Quadrature Amplitude Modulation

Stephen Chapman

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1 Abstract

This paper investigates quadrature amplitude modulation (QAM). We begin with an introduction to what QAM is and how it communicates information. Then the modulation and demodulation schemes are discussed in detail. Digital signal processing, a very current and important development in QAM is discussed at length. The paper concludes with brief discussions on noise and orthogonal QAM multiplexing.

2 Introduction

The purpose of Quadrature Amplitude Modulation (QAM), like most modulation schemes, is to increase the maximum possible bit rate. QAM is based on the combination of amplitude shift keying (ASK) and phase shift keying (PSK). This introduction explains all three with the help of constellation diagrams.

ASK is the simplest modulation scheme, as it just involves modulating the amplitude to give different bit values. In a constellation diagram this is represented by the modulus of the point. In Fig.1, there are four different points with different moduli, signifying that this is a diagram of 4-ASK.

PSK modulates the phase of the signal to transfer information. This results in points on the constellation diagram with equal radius but different angles around the circle. This is shown in Fig. 2

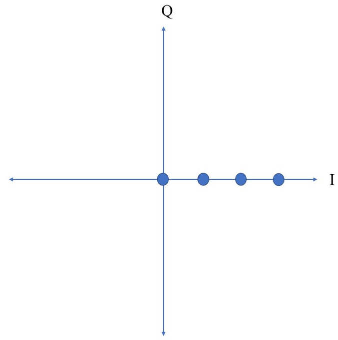

Figure 1: The constellation diagram of 4-ASK. Only the modulus changes.[1]

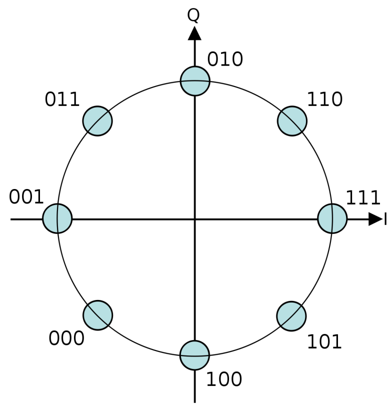

Figure 2: The constellation diagram of 8-PSK. Only the phase changes.[10]

The number of points in the constellation diagram is called the modulation order. It quantifies the number of bits per symbol that can be transmitted. The more points on the constellation diagram, the more bits per symbol is possible. On-off keying (OOK) is a binary form of ASK where there is one point at the origin of the constellation diagram and only one other point. The point at the origin represents a 0 and the other point is a 1. This setup has modulation order of 1 because there is 1 bit/symbol. The 4-ASK in Fig.1 has modulation order 2 because you could assign 00,10,01,11 to the four points so there would be 2 bits/symbol. The 8-PSK diagram shows that there are 3 bits/symbol so it has modulation order 3. From this pattern it is clear that the relationship between the number of points and the modulation order scales with powers of 2. [2]

$$Points = 2^m \tag{1}$$

Where m is the modulation order.

It is also clear that higher modulation orders will allow higher bit rates because one can transmit more bits/symbol. The modulation order is limited by noise. The greater the modulation order, the points will have to be closer together (holding signal power constant) which will increase the noise. To allow more space between points, we can vary both amplitude and phase, called amplitude and phase shift keying (APSK). This creates constellation diagrams with rings. Each ring has different amplitude and contains points of different phase. A more specific case is when amplitude and phase are modulated using IQ modulation, which produces QAM. QAM constellation diagrams have points arranged in a square lattice.

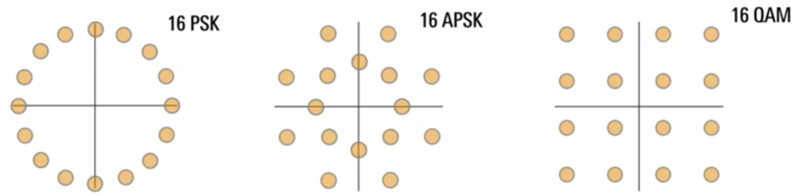

Figure 3: A comparison of three modulation schemes, each has modulation order 4. It is clear that QAM allows for the largest distance between the points, yielding the best signal to noise ratio.[3]

3 IQ Modulation

To create the signal for a QAM system, both the phase and the amplitude of the source must be modulated. This is done using an IQ modulator. IQ modulators consist of two sin waves that are 90° out of phase. The output is the sum of the two waves. Modulating either of the two waves individually results in changes in the output. In this way, modulating the two individual beams allows one to modulate the phase and/or amplitude of the output[4].

