Economics

Suppose an economy consists of three sectors: Agriculture (A), Energy (E), and Manufacturing (M). Each sector has a total annual value of its output: P_A, P_E, P_M.

The goal in this application is to find values for each sector such that the expenditures and profits are balanced. These are called *equilibrium prices*.

1. To find such prices, we first need to know how much of the output of one sector goes into each of the others. When a sector's output returns to itself, it represents the associated expenses to operate that sector.

   Suppose Agriculture sells 10% of its output to Energy and 20% of its output to Manufacturing, with 70% retained. Energy sells 20% of its output to Agriculture and 60% of its output to Manufacturing, with 20% retained. Manufacturing sells 10% of its output to Agriculture and 50% to Energy, with 40% retained.

   We organize such information in an *Exchange Table*. One entry has been filled in for you.

Output			Purchased by:
A	E	M	
			A
.1			E
			M

   Check your work: what should the sum of each column be?

2. At equilibrium, the output of each sector, for instance, P_A, should be the same as its expenses. This creates a system of three equations with three unknowns. Fill in the missing coefficients.

$$\begin{aligned} P_A &= (\quad)P_A + (\quad)P_E + (\quad)P_M \\ P_E &= (0.1)P_A + (\quad)P_E + (\quad)P_M \\ P_M &= (\quad)P_A + (\quad)P_E + (\quad)P_M \end{aligned}$$

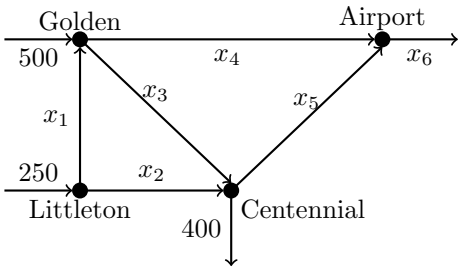
3. Rewrite the system in the form $A\mathbf{x} = \mathbf{0}$.

4. Rewrite the system as an augmented matrix, and solve.

5. Suppose $P_M = 11$ million dollars. What should the output of the other two sectors be for equilibrium?

Network Flow

1. Network models arise in a variety of applications. A *network* consists of a set of nodes with connecting edges with a capacity along which some item may move. Examples include water in pipes, electricity on wires, and here, through traffic of Denver.



2. One of the first assumptions when working with a network is that the flow into the network is the same as the flow out of the network. This network has two places, called *sources*, where traffic flows in. It also has two places where traffic leaves the network. What is the total flow in? What must x_6 equal?

3. The second assumption is that the same holds at each node of the network. That is, the total traffic into Golden is the same as the flow out of Golden, giving

$$500 + x_1 = x_3 + x_4.$$

   Write the equations representing the conservation of flow at the other three nodes.

4. Write an augmented matrix representing the system, then row reduce the matrix to reduced echelon form.

5. Using vectors and the free variables as parameters, describe all solutions to the system.

6. Suppose all flows should head in the direction marked. This means that the corresponding variables $x_1, \ldots, x_5$ must all be nonnegative. In what interval(s) must the free variable(s) lie? Find the solutions when the free variable(s) are as large and as small as possible.

7. If interstate 25, represented as the edge from Golden to Centennial, were completely closed for construction, could this network accurately represent the through traffic in Denver?