

# Fundamentals of Statistics III

## Moving Beyond Mean and SD

# What will we learn

- Quantiles and Box-Whisker Plot
- Histogram
- Measure of Skewness

# Dissect Data with Quantiles

- Quartiles divide the distribution into 4 equal parts.  
Q1: Lower Quartile(25% observations are below Q1)  
Q3: Upper Quartile (25% observations are above Q3)  
Q2 is same as median
- Deciles divide the distribution into 10 equal parts.  
5<sup>th</sup> Decile is same as median
- Percentiles divide the distribution into 100 equal parts.  
50<sup>th</sup> percentile is same as median  
75<sup>th</sup> percentile is same as Q3

# Quantiles in R

*# Import basic\_salary2 data and store in object salary*

```
> quantile(salary$ba,na.rm=T)
```

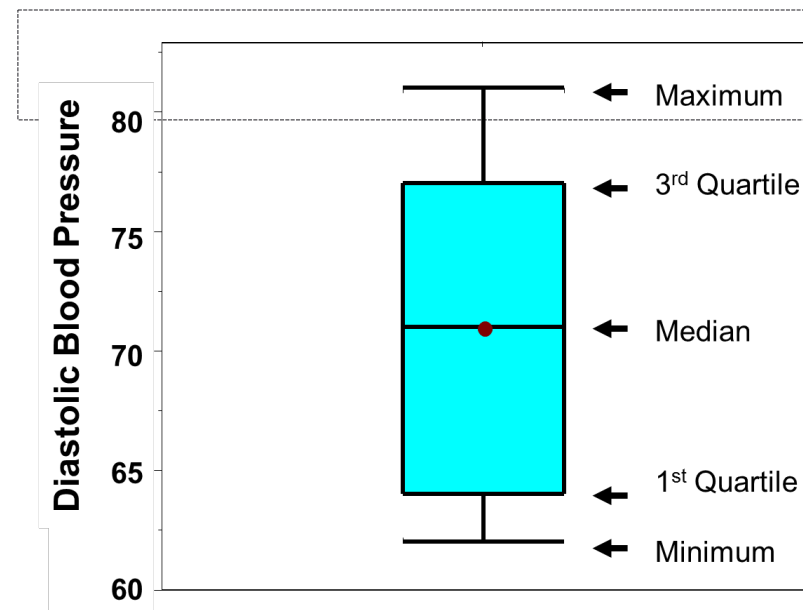
```
0%  25%  50%  75% 100%  
10940 13785 16230 19305 29080
```

```
> quantile(salary$ba,prob=c(0.1,0.5,0.8),na.rm=T)
```

```
10%  50%  80%  
13084 16230 20280
```

# Box-Whisker Plot

- Box and Whisker plot summarizes data graphically using 5 measures: Minimum, Q1, Q2, Q3 and Maximum.
- The body of the box goes from the first quartile (Q1) to the third quartile (Q3).
- The whiskers go from Q1 to smallest non outlier and Q3 to highest non outlier data points.
- The distribution is considered symmetric if median is at the center of the box and whiskers have same length



# Defining Outliers

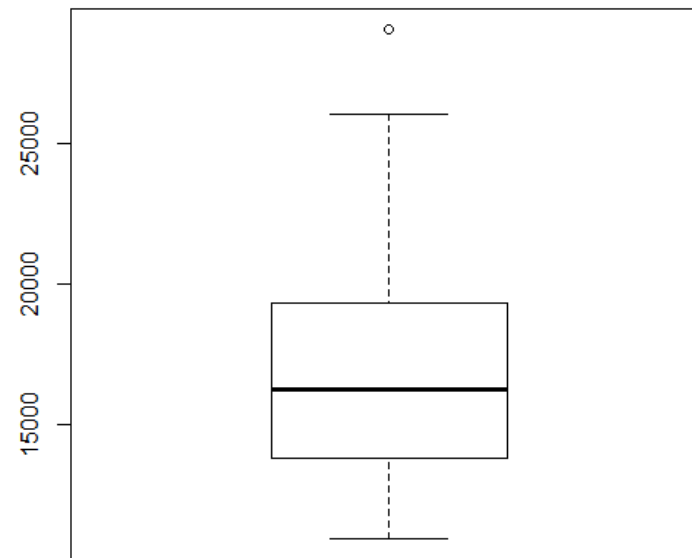
- An outlier is an observation that lies an abnormal distance from other values in a random sample from a population.
- What will be considered abnormal? Before abnormal observations can be singled out, it is necessary to characterize normal observations.
- Non-Outlier observation is