In optical fiber systems, this is done with parallel Mach-Zehnder modulators (MZMs) nested

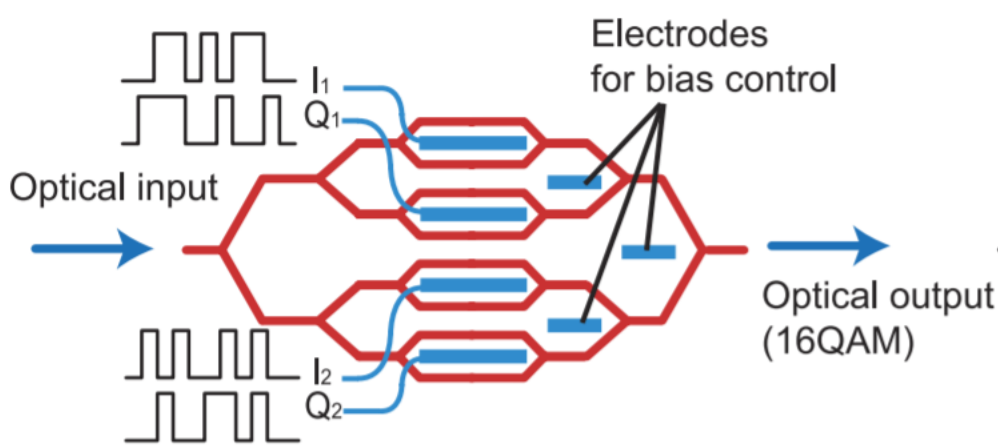

Figure 4: A diagram of an IQ modulator that is used for 16-QAM.[4]

inside a larger MZM. The two arms of the larger MZM are electrically biased to have a 90°

phase difference. This makes one arm the "Q" and one the "I", then the two nested MZM are used to modulate the Q and I beams individually. For 4-QAM only two nested MZMs are used. Higher order QAM necessitates more MZMs nested inside each other. The figure above shows a diagram of an IQ modulator for 16-QAM[4]. Alternatively, 16-QAM could also be achieved with only two nested MZMs, but with electrical signals that modify the paths having four different amplitudes.[2]

4 QAM Detection

Since QAM stores information in both the amplitude and phase, neither direct detection nor normal coherent detection will work. Instead a coherent detection scheme with a 90 degree hybrid device must be used. This device consists of four 3dB couplers. The input to this device is the signal and a heterodyne local oscillator. The device mixes the two inputs to create four outputs[2].

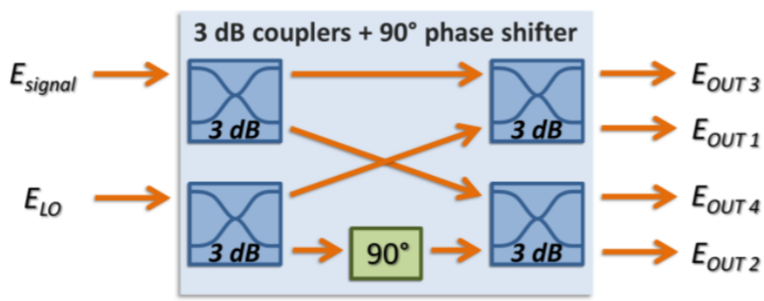

Figure 5: A diagram of an optical 90 degree hybrid device. By mixing the signal with the local oscillator, it is able to recover phase and amplitude information.[5]

The four outputs are related to the signal and local oscillator inputs by the relations

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} E_s + E_{lo} \\ E_s + iE_{lo} \\ E_s - E_{lo} \\ E_s - iE_{lo} \end{bmatrix} \tag{2}$$

