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Optimizing the spatial assignment of schools through a random mechanism towards equal educational opportunity: A resemblance approach

Teqi Dai, Cong Liao, Shaoya Zhao*

Beijing Key Laboratory for Remote Sensing of Environment and Digital City, School of Geography, Faculty of Geographical Science, Beijing Normal University, No. 19, Xinjiekouwai Street, Haidian District, Beijing 100875, China

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ABSTRACT

Although it is generally agreed that public primary education should be equally available to all, a spatial disparity of educational opportunity exists worldwide, for wealthy families can acquire residential locations with better access to the best schools. Random allocation of school places is considered to be an effective approach for breaking such spatial linkages. This study addresses the problem of achieving the optimal spatial equality of educational opportunity by re-assigning school places to demand nodes using a combined mechanism that introduces a random allocation into a proximity-based system. This study proposes the use of a probability distribution for a full description of educational opportunity. A new spatial allocation optimization model towards equal educational opportunity with the maximum school travel distance constraints and capacities constraints was developed to maximize the resemblance between the probability distributions across demand nodes (RES-based model). This model was applied to a case study of primary school allocation in the Shijingshan District of Beijing, China, and was resolved in a heuristic way. The solution was compared with those provided by a capacitated proximity-based model and a model also using the random mechanism but measuring educational opportunity by expectation values (VAR-based model). It was found that the introducing of random mechanism could significantly improve the spatial equality of educational opportunity with a significant loss of spatial efficiency. In addition, the solution provided by the RES-based model was quite different to that provided by the VAR-based model. An improvement in the resemblance of demand node probability distributions might lead to an increase in the variance of expectations. The resemblance approach proposed in this study may also be applied to the optimization of other random objectives.

1. Introduction

In 2017, Beijing announced its new master plan for 2035, in which the goals of Beijing's public education system were stated. For the first time in the history of Beijing's city planning, educational equality was listed before the quality goal or other goals, and reforms to the school assignment mechanism were proposed. This is a typical example of educational reform in China, where the priority has been shifted from efficiency to equality. In the Chinese education system, efficiency has been given precedence for a long time, following Deng Xiaoping's principle of "Efficiency first, give attention to equity". For elementary education, a "bifurcated educational system" was established, which led to very uneven distributions of educational resources (Rosen, 1985; Yang, Ke, Zhan, & Ren, 2014). Moreover, the spatial inequality resulting from the uneven educational resource distribution was exacerbated by China's proximity-based assignment system, which allocated school places according to residential address (Bi & Zhang, 2016; Wen, Xiao, & Zhang, 2017; Wu, Zhang, & Waley, 2015; Xiang, Stillwell, Burns, Heppenstall, & Norman, 2018). The establishment of equal education has been an important strategy for social fairness and justice in China. As the spatial distribution of educational resources is not easy to change quickly (You, 2007), random assignment mechanisms have been encouraged and trialed by some Chinese city governments. Therefore, providing the optimal solution is critical for effective decision making on how to introducing random mechanism into the existing proximity-based system.

Introducing a random mechanism to improve the equality of educational opportunity is a contentious issue that has been faced by cities worldwide. Despite the variations in details, the choice-based system and the proximity-based system are most common allocational systems used elsewhere in the world (Butler & Hamnett, 2007). The issue of spatial inequality of educational opportunity exists in both systems

* Corresponding author.
E-mail address: zhaoshaoya0820@mail.bnu.edu.cn (S. Zhao).

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because of the substantial impacts of geographical access to educational resources. This is not simply because the spatial distribution of educational resources is uneven, but also because the geographical mechanism of reproducing the inequality, i.e., wealthy families tend to obtain a better educational opportunity by location acquisition and the better schools tend to be strengthened further by this spatial selection process (Clark & Maas, 2012; Hamnett & Butler, 2011; Sylvia, 2016; Wei, Xiao, Simon, Liu, & Ni, 2018). Random allocation is an important mechanism that has the potential to break the geographical link between wealth and educational opportunity (Hamnett & Butler, 2013). Although random mechanisms may result in some negative effects, such as the increasing of the distance to school and being resisted by the vested interests, they have been planned or trialed in some cities to improve equality of educational opportunity (Allen, Burgess, & McKenna, 2013). In this study, a spatial optimization model was developed to enable the introduction of a random mechanism into a proximity-based system, with the aim of promoting spatial equality of educational opportunity.

