In [331]:
```
# Question 2

import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import seaborn as sns

# Import data into Pandas Data Frame
data = pd.read_csv('regression.csv', header=None, names=['A','B','C'])
data
```

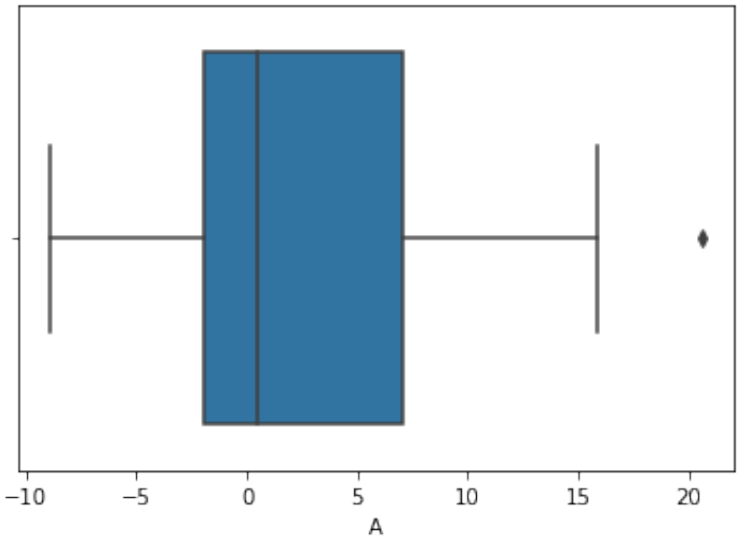
Out[331]:

	A	B	C
0	0.490	-0.180	11.50
1	-1.410	-1.230	11.80
2	0.943	4.510	-3.24
3	3.570	5.070	-23.90
4	-1.700	6.910	-22.10
...	...	...	...
295	6.920	-0.421	33.20
296	11.700	-1.290	108.00
297	9.920	3.690	-126.00
298	11.400	6.290	-315.00
299	10.900	2.570	-97.70

300 rows × 3 columns

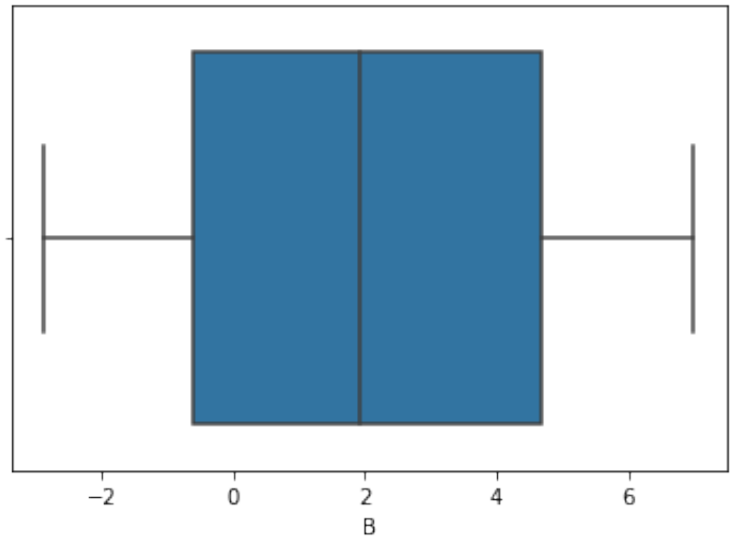
In [332]:
```
# Explore the data

sns.boxplot(x=data['A'])
```

