Graphs' decompositions
and
resolutions of combinatorial problems

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Tree decomposition

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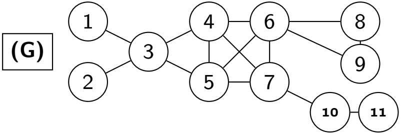

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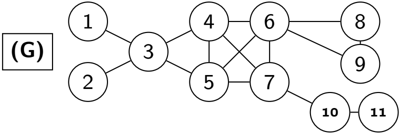

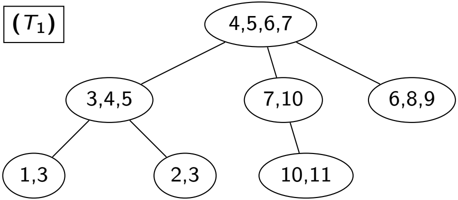

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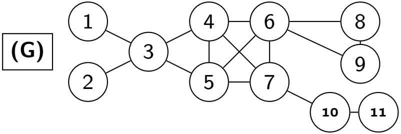

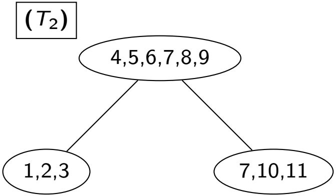

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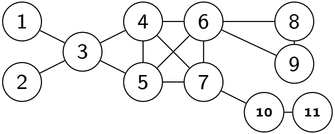
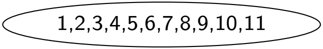


(T_3)



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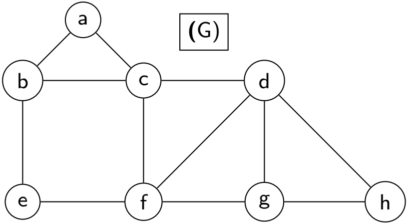

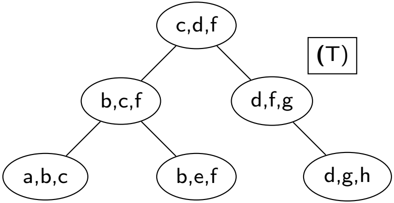

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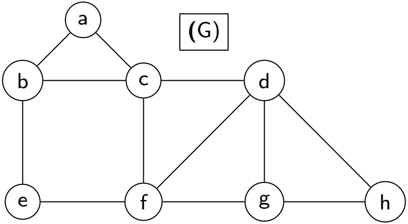

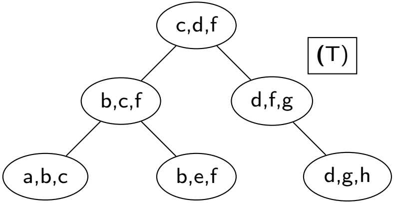

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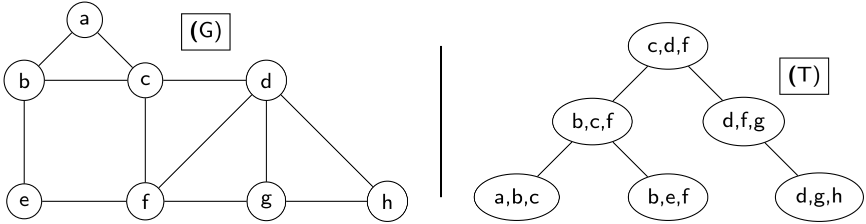


Treewidth(G)=3-1=2

Remark

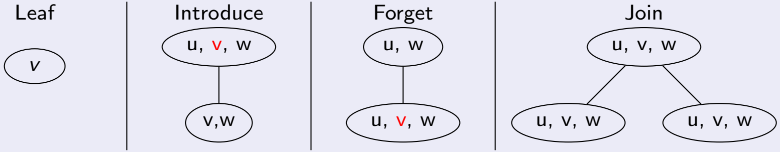
The treewidth of a tree is 1 and if a graph have a treewidth of 1 we can claim that this graph is a forest (i.e. a collection of trees).

Nice tree

Definition (nice tree)

A tree decomposition is *nice* if every node x is one of the following 4 types :

- Leaf : no children, $|B_x| = 1$
- Introduce : 1 child y, $B_x = B_y \cup \{v\}$ for some vertex v
- Forget : 1 child y, $B_x = B_y \backslash \{v\}$ for some vertex v.
- Join : 2 children y_1, y_2 with $B_x = B_{y_1} = B_{y_2}$

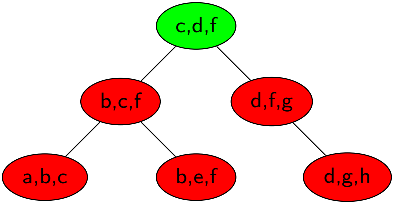


Remark

- *A tree decomposition can be turned into a nice tree decomposition*
- *A nice tree could be very good to simplify a proof or to find an easy program to solve a problem (as we will see later).*

example

This is a decomposition tree saw previously. How could we obtain a nice tree from it ?



