Binary Trees, Binary Search Trees, and Heaps

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CSC220 Programming II – Spring 2016

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  - Corresponds to using all the items in 0 to size-1.

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  - Good for searching.
  - If you are looking for a key, compare it to root item.
  - If it is equal, you have found it.
  - If it is less, look in the left subtree.
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  - For example, at a hospital emergency room serve in order of minutes until death.

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