Then E_1 and E_3 are sent to one balancing diode together. E_2 and E_4 are sent to the another balancing diode together. The equations for the intensity at the two diodes are:

$$I_{13} = E_1 E_1^* - E_3 E_3^* \tag{3}$$

and

$$I_{24} = E_2 E_2^* - E_4 E_4^* \tag{4}$$

We use expressions for the signal and local oscillator fields:

$$E_s = \sqrt{P_s} e^{i(\omega_s t + \phi_s)} * e^{i\phi(t)} * e^{i\phi_n(t)} \tag{5}$$

$$E_{lo} = \sqrt{P_{lo}} * e^{i(\omega_{lo} t + \phi_{lo})} * e^{i\phi_n(t)} \tag{6}$$

In the above expressions the exponential term with ϕ_n denotes the noise of the signal or local oscillator. The term $a(t)e^{i\phi(t)}$ corresponds to the modulated amplitude and phase, i.e. the QAM data imprinted on the signal.[5]

The output electric fields from the 90 degree hybrid are send into balanced photodiodes which can recover the in phase and quadrature currents.

$$I_I = R * \vec{E_1}\vec{E_1}^* - R * \vec{E_3}\vec{E_3}^* = R\sqrt{P_s P_{LO}} a(t) \hat{e_s} \hat{e_{LO}} \cos(\Delta\omega t + \phi_n(t) + \phi_0 + \phi(t)) \tag{7}$$

and

$$I_Q = R * \vec{E_2}\vec{E_2}^* - R * \vec{E_4}\vec{E_4}^* = R\sqrt{P_s P_{LO}} a(t) \hat{e_s} \hat{e_{LO}} \sin(\Delta\omega t + \phi_n(t) + \phi_0 + \phi(t)) \tag{8}$$

These yield the amplitude $a(t)$ and the phase $\phi(t)$.

There are also three analog methods for QAM detection. They are synchronous heterodyne receivers, asynchronous heterodyne receivers, and optical delay modulation techniques. These will not be covered in this paper because current digital techniques have made them obsolete. Instead, this paper will go more in depth on digital signal processing techniques.

5 Digital Signal Processing

The figure below depicts a digital coherent receiver that can detect QAM in both polarizations. A PBS splits the signal into two polarization branches. Each polarization has a 90 degree optical hybrid and four photodiodes which allow recovery of phase and amplitude information. After interacting with the local oscillator, the currents given in equations (7) and (8) are produced. Combining these gives a complex field

$$E_x(t) = R_d \sqrt{P_s P_{LO}} s(t) exp[-i(\omega_{IF} t + \Delta\phi)] + N(t)$$

Where $\Delta\phi = \phi_s - \phi_{LO}$ and $s(t) = \sum_m s_m g(t - mT_s)$ is the symbol stream. s_m is a symbol and T_m is its duration $g(t)$ is the shape of the RZ pulse for a symbol. $N(t)$ is the noise.