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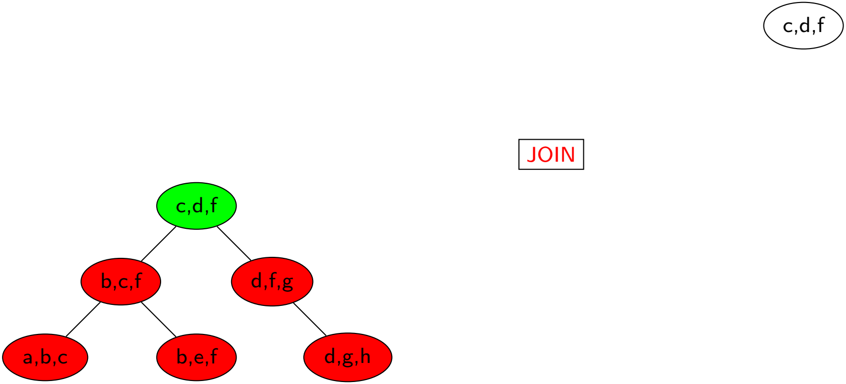

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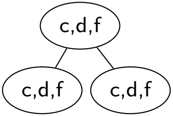

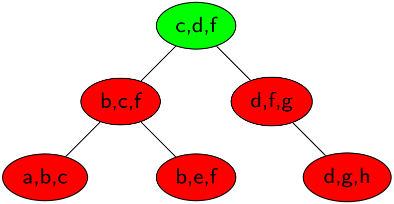

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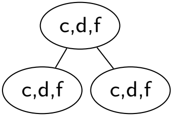

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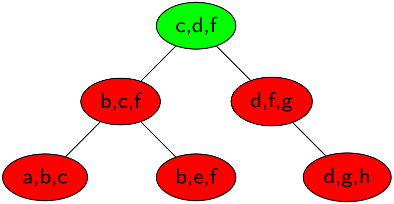

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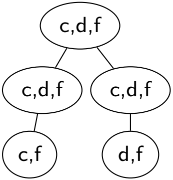

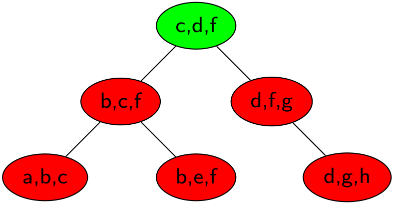

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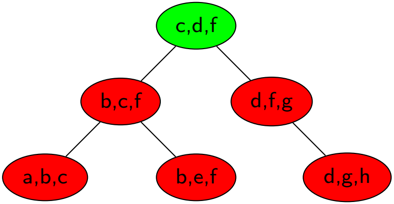

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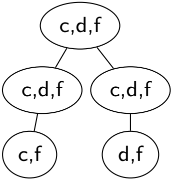

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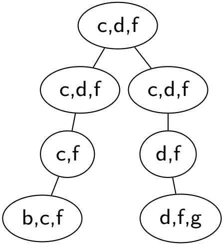

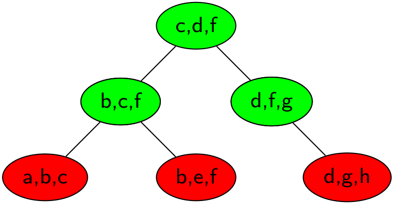

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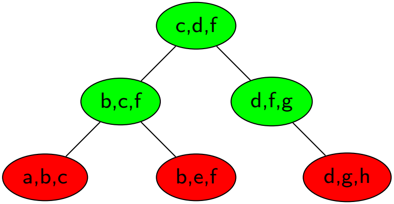
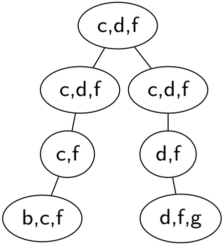


JOIN



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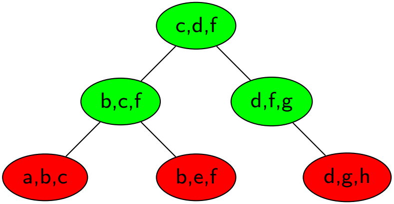

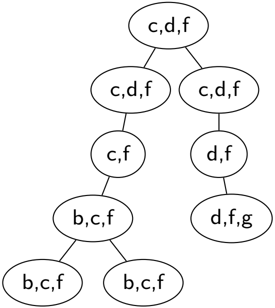

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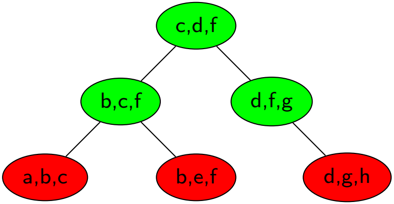

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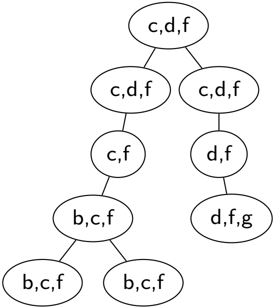

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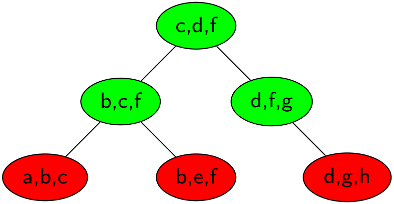

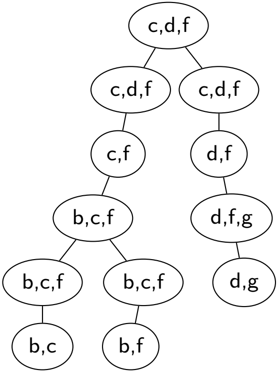

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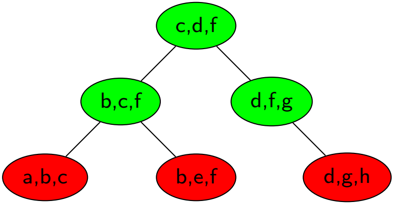