$$\geq Q1 - 1.5 \cdot IQR \text{ and } \leq Q3 + 1.5 \cdot IQR$$

where IQR: Inter-quartile Range =  $Q3 - Q1$

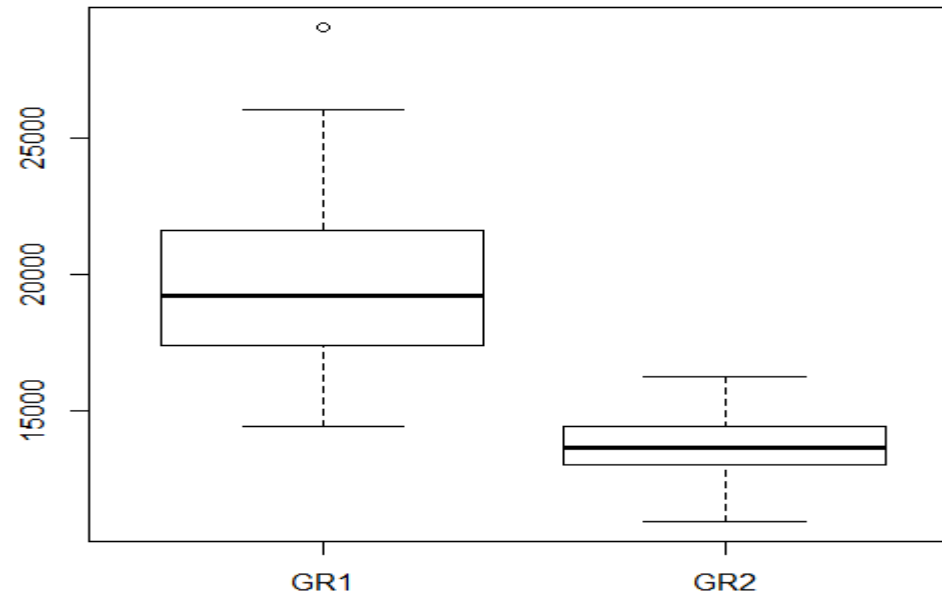
# Box-Whisker Plot in R

```
> boxplot(salary$ba)
```



# Box-Whisker Plot by Grade

```
> boxplot(ba~Grade,data=salary)
```

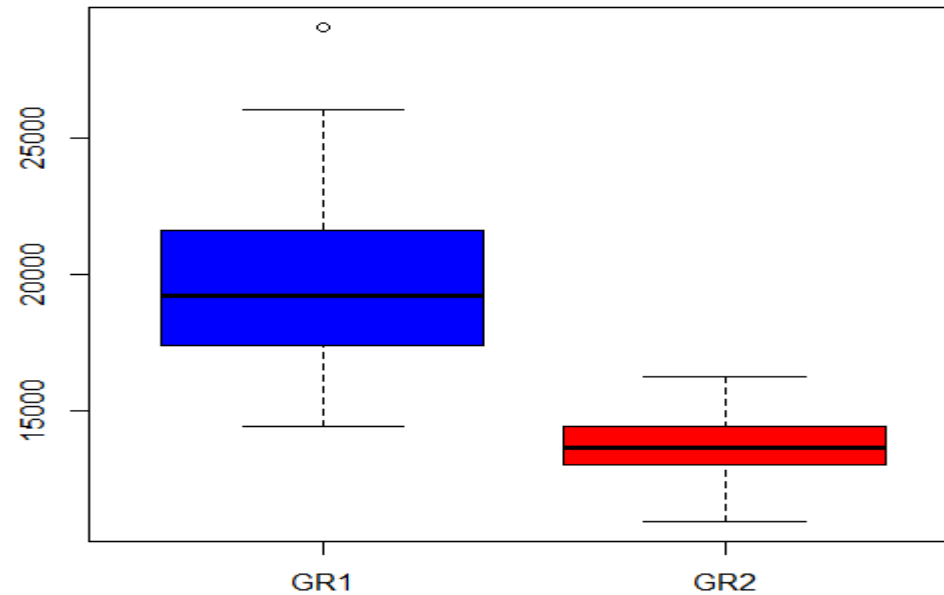


# The distribution of "ba" in C



# Adding Colours

```
> boxplot(ba~Grade,data=salary,col=(c("blue","red")))
```



# Display Observation Number on Outliers

`boxplot()` provides outlier values which can be accessed as follows:

```
>box <- boxplot(ba~Grade,data=salary)
```

```
>box$out
```

```
[1] 29080
```

However, it is better to print observation numbers rather than values.

