



The school boundary stability problem over time

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Abstract

Declining funding for school construction and maintenance, surging student enrollment, school district mandated facilities requirements, and mandated reductions in class sizes have combined to create a classroom space deficit nationwide. In California alone, it is estimated that school capacity falls short of need by more than \$17 billion. The School District Planning Problem (SDPP) may be defined as how to manipulate school capacities and attendance boundaries into feasible alternatives to optimally manage school district resources over the long term. Researchers have worked for more than 30 years on one component of the SDPP involving models that minimize a student allocation cost function (usually travel distance) subject to school capacity and racial balance constraints. Such models have been classified by Schoepfle and Church as generic districting problems. While these models do generate optimal alternatives, their results are often infeasible for implementation and rarely consider long term effects. We present an alternative formulation in this paper called the School Boundary Stability Problem Over Time (SBS) which minimizes the impact of change on the students while including the traditional distance objective to encourage compact boundaries. By minimizing student change, the model has the effect of maximizing the stability of the district's attendance areas over time. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

In an era of decreasing funding for school construction, increasing enrollment, and mandated reductions in classroom loading factors, school district administrators are faced with many complex and sensitive decisions. Such decisions might involve manipulating school

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capacities, adjusting school attendance boundaries, changing the school calendar, adjusting school grade configurations, closing or adding school sites, adding magnet programs, and introducing controlled choice programs. Although there are many components of school planning and management, two of the most visible involve drawing boundaries (student assignments) and adding sites or altering school capacities. Such components form what can be termed the School District Planning Problem (SDPP), defined as the task wherein school district administrators must develop feasible alternatives to best manage school district resources over time. Although all components are important, research on model development has centered on optimizing the assignment of students or tracts to schools. Such models usually attempt to minimize the distance or cost of student assignment subject to constraints on school capacities and ethnic balance. This is certainly a major objective of school administrators, especially given the scale of the current fiscal deficit in school construction. In California alone, the estimated cost to cover the shortage of school construction required to house students at current loading standards is now \$17 billion [6].

The models that focus on the assignment of students to schools have been called Generic Districting Models [28]. For the past 30 years, researchers working on the SDPP have formulated models concentrating mainly on the objective of minimizing a cost or travel distance function. This is certainly a major objective of school administrators, especially given the scale of fiscal deficit in school construction. Unfortunately, these models have not gained widespread use among school district planners. Instead, the norm in school district facility planning has been “pin mapping.” Originally, planners stuck pins in a map of a school district, each pin representing the home location of a student. The planners would then wrap string around the pins to create attendance area boundaries and subsequently count the pins. If the number of students fit the school capacities, and the boundaries “looked good”, the planners could present the alternative to the public. As computer mapping developed, “electronic pin mapping” systems were created for school district planning. Hybrid systems have also been developed, merging mathematical programming models into GIS and/or Spatial Decision Support System (SDSS) applications. Since the mid-1980s, ONPASS (San Jose, CA), EDULOG (Missoula, MT), and Ecotran (Cleveland, OH), amongst others, have been marketing school district boundary analysis systems for the PC with mapping and reporting capabilities [22]. Similar systems have been introduced by Armstrong et al. [1] and Ferland and Guénette [11].

In all these systems, there are algorithms or heuristics that allow the user to create a set of school attendance boundaries such that the students travel the shortest combined distance to their schools. In practice, school district planners using commercial systems generally ignore these capabilities, instead using interactive adjustments from the existing boundary to find feasible alternatives. In so doing, they are looking for solutions that relocate fewer children versus solutions where everyone goes to their closest available school. While it is a major objective to reduce busing costs and walking distances, districts are more concerned with keeping the majority of students at their current school site. The objective is thus less to find the cheapest or most compact overall solution than to find a minimal impact solution that is also compact and low cost. Politically, then, spatial stability is more important than incremental improvements in cost savings.

