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Optimizing the spatial assignment of schools to reduce both inequality of educational opportunity and potential opposition rate through introducing random mechanism into proximity-based system

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ABSTRACT

A serious spatial inequality of educational opportunity was revealed worldwide, for wealthy families can access good schools by buying real estate with good school' enrollment quota. Although the existing studies had revealed that random-based school assignment can significantly improve equality of opportunity allocation, random mechanism was adopted only in few places. Two major resistances of introducing random mechanism exist: the possibility of increased commuting distance to schools and the effected relative beneficiaries. In order to make the random-based allocation more feasible, this study proposes a spatial optimization model to take these two factors into account into proximity-based school assignment system. The proposed multi-objective allocation model, with the constraint conditions of assigning students to 3 closest schools and school capacities, was developed in this study to minimize the spatial disparity of educational opportunity and the potential opposition rate of introducing random mechanism into proximity-based assignment system. The model will be solved by a heuristic algorithm and applied to a case study area of Shijingshan District, Beijing. The results showed that the proposed model could improve spatial equality of educational opportunity significantly, but along with a minor increase on commuting distance to schools. In addition, potential opponents of introducing random mechanism decrease as the weight of parameters related to opposition rate increases in the model, reducing nearly 10% in the best case. Therefore, the solutions provided by proposed model may encounter less resistance in a democratic voting system. However, the results also indicated that there would be some relative beneficiaries who may oppose introducing random mechanism into proximity-based school system even in the best case. This implies that, to achieve equal educational opportunity in the context of proximity-based school system, optimized allocation is needed along with a more even distribution of educational resources.

1. Introduction

In 2016, China's Ministry of Education issued an education guidance, in which a new lottery enrollment mechanism in the stage of compulsory education was proposed. The guidance states that residential neighborhoods in urban areas that are accessible to good schools' enrollment quota and suffers from fierce real estate prices competition in proximity-based school system may correspond to several schools' enrollment quota, rather than only one school. The school allocation for students living in those neighborhoods will be determined by random mechanism eventually. For the first time, random mechanism was attempted to replace the rule of home-to-school distance in the history of proximity-based school system. The policy of introducing random mechanism is consistent with the country's long-term education planning, in which the educational equality was heavily emphasized. The trial of lottery rule is an important strategy for improvement of social equity in China, as the key-ordinary school dichotomy used in the history has made prime educational resources gather to a small number of key schools. Wealthy families obtained prior opportunity of sending their children to attend these schools by buying real estate with key schools' enrollment quota, which lead to the inequality of educational opportunity in the proximity-based system [1–3]. Today, several cities in China, such as

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Beijing and Shanghai have adopted the new rule, breaking the link between the opportunity to attend good schools and family wealth. Providing the optimal solution to introduce random mechanism into proximity-based system is critical for effective decision making.

Although introducing random mechanism to promote equality of educational opportunity has many benefits, introducing random mechanism still faces several challenges. People oppose random-based assignment mechanism for many reasons. Among others, the negative consequences of increased commuting distance to schools and criticism from relative beneficiaries in existing allocation system are two major reasons [4]. In random mechanism, a student could be assigned to a farther school from home under the principle of lottery instead of home-to-school distance. Average commuting distance to schools was comparatively longer in random-based system than that in proximity-based system [5]. While optimal commuting distance to schools has been a central topic in existing optimization models of school planning [6], few prior studies of school assignment models with introducing random mechanism have incorporated the factor of increased commuting distance to schools, nor have they considered the potential resistance of introducing random mechanism.

The most common allocational system used in the world may be classified into choice-based and proximity-based system [7]. Despite the variation in details, the geographical link between family wealth and educational opportunity exist in both systems [8–10]. This study will be carried out under the proximity-based system. To make the assignment model of introducing random mechanism more feasible, this study proposed a multi-objective allocation model with the targets of minimizing spatial disparity of educational opportunity and potential opposition rate of introducing random mechanism, under the constraint conditions of assigning students to 3 closest schools and school capacities. The case study area of Shijingshan District, Beijing will be used to illustrate the method. The reminder of this article is organized as follows. The second section provides a literature review. The third section then formulates the problems and solution. The fourth section presents a case study, including a results and discussion. Finally, the last section summarizes the main findings.