Out[332]: <matplotlib.axes._subplots.AxesSubplot at 0x1a1d35e6d0>



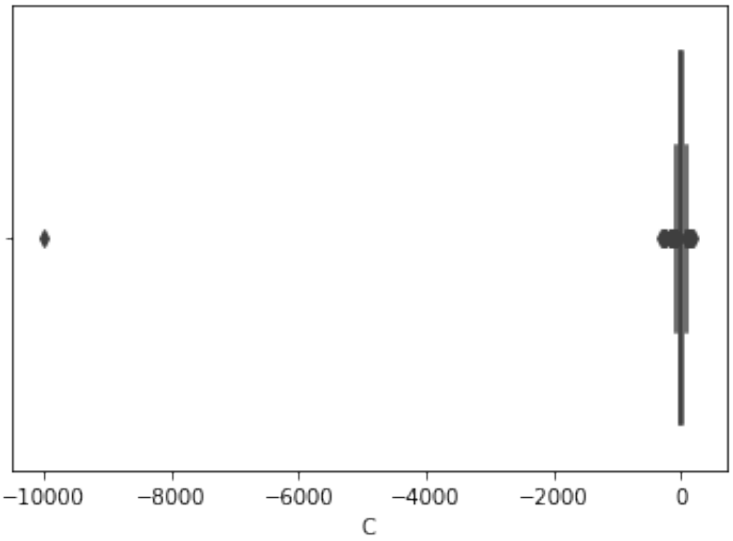
In [333]:
```
sns.boxplot(x=data['B'])
```

Out[333]: <matplotlib.axes._subplots.AxesSubplot at 0x1a1be7ef90>



In [334]:
```
sns.boxplot(x=data['C'])
# Found outliner with boxplot
```

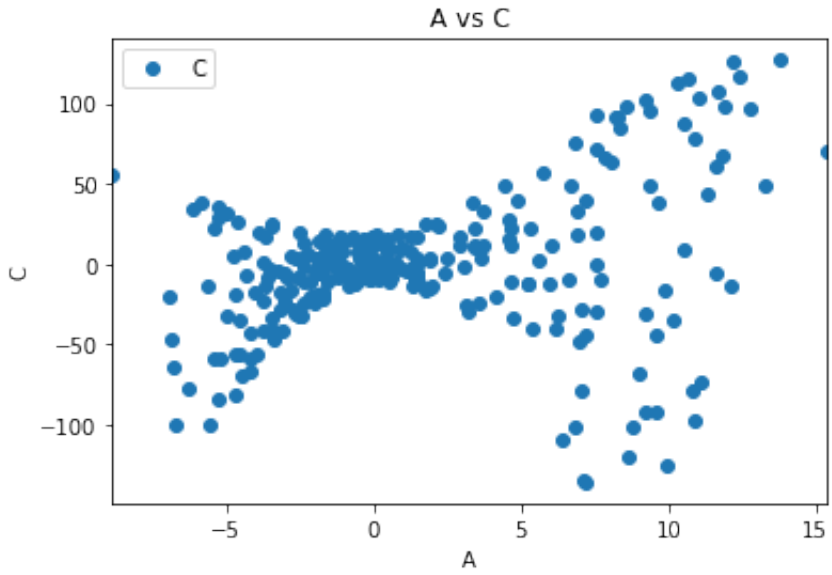
Out[334]: <matplotlib.axes._subplots.AxesSubplot at 0x1a1d407d90>