School planning is a classical geographical problem of spatial optimization (Tong & Murray, 2012). Geographers have been developing models to address the school district planning problem (SDPP) for over 50 years (Yeates, 1963). Various models have been developed and applied by planners or decision makers. According to the focus of the decision making, these models may be classified as the problem of assigning students to schools, the problem of matching the location and capacity of schools, or both problems simultaneously (Teixeira & Antunes, 2007). As a basic component of the SDPP, school assignment optimization models, called as Generic Districting Models, have incorporated many considerations, such as the optimal school size, the school capacity, travel distance to school, the school's social composition, the compactness of school districts, and the stability of school districts (Antunes & Peeters, 2000; Bruno, Genovese, Piccolo, & Sterle, 2014; Delmelle, Thill, Peeters, & Thomas, 2014; Lemberg & Church, 2000). In parallel with studies of other public facilities, most studies have focused on spatial efficiency, and the typical objectives of these studies are minimizing the travel distance of students or minimizing the total cost of schools (Lemberg, 2004; Thomas, 1987). A recent study by Delmelle et al. (2014) identified five key challenging issues in this area, and addressed four of them. The model was further developed and implemented as an open source GIS toolbox (Chen, Thill, & Delmelle, 2018). However, the issue they did not consider was spatial equity.

Among the various objectives and constraints proposed for school planning, equal opportunity is much less often optimized as an objective function in school assignment optimization models (Malczewski & Jackson, 2000). Equity issues are often formulated under the spatial efficiency objectives, and addressed by imposing the maximum distance constraints or the racial balance constraints (Lemberg & Church, 2000; Maxfield, 1972; Schoepfle & Church, 1989, 1991). A few studies have focused on the controlled choice problem, using the objective of maximum choice and constraints on racial balance (Church & Schoepfle, 1993; Müller, Haase, & Kless, 2009). Recently, a bi-objective model on optimal socioeconomic variation and school travel was formulated, and the corresponding standard optimization package was developed (Bouzarth, Forrester, Hutson, & Reddoch, 2018). For the problem of optimizing random assignment towards spatial equality, a new optimization model with an objective function of minimizing accessibility variance (Li, Wang, & Yi, 2017; Wang & Tang, 2013) was introduced to formulate a proximity-based random assignment problem in which the spatial equality was defined as the variance of expectations of educational opportunity (VAR-based model) (Dai, Liu, Liao, & Cai, 2018). However, as expectation was the only one parameter used to depict educational opportunity, the model didn't tell the whole story about educational opportunity. To capture educational opportunity more completely, this study considers possibility distribution to measure educational opportunity, and proposes the resemblance approach for model formulation (RES-based model).

To the best of our knowledge, random mechanism is seldom discussed in previous SDPP studies, and the resemblance of possibility distributions has not been used to depict the equality of educational opportunity, nor been formulated or solved in an optimization model. The remainder of this paper is organized as follows. First, the problem is defined and formulated, and the method used to solve it is described. Then, a case study is discussed and the results are analyzed. For a comparison, we use the case of Shijingshan, a district of Beijing. Finally, a further discussion and the major conclusions of the paper are presented.

2. Formulation of the problem and its solution

2.1. Defining the planning problem

Defining the planning problem is critical for model formulation. School districting problem has been formulated as the transportation optimization problem since Yeates's classic work (1963). Assigning a residential zone (i) to a school (j) was compared to the problem of minimum required commuting (MRC), in which a resident worker in a residential zone (i) was assigned to a job zone (j). This approach of formulating school districting problem as a transportation problem or a network flow problem was followed by many studies (Maxfield, 1972; Franklin & Koenigsberg, 1973; Schoepfle & Church, 1991). However, this approach is also limited due to the special characteristics of school districting problem (Lemberg, 2004). For our problem, it is also helpful, but there are some random features beyond the traditional comparison. When a random allocation is introduced into the proximity-based assignment system, a residence will not be assigned to a single and determinative school, but will rather be assigned to several nearby schools, so that the students at the residence will be allocated randomly to the given school seats with equal possibility.

