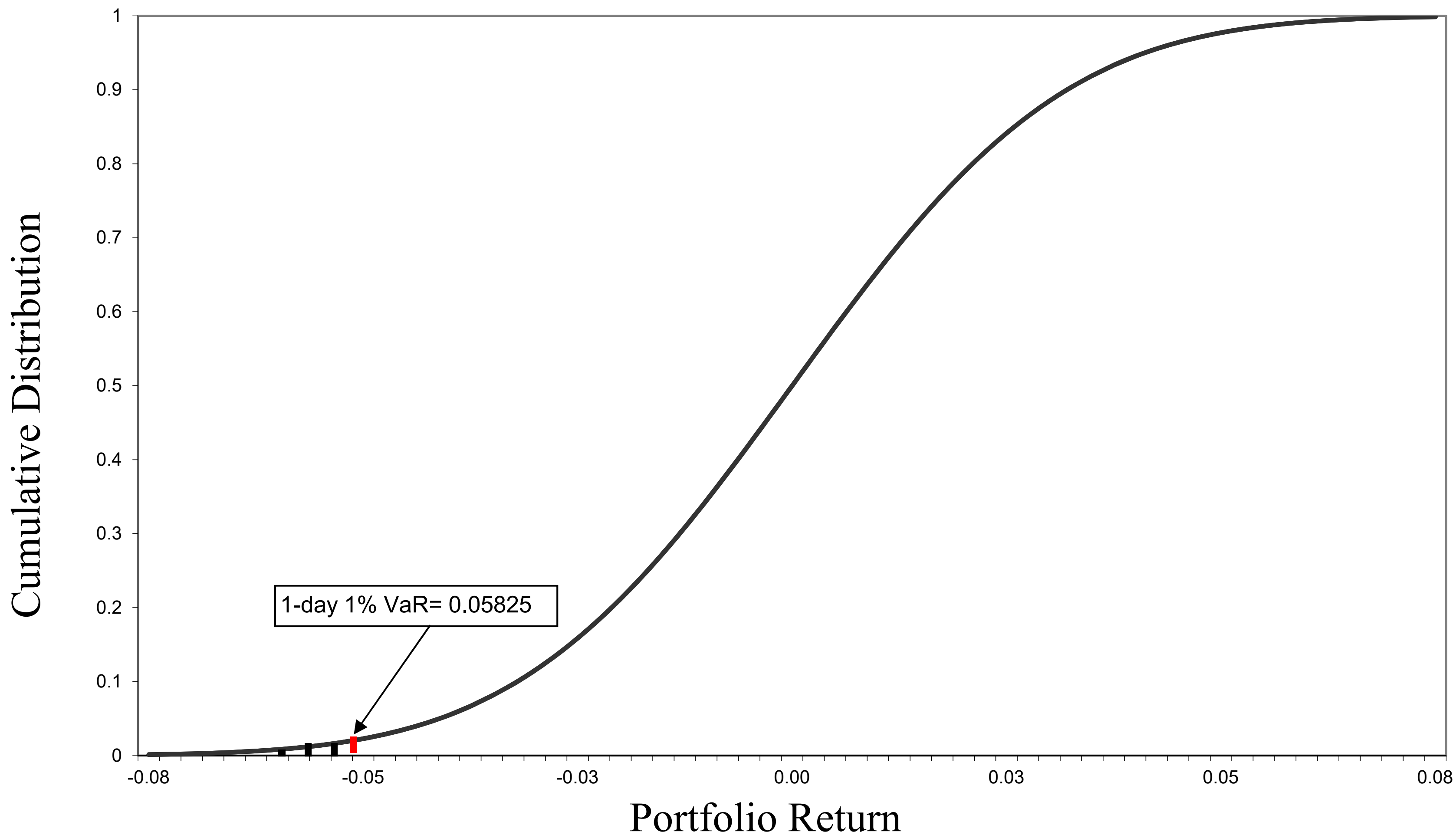


Figure 1.4
Value at Risk (VaR) from the Normal Distribution Return Probability Distribution



Introducing the VaR risk measure

- Consider a portfolio whose value consists of 40 shares in Microsoft (MS) and 50 shares in GE.
- To calculate *VaR* for the portfolio, collect historical share price data for MS and GE and construct the historical portfolio pseudo returns

$$
\begin{aligned}
R_{PF,t+1} &= \ln\left(V_{PF,t+1}\right) - \ln\left(V_{PF,t}\right) \\
&= \ln\left(40S_{MS,t+1} + 50S_{GE,t+1}\right) - \ln\left(40S_{MS,t} + 50S_{GE,t}\right)
\end{aligned}
$$