If stability is important in space, it is equally important over time. When a school board is

forced to address the SDPP, it is taking a risk. Since boundary changes, schedule changes, and site decisions impact students, parents, traffic patterns, and/or neighborhoods, community harmony is at risk. Clearly, such unrest can result in loss of office. School boards thus have a major incentive to make a decision (when they are forced to do so) in favor of an alternative that will put off as long as possible the need to make this type of public decision again. In response to such conditions, the current paper introduces a new formulation for the districting component of the SDPP wherein the objective of school district stability is added. We refer to this as the School Boundary Stability Problem Over Time (SBS). As the name implies, the problem involves finding long-term stable solutions that keep change to a minimum.

2. Modeling the school boundary problem

For more than 30 years, researchers have been applying optimization techniques to the districting component of the SDPP. These efforts have been a function of available mathematical techniques and computational resources as well as the pressing needs of the school districts at the time. From the 1960s through the 1990s, there has been revolutionary progress in optimization techniques and computer power. In the 1960s, computers were slow and expensive. Researchers thus sought fast codes and simple implementations, especially for large and complex problems like the SDPP. At the same time, suburban growth and increased enrollment occurred as children of the “Baby Boom” (1946–1964) moved through school systems. These conditions made school capacity and student allocation major problems. A natural approach to the problem was to formulate the SDPP as a classic transportation problem, taking advantage of new higher speed codes for the low speed and expensive computers.

In the late 1960s and early 1970s, desegregation became a major issue, especially within urban school districts. This era also saw the development of faster and more affordable mainframe computers, allowing even moderate sized school districts timeshared access. These times also saw the development of the first readily available linear programming solvers. Researchers built on the original problem formulations, moving from a transportation structure to an expanded linear programming (LP) formulation with added desegregation constraints. The size and scope of the SDPP for large school districts was still beyond the capability of LP solvers of the period, especially given the cost of time on existing computers. Research continued on improved transportation formulations that could solve larger problems within existing time and cost constraints.

In the 1980s and early 1990s, desegregation became less important nationwide as housing discrimination diminished and the courts became more conservative. Revised problem formulations thus added appropriate objectives and constraints to the traditional formulation. Computing resources became cheaper and much more powerful as personal computers and workstations replaced earlier mainframes. It is only recently that there are available sufficiently fast tools in the form of computers, LP–IP (integer programming) solvers, and heuristic techniques to solve the traditional school districting problem (as formulated in the 1960s) for large school districts.

2.1. *Transportation approaches — student allocation as commodity flows*

The first application of optimization to school districting was introduced by Garrison [13]. He suggested the possibility of using the “transportation problem” to analyze the spatial structure of a school district. Yeates [34] was the first to apply the transportation problem as a policy analysis tool in analyzing the efficiency of school busing in a Wisconsin school district. As a result of his model, Yeates was able to demonstrate an 18% difference between the transport distances in the district under study and the optimal configuration. Optimality for Yeates was that allocation wherein students traveled the shortest distance subject to assignment and capacity constraints. By formulating the problem as a transportation problem, Yeates was able to take advantage of a specialized code to find an optimal solution using little computer time. This was important when the cost savings for improved school busing measured in the thousands of dollars and the costs of timeshared computer time could be measured in hundreds of dollars per run.

As desegregation became a driving issue for school district planners and administrators, Belford and Ratliff [2] were the first to apply a network flow model to a school districting problem with ethnic minority assignment constraints. This work expanded on the earlier LP-based efforts of Clarke and Surkis [9]. Again, it was concern over the size and complexity of the school districting problem that prompted the implementation of a network flow formulation. Other formulations of school districting as a network flow or classic transportation problem include those of Maxfield [24], Franklin and Koenigsburg [12], McDaniel [25], Jennergren and Obel [18], Woodall et al. [33], and Schoepfle and Church [29,30].