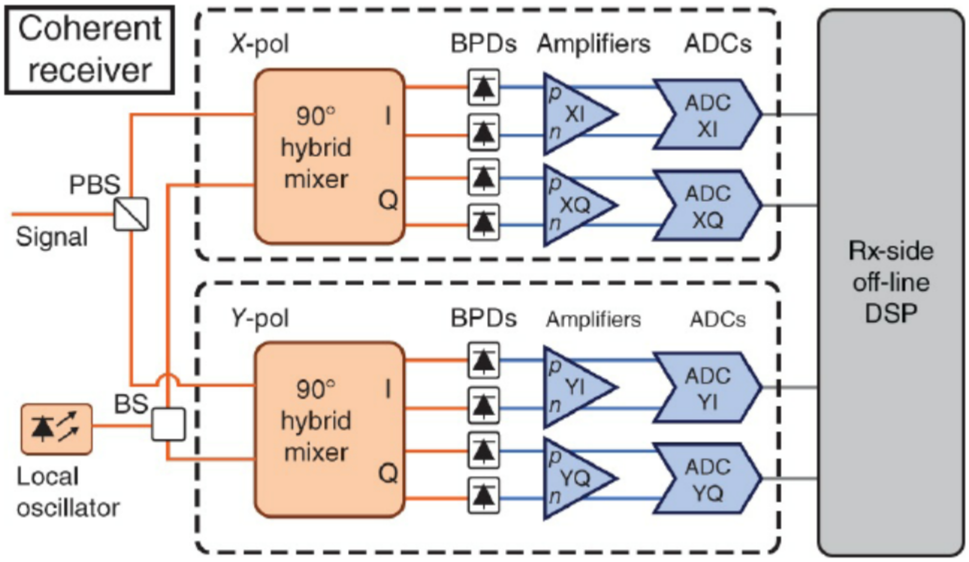

Figure 6: A diagram of a full detector for PDM QAM systems. After splitting up polarizations, there are two 90 degree optical hybrids which allow for QAM detection. Then digital signal processing does the rest.[5]

The fields are digitized by four ADCs. The digitized signal can be written as

$$y_n = x_n exp[-i(\omega_{IF} t_n + \Delta\phi_n)] + N_n \tag{9}$$

Then the DSP chip has three main functions: removal of the intermediate frequency, compensation of fiber dispersin, reductoin in nonlinear degradation.

5.1 Removal of Intermediate Frequency

The signal needs to be demodulated to the baseband. The intermediate frequency is determined by analyzing the product of two successive symbols y_n and y_{n-1}

$$y_n y_{n-1}^* = |x_n x_{n-1}^*| exp[-i(\frac{\omega_{IF}}{R_s} + \theta_n + \Delta\phi_n - \Delta\phi_{n-1})] + N_n' \tag{10}$$

Where $\theta_n = Arg(x_n x_{n-1}^*)$ isthe relative phase of the symbols. Assuming no noise, θ_n becomes a multiple of 2π and we find that

$$\omega_{IF} = -R_s Arg[(y_n y_{n-1}^*)^M]$$

Where M corresponds to M-PSK[2].

5.2 Compensation of GVD and PMD

Before digital signal processing, GVD could be corrected for using a filter in the frequency domain with a specific transfer function. Digitally, the fourier transform of the transfer function is taken to yield its impulse response function. For accumulated dispersion d, the impulse response function would be

$$h(t) = \sqrt{\frac{2\pi}{id}} exp(\frac{it^2}{2d}) \tag{11}$$

This impulse response has an infinite duration, but in the digital domain it can still be implemented using a finite-impulse-response filter and multiple tapped delay lines. A tapped delay line is where a delay line has points that take the output and possibly scale it, then each of these sampled outputs are summed together to create the final output. This is depicted in Fig. 7. The more dispersion, the more taps will be needed[7].

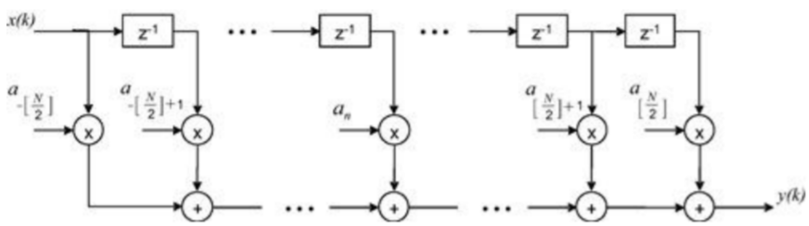

Figure 7: An FIR filter diagram. Each of the vertical lines is a 'tap'. They all sum together to create the output y_k.[7]

5.3 Managing Nonlinear Effects

A recent technique called digital backward propagation can theoretically account for all linear and nonlinear degredations simultaneously. It numerically propagates the digitized electric field backward through the fiber using the known values of all the fiber's parameters. The nonlinear Schrodinger equation is used to model the propagation of pulses in afiber, but it cannot account for the random birefringence along the fiber.The Manakov approximation (a.k.a. dispersion-folded) assumes that nonlinear effects act equally on both polarizatinos. The DBP is pased on this approximation, which gives the equation:

$$\frac{\partial}{\partial z} E_x = -\frac{\alpha}{2} E_x + \frac{i\beta_2}{2} \frac{\partial^2}{\partial t^2} E_x - i\gamma \frac{8}{9} (|E_x|^2 + |E_y|^2) E_x \tag{12}$$

and an identical equation for E_y. α is the attentuation coefficient. γ is the nonlinear coefficient. This equation cannot be solved analytically, so it is done numerically using the split-step method. To see how these steps are applied, we first recognize that this equation consists of a linear and a nonlinear part.