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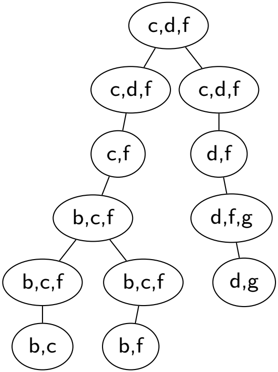

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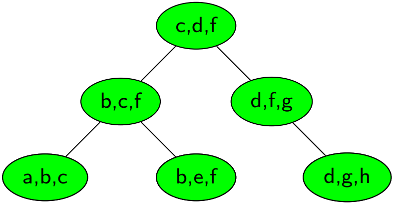

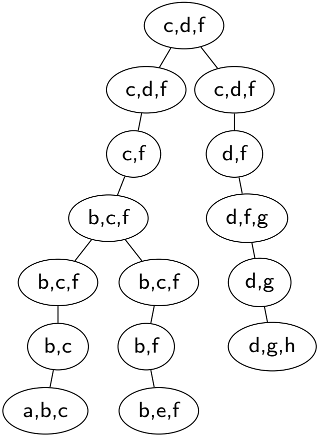

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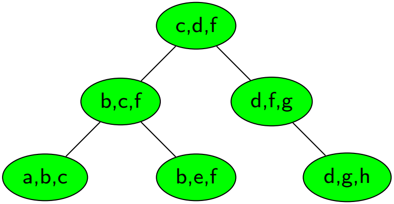

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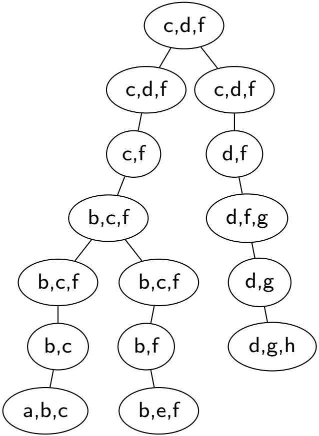

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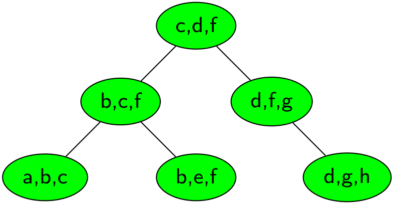

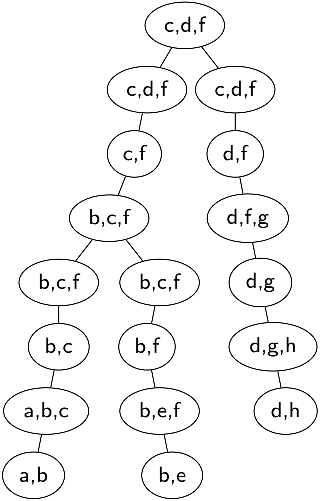

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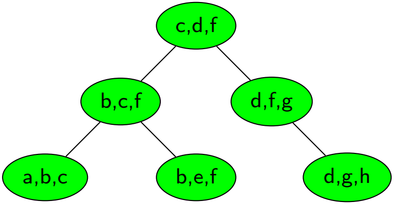

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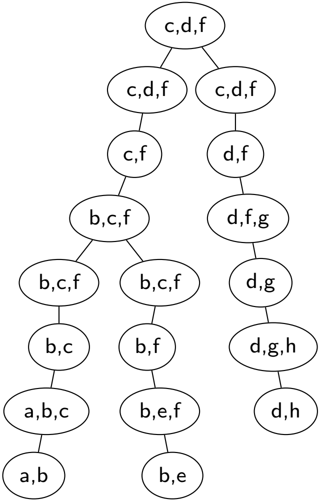

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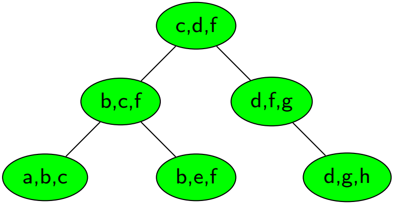

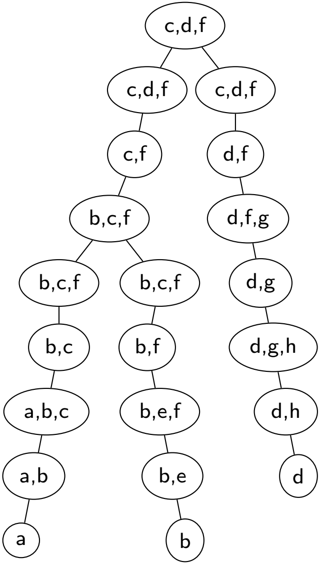

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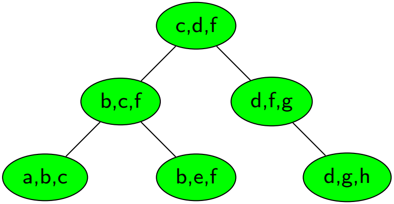

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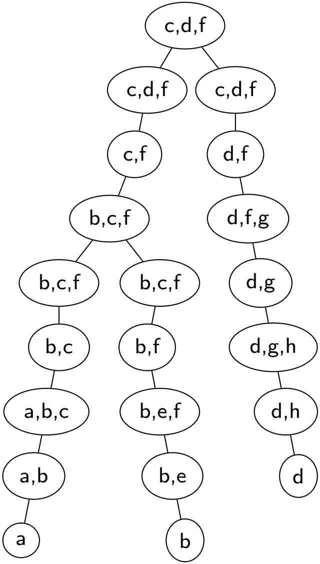

the problem

Problem (k-color)

problem : *Let (G) be a graph and k an integer. We want to know if it is possible to draw each vertice of the graph such that two neighbors have never the same color and with only k color.*
This problem is a problem of decision which is NP.