Importantly, there are drawbacks to the classical network formulation of the school districting problem. It thus requires fixed capacity and ethnic balancing levels. In school districting, planners do not usually wish to assign a specific number of students to a school. More desirable is some range under the site capacity of total enrollment and some range between upper and lower bounds of minority group assignment of the school site. Schoepfle and Church [29,30] address this problem by creating a hybrid heuristic that iteratively solves network models whose inputs and outputs (optimal school sizes and optimal race balances) complement one another. They also present a generalized network formulation with side constraints to address this problem. An additional drawback of the network flow formulation of the SDPP is that it treats students as commodities. While efficiency is a major objective in fleet delivery of cargo from warehouses to depots, it is not the most important aspect to reallocating children. The precedent for treating school children as deliverable commodities, while a breakthrough for Yeates [34], is less than ideal today.

3. The generic school boundary problem

Schoepfle and Church [29] introduced the Generic School Districting Problem (GdiP), referring to the class of school boundary problems allocating students to schools while minimizing a cost or distance function, subject to school capacity and racial balancing. Within this general class of formulations, there are three subclasses of models. The first includes linear

models wherein the decision variables are based on the number of students of a specific race assigning from a tract to a school. Tracts can be “split by race,” allowing students from different ethnic groups in the same tract to be assigned to different schools. A key advantage of this approach is that it allows for more flexible solutions to desegregation problems. An important negative by-product of the model is that in solving the school desegregation problem, difficult social and political considerations arise.

The second subclass includes linear models wherein the decision variables are based on the fraction of population in a tract assigned to a school. Tracts are “split by space”, i.e. split between schools into smaller racially homogenous pieces. This method reduces the undesirable social and political aspects of the first subclass, but creates negative political aspects in the more fragmented boundaries resulting from splits by space.

The third subclass includes integer models wherein the decision variables are based on the allocation of whole student populations in tracts to schools. This approach minimizes the negative social and political aspects of splitting tracts, but increases the computational difficulty of the models, especially for larger school districts. Formulation of the third subclass of the GdiP (first introduced by Liggett [23] is as follows:

Minimize the sum of travel costs or distances of the population of each tract i in their assignment to school j .

$$\min Z = \sum_i \sum_j c_{ij} A_i X_{ij} \quad (1)$$

Such that:

The sum of students at any school shall not exceed the school capacity.

$$\sum_i A_i X_{ij} \leq CAP_j \text{ for each } j \quad (2)$$

All students from each tract will be assigned to only one school.

$$\sum_j X_{ij} = 1 \text{ for each } i \quad (3)$$

Minority group enrollment at each school will not violate district-specified target percentages. These racial percentage constraints could also be formulated as minority population capacity bounds and, in some instances, may be formulated for multiple thresholds for more than one targeted minority population.

$$\sum_i (R_{\text{low}} A_i - M_i) X_{ij} \leq 0 \text{ for each } j \quad (4)$$

$$\sum_i (M_i - R_{\text{high}} A_i) X_{ij} \leq 0 \text{ for each } j \quad (5)$$

Integer restrictions on decision variables.

$$X_{ij} = 1, 0 \text{ for each } i, j \quad (6)$$

where i = index of neighborhoods, j = index of schools, A_i = number of students in tract i , M_i = number of minority students in tract i , CAP_j = capacity of school j , c_{ij} = transport cost or distance from tract i to school j , R_{low} = fractional lower bound of minority enrollment, R_{high} = fractional upper bound of minority enrollment and $X_{ij} = 1$ if tract i assigned to school j , 0 otherwise.

Formulations that expanded or improved upon mathematical programming models of school districting include those of Ploughman et al. [27], Heckman and Taylor [15], Trifon and Livnat [32], Holloway et al. [17], McKeown and Workman [26], Knutson et al. [19], Bruno and Anderson [5], Bovet [3], Henig and Gerchak [16], and Greenleaf and Harrison [14]. Others have incorporated additional objectives into the GdiP creating multiobjective or goal programming formulations, e.g. Sutcliffe et al. [31] and Brown [4] who sought equalized distribution by sex and reading ability, and Diamond and Wright [10] and Church and Murray [7] who optimized for compact attendance areas, minimal hazards, minimal dislocation, and equivalent sized schools.

