

1 Problem

$$\begin{aligned}\min_x f(x) \\ g(x) &\leq 0 \\ h(x) &= 0\end{aligned}$$

Also, we are given:

$$\begin{aligned}\Delta_0 &\leftarrow 1, \\ 0 &< \gamma_0(=.1) < \gamma_1(=.5) \leq 1 \leq \gamma_2(=2), \\ 0 &< \eta_1(=.9) \leq \eta_2(.9), \\ \gamma_\theta &(=10^{-4}) \in (0,1), \\ \kappa_\Delta &(=.7) \in (0,1], \\ \kappa_\theta &(=10^{-4}) \in (0,1), \\ \kappa_\mu &(=100) > 0, \\ \mu &(=.01) \in (0,1), \\ \psi &(=2) > \frac{1}{1+\mu}, \\ \kappa_{tmd} &=.01\end{aligned}$$

2 Original Algorithm

Algorithm 1 Filter Trust Region Search

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1: procedure TRUST REGION FILTER
2:   initialize
3:    $k = 0$ 
4:   choose an  $x_0$ 
5:   while  $k < \maxit$  do
6:     Compute  $m_k, g_k = \nabla m_k(x_k), c_k, A_k, f_k = f(x_k), \mathcal{A}, \theta_k$ 
7:     Solve:
8:       
$$\begin{aligned} \nabla^2 m_k(x_k) + A_k^T \lambda &= g_k \\ A_k x &= c_k \end{aligned}$$

9:      $H_k \leftarrow \nabla^2 m_k(x_k) + \sum \lambda_i \nabla^2 c_{i k}$ 
10:     $\chi_k \leftarrow |\arg \min_t \{ \langle g_k + H_k n_k, t \rangle \mid A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \geq 0 \wedge \|t\| \leq 1 \}|$ 
11:    if constraint violation  $= 0 \wedge \chi = 0$  then success
12:     $n_k \leftarrow -A_{\mathcal{A}}^T [A_{\mathcal{A}} A_{\mathcal{A}}^T]^{-1} c_{\mathcal{A}}$ 
13:    if Feasible region  $\neq \emptyset \wedge \|n\| \leq \kappa_{\Delta} \Delta_k \min\{1, \kappa_{\mu} \Delta_k^{\mu}\}$  then
14:       $t_k \leftarrow \arg \min_t \{ (g_n + H_n)^T t + \frac{1}{2} t^T H t \mid c_{eq} + A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \geq 0 \wedge \|n_k + t\| < \Delta_k \}$ 
15:      they find this by backtracking on a curvilinear search of the projected gradient
16:       $s_k \leftarrow t_k + n_k$ 
17:      if  $x_k + s_k$  is acceptable:  $\theta(x_k + s_k) \leq (1 - \gamma_{\theta})\theta' \vee f(x_k + s_k) \leq f' - \gamma_{\theta}\theta'\psi(f', \theta') \in \text{Filter}$  then
18:        if  $m_k(x_k) - m_k(x_k + s_k) \geq \kappa_{\theta} \theta_k^{\psi}$  then
19:          
$$\rho = \frac{f(x_k) - f(x_k + s_k)}{m_k(x_k) - m_k(x_k + s_k)}$$

20:          if  $\rho < \eta_1$  then
21:            reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ 
22:             $k \leftarrow k + 1$ 
23:            continue
24:          else if  $\rho > \eta_2$  then
25:            increase  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\Delta_k, \gamma_2 \Delta_k]$ 
26:          else
27:            add  $x_k$  to filter
28:             $x_{k+1} \leftarrow x_k + s$ 
29:          else
30:            reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ 
31:             $k \leftarrow k + 1$ 
32:            continue
33:        else
34:          add  $x_k$  to filter
35:          compute new  $r$  (restoration step) and  $\Delta_{k+1}$ 
36:          if impossible to restore then fail
37:           $x_{k+1} \leftarrow x_k + r$ 
38:           $k \leftarrow k + 1$ 

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3 My Algorithm

Algorithm 2 Filter Trust Region Search

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1: procedure TRUST REGION FILTER
2:   initialize
3:    $k = 0$ 
4:   choose an  $x_0$ 
5:   while  $k < \text{maxit}$  do
6:     ensure poisedness, possibly adding points to the model
7:     Compute  $m_k, g_k = \nabla m_k(x_k), c_k, A_k, f_k = f(x_k), \mathcal{A}, \theta_k$ 
8:     Solve:
9:       
$$\begin{matrix} \nabla^2 m_k(x_k) + A_k^T \lambda = g_k \\ A_k x = c_k \end{matrix}$$

10:     $H_k \leftarrow \nabla^2 m_k(x_k) + \sum \lambda_i \nabla^2 c_{i,k}$ 
11:     $\chi_k \leftarrow |\arg \min_t \{ \langle g_k + H_k n_k, t \rangle \mid A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \leq 0 \wedge \|t\| \leq 1 \}|$ 
12:    if constraint violation  $= 0 \wedge \chi = 0$  then
13:      if  $\Delta_k < \text{tol}$  then
14:        reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$  continue
15:      success
16:       $n_k \leftarrow \arg \min_n \{ \|t\|^2 \mid c_{eq} + A_{eq} n = 0 \wedge c_{ineq} + A_{ineq} n \leq 0 \wedge \|n\|^2 \leq \Delta_k \}^2$ 
17:      if Feasible region  $\neq \emptyset \wedge \|n\| \leq \kappa_\Delta \Delta_k \min\{1, \kappa_\mu \Delta_k^\mu\}$  then
18:         $t_k \leftarrow \arg \min_t \{ (g_n + H_n)^T t + \frac{1}{2} t^T H t \mid c_{eq} + A_{eq} t = 0 \wedge c_{ineq} + A_{ineq} t \leq 0 \wedge \|s\| \leq \Delta_k \}$ 
19:         $s_k \leftarrow t_k + n_k$ 
20:        // Here we evaluate new  $c$  and  $f$  at  $x_k + s_k$ 
21:        if  $x_k + s_k$  is acceptable:  $\theta(x_k + s_k) \leq (1 - \gamma_\theta) \theta' \vee f(x_k + s_k) \leq f' - \gamma_\theta \theta' \forall (f', \theta') \in \text{Filter}$  then
22:          if  $m_k(x_k) - m_k(x_k + s_k) \geq \kappa_\theta \theta_k^\psi$  then
23:             $\rho = \frac{f(x_k) - f(x_k + s_k)}{m_k(x_k) - m_k(x_k + s_k)}$ 
24:            if  $\rho < \eta_1$  then
25:              reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ 
26:               $k \leftarrow k + 1$ 
27:              continue
28:            else if  $\rho > \eta_2$  then
29:              if  $\|s\| < \frac{\Delta_k}{2}$  then
30:                reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ 
31:                else
32:                  increase  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\Delta_k, \gamma_2 \Delta_k]$ 
33:                else
34:                  add  $x_k$  to filter
35:                   $x_{k+1} \leftarrow x_k + s$ 
36:                else
37:                  reduce  $\Delta$ :  $\Delta_{k+1} \leftarrow \text{some} \in [\gamma_0 \Delta_k, \gamma_1 \Delta_k]$ 
38:                   $k \leftarrow k + 1$ 
39:                  continue
40:                else
41:                  add  $x_k$  to filter
42:                  compute new  $r$  (restoration step) and  $\Delta$ 
43:                  if impossible to restore then fail
44:                   $x_{k+1} \leftarrow x_k + r$ 
45:                   $k \leftarrow k + 1$ 

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