Considering the random feature at the individual scale, our problem may be better clarified as following the recent study of simulation of commuting trips by a Monte Carlo approach (Hu & Wang, 2015). A Monte Carlo method generates random numbers according to a certain probability distribution function. It can provide a powerful framework for spatial analysis, and has been increasingly applied in several areas (Gao, Wang, Gao, & Liu, 2013; Luo, McLafferty, & Wang, 2010; Watanatada & Ben-Akiva, 1979). In Hu & Wang's work (2015), it was used to simulate actual urban commuting at the individual scale, i.e., the commuter at a census tract will be choosing randomly to generate a trip while following the distribution of actual commuting (the actual assignment) at the scale of census tract. In our case, the student at a residence will also be chosen randomly, but the distribution of assignment is unknown, which will solve by the optimization model towards maximum equality of educational opportunity. Specifically, the educational opportunity of the residence will be decided by a lottery pool, which consists of seats from the given schools. Different combinations of school places can result in differences in the educational opportunity. A proper assignment of school places may improve the equality of the random allocation, which corresponds the actual and known assignment in Hu & Wang's study (2015).

In such framework, the first task is to define the objective function, which requires defining equal opportunity in a mathematical form. The notion of equal opportunity is a rather complicated and ambiguous concept in terms of philosophy or politics (Arneson, 2018), and spatial equity has also been variously discussed (Hay, 1995; Marsh & Schilling, 1994; Morrill & Symons, 1977; Mulligan, 1991; Talen, 2001; Truelove, 1993). This study was not devoted to the issue of defining equity. For the equal opportunity of elementary education, we followed Le Grand's definition: "*equity will be achieved if all members of a population, regardless of where they live, have the same choice sets*" (Hay, 1995). Similarly, the spatial equality of educational opportunity has also been defined to "*require both parity of esteem across the field of provision and that everybody has broadly similar opportunities*" (Hamnett & Butler, 2011). Thus, we considered how to measure the resemblance of the choice sets in a mathematical form to formulate the spatial equality of educational opportunity as an objective.

Various mathematical indices can be used for equality and variance is a popular one (Ogryczak, 2009). However, the index of expectation was not sufficient for measuring the choice set. This is because a complete description for educational opportunity or the choice set should be a probability distribution function, which is composed of the probability of attending each school, whereas expectation is only one of the parameters used for the description of the probability distribution. For example, two different probability distributions may have the same expectation. In this situation, the educational opportunities are quite different due to the higher risk of entering a poor school or the greater chance of entering a good school. When the probability distribution is used to define educational opportunity, the resemblance of the probability distributions is a suitable measurement to formulate spatial equality. The more similar the probability distributions at different locations are, the more spatial equality of educational opportunity will be obtained. However, this resemblance approach triggers a new problem, i.e., how to measure the resemblance of the probability distributions. Mathematical indices are generally used to measure the similarity between two distributions, such as simple correlation coefficients. Following the concept used in the Gini index of calculating the accumulated differences between the actual distribution and an absolutely equal one, we calculated the total difference between assigned possibility distributions with the absolute equal possibility distribution. The details are presented in the following steps.

First, we define the absolute equality of educational opportunity, i.e., all students have the same opportunity. This will be achieved if all school places are assigned to every student by equal possibility, without consideration of the distance constraint. Therefore, the possibility of entering school j, denoted by A_j, can be calculated by the ratio of school places to the whole population, as shown in Eq. (1). Then the absolute equality of educational opportunity can be depicted by a n-dimension vector as A (A_1, …, A_n),

$$A_j = \frac{S_j}{N} = \frac{S_j}{\sum_{i=1}^{m} H_i}, \tag{1}$$

where i and I are the index of m demand nodes and the set of demand nodes, j and J are the index of n candidate schools and the set of candidate schools, S_j is the total number of places in school j, N is the total number of students in the whole area, and H_i is the number of students at demand node i.

Second, the actual possibility distribution at demand location i is calculated in the same manner. The actual possibility distribution at demand location i is written as P_i. It is also a n-dimension vector. In a proximity-based system, school places beyond a certain travel distance will not be assigned to students. Thus, P_{ij}, the probability of being assigned to school j at demand location i, can be calculated by Eq. (2):

$$P_{ij} = \frac{X_{ij}}{H_i}, \tag{2}$$

where X_{ij} is a non-negative integer, which represents the number of school places assigned to demand node i from school j.