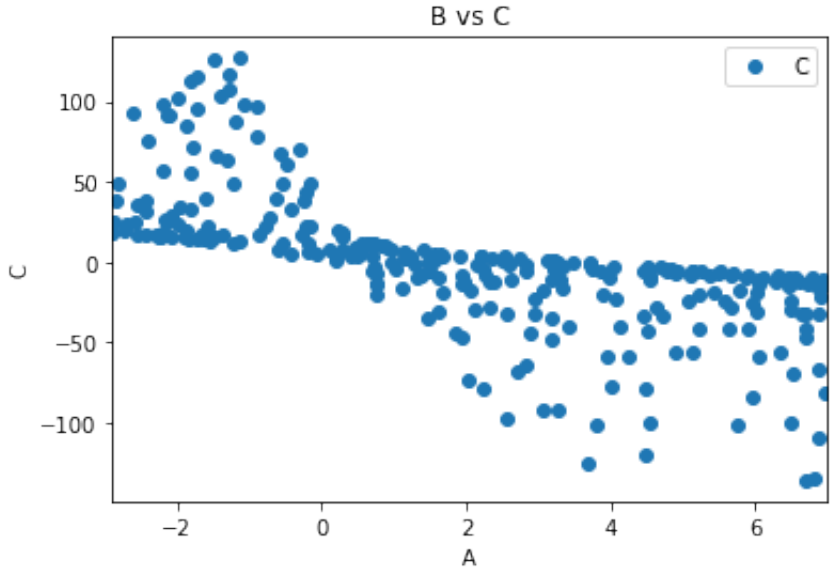
In [335]:
```
# Remove outliners.
Q1 = data.quantile(0.25)
Q3 = data.quantile(0.75)
IQR = Q3 - Q1
data = data[~((data < (Q1 - 3 * IQR)) | (data > (Q3 + 3 * IQR))).any(a
data.shape
```

Out[335]: (278, 3)

In [336]:
```
data.plot(x='A', y='C', style='o')
plt.title('A vs C')
plt.xlabel('A')
plt.ylabel('C')
plt.show()
# Heteroscedasticity needs to be addressed.
```

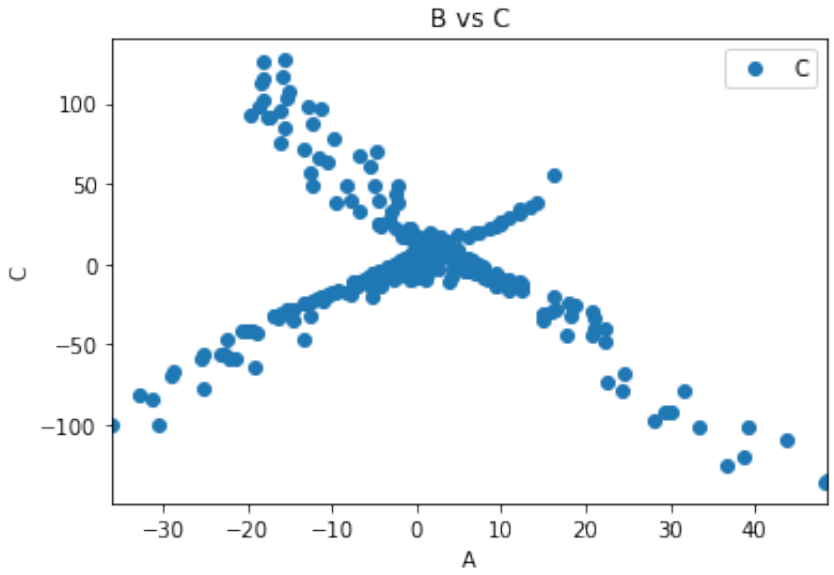


In [337]:
```
data.plot(x='B', y='C', style='o')
plt.title('B vs C')
plt.xlabel('A')
plt.ylabel('C')
plt.show()
# Heteroscedasticity needs to be addressed.
```



In [338]:
```
data['A*B'] = data['A'] * data['B']
data.plot(x='A*B', y='C', style='o')
plt.title('B vs C')
plt.xlabel('A')
plt.ylabel('C')
plt.show()
```

```
/Users/shengchengchen/opt/anaconda3/lib/python3.7/site-packages/ipyke
rnel_launcher.py:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas
-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
(http://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html
#returning-a-view-versus-a-copy)
  """Entry point for launching an IPython kernel.
```



In [339]:
```
data['|A|*B'] = data['A'].abs() * data['B']
del data['A*B']
data
```