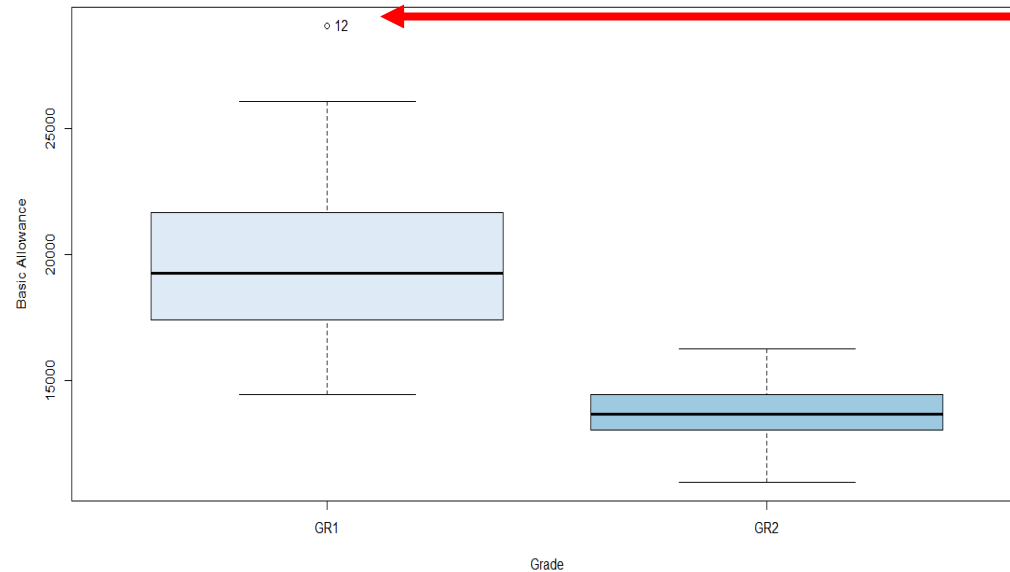
`Boxplot()` from package `car` solves this problem.