Then, we introduce the notion of PA-distance. We define the abstract distance in n-dimensional space between P_i and A as *PA-distance* (d_{PA}), which represents the distance or the deviation between the actual educational opportunity and the educational opportunity of absolute equality. As both P_i and A are n-dimension vectors, the *PA-distance* can be calculated by Eq. (3):

$$d_{PA,i} = \sqrt{\sum_{j=1}^{n} (P_{ij} - A_j)^2}. \tag{3}$$

Based on the *PA-distance*, the resemblance between probability distributions can be measured by calculating the sum of PA-distances from all P_i to A. Thus, the resemblance of all P_i values can be converted into a distance calculation in multi-dimensional space. The smaller the total PA-distance is, the more similar the probability distributions are. Interestingly, considering this problem in n-dimensional space, it is a problem of minimizing total distance, which is similar to the objective of the p-median problem.

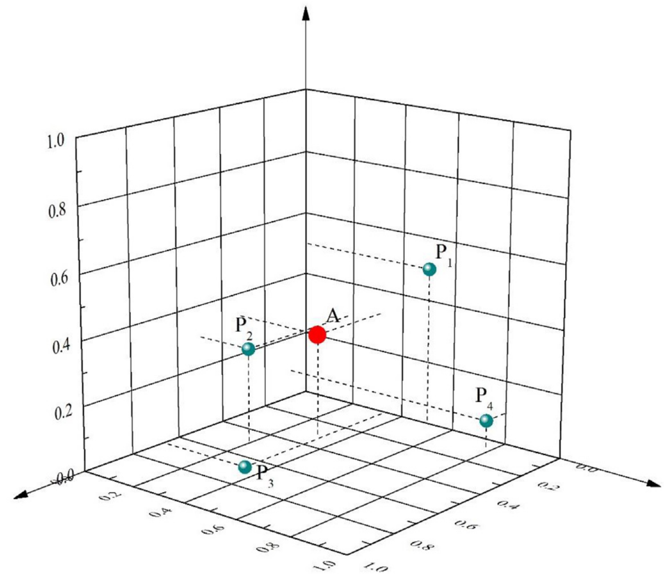

Fig. 1. The four students and three schools example. Each axis represents the possibility of entering the corresponding school. The red point represents absolute equality (A). The green points represent the actual possibility distributions of the students (P_i). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

We used a four students and three schools example to illustrate the concept, as shown in Fig. 1. The educational opportunity of a student is composed of three possibilities to enter three corresponding schools. According to a student's three possibilities, his/her educational opportunity can be located in a three-dimensional space, which is shown as a green point in Fig. 1. If the opportunity of entering a particular school is assigned equally to the students, then the absolute equality of educational opportunity can be set in that space as the red point. The distances between the values of A and P_i are the *PA-distances* defined above. The lower the sum of distances is, the closer is the situation to the goal of absolute equality.

2.2. RES-based model formulation

Based on the definition above, the objective function is to minimize the total population-weighted PA-distances, which can be written as;

$$Minimize \sum_{i=1}^{m} H_i d_{PA,i}. \tag{4}$$

Using the definitions in Eqs. (1) to (4), the objective of the model is written as follows:

$$Minimize \sum_{i=1}^{m} H_i \sqrt{\sum_{j=1}^{n} \left(\frac{X_{ij}}{H_i} - \frac{S_j}{\sum_{i=1}^{m} H_i} \right)^2} \tag{5}$$

Subject to

$$H_i = \sum_{j=1}^{n} X_{ij},\ \forall\ i \in I,\ \forall\ j \in J, \tag{6}$$

$$C_{max,j} \geq \sum_{i=1}^{m} X_{ij},\ \forall\ i \in I,\ \forall\ j \in J, \tag{7}$$

$$C_{min,j} \le \sum_{i=1}^{m} X_{ij}, \forall\ i \in I, \forall\ j \in J, \tag{8}$$

$$X_{ij} = 0\ if\ d_{ij} > D_{max}, \forall\ i \in I, \forall\ j \in J, \tag{9}$$

$$X_{ij} \in Z_0^+, \forall\ i \in I, \forall\ j \in J, \tag{10}$$

where $C_{\max,j}$ and $C_{min,j}$ are the maximum and minimum capacities of school j respectively; d_{ij} represents the geographical distance between demand node i and school j; and D_{max} is the maximum geographical distance to school for any student. Constraint (6) ensures that all students are required to enter school. Constraints (7) and (8) require that each school has a student population within its capacity. Constraint (9) ensures that the travel distance from demand i to school j is less than $D_{\max}$. Constraint (10) is the integrality constraint which requires X_{ij} is a non-negative integer.