tree-width : *k-color is possible for a graph (G) if and only if* $k \geqslant treewidth(G)$.

nice tree : *a nice tree of (G) give a way to find a k-coloration of (G) (if* $k \geqslant treewidth(G)$*).*

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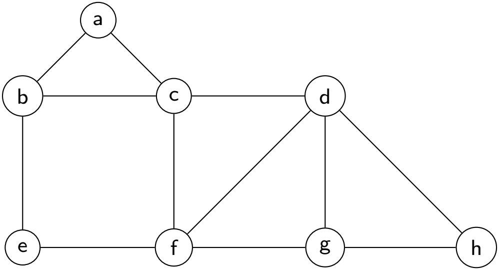
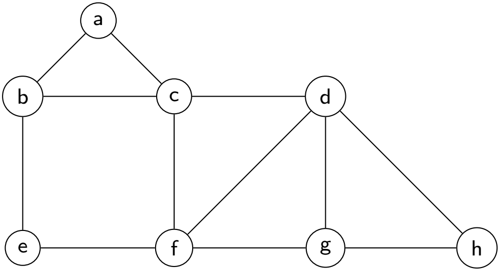
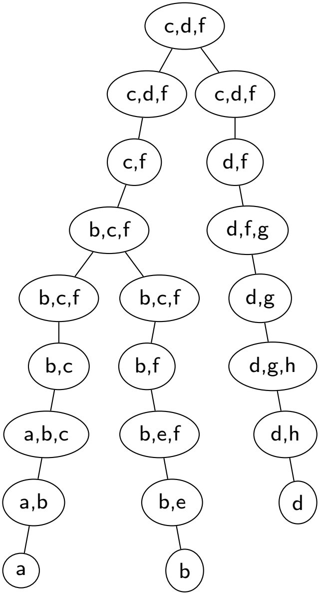
Illustration : we will use the previously trees to solve the problem with this graph :



nice tree of the graph

- treewith(G)=3 : we can solve the problem with 3 colors
- we can fixe a color for a "Leaf"
- when we meet an "Introduce" we add a color
- when we meet a "Forget" we can claim that the vertex which has disappeared won't come back (propertie of the decomposition) and so we can reuse the color.





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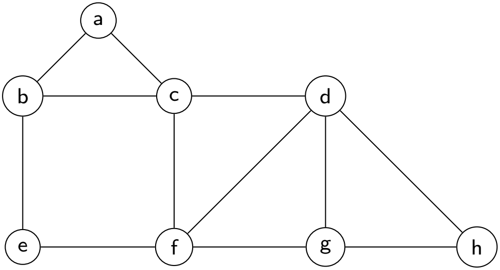

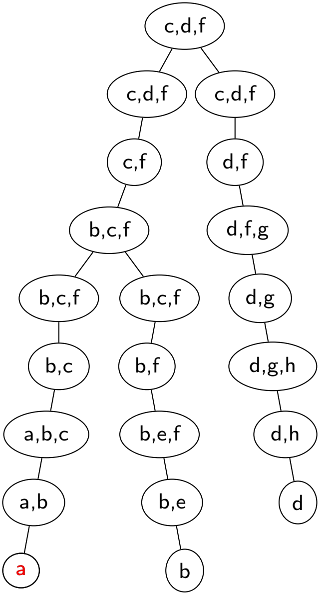

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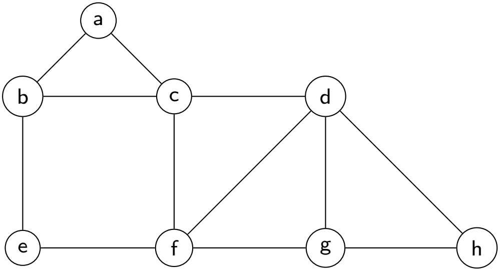

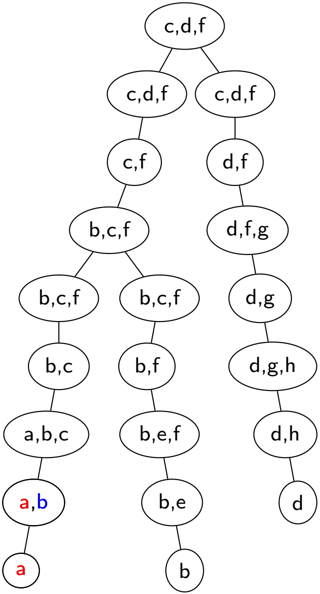

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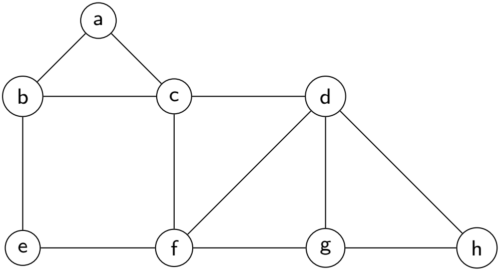