In a related development, school choice has been proposed as an alternative to traditional districting. In 1993, Church and Schoepfle [8] thus developed a set of formulations to optimize the allocation of students in a school district using controlled choice based on the Cambridge, MA desegregation plan. These formulations maximize the sum of the preferences of students allocated to their 1st, 2nd, and 3rd choice of schools, while constraining for racial balance and capacity at each site. It remains to be seen whether choice in public schools will come into common use in American school districts. At present, there are political and ideological barriers to school choice and the current demographic and financial capacity crisis has put such reform efforts on the “back burner”.

4. The school boundary stability problem over time

In order to develop a formulation of the SDPP that can be used by school districts in spatial decision support systems, we must pay special attention to the feasibility of approval and implementation of alternatives generated by the model. While cost savings and compact attendance areas are important, it is arguably more important to minimize the impact of change (maximize the stability) of the students in their schools. It is the changes in student assignment that directly impact the public. Indeed, many parents will be disturbed if their children are moved from their current school, even if such a move reduces travel distance. The impact of change in schools on the students themselves is also significant in terms of disruption. It is equally important that, at the end of the decision process, the chosen solution delays the need for another redistricting for as long as possible.

It is noteworthy that minimizing disruption has been used in objective functions by Diamond and Wright [10] and Church and Murray [7] while constraining over time using projected enrollment has been used in various models for opening and closing school sites [14,16,27,32]. Our formulation departs from previous work in that it combines these considerations while placing less emphasis on the cost function as found in the classic problem.

This is an important distinction as the attempt here is to push the modeling into identifying politically feasible solutions.

The SBS differs from the GdiP in that, rather than only minimizing a cost function, it minimizes both a cost function and the number of students moved from their current locations. This creates an objective of maximizing spatial stability for the students within their neighborhoods. The cost function continues to encourage compact boundaries for school attendance areas while being concerned with moving the current population. This allows for, and encourages, the movement of vacant lots between school sites for purposes of future development. Politically, it is much easier to move children who are not yet in one's schools.

Minimizing the movement of current school student assignments is constrained by the ability of each school's capacity to hold current and future enrollments. In order to use this model, a school district must be able to project enrollment by neighborhood. To do so, school districts often use a modified cohort survival method [21]. This involves a three-step process whereby students are advanced one grade. A cohort survival coefficient is then applied to students by grade and attendance area to estimate birth rates, net migration, dropouts, and retentions. The third step adds students generated by new home construction and multiplies by appropriate dwelling yield factors. For a solution to be feasible, it must satisfy the capacity constraints of each school site for the base year and for all projected years' enrollments. The formulation allows districts to use different capacities at each site over time in order to simulate alternative schedules, or to add or move portable buildings. Since most school districts do not have accurate projections of minority group distributions over time, we constrain ethnic balance only during the base year. This also reduces the complexity of the problem.

One potential drawback of the proposed formulation is that students could be moved in the base year to preempt future over-enrollment conditions. While moving students before they need to be moved is a concern, this is likely to occur only when there are significant long-term shifts in districtwide cohorts. In such cases, it might be wise to reduce the number of years of the timed capacity constraint. When the boundary shifts are driven by ethnic imbalances in base-year populations or by new residential construction, unnecessary base year moves are less likely. This is the case because desegregation moves are only targeted for the base year, and residential growth moves will emphasize the movement of vacant tracts. The number of unnecessary preemptive moves should be far less, however, than in the GdiP model where many unnecessary moves are generated solely to reduce transportation distances.

Our first formulation of the SBS involves adaption of the integer formulation of the GdiP into a multiobjective problem that includes both the traditional cost or distance objective and a student stability objective. The cost and stability objectives are weighted (with W_1 and W_2 , respectively) to allow tradeoffs between the two. The stability objective is achieved in the assignment model by adding an E_{ij} term that represents whether tract i is currently assigned to school j . This formulation is extended into time by including both future student populations and school capacities. The A_i term representing the student population of tract i is thus transformed into A_{it} which represents the student population of tract i in year t . The term A_{i1} represents the tract i student population in the base year (year 1). School capacity is also extended into time so that the decision maker has the ability to adjust future school capacities. This allows the model user to develop a trial boundary solution, to budget future school capacity adjustments, and perhaps to avoid some boundary changes altogether.