The planning model using a random mechanism described above is a problem of integer quadratic programming. A heuristic algorithm is used to solve this hard problem. Following the method used to solve the VAR-based model (Dai et al., 2018), particle swarm optimization (PSO) algorithm was programmed and solved in C# language.

2.3. Comparison scenarios

For comparison, the previous VAR-based model based on a proximity-based random mechanism and the modified p-median model based on a pure proximity-based mechanism (DIS-based model) were used to illustrate the different solutions. According to the VAR-based model, educational opportunity of a given demand location is defined by the expectation index which is the average weighted quality of assigned school places. Then EA-deviation (denoted as $d_{EA,i}$) is introduced to present the absolute deviation of expectation at the demand node i from the average weighted educational opportunity of the whole area (denoted as a), which can be calculated by Eq. (11):

$$d_{EA,i} = \left| \sum_{j=1}^{n} \frac{X_{ij}}{H_i} Q_j - a \right|, \tag{11}$$

where a is the mean value of weighted Q in the study area, Q_j is the score of school j's quality.

And the objective is to minimize the variance of the expectation values of all demand locations. The model is formulated as follows:

$$Minimize\ \sum_{i=1}^{m} H_i d_{EA,i}{}^2 \tag{12}$$

Subject to (6)–(10).

In the DIS-based model, the students are assigned to the nearest school if the capacity permits. The formulation of our model was based on the capacitated p-median model. The objective is set as minimizing the total travel distance to school. In addition, as this study focused on allocation of school places, the school number (p) and their locations were given according to the actual situation. The model was formulated as follows:

$$Minimize\ \sum_{i=1}^{m} \sum_{j=1}^{n} H_i d_{ij} Y_{ij} \tag{13}$$

Subject to

$$\sum_{i} Y_{ij} = 1, \forall\ i \in I, \forall\ j \in J, \tag{14}$$

$$C_{max} \ge \sum_{i=1}^{m} H_i Y_{ij}, \forall\ i \in I, \forall\ j \in J, \tag{15}$$

$$C_{min} \le \sum_{i=1}^{m} H_i Y_{ij}, \forall\ i \in I, \forall\ j \in J, \tag{16}$$

$$Y_{ij} = 0\ if\ d_{ij} > D_{max}, \forall\ i \in I, \forall\ j \in J, \tag{17}$$

$$Y_{ij} \in \{0, 1\}, \forall\ i \in I, \forall\ j \in J, \tag{18}$$

where constraint (14) ensures that each demand node was assigned to one and only one school. Constraints (15) and (16) are capacity constraints. Constraint (17) is the maximum travel distance constraint. Constraint (18) states that variable Y is binary.

3. Case study

3.1. Data preparation

The models were applied in a case study of Shijingshan District, located in the west of Beijing. The district covers an area of about 84 km^2. In this study, it was divided into 303 blocks at the level of residential quarters. The centroids of the residential quarters were used as demand locations, and public primary schools were used as supply locations. The student population was used as the demand size. In 2012, there were 31 schools with a total demand of 2801 students. The school's educational quality was measured by the proportion of senior professional teachers among the teaching staff employed at each school, of which the mean value for the whole area was 0.58. Data was obtained from the Shijingshan Education Yearbook or a survey conducted by the local government. The road network data was used to compute the travel distances from each demand node to each school site. Following a previous study (Burgess, Greaves, Vignoles, & Wilson, 2011), 3 km was set as the maximum travel distance to school.

3.2. Results

The models were applied to the case study, and the results are presented and compared in the following section. Fig. 2 shows the assignment of school places to demand nodes by the three models (i.e., DIS-based model, VAR-based model, and RES-based model). When a random mechanism was introduced, the demand nodes were assigned to multiple schools, while in the traditional proximity-based mechanism the demand nodes were assigned to one deterministic school. The spatial pattern by the RES-based model was more disperse than the pattern by the VAR-based model. The average number of assigned schools for all demand nodes by the VAR-based model was 3.86, while that by the RES-based model was 4.19. Compared with the actual school size, 23 schools experienced an increase in school size by the DIS-based model, 24 schools by the VAR-based model, and 27 schools by the RES-based model.