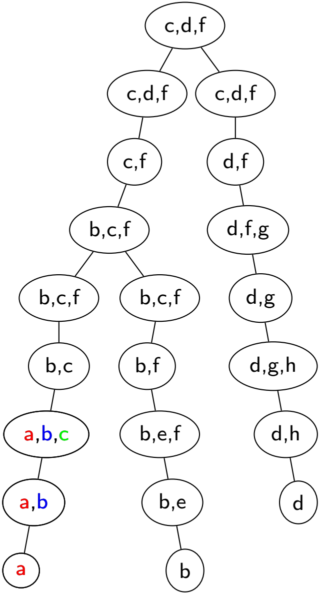

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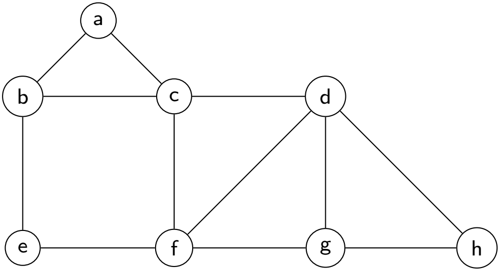

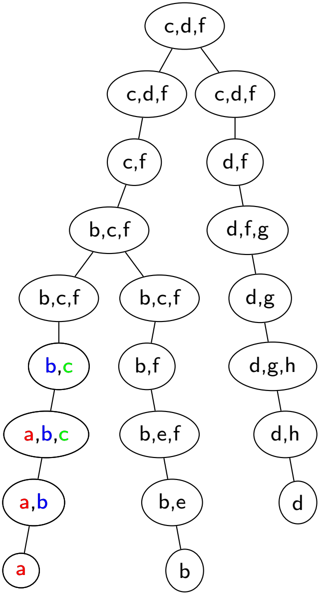

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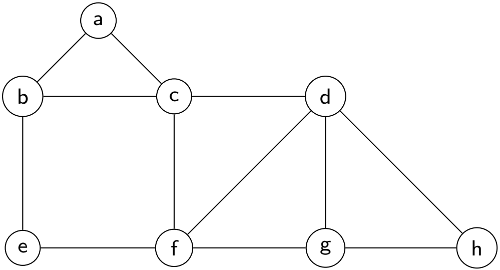

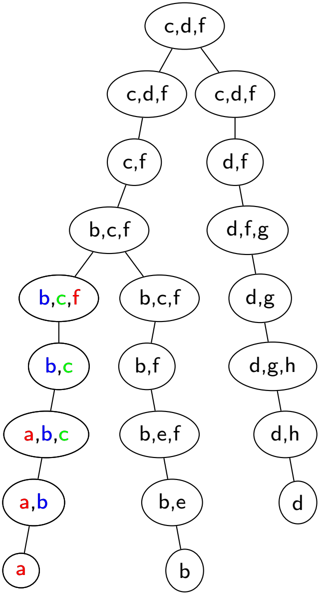

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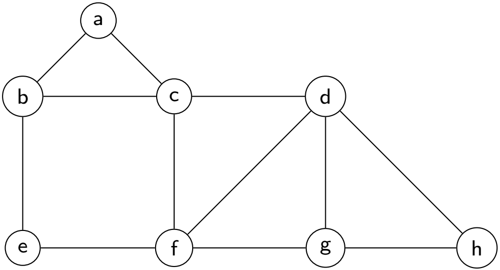

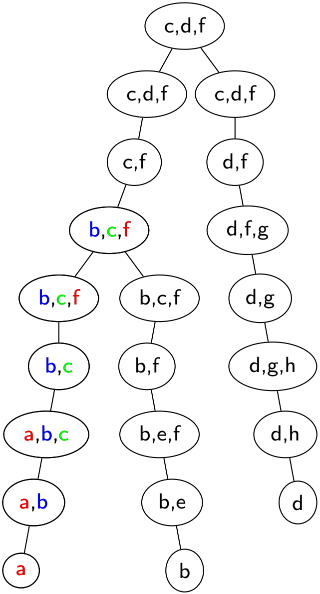

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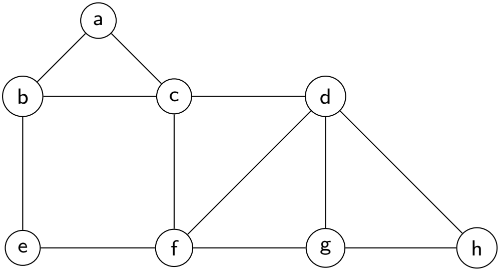

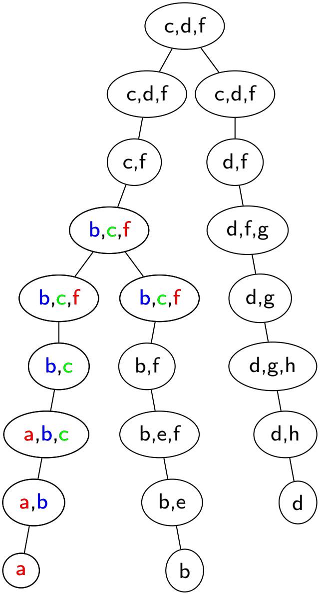

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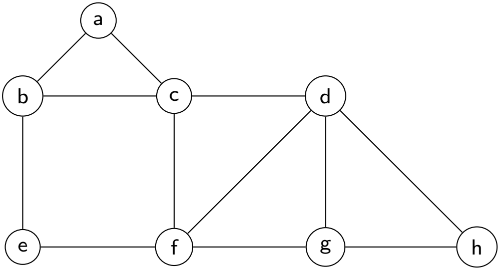

