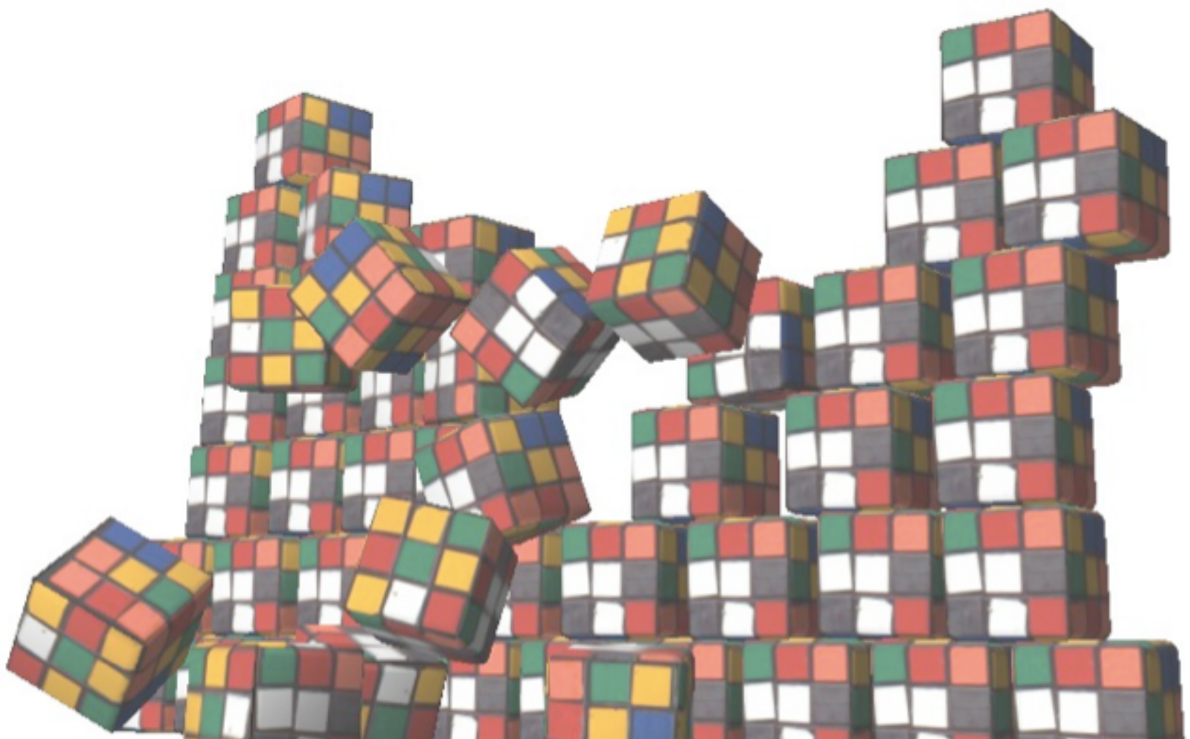
Rigid Body Simulations

Thuerey / Game Physics



Derivation of Impulse Equations

- Re-cap, total impulse in 2D:

$$J = \frac{-(1+c)\mathbf{v}_{rel} \cdot \mathbf{n}}{\frac{1}{M_a} + \frac{1}{M_b} + (\mathbf{x}_a \times \mathbf{n})^2 / I_a + (\mathbf{x}_b \times \mathbf{n})^2 / I_b}$$

- Acting on linear velocities of bodies A and B via:

$$\mathbf{v}'_a = \mathbf{v}_a + J_{lin}\mathbf{n}/M_a \quad , \quad \mathbf{v}'_b = \mathbf{v}_b - J_{lin}\mathbf{n}/M_b$$

First: Linear Impulse Only

- Relative velocity: $\mathbf{v}_r = \mathbf{v}_a - \mathbf{v}_b$ (1)
- Pre collision velocity $\mathbf{v}_r$, post collision $\mathbf{v}_r'$
- Coefficient of restitution should act via: $\mathbf{v}_r' = -c\mathbf{v}_r$ (2)
- Eq. (1&2) along normal direction: $(\mathbf{v}_a' - \mathbf{v}_b') \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$ (3)
- Impulse acts as: $\mathbf{v}_a' = \mathbf{v}_a + J_{lin}\mathbf{n}/M_a$, $\mathbf{v}_b' = \mathbf{v}_b - J_{lin}\mathbf{n}/M_b$
- Insert into (3):

$$(\mathbf{v}_a + J_{lin}\mathbf{n}/M_a - \mathbf{v}_b + J_{lin}\mathbf{n}/M_b) \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$$