2. Literature review

School assignment has widely been studied in the field of School District Planning Problem (SDPP) [11,12]. As commuting distance to schools is an important factor in school assignment, it is a classical geography problem of spatial optimization [13].

A large percent of spatial optimization models has been developed to assign students to schools. Distance or accessibility to schools was often used as an objective in optimization models to achieve the optimal efficiency of attending school. The closest assignment model aiming at minimizing commuting distance was often adopted [14–16]. To make the model more applicable, controlling the number of students with non-closest assignments was considered in the literature of school facility location planning, as assigning all students to their closest school was generally unrealistic under some constraints, such as school capacities [17,18]. Besides optimal distance, the models with the constraints of compactness and contiguity of school districts were developed [19,20]. These models make the results of school district calculated by the model more manageable. In addition, other factors, such as stability of school district over time, and expected utility of school choice, were also taken into account in existing models [21–23].

The school assignment models toward equal opportunity has been another focus of attention in SDPP. Educational opportunity provided by public primary or secondary schools is usually deemed as a public service provided by governments in modern educational system. Providing school allocation solution for equal opportunity is critical for an accepted school allocation system by the public. Optimization models involving equity issues in early studies focused on the equality of commuting distance to schools by adding the constraints of maximum commuting distance or the objective of minimizing the maximum distance in the models [24,25]. To achieve political equality of attending school, the objective of racial balance in schools was designed into optimization models [26,27]. The equality of educational opportunity has been seriously considered in recent decades [28]. There are often obvious differences in educational opportunities offered by different schools. Considering the relationship between socioeconomic states and accessibility to schools, a model to balance the socioeconomic compositions between schools along with minimizing total commuting distance was developed [29].

Random mechanism is regarded as an equal mechanism to break the geographical link between household wealth and educational opportunity in school allocation [30]. The ex-ante equal opportunity for each student achieved by random mechanism could reduce the severe spatial disparity of school allocation and reproduction of inequality occurred in some widely used allocation system in the world, as the students with higher socio-economic status are more likely to be accepted by good schools [31]. Random mechanism was introduced to allocate over-demanded places of schools for the students with same priority order to attend school in choice-based system. The case to replace the home-to-school distance rule with random mechanism was adopted in a few cases, and the equality of educational opportunity in these cases has been improved [32]. However, the lotteries were usually implemented after the prioritization of geographic proximity [33,34]. For optimization problem of introducing random mechanism into proximity-based enrollment system, an model of minimizing of expectations of educational opportunity between locations with demand for school places was developed to introduce random mechanism into proximity-based system, and results indicated that spatial equality of educational opportunity was encouraging [35].

However, the optimization model in existing studies of introducing random mechanism have not fully considered increased commuting distance to schools and potential resistances of relative beneficiaries in proximity-based system. The requirement of commuting distance to schools was generally simplified by adding the constraint condition of maximum commuting distance in existing models. However, the maximum commuting distance to schools was a relatively relaxed condition. Within maximum commuting distance, some residences may encounter up to 10 or more schools [36]. Most parents are usually unwilling to send their children to comparatively further schools. Closer commuting distance to schools was still preferred by most families [37]. The result of increased commuting distance could reduce parents' support rate of introducing random mechanism. In addition, introducing random mechanism into proximity-based system may encounter potential resistance from relative beneficiaries in proximity-based system, such as those groups benefiting from capitalization of schools' educational quality [38]. These relative beneficiaries would oppose introducing random-based allocation mechanism.

The consideration of increased commuting distance and potential resistance of relative beneficiaries will make the model of introducing random mechanism into proximity-based system more feasible. In this study, a multi-objective optimization model towards equal educational opportunity and declined potential opposition rate was proposed to introduce random mechanism into proximity-based system, under the constraint conditions of 3 closest schools and school capacities.

3. Formulation of the problem and its solution

3.1. Defining the planning problem

The planning problem is to find a solution for school assignment to minimize both spatial disparity of educational opportunity and potential resistance from relative beneficiaries by introducing random mechanism. The first task is to define the mathematical formulation of objective function, which requires defining equal opportunity and the potential resistance of introducing random mechanism.

Defining equal opportunity is rather complicated. Different philosophies and political schools have different standpoint on equal opportunity [39]. The spatial viewpoint of equity has been discussed variously [40–43]. In this study, we followed Tou's definition of spatial equity [44]: "all residents should be equally treated, wherever they live". Similarly, the spatial equality of educational opportunity has also been defined to "all children would be educated equally irrespective of their background or where they lived" [7]. Thus, the equal opportunity will be achieved if all school places are assigned to every student by equal possibility. We measured the disparity of educational opportunity between different locations.

