CSE 20 Winter 2022
Homework 3
Due date: Wednesday, Jan 26, 2021 at 11:59pm

In this assignment,

You will consider how circuits and logic can be used to represent mathematical and technical claims. You will use propositional and predicate logic to evaluate these claims.

In this class, unless the instructions explicitly say otherwise, you are required to justify all your answers.

1. For each part of this question you will use the following input-output definition table with four inputs x_3, x_2, x_1, x_0

x_3	x_2	x_1	x_0	*out*
1	1	1	1	0
1	1	1	0	0
1	1	0	1	0
1	1	0	0	1
1	0	1	1	0
1	0	1	0	0
1	0	0	1	0
1	0	0	0	0
0	1	1	1	0
0	1	1	0	1
0	1	0	1	0
0	1	0	0	0
0	0	1	1	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

   (a) (4 points) Construct a compound proposition that implements this input-output table

   (b) (4 points) Draw a combinatorial circuit corresponding to this compound proposition.

   (c) (9 points) For each of the following functions, consider whether this input-output table (and the logic circuit from part (a)) implements the rule for the function. If yes, explain why using the definitions of the operations involved and consider **all** possible inputs. if not, provide a specific example where the value of the function does not match the output of the definition table

   [Note: these functions are known as *predicates* since their codomain is the set $\{T, F\}$.]

   i. $P_1 : \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \to \{T, F\}$ given by

$$P_1(\ (x_3, x_2, x_1, x_0)\ ) = \begin{cases} T & \text{when } x_0 + x_1 + x_2 + x_3 = 2 \\ F & \text{otherwise} \end{cases}$$

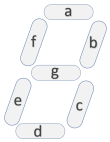
   ii. $P_2 : \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \to \{T, F\}$ given by

$$P_2(\ (x_3, x_2, x_1, x_0)\ ) = \begin{cases} T & \text{when } (x_3x_2x_1x_0)_{2,4} \textbf{ mod } 6 = 0 \\ F & \text{otherwise} \end{cases}$$

   iii. $P_3 : \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \to \{T, F\}$ given by

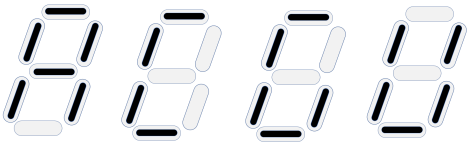
$$P_3(\ (x_3, x_2, x_1, x_0)\ ) = \begin{cases} T & \text{when } (x_3x_2x_1x_0)_{2,4} = [x_3x_2x_1x_0]_{s,4} \\ F & \text{otherwise} \end{cases}$$

2. Simple digital devices like calculators, digital clocks, etc. use a display system called 7-segment display. It works by lighting up a combination of the 7 segments for each digit.



Suppose you need to display the bases of an RNA string using the 7-segment display in the following way.

Pictured below are the combinations for $\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}$:



You can encode these 4 letters with only 2 bits: x_1, x_0.

Consider the function: $R : \{0,1\} \circ \{0,1\} \to \{\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}\}$ given by:

$R(00) = \texttt{A}$
$R(01) = \texttt{C}$
$R(10) = \texttt{G}$
$R(11) = \texttt{U}$

(a) (7 points) Fill in the "truth table" for a circuit that displays the characters $\texttt{A}, \texttt{C}, \texttt{G}, \texttt{U}$ in 7-segment display based on the result of the function $R(x_1x_0)$.

x_1	x_0	a	b	c	d	e	f	g
0	0							
0	1							
1	0							
1	1							

(b) (7 points) Draw the circuit. *(you may rearrange your outputs circuits and duplicate your inputs to make your circuit neater. Your circuit may be a drawing.)*

3. Write out the truth tables for the following compound propositions:

(a) (4 points)

$$(p \to q) \oplus \neg(p \wedge \neg r)$$

(b) (4 points)

$$(p \vee \neg q) \to (\neg r \to q)$$

(c) (4 points)

$$\neg p \to (\neg q \to \neg r)$$

4. (12 points) A cat, a dog, a frog and a pig are all accused of robbing the bank.

The police have deduced the following:

- If the cat is not guilty then the dog is guilty.
- The cat is not guilty if and only if the frog is guilty.
- The dog is not guilty implies that the pig is guilty.
- The pig is guilty if the cat is not guilty.

(a) Determine if the police

- Have enough information to be certain who was involved in the robbery and who was not involved in the robbery.
- Do not have enough information to be certain who was involved in the robbery and who was not involved in the robbery.
- Have conflicting information (in other words, the statements are not consistent.)

(To show your work, either construct a truth table or use logical equivalences to justify your answer.)

(b) Now, suppose that the pig comes forward and admits to being involved with the robbery.

(Same as before:)

With this new information, determine if the police

- Have enough information to be certain who was involved in the robbery and who was not involved in the robbery.
- Do not have enough information to be certain who was involved in the robbery and who was not involved in the robbery.
- Have conflicting information (in other words, the statements are not consistent.)

(To show your work, either construct a truth table or use logical equivalences to justify your answer.)