# Display Observation Number on Outliers

```
> install.packages("car")
```

```
> library(car)
```

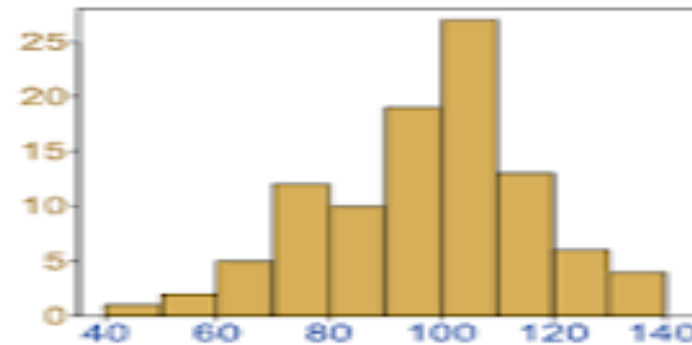
```
> Boxplot(ba~Grade,data=salary)
```



12<sup>th</sup>  
observation  
has ba  
29080

# Histogram

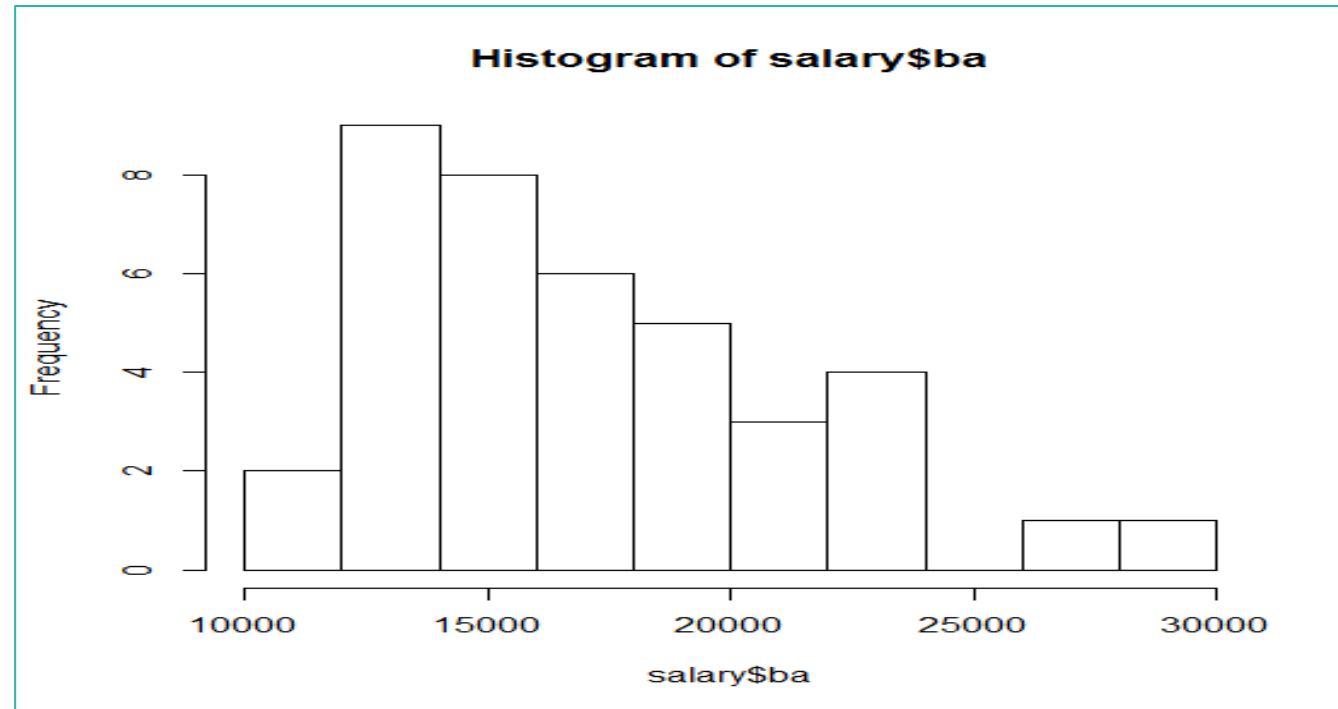
- Histogram is useful in visualizing a distribution.
- To construct a histogram, the first step is to "bin" the range of values—that is, divide the entire range of values into a series of intervals—and then count how many values fall into each interval(frequency)



The number of bins  $k$  can be assigned directly or can be calculated from a suggested bin width  $h$  as:  $(\max Y - \min Y)/h$  where  $Y$  is a variable of interest.

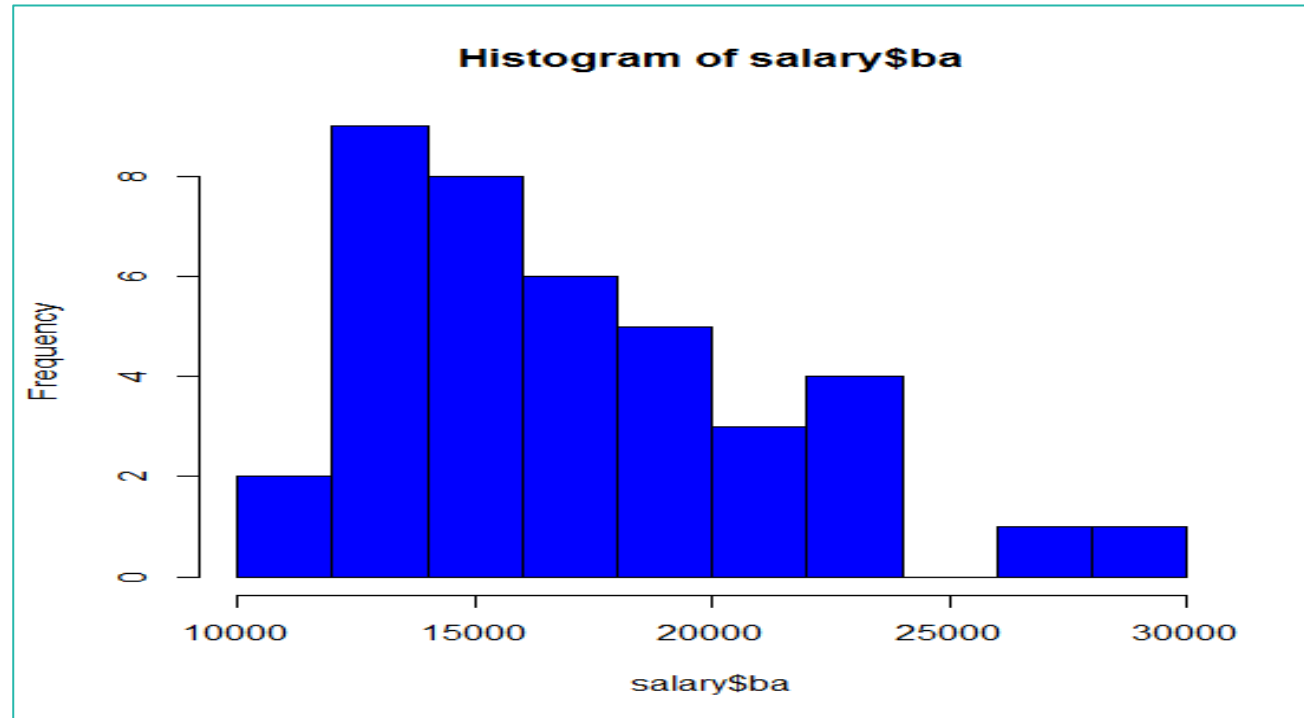
# Histogram in R

```
> hist(salary$ba)
```