$$\frac{\partial \mathbf{E}}{\partial z} = (\hat{D} + \hat{N})\mathbf{E} \tag{13}$$

where $\mathbf{E} = [E_x E_y]^T$ and the linear part, $\hat{D}$, and nonlinear part, $\hat{N}$ are given by

$$\hat{D} = \frac{i\beta_2}{2}\frac{\partial^2}{\partial t^2}$$

and

$$\hat{N} = -i\gamma\frac{8}{9}\mathbf{E^H}\mathbf{E} - \frac{\alpha}{2}$$

Where the superscript H denotes the hermitian transpose. Eq. (13) can be split into two equations by splitting the electric field vecxtor into linear and nonlinear parts and assuming that the nonlinear part is much smaller in magnitude. Then

$$\frac{\partial E_l}{\partial z} = \hat{D}E_l, \qquad \frac{\partial E_{nl}}{\partial z} = \hat{D}E_{nl} + \hat{N}E_l \tag{14}$$

These can be solved with different step sizes.[2] Although DSP is a revolutionary technology, it isn't perfect. The split-step method is an $Nlog(N)$ computation problem that requires time to perform. In many telecommunications there is not much time for computation because system lag can be detrimental. This issue is getting smaller as time goes on because computers are becoming more powerful and more efficient and accurate methods of DBP are being developed[6].

6 Noise

The BER of M-ary QAM is given by

$$BER = \frac{\sqrt{M}-1}{\sqrt{M}log_2\sqrt{M}}erfc\sqrt{(\frac{3log_2M}{2(M-1)})\frac{E_b}{N_0}} \tag{15}$$

Where E_b and N_0 are energy and noise power per bit[7]. The high amount of information transferred and the low BER means that QAM is able to come closer to the Shannon limit than other modulation methods. This is depicted in the figure below:

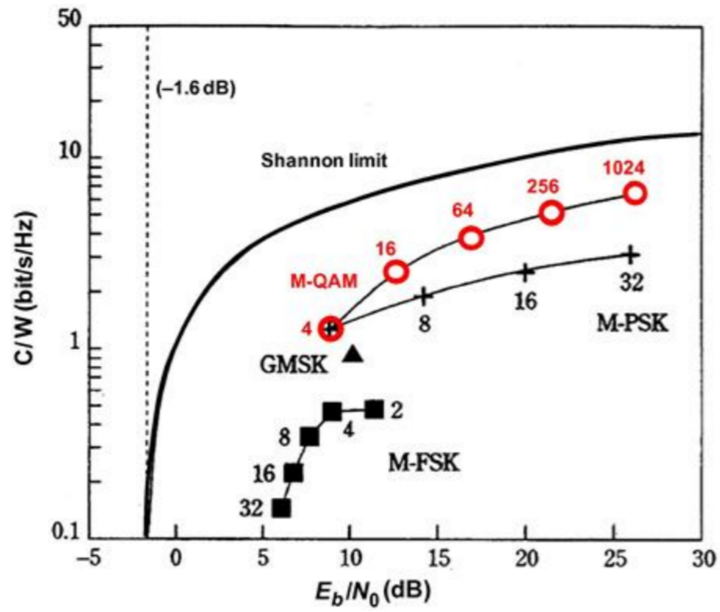

Figure 8: A diagram depicting how close to the Shannon limit different modulation schemes get. C/W is the capacity per bandwidth, or the spectral efficiency of the communication.[7]

7 Orthogonal QAM

7.1 OFDM-QAM

In OFDM the subcarriers are orthogonal, which improves the spectral efficiency. OFDM is also advantagous because the multiple subcarriers transmit the bits in parallel, so the symbol rate of each subcarrier is reduced. This allows electronic components such as ADCs and DSP to be able to keep up with the symbol rate. OFDM-QAM was first demonstrated in 2007, and since then the possible spectral efficiency and bit rate have been steadily increasing.[2] The baseband OFDM signal is given by

$$S(t) = \sum_{n=0}^{N-1} a_n(t)\cos(2\pi n\frac{t}{T}) - b_n(t)\sin(2\pi n\frac{t}{T}) \tag{16}$$