When introducing random mechanism into proximity-based assignment system, a location with demand for school places might receive educational opportunities (school places) from several schools within a given maximum commuting distance [35]. Due to the limitation of uneven spatial distribution of schools, every location with demand for school places will face different number of schools within the range of maximum commuting distance. Spatial equality of educational opportunity should minimize the differences of educational opportunities between different locations. The allocation results of different school enrollment quotas for every location with demand for school places will affect the results of spatial equality. The educational opportunity available in every location will formulate a lottery pool, and lottery will be used to allocate the students in the location to a specific school.

The equal opportunity can be represented by minimizing the variance of education opportunities between locations with demand for school places. The variance of education opportunities can be calculated by Eq. (1).

$$F_1(\mathrm{x}) = \sum_{\mathrm{i=1}}^{\mathrm{m}} H_i \left(\sum_{j=1}^{n} \frac{Y_{ij} X_{ij}}{H_i} Q_j - a \right)^2 \tag{1}$$

where H_i represents the number of students in demand location i; Y_{ij} is a binary variable, which is equal to 1 only if demand location i obtains at least one school place from school j within maximum commuting distance and otherwise Y_{ij} is 0; X_{ij} is a positive integer, which represents the number of school places in school j assigned to location i; Q_j represents the educational opportunity of school j; a represents the average weighted educational opportunity of the whole area.

It is hard to define the potential resistance of introducing random mechanism. There are many reasons for some people to resist random mechanism, such as denying the link between the opportunity to attend good schools and family wealth associated with residential location, increased commuting distance and relative beneficiaries [4]. In this study we considered increased commuting distance and relative beneficiaries. A student who will receive a lower educational opportunity under random mechanism than proximity-based mechanism would be regarded as a potential relative beneficiary in existing assignment system.

Generally speaking, wealthy families who can afford to buy properties that comes with good school's enrollment quota are more likely to become relative beneficiaries in proximity-based assignment system [45,46]. Students or parents might oppose random mechanism when the assigned school quality after lottery is lower than the school quality in proximity-based assignment system. Therefore, in this study, whether a student i opposes introducing random mechanism, denoted by binary variable O_i, can be represented by the result of the potential school quality by random-based assignment minus the available educational quality by proximity-based assignment, as shown in Eq. (2).

$$O_i = S_i - S_p \tag{2}$$

where S_i and S_p are the scores of educational quality that a student can obtain in random mechanism and proximity-based mechanism. If O_i is less than 0, O_i is equal to 0, which means the student i may oppose random mechanism. Otherwise, it is 1. Then the number of opponents can be computed by the sum of O_i, as shown in Eq. (3). N means the total number of students.

$$F_2(\mathrm{x}) = \sum_{i=1}^{i=N} O_i \tag{3}$$

The increased commuting distance is considered as a constraint condition. In some previous works, the constraint condition of maximum commuting distance was used to reduce the negative effects of long commuting distance to schools. In most cases, 3 km or 5 km are common threshold to evaluate distance to schools [47]. To reduce the commuting distance to schools caused by random mechanism, this study limited the number of schools accessed by demand location. Referred to the number of preferences in some free choice systems [32], the number of schools is limited to 3, which are the closest schools within the maximum commuting distance in this study. The following constraint condition is added to the model.

$$Y_{ij} = 1, \quad \forall \ Y_{ij} = (1th, 2th, 3th \ closest \ school) \tag{4}$$

where Y_{ij} is a binary variable, which is equal to 1 only if demand node i obtains at least one school place from three closest schools j within maximum commuting distance and otherwise Y_{ij} is 0.