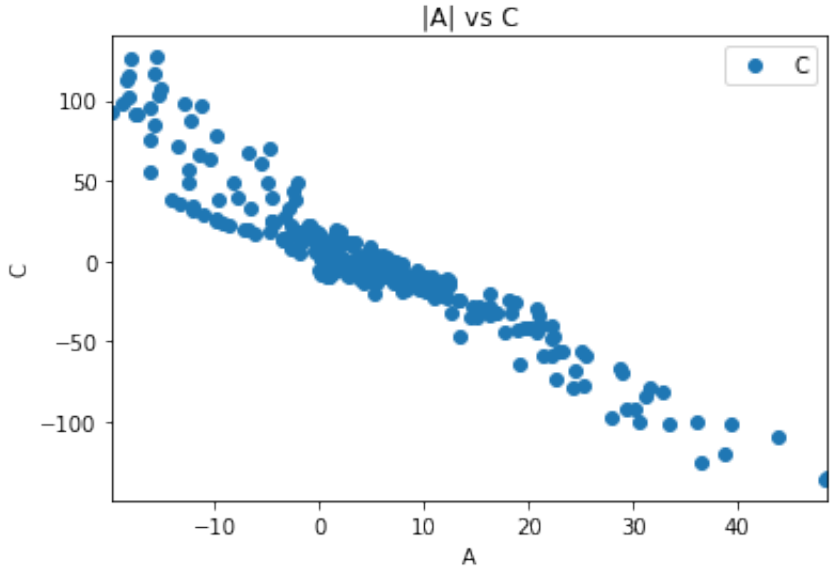
```
/Users/shengchengchen/opt/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:1: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
(http://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)
  """Entry point for launching an IPython kernel.
```

Out[339]:

	A	B	C	|A|*B
0	0.490	-0.180	11.50	-0.08820
1	-1.410	-1.230	11.80	-1.73430
2	0.943	4.510	-3.24	4.25293
3	3.570	5.070	-23.90	18.09990
4	-1.700	6.910	-22.10	11.74700
...	...	...	...	...
292	7.230	6.690	-136.00	48.36870
295	6.920	-0.421	33.20	-2.91332
296	11.700	-1.290	108.00	-15.09300
297	9.920	3.690	-126.00	36.60480
299	10.900	2.570	-97.70	28.01300

278 rows × 4 columns

In [340]:
```
data.plot(x='|A|*B', y='C', style='o')
plt.title('|A| vs C')
plt.xlabel('A')
plt.ylabel('C')
plt.show()
# Linear relationship between
```



In [341]:
```
# Random shuffle the dataset for train test split.
data = data.iloc[np.random.RandomState(seed=42).permutation(len(data))]
data
```

30	-2.81	-0.0679	4.81	-0.190799
126	-3.97	6.3300	-56.80	25.130100
199	-4.43	-0.5990	7.63	-2.653570
142	-5.82	-2.4400	38.70	-14.200800
269	9.39	-1.7200	94.80	-16.150800
...	...	...	...	...
188	-3.54	5.6200	-42.10	19.894800
71	3.61	1.6900	3.36	6.100900
106	4.66	-0.2260	22.00	-1.053160
289	7.13	6.8200	-135.00	48.626600
102	-2.03	6.5000	-24.30	13.195000

278 rows × 4 columns

In [342]:
```
# Split the data set to 70% training and 30% testing
data_train = data.head(int(len(data)*0.7)).reset_index()
data_test = data.tail(len(data)-int(len(data)*0.7)).reset_index()
print(data_train)
print(data_test)
```

```
     index       A       B      C       |A|*B
0       30 -2.8100 -0.0679   4.81  -0.190799
1      126 -3.9700  6.3300 -56.80  25.130100
2      199 -4.4300 -0.5990   7.63  -2.653570
3      142 -5.8200 -2.4400  38.70 -14.200800
4      269  9.3900 -1.7200  94.80 -16.150800
..     ...     ...     ...    ...        ...
189    153 -0.3030 -1.7100  14.60  -0.518130
190     23 -5.2700 -2.0900  28.90 -11.014300
191    238  7.0100  3.1800 -48.20  22.291800
192    233  7.0500  4.4800 -78.40  31.584000
193     81  0.0713  5.6900  -6.93   0.405697

[194 rows x 5 columns]
    index       A      B       C      |A|*B
0      39  -0.409  2.500    1.53   1.02250
1     201  10.700 -1.710  115.00 -18.29700
2     265   5.260  2.370  -12.80  12.46620
3      47   2.170 -1.990   24.10  -4.31830
4      94  -2.180  3.290  -10.50   7.17220
..    ...     ...    ...     ...       ...
79    188  -3.540  5.620  -42.10  19.89480
80     71   3.610  1.690    3.36   6.10090
81    106   4.660 -0.226   22.00  -1.05316
82    289   7.130  6.820 -135.00  48.62660
83    102  -2.030  6.500  -24.30  13.19500

[84 rows x 5 columns]
```