# Adding Colour in Histogram

```
> hist(salary$ba,col="blue")
```



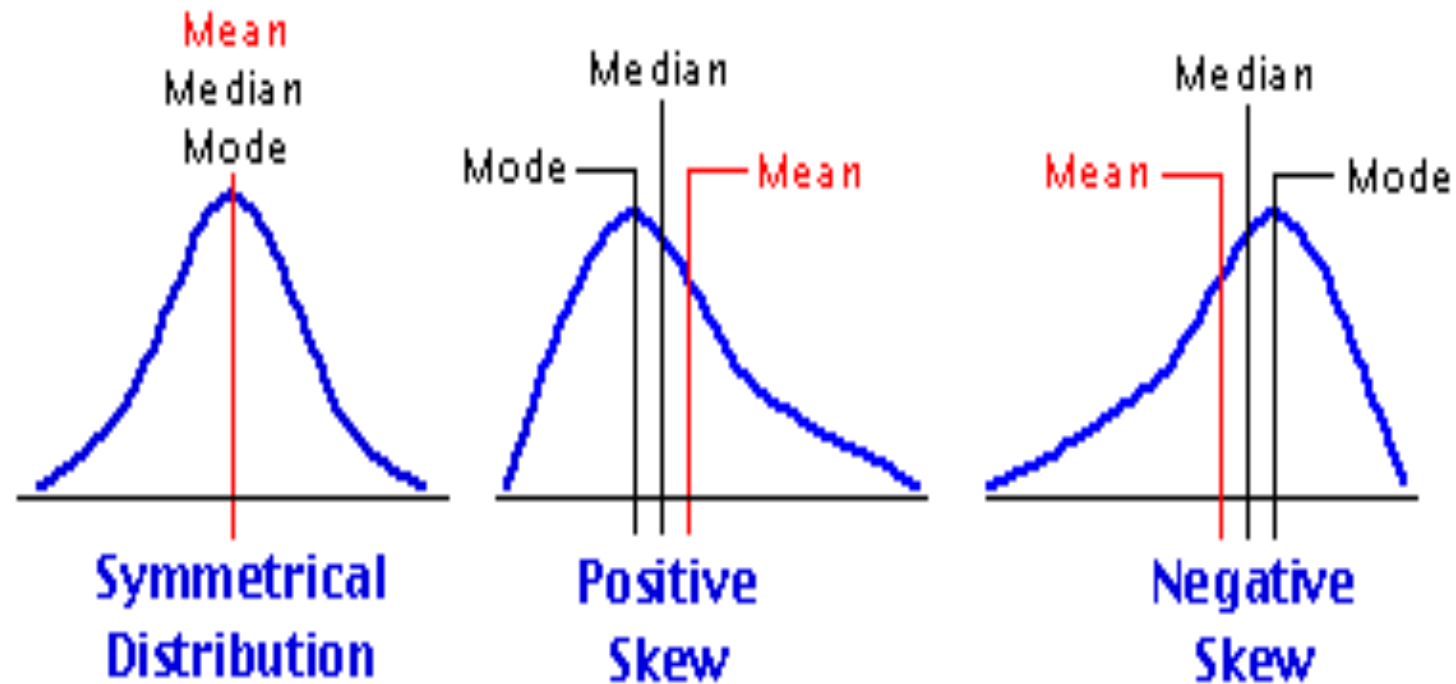
# Try this code

```
hist(salary$ba,col="blue",breaks=c(10000,15000,20000,25000,30000))
```