3.2. Model formulation

Based on the definition in Eqs. (1) and (3), the objective function of the CVAR-based model can be written as:

$$Minimize \ \{F_1(\mathrm{x}), \ \ F_2(\mathrm{x})\} \tag{5}$$

The goal is to simultaneously minimize $F_1(\mathrm{x})$ and $F_2(\mathrm{x})$, subject to constraint conditions in equation 10–17 listed in the appendix. To balance the relation between different objectives, the objective function can be converted into a single-criterion form through the linear combination of objectives with corresponding weights [48]. The Utopia point is used to achieve consistency of benchmarks in different goals. The Utopia point ($f_1(X)^*$, $f_2(X)^*$) is obtained by minimizing each of the objective functions under constraint conditions. Then an evaluation function is constructed according to the distance between the actual point and the Utopia points. The objective function can be transformed to the following form:

$$Minimize \ \ \sum_{k=1}^{q} w_k \left[\frac{f_k(X) - f_k(X^*)}{f_k(X^*)} \right] \tag{6}$$

Using the definitions in Eqs. (1) and (3), the objective of the model is written as follows:

$$Minimize \ \ \alpha * \left[\frac{\sum_{\mathrm{i=1}}^{\mathrm{m}} H_i \left(\sum_{j=1}^{n} \frac{Y_{ij} X_{ij}}{H_i} Q_j - a \right)^2 - f_1(X^*)}{f_1(X^*)} \right] + \beta * \left[\frac{\sum_{i=1}^{i=N} O_i - f_2(X^*)}{f_2(X^*)} \right] \tag{7}$$

The two parameters represent the importance of the objectives, and the sum of two parameters need to be 1. In particular, when the parameter β in CVAR-based model is set at 0, the function form of CVAR-based model can be transformed into the form of minimizing the variance of educational opportunities. Similar to many other integer quadratic planning in school assignment problem, the problem is hard to be solved in a given polynomial time. Hence, heuristic methods are used to deal with the problem. Particle swarm optimization algorithm was programmed and solved in C# language. In a few cases, the students need to be assigned to the schools that is not within the 3 closest schools to make the model solvable. In these cases, the constraint of maximum commuting distance is still required. Following a previous study [36], 3 km was set as the maximum commuting distance to schools.

3.3. Comparison scenarios

For comparison, the model of closest assignment was used to compare with the results of introducing random mechanism. In closest assignment model, the students are assigned to the closest school if the capacity permits (proximity-based model). The formulation of model was based on the modified *p*-median model, with the objective of minimizing the total commuting distance to schools [16]. The model is formulated as follows:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} N_i d_{ij} Y_{ij} \tag{8}$$

The details of the model are listed in equation 18–24 in the appendix.

4. Case study

4.1. Data preparation

The models were applied in a case study area of Shijingshan District, Beijing. The case study area consisted of 303 residential quarters in 2013, with an area of 84 square kilometers. The centroids of the residential quarters were used as demand locations, and the student population of 2801 in first grade of primary school was used as demand size. The location of 31 public primary schools in case study area were used as supply locations. Fig. 1 shows the overview of the study area.

The educational opportunity is an abstract concept which can be appropriate measure by some manifest variables. In this study, the educational opportunity is inferred appropriately by educational quality that a school can provided. There are several indicators to evaluate school quality [49,50], such as teacher quality, student performance, hardware conditions, etc. Among them, the quality of teachers is one widely used indicator in previous studies [51]. It was also used to measure educational opportunity in this study. The school quality is measured by the percentage of senior professional teachers among the teaching staff employed at each school.

Data of schools, students, and teachers was obtained from the Shijingshan District Education Yearbook and a survey conducted by the local government. The road network was used to obtain the commuting distance between demand locations and schools. The mean value of the schools' educational quality for the whole area was 0.58. The maximum educational quality score in the study area was 0.82, and the minimum was 0.23.

4.2. Results

4.2.1. Comparison of CVAR-based model and proximity-based model

The models were applied to the case study area, and the results are presented and compared in the following section. Fig. 2 shows the assignment of school places to demand locations by the two models. In CVAR-based model, the value of parameter α and β should first be set. Here, to illustrate the method, the α and β is set at 0.5, which represents that the two objectives are equally important. The values served as a base for comparison and discussion. In the CVAR-based model, the demand nodes can receive school places from several schools, rather than one deterministic school as in proximity-based system. In the CVAR-based model, the average number of optional schools for all demand nodes was 2.36, which means that many demand locations do not access three or more alternative schools within the maximum commuting distance.