5. Formulation of the school boundary stability problem over time

The proposed mathematical formulation (beginning with the objective function) of the SBS is as follows:

Minimize the weighted sum of travel costs/distances of the population of tract i to school j (first term), and the number of students assigned to schools other than their current one (second term).

$$\min Z = W_1 \sum_i \sum_j c_{ij} X_{ij} A_{it_1} + W_2 \sum_i \sum_j E_{ij} X_{ij} A_{it_1} \quad (7)$$

Such that:

The number of students at any school shall not exceed the school's capacity in any year. Adding a time subscript to capacity allows the administrator to add, subtract, or move capacities between schools.

$$\sum_i A_{it} X_{ij} \leq CAP_{jt} \text{ for each } j, t \quad (8)$$

All students from each tract will be assigned to only one school.

$$\sum_j X_{ij} = 1 \text{ for each } i \quad (9)$$

Minority group enrollment at each school in the base year will not violate district-specified target percentages. These constraints could also be formulated as minority population capacity bounds and, in some instances, may be formulated for multiple thresholds for more than one targeted minority population.

$$\sum_i (R_{\text{low}} A_{it_1} - M_i) X_{ij} \leq 0 \text{ for each } j \quad (10)$$

$$\sum_i (M_i - R_{\text{high}} A_{it_1}) X_{ij} \leq 0 \text{ for each } j \quad (11)$$

Integer restrictions on decision variables.

$$X_{ij} = 1, 0 \text{ for each } i, j \quad (12)$$

where i = index of neighborhoods, j = index of schools, t = index of years, A_{it} = number of students in tract i in year t , A_{it_1} = number of students in tract i in year 1 (the base year), M_i = number of minority students in tract i in the base year (year 1), CAP_{jt} = capacity of school j in year t , c_{ij} = transport cost or distance from tract i to school j , R_{low} = fractional lower bound on minority enrollment, R_{high} = fractional upper bound on minority enrollment, X_{ij} = decision variable indicating assignment of tract i to school j (1 if assigned, 0 if not) and E_{ij} = decision variable indicating that tract i is currently not assigned to school j (1 if not currently assigned, 0 if currently assigned).

The proposed SBS model of (7)–(12) can also be cast as a problem of reassignment. Although at first this may seem less than intuitive, it is mathematically equivalent to the SBS model while being more compact. To create this equivalent model, it is necessary to recast the decision variables X_{ij} . Thus, X_{ij} now indicates whether tract i is reassigned from its currently assigned school to another school j . The advantage here is that rather than assigning from “scratch”, the district is reassigned from a base boundary. This also reduces the number of decision variables from $i \times j$ to $i \times (j - 1)$. In terms of the objective function, this changes the travel cost/distance from total cost or distance of travel in (1) to the decrease (or increase) in travel cost or distance over that of the base boundary in (7). Constraints (14), (16), and (17) given below, also reflect the change in the decision variables. The term N_j refers to the set of tracts currently assigned to school j . The population in the reassigned solution is represented by the population of the base-year school attendance area in year t (A_{it}) plus the net change in population due to the shift of boundaries in year t . This net change is the sum of the populations of the tracts moved into school j ($\sum_{i \notin N_j} A_{it} X_{ij}$) minus the sum of the populations of tracts reassigned from school j to some other school k ($\sum_{i \in N_j} \sum_{j \neq k} A_{it} X_{ik}$).

6. Reformulation of the school boundary stability problem over time

The proposed mathematical formulation of the SBS recast as a reassignment problem is given as (13)–(18), below:

Minimize the weighted sum of the travel costs/distances of the population moved \times costs of the moves (first term) and the population moved (second term).

$$\min Z = W_1 \sum_i \sum_j c_{ij} X_{ij} A_{it_1} + W_2 \sum_i \sum_j X_{ij} A_{it_1} \quad (13)$$

such that:

No school site may exceed enrollment capacity in any given year.