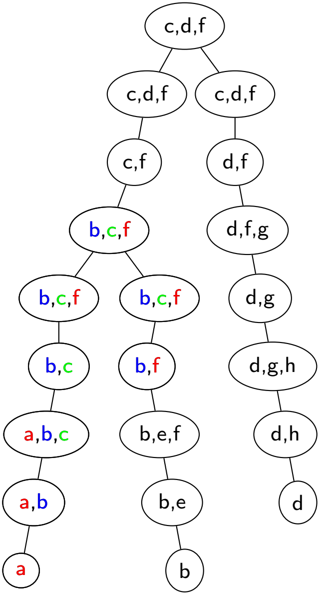

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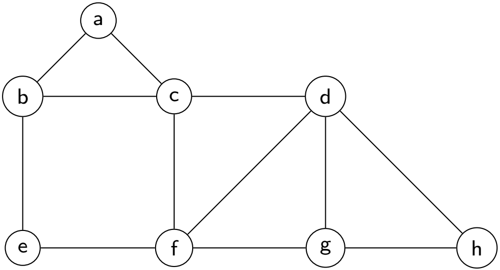

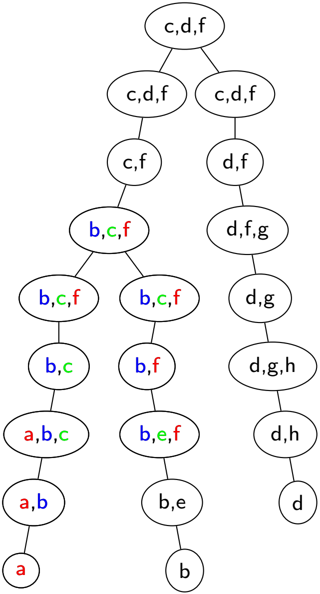

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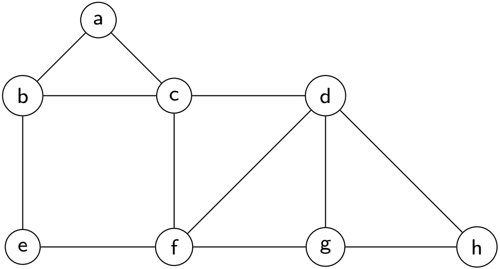

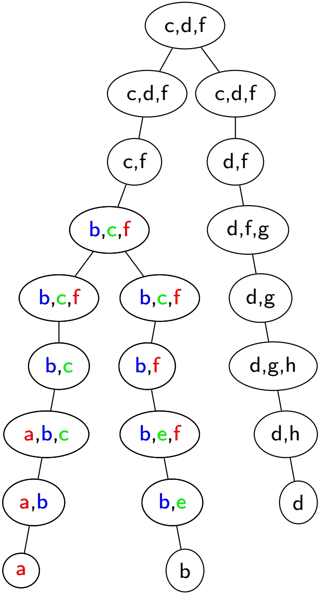

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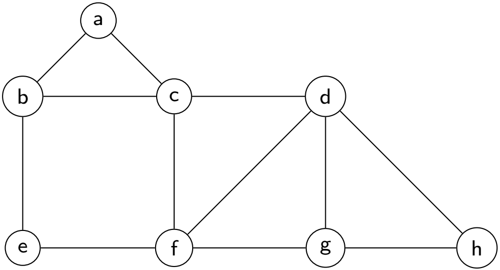

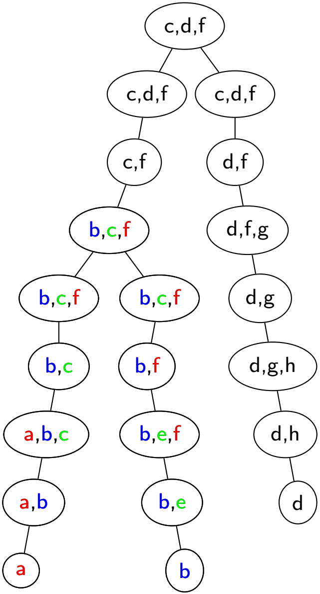

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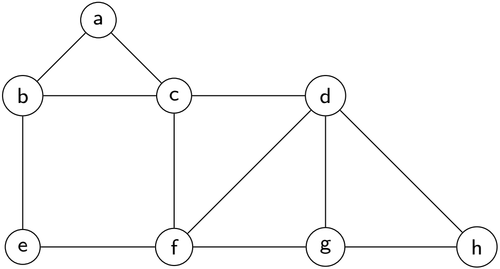

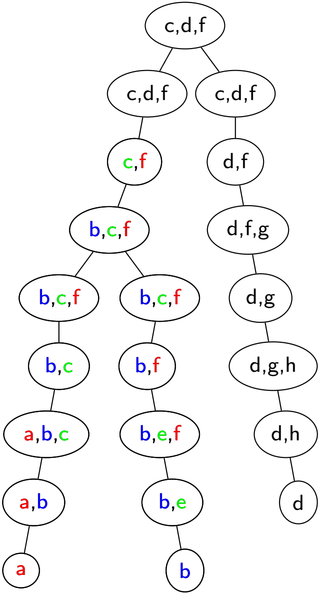

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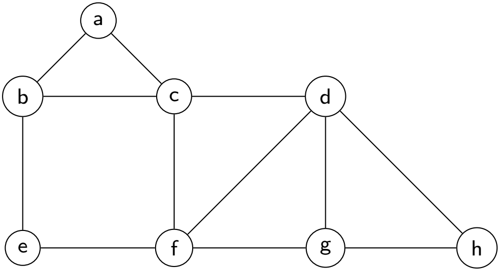