The main features of the solutions are summarized in Table 1. Clearly, introducing random mechanism can improve the equality of educational opportunity significantly. In proximity-based assignment, the variance value of expectation of educational opportunity was 29.66. In the CVAR-based model, the value was 12.80. The variance of the CVAR-based models was 43.16% of that in proximity-based mechanism. The average commuting distance to schools of all students was 0.96 km

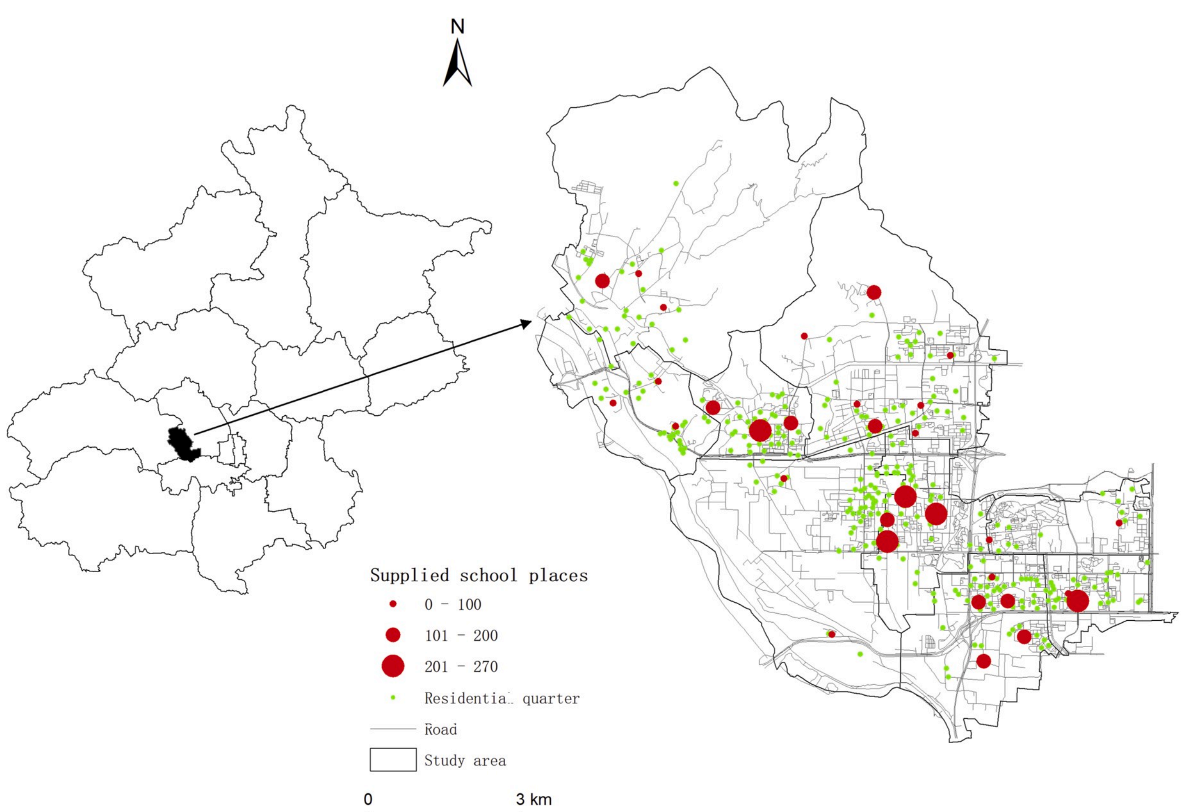

Fig. 1. Overview of the study area.

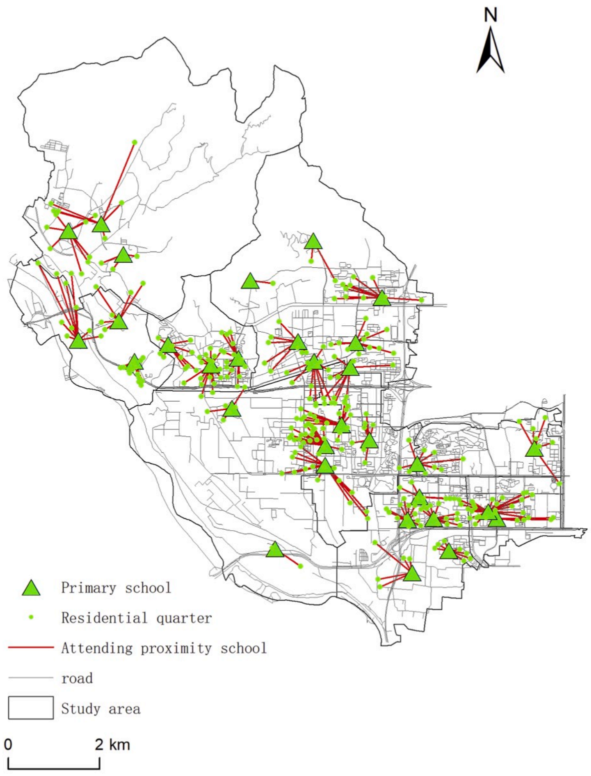

Fig. 2. The school assignment solutions provided by the proximity-based model (left) and the CVAR model (right).