The main features of the solutions are summarized in Table 1. Clearly, the introduction of a random mechanism does improve the equality of educational opportunity compared with the pure proximity-based mechanism. As expected, the best equality measured by d_{PA} was obtained by the RES-based model. The average d_{PA} in the 31-dimentional space by the RES-based model was 0.45, which was only about 45.88% of that in a pure proximity-based mechanism, and which was also significantly lower (by about 95.33%) than that obtained by the VAR-based model. There were also significant decreases measured by the variance of expectations when compared with the DIS-based model. The variance of the DIS-based model was 2.23 times higher than that of the VAR-based model and 1.84 times higher than that of the RES-based model. Comparing the two models after introducing the random mechanism, there was a trade-off between the average PA-distance and the variance of expectations. From the VAR-based model to the RES-based model, the average d_{PA} declined by 4.67%, from 0.47 to 0.45, while the variance increased by 13.73%, from 10.79 to 12.27.

One shortcoming of introducing the random mechanism is a possible increase in travel distance. The average travel distance to school of all students was only 0.82 km using the DIS-based model. With the introduction of a lottery, this distance increased to 1.93 km using the

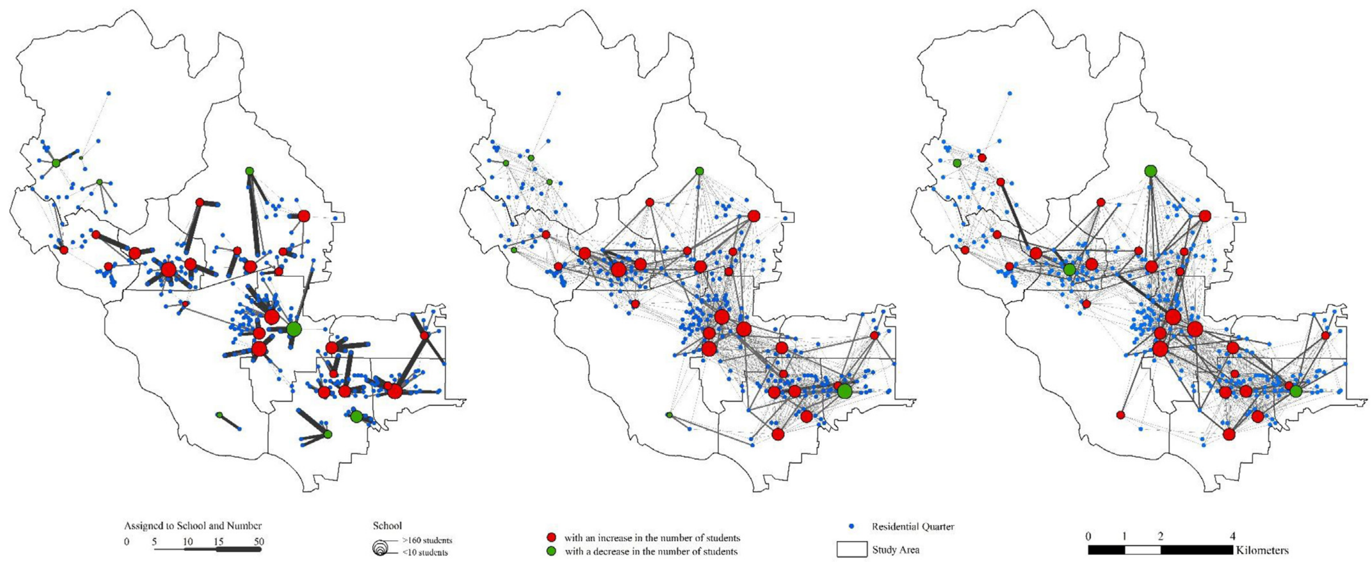

Fig. 2. The school assignment solutions provided by the DIS-based model (left), the VAR-based model (middle), and the RES-based model (right).

Table 1
A comparison of the major indicators between the three models.