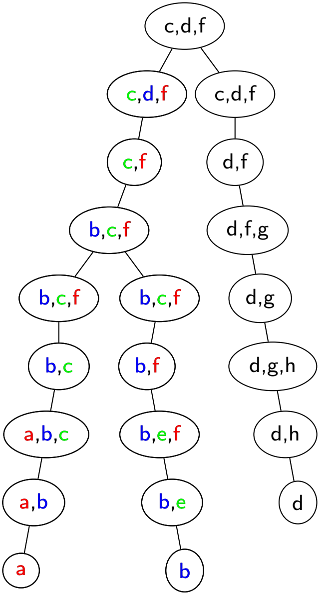

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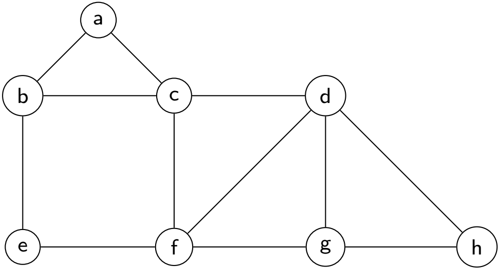

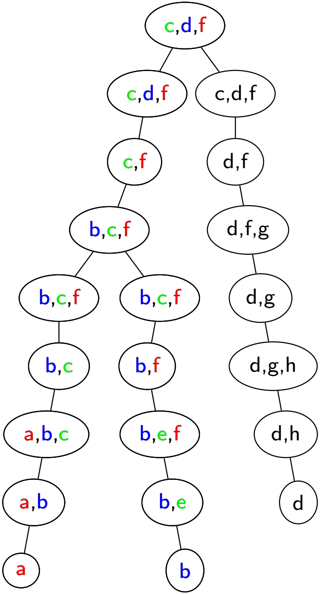

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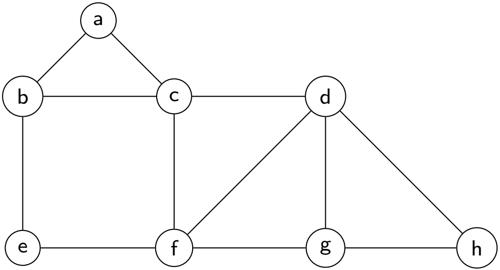

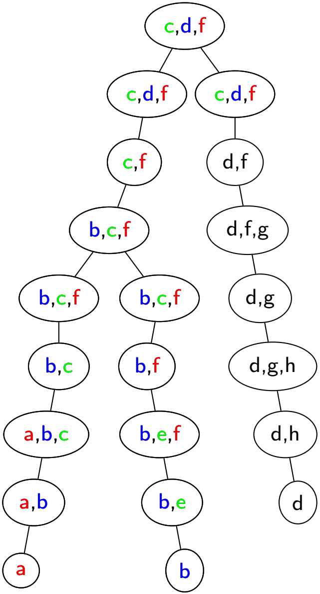

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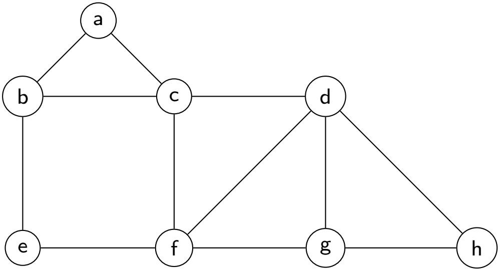

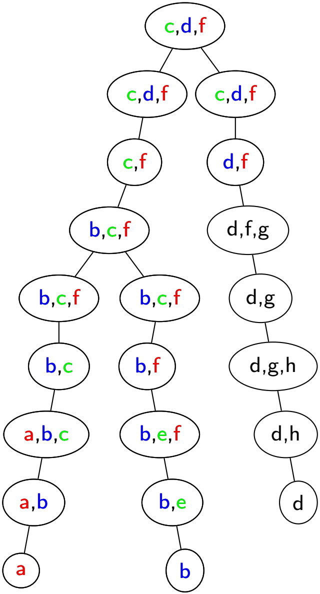

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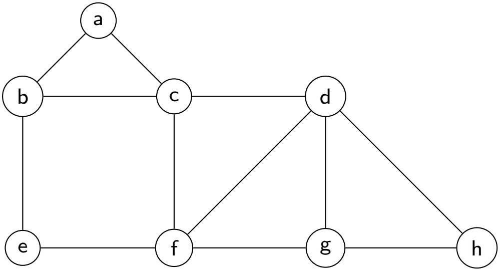

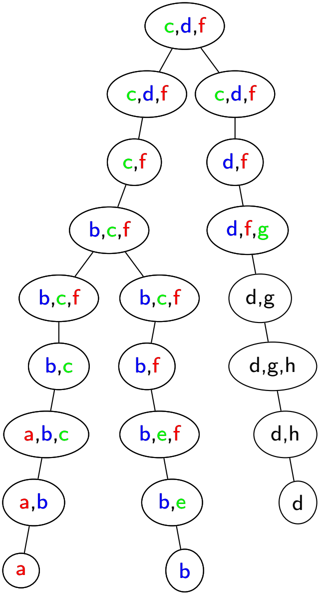

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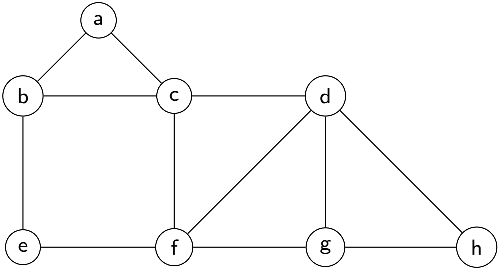