Table 1
A comparison of the major results of the models.

	Absolute value		Relative value (Proximity = 100)	
	Proximity-based model	CVAR-based model	Proximity-based model	CVAR-based model
Variance of expectation	29.66	12.80	100	43.16
Average commuting distance	0.82	0.96	100	117
Opposition rate	/	24.78%	/	/

using CVAR-based model, which was slightly higher than 0.82 km by proximity-based mechanism. With the constraint condition of 3 closest schools, the average commuting distance to schools did not increase much. Note that all of distances were lower than the actual average distance of 4.3 km obtained from an official survey.

Table 2 illustrates the percentages of students assigned to their closest school, second and third closest schools. In proximity-based assignment, a majority of students were assigned to their closest school considering the capacity constraint of schools, about 85.49%. In CVAR-based model, the value was 42.27%. With the constraint condition of 3 optional schools, the school opportunity was allocated to more non-closest schools by random mechanism. The assignment results were more balanced in CVAR based model than the proximity-based model.

Table 2
The percentages of students assigned to their closest three schools.

Methods	1st	2nd	3rd	Total(%)
CVAR-based model	42.27	32.52	24.38	99.17
Proximity-based model	85.49	10.89	3.30	99.68

The possibility of being assigned to a non-closest school has increased. The student proportion of the nearest 3 schools were 42.27%, 32.52%, 24.38% in CVAR-based model, while that was 85.49%, 10.89% and 3.30% in proximity-based model. In CVAR-based model, the uncertainty caused by random mechanism is an important way to break the link between school opportunity and household wealth.

Sensitivity analysis of parameter α and. β

A sensitivity analysis is carried out by setting the α and β-value ranging from 0 to 1, with a step size of 0.2. Table 3 shows the results of sensitivity analysis for parameter α and β. The variance values of expectation were mainly distributed between 12 and 13. Compared with the proximity-based assignment, equality of educational opportunity can be improved significantly by CVAR-based model. With the constraint condition of 3 closest schools, the average commuting distance was all around 1 km. The increased commuting distance to school is slightly higher compared with the value in proximity-based system and the fluctuation of the average commuting distance is not obvious. This means that by controlling the number of assigned schools, the increase in school distance by introducing random mechanism can be effectively reduced.

One important observation from Table 3 is that the opposition rate decreased with the increase of parameter β. 30.74% of all students may oppose random mechanism when β was set at 0 in CVAR-based model, i.

Table 3
The sensitivity analysis of parameter.β

Parameter α	Parameter β	Variance of expectation	Average commuting distance (km)	Opposition rate (%)
1	0	12.29	0.96	30.74
0.8	0.2	20.22	0.98	27.56
0.6	0.4	12.66	1.00	24.78
0.4	0.6	12.87	1.01	24.49
0.2	0.8	12.88	0.95	23.13
0	1	12.81	0.98	21.64

e., without considering the objective of minimizing opposition rate. The minimum opposition rate of decreased average educational quality by CVAR-based model is 21.64%, about 606 students when β was set at 1.

Fig. 3 shows the assignment results of different parameters in CVAR-based model. School opportunities were allocated to more demand locations than proximity-based model. The boldest lines in the figures indicate that more than 13 students from a demand location were allocated to the corresponding schools. The thinnest lines represent fewer than five students were assigned to the corresponding schools. Although different parameters setting lead to different allocation results, the overall assignment pattern has some similarities. Despite the variation of parameters, some school places of schools would be assigned to designated demand points under the constraint condition of 3 closest schools. However, in random-based allocation mechanism, each student does not know which of the three schools he/she will be assigned to.