Introducing the VaR risk measure

- The stock prices include accrued dividends and other distributions

- Constructing a time series of past portfolio pseudo returns enables us to generate a portfolio volatility series using for example the RiskMetrics approach where

$$\sigma^2_{PF,t+1} = 0.94\sigma^2_{PF,t} + 0.06R^2_{PF,t}$$

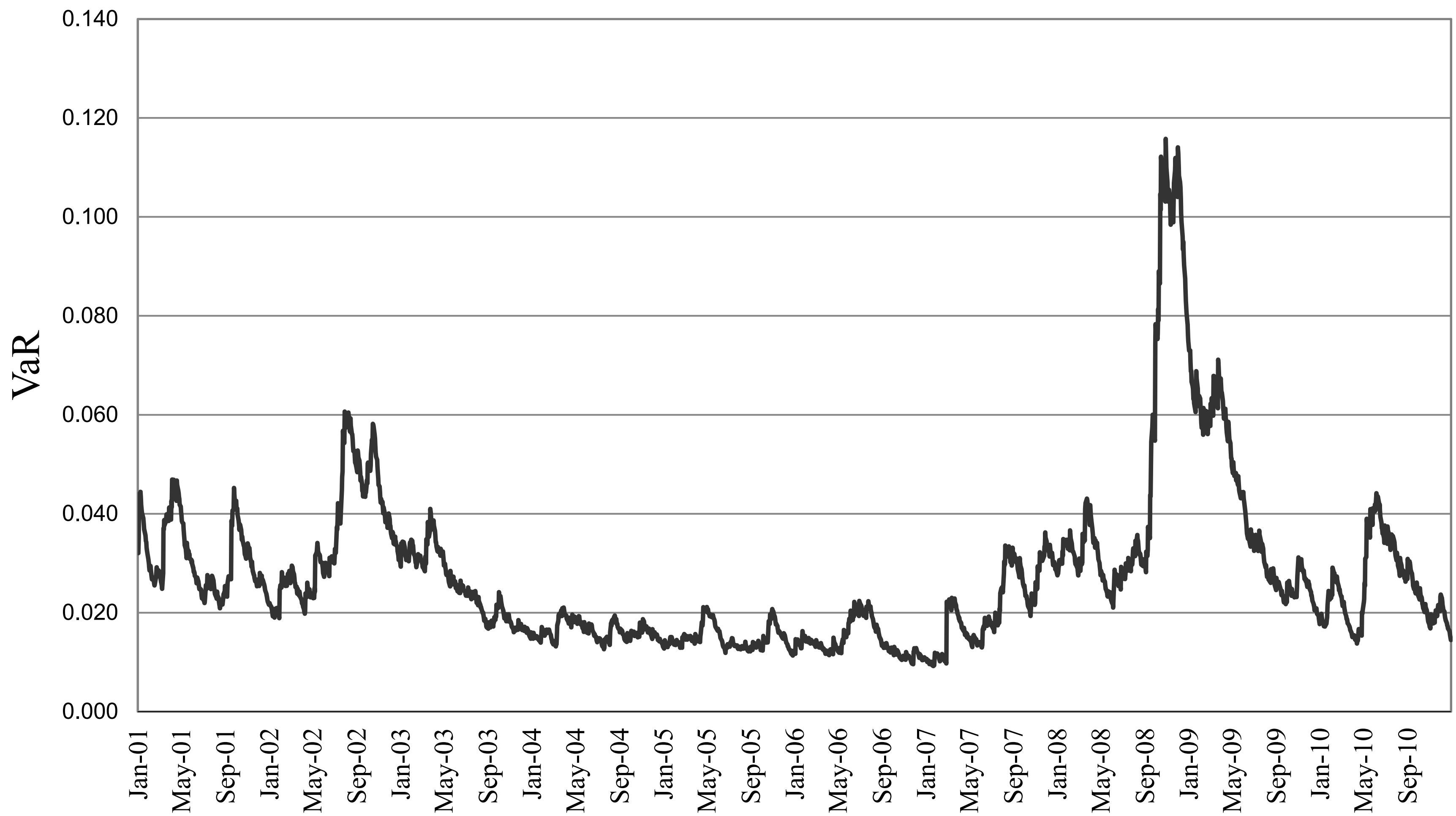
Introducing the VaR risk measure

- We can now directly model the volatility of the portfolio return, $R_{PF,t+1}$, call it $\sigma_{PF,t+1}$, and then calculate the *VaR* for the portfolio as

$$VaR_{t+1}^{p} = -\sigma_{PF,t+1}\Phi_{p}^{-1}$$

- We assume that the portfolio returns are normally distributed

Figure 1.5
1-day, RiskMetrics 1% VaR in S&P500 Portfolio
Jan 1, 2001 - Dec 31, 2010



Drawbacks of VaR

- Extreme losses are ignored - The *VaR* number only tells us that 1% of the time we will get a return below the reported *VaR* number, but it says nothing about what will happen in those 1% worst cases.
- *VaR* assumes that the portfolio is constant across the next K days, which is unrealistic in many cases when K is larger than a day or a week.
- Finally, it may not be clear how K and p should be chosen.