# What is Skewness?

- Skewness is a measure of 'lack of symmetry' of the data.
- *positive skew*: The right tail is longer; the mass of the distribution is concentrated on the left
- *negative skew*: The left tail is longer; the mass of the distribution is concentrated on the right.
- If the distribution is symmetric, then the mean is equal to the median, and the distribution has zero skewness.
- The Normal distribution is symmetric distribution.

# Visualizing Skewness





# How to Calculate Skewness?

- The Pearson measure of skewness is defined as  
(mean – mode) / standard deviation
- Another form of the Pearson measure of skewness is  
3 (mean – median) / standard deviation

- The Bowley coefficient of skewness is

$$\frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1} = \frac{Q_1 - 2Q_2 + Q_3}{Q_3 - Q_1},$$

# Skewness Based on Third Moment

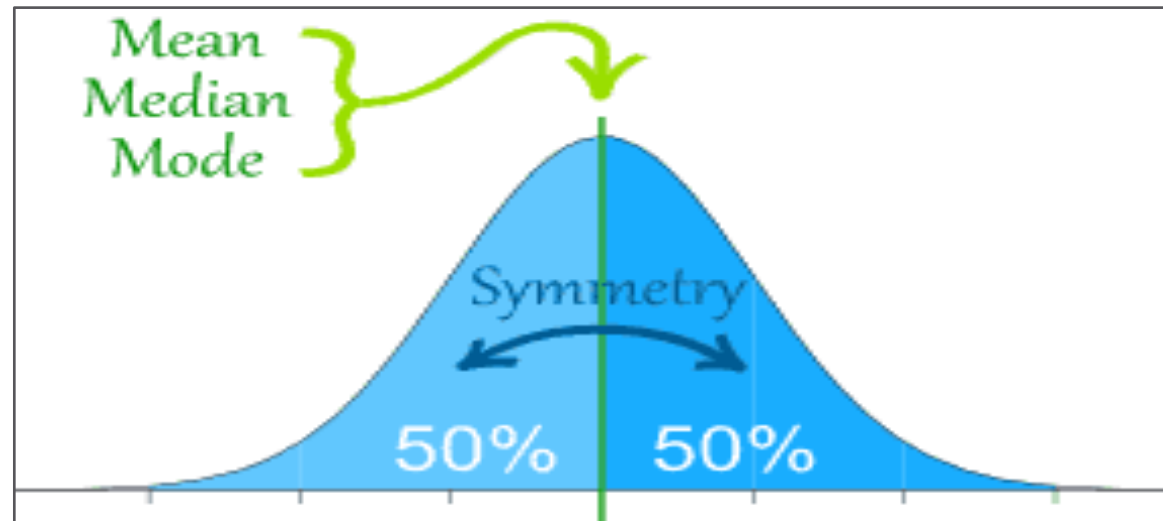
- The most widely used measure of skewness is based on the third moment.

$$\frac{n}{(n-1)(n-2)} \sum \left( \frac{x_j - \bar{x}}{s} \right)^3$$

- Any threshold or rule of thumb is arbitrary, but here is one: If the skewness is greater than 1.0 (or less than -1.0), the skewness is substantial and the distribution is far from symmetrical. Value 'zero' indicates symmetric distribution.

# Normal Distribution

- Commonly used distribution for continuous variables.
- Also known as the Gaussian distribution.
- Normal curve is a symmetric bell-shaped curve.
- Many statistical methods assume that population is normally distributed.



# Using R to Measure Skewness

```
> install.packages("e1071")
```

```
> library(e1071)
```

```
# Use basic_salary2 data
```

```
> skewness(salary$ba,na.rm=T,type=2)
```

```
[1] 0.9033507
```

Note that type=2 uses moment based formula as discussed.  
Most softwares use the same formula.

# Skewness by Grade

```
> library(e1071)
```

```
> f<-function(x) skewness(x,na.rm=T,type=2)
```

```
> aggregate(ba~Grade,data=salary,FUN=f)
```

	Grade	ba
1	GR1	0.85500651
2	GR2	0.08682743