First: Linear Impulse Only

- From previous slide: $(\mathbf{v}_a + J_{lin}\mathbf{n}/M_a - \mathbf{v}_b + J_{lin}\mathbf{n}/M_b) \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$
- Pull out $\mathbf{v}_r$ and $\mathbf{n}$: $J_{lin}(1/M_a + 1/M_b)\mathbf{n} \cdot \mathbf{n} = -(1 + c)\mathbf{v}_r \cdot \mathbf{n}$
- Gives linear Impulse: $$\boxed{J_{lin} = \frac{-(1+c)\mathbf{v}_{rel} \cdot \mathbf{n}}{\frac{1}{M_a} + \frac{1}{M_b}}}$$

Impulse with Angular Velocities

- Full post collision velocity given by:

$$\mathbf{v}'_a = \mathbf{v}_a + J\mathbf{n}/M_a + \left(\mathbf{w}_a + I_a^{-1}(\mathbf{x}_a \times J\mathbf{n})\right) \times \mathbf{x}_a \ ,$$

$$\mathbf{v}'_b = \mathbf{v}_b - J\mathbf{n}/M_b + \left(\mathbf{w}_b - I_b^{-1}(\mathbf{x}_b \times J\mathbf{n})\right) \times \mathbf{x}_b$$

(Like torque)

- Insert into eq. (3), which was $(\mathbf{v}'_a - \mathbf{v}'_b) \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$:
- $\left(\mathbf{v}_a + J\mathbf{n}/M_a - \mathbf{v}_b + J\mathbf{n}/M_b + \left(\mathbf{w}_a + I_a^{-1}(\mathbf{x}_a \times J\mathbf{n})\right) \times \mathbf{x}_a - \left(\mathbf{w}_b - I_b^{-1}(\mathbf{x}_b \times J\mathbf{n})\right) \times \mathbf{x}_b\right) \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$
- Next, re-arrange and pull out J …

Impulse with Angular Velocities

- $\left(\mathbf{v}_a + J\mathbf{n}/M_a - \mathbf{v}_b + J\mathbf{n}/M_b + \left(\mathbf{w}_a + I_a^{-1}(\mathbf{x}_a \times J\mathbf{n})\right) \times \mathbf{x}_a - \left(\mathbf{w}_b - I_b^{-1}(\mathbf{x}_b \times J\mathbf{n})\right) \times \mathbf{x}_b\right) \cdot \mathbf{n} = -c\mathbf{v}_r \cdot \mathbf{n}$

- Move $\mathbf{v}_r$ to the right side and also collect $\mathbf{w} \times \mathbf{x}$ terms:

  $$\mathbf{v}_r = \mathbf{v}_a + \mathbf{w}_a \times \mathbf{x}_a - \mathbf{v}_b - \mathbf{w}_b \times \mathbf{x}_b$$

- In 2D the inertia tensor is a scalar

- Thus we can simplify: $\left(I^{-1}(\mathbf{x} \times \mathbf{n}) \times \mathbf{x}\right) \cdot \mathbf{n} = I^{-1}(\mathbf{x} \times \mathbf{n})^2$

- Giving: $J = \dfrac{-(1+c)\mathbf{v}_{rel} \cdot \mathbf{n}}{\frac{1}{M_a} + \frac{1}{M_b} + (\mathbf{x}_a \times \mathbf{n})^2/I_a + (\mathbf{x}_b \times \mathbf{n})^2/I_b}$

Impulse with Angular Velocities

- In 3D, we need to keep both cross product terms
- Full impulse equation:

$$\boxed{J = \frac{-(1+c)\mathbf{v}_{rel} \cdot \mathbf{n}}{\frac{1}{M_a} + \frac{1}{M_b} + \left(I_a^{-1}(\mathbf{x}_a \times \mathbf{n}) \times \mathbf{x}_a\right) \cdot \mathbf{n} + \left(I_b^{-1}(\mathbf{x}_b \times \mathbf{n}) \times \mathbf{x}_b\right) \cdot \mathbf{n}}}$$

So much for 2D...