$$-\sum_{i \in N_j} \sum_{k \neq j} A_{it} X_{ik} + \sum_{i \notin N_j} A_{it} X_{ij} \leq CAP_{jt} - \sum_{i \in N_j} A_{it} \text{ for each } j, t \quad (14)$$

No neighborhood may be assigned to more than one school.

$$\sum_{j \notin K_i} X_{ij} \leq 1 \text{ for each } i \quad (15)$$

No school may have a minority enrollment below a lower bound in the base year.

$$-\sum_{i \in N_j} \sum_{k \neq j} ((R_{\text{low}} * A_{it_1}) - M_i) X_{ik} + \sum_{i \in N_j} (M_i - (R_{\text{low}} * A_{it_1})) X_{ij} \geq R_{\text{low}} \sum_{i \in N_j} A_{it_1} - \sum_{i \in N_j} M_i \quad (16)$$

for each j .

$$-\sum_{i \in N_j} \sum_{k \neq j} ((R_{\text{high}} * A_{it_1}) - M_i) X_{ij} + \sum_{i \notin N_j} (M_i - (R_{\text{high}} * A_{it_1})) X_{ij} \leq R_{\text{high}} \sum_{i \in N_j} A_{it_1} - \sum_{i \in N_j} M_i \quad (17)$$

for each j

Neighborhoods cannot not be split and moves shall be positive (nonnegative).

$$X_{ij} = 1, 0 \text{ for each } i, j \neq K_i \quad (18)$$

where i = index of planning areas, j = index of school sites, t = index of time (in years), X_{ij} = decision variable indicating that planning area i is reassigned to school j (1 is reassigned, 0 is not), N_j = set of planning areas currently assigned to school j , A_{it} = population of planning area i in year t , A_{it_1} = number of students in tract i in year 1 (the base year), M_i = minority population of planning area i in the base year, c_{ij} = cost or distance of travel to change the assignment of planning area i from the current school site to school site j , $K_i = \{j | \text{tracts } i \text{ currently assigned to school } j\}$, CAP_{jt} = maximum capacity of school j in year t , R_{low} = fractional lower bound on minority enrollment at any school, R_{high} = fractional higher bound on minority enrollment at any school, W_1 = weight on the cost/distance of a change in assignment and W_2 = weight on the population of a change in assignment

It is important to note that a change (represented by X_{ij}) that is not politically viable may be eliminated by removing the variable from the problem formulation. In fact, it is possible to reduce the problem size by first removing all possible reassignment variables that are not politically acceptable. In the next section, we present an application of the proposed model to a school district containing 250 tracts and 9 schools.

7. An example

The following example shows the impact of maximizing stability of students in a school district on the feasibility of a solution to the SDPP. We use data from 1992 enrollments in the Santa Barbara (CA) Elementary School District [20]. This district has 9 school sites and 250 neighborhoods, and, at the time, was at or near capacity at several sites. One school site (school #3) had extra capacity. The district had desegregation problems with the majority (Hispanic) population, concentrated on the east side of the district, and the minority (Non-Hispanic Caucasian) population concentrated to the northwest. The district minimum and maximum minority thresholds were set at 10% and 55%, respectively. School #3 was under the limit for minority enrollment and thus required the reallocation of students in the base year. We used the district's 5-year enrollment projection which showed that demographics were shifting spatially and that schools 4, 5, 6, and 7 would be over enrolled in the future (see Table 1 and Fig. 1). Elements in the table that involve a violation of capacity in the projections are highlighted with shaded cells and bold text. In the current example with the 1992 base boundary, school #5 is projected to be over-enrolled in years 1993–1997, school #7 in years 1996 and 1997, and schools #4 and #6 in 1997 only.