Here a_n and b_n are the symbol data and T is the symbol period. N is the number of subcarriers. The subcarriers are spaced so that the oscillating tail of each subcarrier intersects at the zero level. The data can be demodulated by using the orthogonality, for example, dtat a_k can be demodulated by:

$$\frac{2}{T}\int_{t0}^{t0+T} S(t)\cos(2\pi k\frac{t}{T})dt = \frac{2}{T}\sum_{n=0}^{N-1} a_n(t_0)\int_{t0}^{t0+T}\cos(2\pi n\frac{t}{T})\cos(2\pi n\frac{t}{T})dt...$$
$$-b_n(t_0)\int_{t0}^{t0+T}\cos(2\pi n\frac{t}{T})\sin(2\pi n\frac{t}{T})dt$$
$$= a_k(t_0)$$

To combine QAM and OFDM, the data is first encoded in QAM format. The data is then divided into N subcarriers and converted into a parallel sequence using the inverse fourier transform (IFFT). Then it is put into OFDM. At the receiver end, the OFDM is first demodulated using FFT and then the QAM can be demodulated into binary data.

7.2 OTDM-QAM

Orthogonal Time Division Multiplexing (OTDM) is another method for QAM to surpass the speed limitations imposed by electrical signal processing components. The first OTDM-QAM system was a 16-QAM system experimentally demonstrated in 2011.[8] In OTDM-QAM systems, the QAM signal is generated as an RZ optical pulse. This is achieved with a coherence CW laser followed by an RZ pulse carving.
After QAM modulation, the signal is OTDM multiplexed. At the receiver a local oscillator

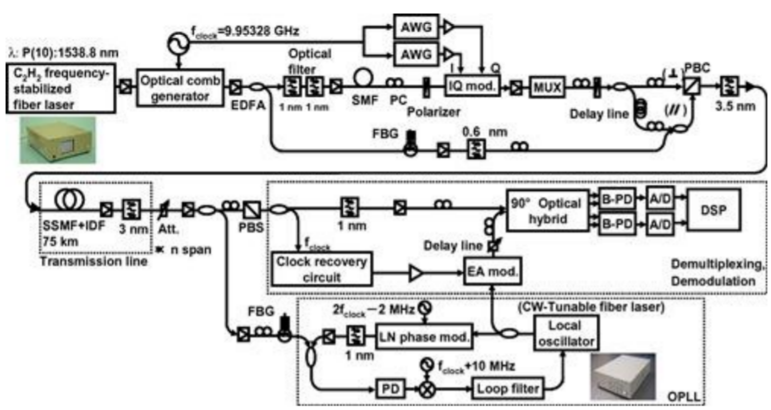

Figure 9: A diagram of a 400 Gbit/s OTDM 32-QAM setup. The OTDM makes the system significantly more complicated that a usual 32-QAM system.[7]

is used for coherent detection.[7] These components can be seen in Fig.9. They use an optical comb generator to create RZ pulses before the IQ modulator. At the receiver end a local oscillator inside an optical phase locked loop (OPLL) must be used in addition to the usual QAM detection system discussed above.

8 Conclusion

Due to the novelty of this topic, there are few textbooks that cover these topics in depth. One of the difficult parts of writing this paper was deciding how in depth to discuss each topic. One could write a lengthy paper on the advancements in digital signal processing alone. If one looks deeper, the math behind some of the discussed techniques can become very difficult. The many different components that make up QAM systems all have current research being performed to improve their capabilities.
As the demand for data transmission continues to grow, the demand for more technologically advanced fibers will grow as well. The current single fiber data transmission record is over 10 Petabits per second, which is an astonishingly high number. This was achieved with a 256-QAM WDM multi-core multi-mode fiber.[9] This record will surely not stand for long, though, as telecommunications is a very dynamic field and developments are being made every day. In the future we will see terabit fibers used commercially, which will be made possible in part by QAM.

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