5. Discussion and conclusions

The problem of unequal educational opportunity is concerned not only in proximity-based assignment system but also in other school assignment systems. This study described a model with introducing random mechanism into proximity-based assignment system to assign students to schools, to minimize the spatial disparity of educational opportunity and potential resistance from relative beneficiaries. The motivation for the study comes from the obvious disparity between the positive effect of introducing random mechanism on educational equality but few cases of adopting random mechanism. The model is a bi-objective optimization model that can be solved by a heuristic algorithm. A case study area, Shijingshan district, Beijing, was applied and the results were analyzed. The results of the random-based model and the proximity-based models were compared and contrasted.

The study indicated that introducing random mechanism into proximity-based system could significantly improve the spatial equality of allocation of educational opportunity, with a slight increase of average commuting distance to schools. The commuting distance can be reduced by limiting the number of non-closest schools in random-based model, which can alleviate the public's concern on long commuting distance to schools of introducing random mechanism. Considering the remarkable decline in commuting distance to schools (about 44%), the loss of equality of educational opportunity (about 14%) is acceptable in the CVAR-based model.

Random mechanism could be opposed by some relative beneficiaries. These relative beneficiaries generally access schools with high educational quality by buying real estate with good schools' enrollment quota, which prone to produce residential segmentation based on family wealth [3,38]. In our case study, 30.74% of the total students would oppose the CVAR-based model. In assignment results by random mechanism, they may be assigned to schools with lower educational quality [1]. After adding the objective of opposition rate, the opposition rate of the total students dropped to 21.64% in CVAR-based model. This would be a great progress. However, there are still some students who may resist random mechanism. These students were generally the beneficiaries in proximity-based assignment system. Except for those who reject the decline in educational quality, those people who are risk averters may also resist adopting random mechanism. Therefore, improving equality of educational opportunity by introducing random