	Absolute value			Relative value (DIS = 100)		
	DIS	VAR	RES	DIS	VAR	RES
Average d_{PA}	0.97	0.47	0.45	100	48.15	45.88
Variance of expectations	34.89	10.79	12.27	100	30.93	35.18
Average distance to school (km)	0.82	1.93	1.95	100	235.78	237.86

VAR-based model and 1.95 km using the RES-based model, i.e., about 1.36 and 1.38 times higher, respectively. However, both distances were still lower than the actual average distance of 4.3 km obtained from an official survey. This indicates that the costs of travelling this distance would be acceptable for the public. It implies that a random mechanism does significantly increase the travel distance, but it can be controlled within a tolerable limit ($D_{\max}$).

From the perspective of assigning more students to their closest school, the introducing of random mechanism has another shortcoming. Table 2 showed the results of the percentages of students sent to their closest school, the second closest school, etc. With the introducing of random mechanism, there was a sharp contrast of the percentages of closest assignment. In the DIS-based model, not all (due to the capacity constraints) but a majority of students were sent to their closest school. The percentage reached about 74.29%. This percentage decreased significantly by the VAR-based model or by the RES-based model. The majority of students was sent to their non-closest school, about 85.37% by the VAR-based model or about 74.47% by the RES-based model.

The assignments obtained using the RES-based model were significantly different from those obtained using the VAR-based model. Compared with the solution provided by the VAR-based model, the combinations of school places obtained by the RES-based model at most demand nodes had lower d_{PA} values and higher d_{EA} values, which were defined as the deviation of expectation of the average educational opportunity of the whole area (value of a) at the demand node. This is clearly shown in Fig. 3 where all demand nodes are plotted as circles using the values obtained from both models. The size of the demand node is represented by the radius of the circle. For the measurement by d_{PA}, 50.83% of demand nodes with 61.09% of students were located in the top-left side. This means that most demand nodes would be closer to the absolutely equal distribution of educational opportunity (n-dimension vector A). In contrast, for the index of d_{EA} the spatial pattern was quite different, with 54.79% of demand nodes with 53.55% of students in the bottom-right side of the graph. This indicates that for the RES-based model, not only were most demand nodes closer to point A, but also over half of the nodes were closer to the average expectation of educational opportunity.

For RES-based model, there exists a trade-off between spatial equality and the travel distance to school. As mentioned above, the absolute equality of A can be achieved by removing the distance threshold $D_{\max}$. The equity can be improved further by a relaxation of the maximum distance constrain. The $D_{\max}$ constraint of 5 km was used to solve the RES-based model. The average d_{PA} decreased by 20.87%, from 0.45 to 0.35, and the variation of educational quality decreased by 28.32%, from12.27 to 8.80. The improvement in the equality of educational opportunity resulted in a 59.63% increase in the average travel distance to school. This was still lower than the actual average distance travelled of 4.3 km.

4. Discussion and conclusions

China has a heavy historical burden of educational inequality, and therefore introducing a random mechanism into the current proximity-based system of school allocation has been proposed as an important educational reform measure. By focusing on configuring the optimal combination of school places at the demand nodes through a lottery, with the setting of a maximum school travel distance, this study

Table 2
The percentages of students assigned to their closest school or other non-closest schools (%).

Methods	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	13th	14th	Total
DIS	74.29	14.46	7.53	0.82	2.28	0.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	100.00
VAR	14.64	15.14	12.53	9.68	9.89	8.53	7.93	7.00	5.36	3.46	2.53	2.43	0.64	0.25	100.00
RES	15.53	14.10	12.50	9.39	9.35	10.39	7.89	6.89	5.75	2.96	2.53	2.03	0.43	0.25	100.00