A custom code (SBSMPS) was developed to set up an SBS problem using the general MPS format, a widely accepted standard for defining LP and LP/IP problems. This special problem

Table 1
1992 Base assignment enrollments

School	Capacity	% Minority	1992	1993	1994	1995	1996	1997
1	698	35.6	627	641	653	663	651	654
2	592	18.8	573	562	581	580	576	569
3	1018	6.0	737	744	733	702	707	717
4	726	24.5	706	699	702	721	718	750
5	606	12.6	586	612	634	645	647	623
6	619	40.3	586	604	602	605	605	620
7	686	49.2	591	624	657	679	714	739
8	593	50.9	550	548	539	541	520	526
9	527	29.6	496	511	526	522	531	535
Σ	6065	28.9	5450	5545	5627	5658	5668	5733
Average Distance from Home to School: 1.04 Miles						No Moves — Base Case		

file was then read into an LP/IP solver program. The SBSMPS program prompted for a set of parameters including the weight on distance change, the weight on population change, the upper and lower ethnic enrollment bounds, and the number of years of enrollment projections. The data required to generate the MPS file included student enrollments (by neighborhood by year), school capacities, ethnic populations (by neighborhood for the base year), the baseline assignment of neighborhoods to schools (the attendance area boundaries), and a districtwide transportation network (link distances). To generate optimal solutions, we used the CPLEXTM Mixed Integer Optimizer (Version 5.0) on a Dell Optiplex GXPro. We ran four alternatives:

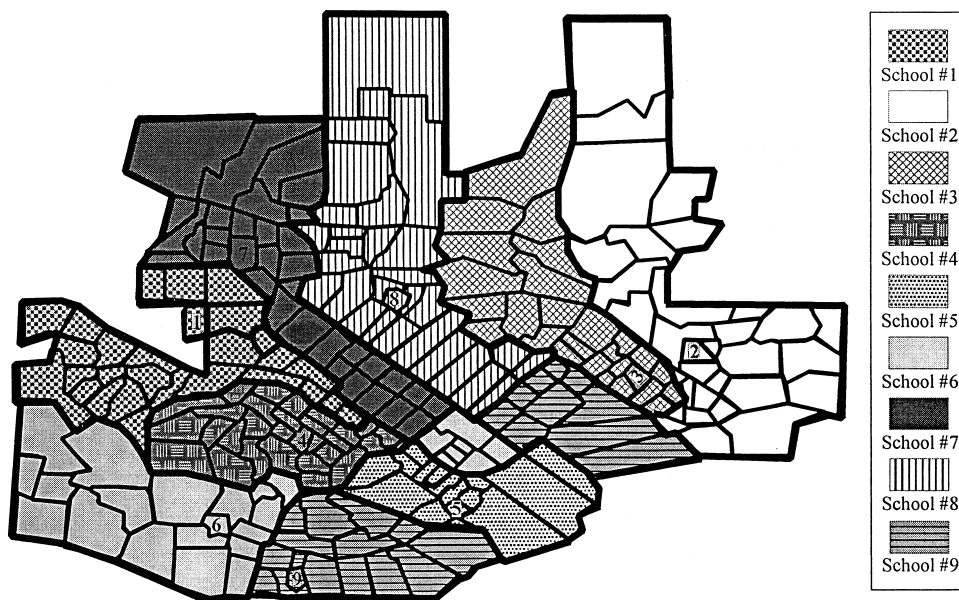


Fig. 1. 1992 Base attendance boundaries.

1. A standard GdiP optimization minimizing distance traveled by students from home to school subject to capacity and ethnic balance constraints (no capacity constraints in subsequent time periods).
2. The same GdiP optimization with the addition of capacity constraints over the next five years.
3. An SBS optimization maximizing student stability (minimizing student moves) in the base year subject to ethnic balance constraints and capacity constraints over time.
4. A multiobjective SBS optimization with 50% weight on minimizing student distance traveled and 50% weight on maximizing student stability in the base year again subject to ethnic balance constraints and capacity constraints over time.

The first alternative showed marked improvement in average distance traveled, but required redistricting the following year due to capacity violations (see Table 2). This scenario represents the classic districting problem where enrollment constraints over time are not included. In the current example, the district is within capacity and minority constraints in the base year (1992) as enrollment has flowed into school #3 bringing its minority enrollment up to 10%. Notice that this optimal solution creates instability rather than resolving it. Clearly, capacity constraints on projected enrollments are advised. The optimal solution still moves close to half the students in the district and would over-enroll a third of the schools the following year. The solution would also result in more fragmented and uneven attendance area boundaries (see Fig. 2) than in the 1992 Base Boundary (see Fig. 1).