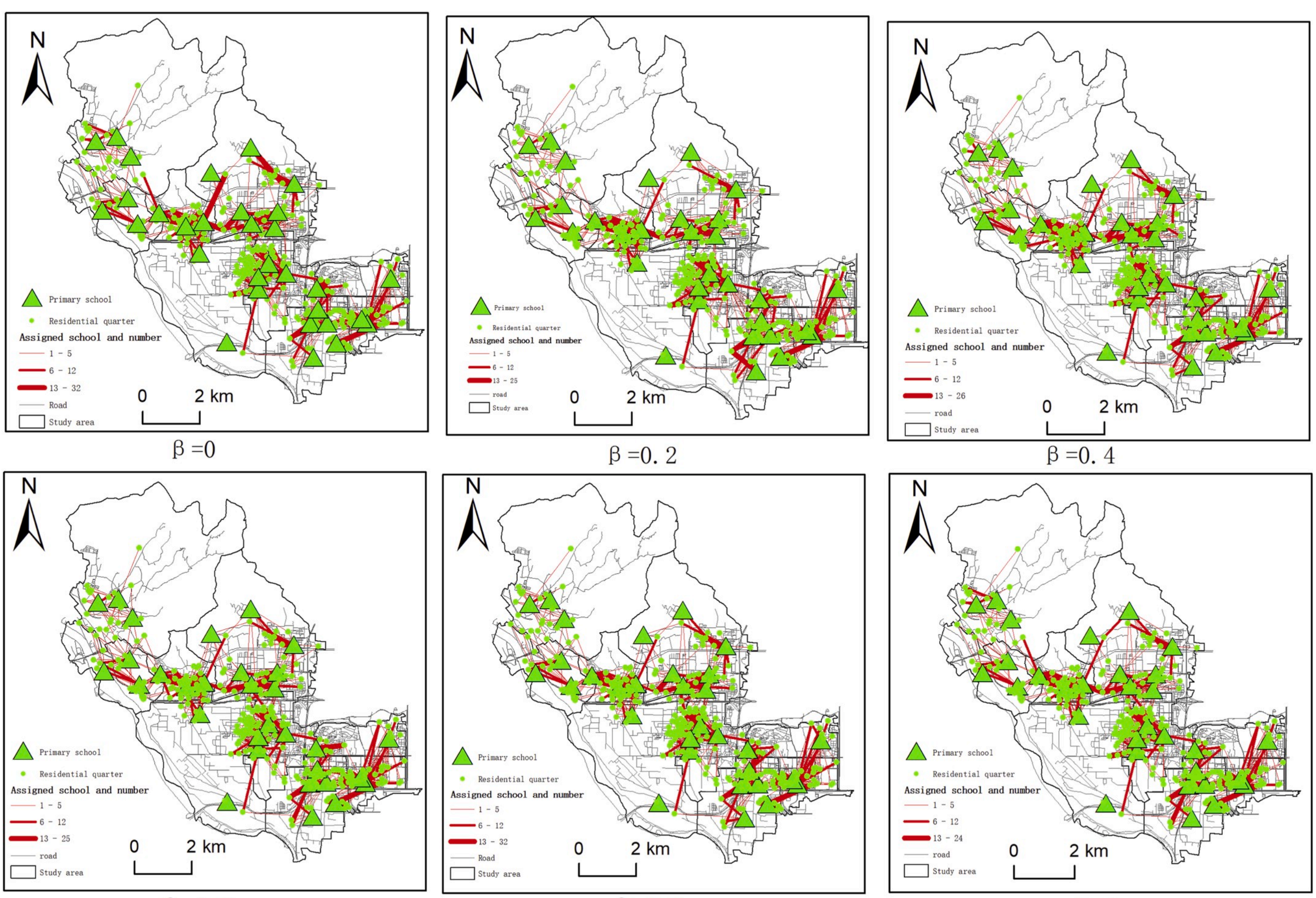

Fig. 3. Assignment results of different parameter setting of CVAR-based model.

mechanism into proximity-based enrollment system still face great challenges.

Some further research is needed to this study. The expression of educational opportunity might be more complex than the index used in this study. A multi-period location-allocation model could be considered for decision maker to adapt the changed student needs.

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Declaration of competing interest

None.

CRediT authorship contribution statement

Cong Liao: Conceptualization, Methodology, Writing - original draft. **Bronte Scheuer:** Investigation, Validation, Writing - review & editing. **Teqi Dai:** Resources, Formal analysis. **Yuan Tian:** Supervision, Project administration, Funding acquisition.

Appendix

The details of the models used in this study.

(1) The objective function and the constraints of the CVAR-based model

$$Minimize\ \{F_1(\mathrm{x}),\ \ F_2(\mathrm{x})\} \tag{9}$$

Subject to:

$$\sum_{j=1}^{n} Y_{ij}X_{ij} = H_i \tag{10}$$

$$\sum_{i=1}^{m} Y_{ij}X_{ij} > C_{min} \tag{11}$$

$$\sum_{i=1}^{m} Y_{ij}X_{ij} < C_{max} \tag{12}$$

$$d_{ij}*Y_{ij} \le D_{max} \tag{13}$$

$$0 \le X_{ij} \le H_i \tag{14}$$

$$X_{ij} \in Z^n \tag{15}$$

$$Y_{ij} \in \{0,1\}\ \ \forall i \in I,\ \forall j \in J \tag{16}$$

$$Y_{ij} = 1,\ \forall\ Y_{ij} = (1th, 2th, 3th\ closest\ school) \tag{17}$$

where d_{ij} represents the distance between node *i* and node *j*; $\mathrm{Y_{ij}}$ is a binary variable, which is equal to 1 only if demand node *i* obtains at least one school place from school *j* within maximum school distance and otherwise Y_{ij} is 0; X_{ij} is a positive integer, which represents the number of school places assigned to node *i* from school *j*.

(2) The objective function and constraints of modified *p*-median model

$$\text{Minimize}\quad \sum_{i=1}^{m}\sum_{j=1}^{n} N_i d_{ij} Y_{ij} \tag{18}$$

subject to

$$H_i = \sum_{j=1}^{n} X_{ij} \times Y_{ij} \tag{19}$$

$$C_{max} \ge \sum_{i=1}^{m} X_{ij} \times Y_{ij} \tag{20}$$

$$C_{min} \le \sum_{i=1}^{m} X_{ij} \times Y_{ij} \tag{21}$$

$$D_{max} \ge d_{ij}*Y_{ij} \tag{22}$$

$$X_{ij} \in Z^n \tag{23}$$

$$Y_{ij} \in \{0,1\}\ \ \forall i \in I,\ \forall j \in J \tag{24}$$

where equation (16) is the objective function. Constraint (17) ensures that all students are required to attend school. Constraints (18) and (19) are capacity constraints. Constraint (20) is the constraint of maximum commuting distance. Constraint (21) is the integrality constraint. Constraint (22) states that variable Y is binary.

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