Map, Jumble, and Singly-Linked List with Dummy

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CSC220 Programming II – Spring 2016

Map

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  - It will have a null key and value.

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- The beginning of an implementation is here.

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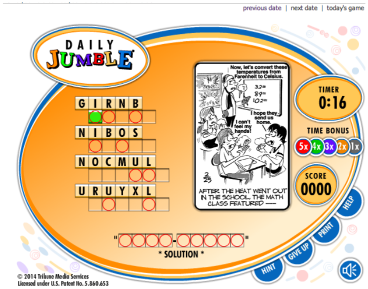
previous date | next date | today's game
DAILY JUMBLE
GIRNB
NIBOS
NOCMUL
URUYXL
Now, let's convert these temperatures from Farenheit to Celsius.
32= 89= 102=
I hope they send us home.
I can't feel my hands!
AFTER THE HEAT WENT OUT IN THE SCHOOL, THE MATH CLASS FEATURED ——
* SOLUTION *
TIMER 0:16
TIME BONUS 5x 4x 3x 2x 1x
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HELP
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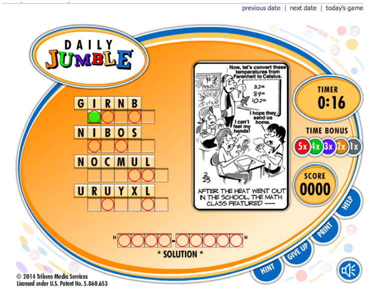

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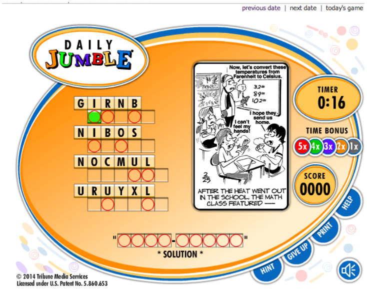
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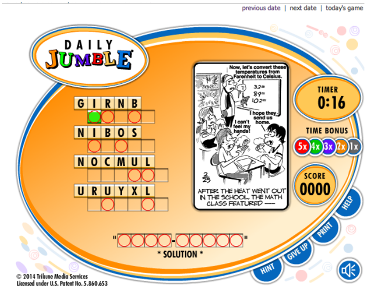
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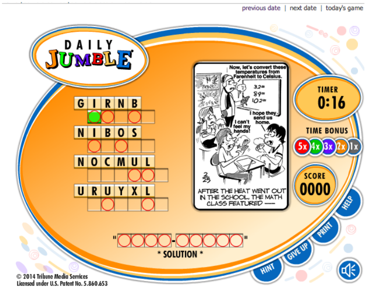
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  - How can a Map help us to do that?

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  - But the number of orderings is 8! = 40,320, bad!.

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  - But we won't do that this time.

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  - The linked list lets us add quickly once we get there,
  - but it takes a while to get there.
- We need a faster way.

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  - So what? It's still $\mathrm{O}(n)$.

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See an example lookup here.

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