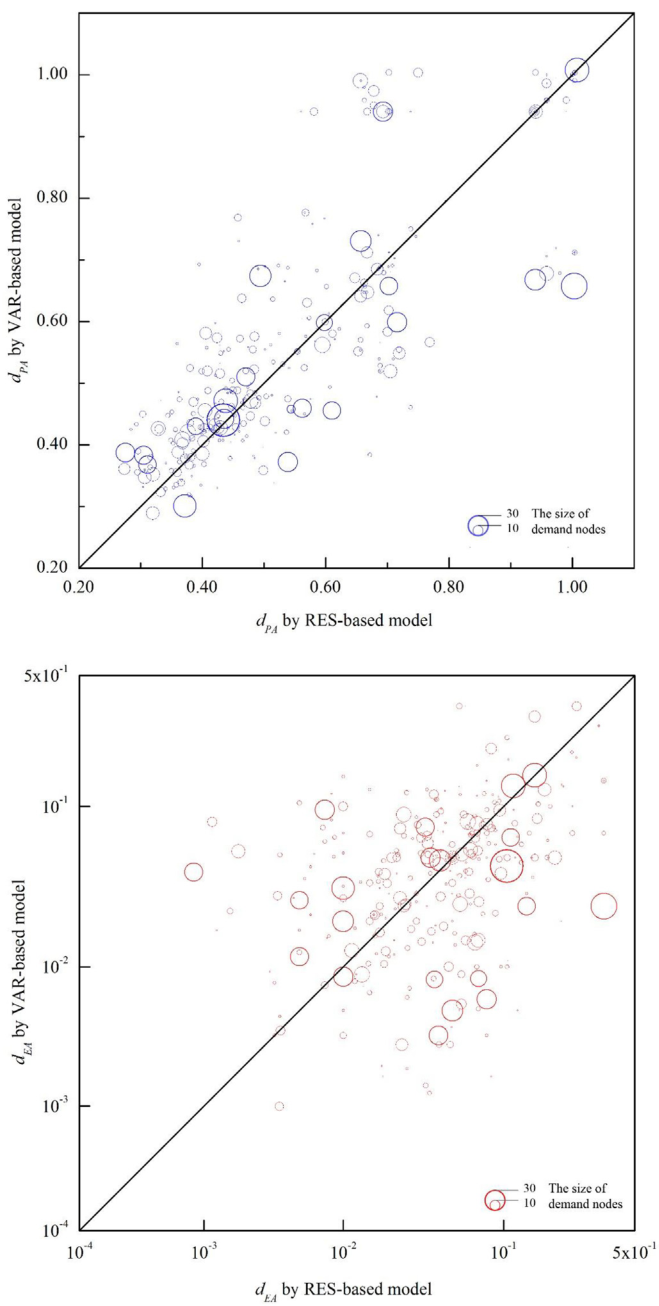

Fig. 3. Comparison of d_{PA} and d_{EA} by the VAR-based and RES-based models.

proposes a new model, the RES-based model, to ensure the maximum equality of educational opportunity. The possibility distribution was used to depict the full picture of educational opportunity, which resulted in the variance index being unsuitable for the new objective of spatial equality. In this study, the objective in terms of the maximum resemblance of all possibility distributions was converted into a problem of minimum total distances in a multi-dimensional space. The model was solved by the PSO method and illustrated by a case study.

The results of the case study indicated that introducing a lottery into a proximity-based school assignment system could significantly improve the spatial equality of educational opportunity. Compared with the pure proximity-based mechanism, the average PA-distance obtained by the RES-based model declined significantly, with an acceptable increase in the average travel distance to schools. Therefore, such random solutions might be adopted by China's centralized decision-making system, which faces serious educational inequality and has started experimental reforms of introducing random assignment in some cities. However, the ratios of assigning to the closest schools or the second closest schools were rather low, which might weaken the supporting ratio. Relaxing the constraint of the maximum travel distance, the spatial equality of educational opportunity could be significantly further improved.

In general, the RES-based model provides the optimal spatial scenario for spatial equality, and may lead to some further research. For example, as the case study indicated that the RES-based model would decease spatial efficiency such as travel distance and the closest assignment, the further research is needed to use the RES-based objective as part of a more realistic study with wider objectives in a multi-objective and multi-period location-allocation model. And as it conflicts with the existing interest in school quality capitalization, the implementation of this model will depend on political decisions. Thus, future studies need to consider the acceptable support ratio for the decision maker. In practice, it is of interest to develop open computational toolbox for users. In addition, the resemblance approach used in this paper may also be modified and applied to other optimization problems towards spatial equality.

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Teqi Dai is a lecturer in Beijing Normal University, mainly focuses on Transport geography and spatial optimization.

Cong Liao is a Ph.D. in Beijing Normal University, he researches on spatial optimization.

Shaoya Zhao is a Master Degree Candidate in the Beijing Normal University, her research field is about spatial analysis.