In [343]:
```
# Train the regression model.
X = data_train.drop('C', axis=1).values
y = data_train['C'].values
ones = np.ones(shape=X.shape[0]).reshape(-1, 1)
X = np.concatenate((ones, X),axis=1)

# Solve the coefficients for ordinary least square regression.
beta = np.linalg.inv(X.transpose().dot(X)).dot(X.transpose()).dot(y)
print(beta)
```

```
[ 1.18478845e+01  1.45314711e-03  1.96248283e+00  7.09852620e-01
 -3.37211710e+00]
```

In [344]:
```
beta0 = beta[0]
beta_coeff = beta[1:]
rows, _ = data_train.shape
y_pred = []
A_abs_B = data_train.drop('C', axis=1)

# Make predictions
for row in range(rows):
    y_pred.append(beta0 + beta_coeff.dot(A_abs_B.values[row, :]))
data_train['C_pred'] = np.ravel(y_pred)

# Reorder columns
data_train = data_train[['A', 'B', '|A|*B', 'C', 'C_pred']]
data_train
```

Out[344]:

	A	B	|A|*B	C	C_pred
0	-2.8100	-0.0679	-0.190799	4.81	6.972100
1	-3.9700	6.3300	25.130100	-56.80	-76.008349
2	-4.4300	-0.5990	-2.653570	7.63	11.966209
3	-5.8200	-2.4400	-14.200800	38.70	46.787301
4	9.3900	-1.7200	-16.150800	94.80	83.907937
...	...	...	...	...	...
189	-0.3030	-1.7100	-0.518130	14.60	12.008931
190	-5.2700	-2.0900	-11.014300	28.90	37.196940
191	7.0100	3.1800	22.291800	-48.20	-46.962490
192	7.0500	4.4800	31.584000	-78.40	-77.302835
193	0.0713	5.6900	0.405697	-6.93	14.776518

194 rows × 5 columns

In [345]:
```
# Coefficient of Determination.
# Compute Sum of Sqaured Error and Sum of Sqaured Total.
SSE = ((data_train['C_pred'] - data_train['C'])**2).sum()
SST = ((data_train['C'] - data_train['C'].mean())**2).sum()
# Compute r squared to measure performance.
r_sq = 1 - SSE/SST
print(r_sq)
# R squared for training set is high.
```

```
0.9384794124804481
```

In [346]:
```
rows, _ = data_test.shape
y_pred = []
A_abs_B = data_test.drop('C', axis=1)

# Make predictions.
for row in range(rows):
    y_pred.append(beta0 + beta_coeff.dot(A_abs_B.values[row, :]))
data_test['C_pred'] = np.ravel(y_pred)

SSE = ((data_test['C_pred'] - data_test['C'])**2).sum()
SST = ((data_test['C'] - data_test['C'].mean())**2).sum()
# Compute r squared to measure performance.
osr_sq = 1 - SSE/SST
print(osr_sq)
```

```
0.9359914968357479
```

In [347]:
```
# The out of sameple R^2 (test set) is 0.936 which is only slightly lo
# Therefore, overfitting is NOT a major issue for the model.
# 0.936 OS-R^2 is high, thus C can be predicted by A and B.
```