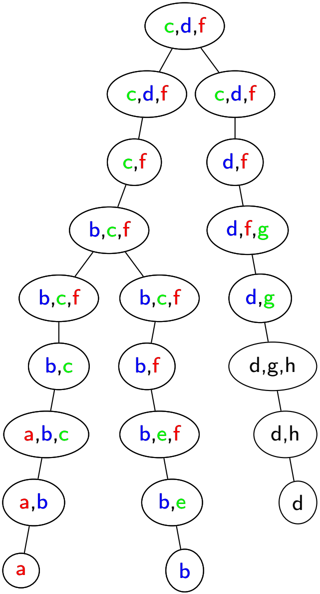

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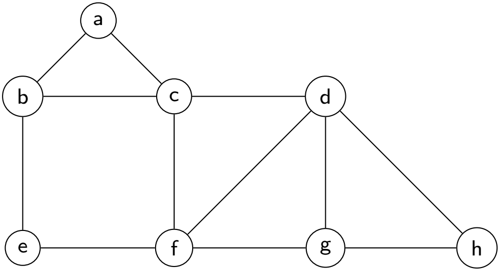

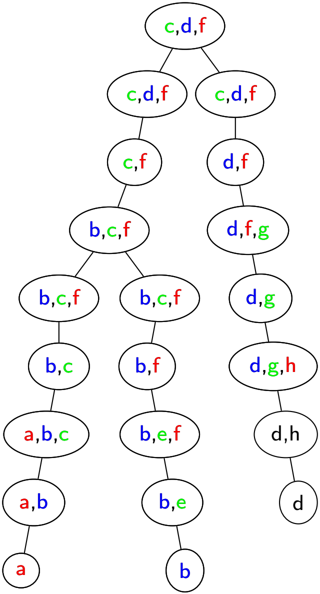

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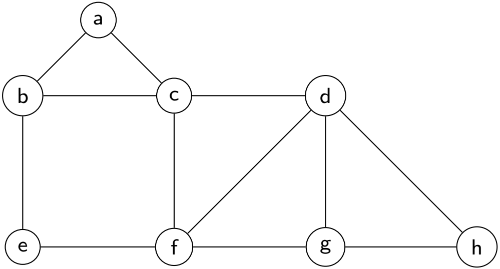

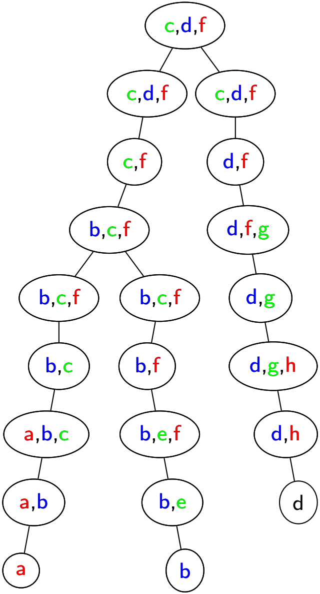

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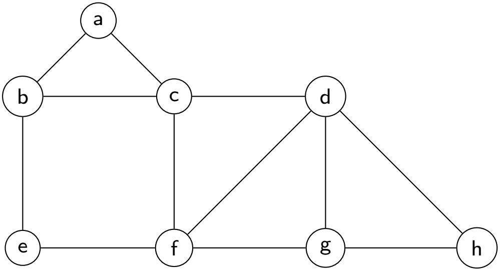

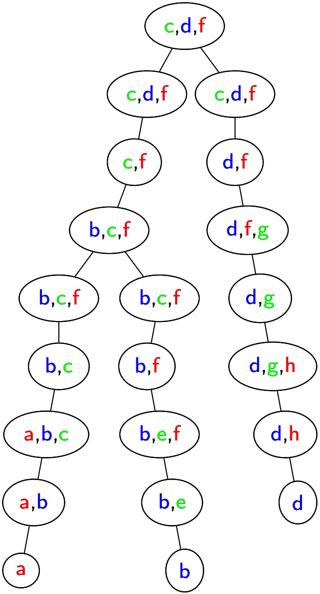

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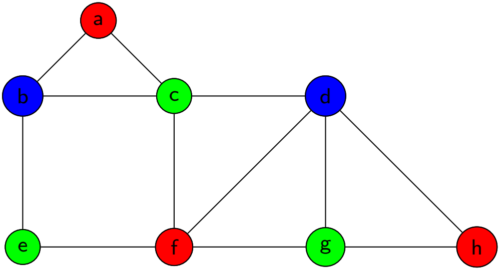

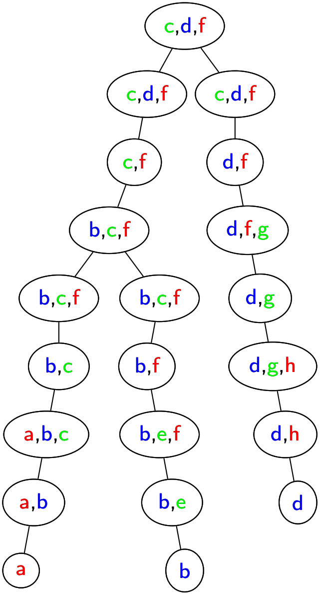

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Bibliography