

1 Problem

$$\begin{aligned} \min_x & f(x) \\ & g(x) \leq 0 \\ & h(x) = 0 \end{aligned}$$

2 Algorithm

Algorithm 1 Filter Line Search

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1: procedure FILTER LINE SEARCH
2:   loop:
3:     while True do
4:        $ineq \leftarrow m_g(x^k)$ 
5:        $active \leftarrow [i] : m_g(x^k)_i \geq -\tau$ 
6:        $c \leftarrow [m_h(x^k); m_g(x^k)_{active}]$ 
7:        $A \leftarrow [\nabla m_h(x^k); \nabla m_g(x^k)_{active}]$ 
8:        $G \leftarrow \nabla m_f(x^k)$ 
9:        $H \leftarrow \nabla^2 f(x^k)$ 
10:       $\theta \leftarrow \theta(x^k)$ 
11:      if check stopping criteria with model functions at  $x^k$  then
12:        if  $r > \tau$  then
13:           $r \leftarrow \gamma_{dec} r$ 
14:          improve model
15:        continue.
16:      return success
17:       $KKT \leftarrow [H, A^T; A, 0]$ 
18:       $rhs \leftarrow [G; c]$ 
19:      if  $cond(KKT) > maxcondition$  then
20:        restore feasibility
21:      continue
22:       $[d; \cdot] = -KKT^{-1} rhs$ 
23:       $\alpha_{min} =$  compute minimum alpha
24:       $\alpha = 1$ 
25:       $accept = False$ 
26:      while  $not accept$  do
27:         $m \leftarrow \alpha G^T d$ 
28:        Variant: clip to trust region
29:        if  $\alpha < \alpha_{min}$  then
30:          restore feasibility
31:          goto loop
32:         $x_{new} = x + \alpha * d$ 
33:         $\theta_{new} = \theta(x_{new})$ 
34:         $f_{new} = m_f(x_{new})$ 
35:        if new inequality constraint becomes active then
36:           $\alpha = \tau \alpha$ 
37:          continue
38:        plot model
39:        if  $[f_{new}, \theta_{new}]$  is dominated then
40:           $\alpha = \tau \alpha$ 
41:          continue
42:         $f_{type} \leftarrow m < 0 \wedge (-m)^s f \alpha^{1-s} f > \delta \theta^s \theta$ 
43:        if  $f_{type}$  then
44:          if  $f_{new} \leq f + \eta_f m$  then
45:             $accept \leftarrow True$ 
46:          else
47:             $\alpha = \tau \alpha$ 
48:            continue
49:          else
50:            check equation 8
51:             $\alpha = \tau \alpha$ 
52:        if  $accept$  then
53:
54:         $\rho \leftarrow$  compute rho
55:        if  $\rho$  is small then
56:          if  $\|x_{new} - center\| < \frac{1}{2} r$  then
57:             $r \leftarrow \gamma_{dec} r$ 
58:          else
59:             $r \leftarrow \gamma_{inc} r$ 
60:            improve the model
61:          else if  $\rho$  is medium then
62:            if lambda poised then
63:               $r \leftarrow \gamma_{dec} r$ 
64:            improve the model
65:          else continue
66:          if  $(1 - \gamma_\theta) * \theta_{new} > tol$  then add to filter:  $[f_{new} - \gamma_f \theta_{new}, (1 - constants.gamma_\theta) \theta_{new}]$ 
67:           $x^{k+1} \leftarrow x_{new}$ 
68:          construct model functions  $m_f, m_h, m_g$ 
69:           $i \leftarrow i + \max(delta_1(string(i)), delta_2(j))$ .
70:          goto top.

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