1 Problem

$$
\begin{aligned}
\min_x & f(x) \\
& g(x) \leq 0 \\
& h(x) = 0
\end{aligned}
$$

Also, we are given:

$$
\begin{gathered}
\Delta_0 \leftarrow 1, \\
0 < \gamma_0(= .1) < \gamma_1(= .5) \leq 1 \leq \gamma_2(= 2), \\
0 < \eta_1(= .9) \leq \eta_2(.9), \\
\gamma_\theta(= 10^{-4}) \in (0, 1), \\
\kappa_\Delta(= .7) \in (0, 1], \\
\kappa_\theta(= 10^{-4}) \in (0, 1), \\
\kappa_\mu(= 100) > 0, \\
\mu(= .01) \in (0, 1), \\
\psi(= 2) > \frac{1}{1 + \mu}, \\
\kappa_{tmd} = .01
\end{gathered}
$$

2 Original Algorithm

Algorithm 1 Filter Trust Region Search

procedure TRUST REGION FILTER
  initialize
  $k = 0$
  choose an x_0
  while $k < maxit$ **do**
    Compute $m_k, g_k = \nabla m_k(x_k), c_k, A_k, f_k = f(x_k), \mathcal{A}, \theta_k$
    Solve:

$$\begin{aligned} \nabla^2 m_k(x_k) + A_k^T \lambda &= g_k \\ A_k x &= c_k \end{aligned}$$

    $H_k \leftarrow \nabla^2 m_k(x_k) + \sum \lambda_i \nabla^2 c_{ik}$
    $\chi_k \leftarrow |\arg\min_t \{\langle g_k + H_k n_k, t\rangle | A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \geq 0 \wedge \|t\| \leq 1\}|$
    if constraint violation $= 0 \wedge \chi = 0$ **then success**
    $n_k \leftarrow -A_{\mathcal{A}}^T [A_{\mathcal{A}} A_{\mathcal{A}}^T]^{-1} c_{\mathcal{A}}$
    if Feasible region $\neq \emptyset \wedge \|n\| \leq \kappa_\Delta \Delta_k \min\{1, \kappa_\mu \Delta_k^\mu\}$ **then**
      $t_k \leftarrow \arg\min_t \{(g_n + H_n)^T t + \frac{1}{2} t^T H t | c_{eq} + A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \geq 0 \wedge \|n_k + t\| < \Delta_k\}$
      they find this by backtracking on a curvilinear search of the projected gradient
      $s_k \leftarrow t_k + n_k$
      if $x_k + s_k$ is acceptable: $\theta(x_k + s_k) \leq (1 - \gamma_\theta)\theta' \vee f(x_k + s_k) \leq f' - \gamma_\theta \theta' \forall (f', \theta') \in$ Filter **then**
        if $m_k(x_k) - m_k(x_k + s_k) \geq \kappa_\theta \theta_k^\psi$ **then**
          $\rho = \frac{f(x_k) - f(x_k + s_k)}{m_k(x_k) - m_k(x_k + s_k)}$
          if $\rho < \eta_1$ **then**
            reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$
            $k \leftarrow k + 1$
            continue
          else if $\rho > \eta_2$ **then**
            increase Δ: $\Delta_{k+1} \leftarrow$ some $\in [\Delta_k, \gamma_2 \Delta_k]$
        else
          add x_k to filter
        $x_{k+1} \leftarrow x_k + s$
      else
        reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$
        $k \leftarrow k + 1$
        continue
    else
      add x_k to filter
      compute new r (restoration step) and Δ_{k+1}
      if impossible to restore **then fail**
      $x_{k+1} \leftarrow x_k + r$
    $k \leftarrow k + 1$

3 My Algorithm

Algorithm 2 Filter Trust Region Search

procedure TRUST REGION FILTER
initialize
$k = 0$
choose an x_0
while $k < maxit$ **do**
ensure poisedness, possibly adding points to the model
Compute $m_k, g_k = \nabla m_k(x_k), c_k, A_k, f_k = f(x_k), \mathcal{A}, \theta_k$
Solve:
$$\begin{aligned} \nabla^2 m_k(x_k) + A_k^T \lambda &= g_k \\ A_k x &= c_k \end{aligned}$$
$H_k \leftarrow \nabla^2 m_k(x_k) + \sum \lambda_i \nabla^2 c_{i\,k}$
$\chi_k \leftarrow |\arg\min_t \{\langle g_k + H_k n_k, t\rangle | A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \leq 0 \wedge \|t\| \leq 1\}|$
if constraint violation $= 0 \wedge \chi = 0$ **then**
if $\Delta_k < tol$ **then**
reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ continue
success
$n_k \leftarrow \arg\min_n \{\|t\|^2 | c_{eq} + A_{eq} n = 0 \wedge c_{ineq} + A_{ineq} n \leq 0 \wedge \|n\|^2 \leq \Delta_k\}^2$
if Feasible region $\neq \emptyset \wedge \|n\| \leq \kappa_\Delta \Delta_k \min\{1, \kappa_\mu \Delta_k^\mu\}$ **then**
$t_k \leftarrow \arg\min_t \{(g_n + H_n)^T t + \frac{1}{2} t^T H t | c_{eq} + A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \leq 0 \wedge \|s\| \leq \Delta_k\}$
$s_k \leftarrow t_k + n_k$
// Here we evaluate new c and f at $x_k + s_k$
if $x_k + s_k$ is acceptable: $\theta(x_k + s_k) \leq (1 - \gamma_\theta)\theta' \vee f(x_k + s_k) \leq f' - \gamma_\theta \theta' \forall (f', \theta') \in$ Filter **then**
if $m_k(x_k) - m_k(x_k + s_k) \geq \kappa_\theta \theta_k^\psi$ **then**
$\rho = \frac{f(x_k) - f(x_k + s_k)}{m_k(x_k) - m_k(x_k + s_k)}$
if $\rho < \eta_1$ **then**
reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$
$k \leftarrow k + 1$
continue
else if $\rho > \eta_2$ **then**
if $\|s\| < \frac{\Delta_k}{2}$ **then**
reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$
else
increase Δ: $\Delta_{k+1} \leftarrow$ some $\in [\Delta_k, \gamma_2 \Delta_k]$
else
add x_k to filter
$x_{k+1} \leftarrow x_k + s$
else
reduce Δ: $\Delta_{k+1} \leftarrow$ some $\in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$
$k \leftarrow k + 1$
continue
else
add x_k to filter
compute new r (restoration step) and Δ
if impossible to restore **then fail**
$x_{k+1} \leftarrow x_k + r$
$k \leftarrow k + 1$