The second alternative adds capacity constraints over time to the established GdiP model. Under this scenario, there was some improvement in average distance traveled with no capacity violations over the 5-year planning period (see Table 3). Notice the enrollment figures in bold typeface representing the projected school enrollments at 100% of site capacity (school #4–1992, school #2–1994, school #8–1995, and school #3–1996). These are of concern given the uncertainty of enrollment projections. The school planner may choose to reduce school capacities in order to create a “safety margin” for potential under-projections. The optimal solution with our full capacity constraint set would move more than half the students in the

Table 2
GdiP: minimizing distance in the base year

School	Capacity	% Minority	1992	1993	1994	1995	1996	1997
1	698	52.0	398	407	419	415	419	425
2	592	13.1	589	603	612	603	603	618
3	1018	10.0	1018	1007	1019	1004	1003	982
4	726	19.7	726	720	724	737	728	752
5	606	11.1	604	646	672	693	704	706
6	619	54.1	436	442	430	427	418	410
7	686	52.3	562	582	609	641	658	686
8	593	44.6	592	605	612	618	609	616
9	527	35.2	525	533	530	520	526	538
Σ	6065	28.9	5450	5545	5627	5658	5668	5733
Average Distance from Home to School: 0.85					Number of Students Moved: 2285			

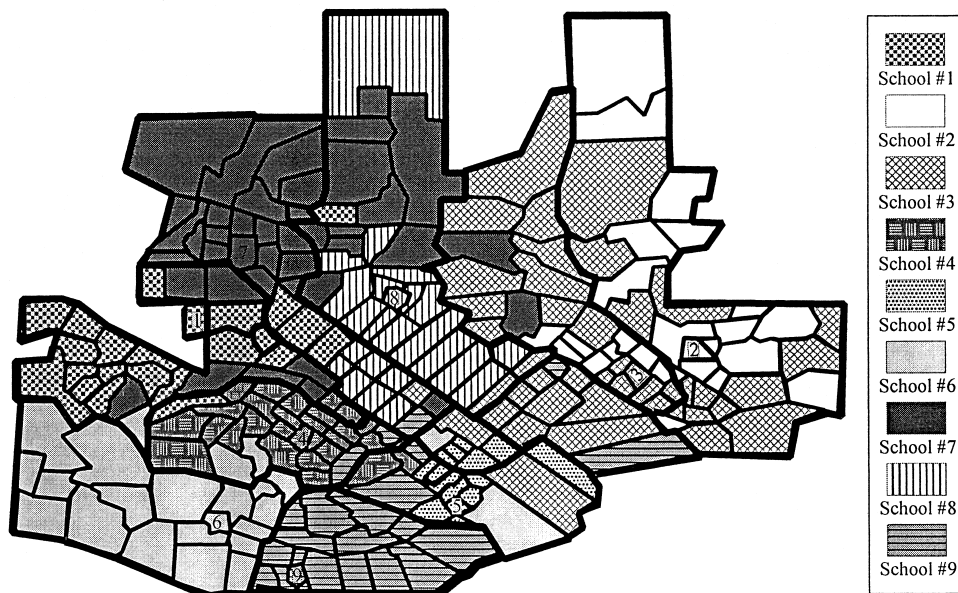


Fig. 2. GdiP — minimizing distance in the base year only.

district and result in even more fragmented and uneven attendance area boundaries than under the first alternative (see Figs. 2 and 3).

The third alternative is based solely on maximizing stability (minimizing student moves). The resulting solution, unlike the first and second alternatives, showed no improvement in average distance traveled over the base boundary. But, like the second alternative, this solution realized no capacity violations over the 5-year planning period (see Table 4). Further, it moved less than 2% of the students in the district and resulted in less fragmented and uneven attendance area boundaries than under the distance-oriented alternatives (see Figs. 2, 3, and 4). While

Table 3
GdiP: minimizing distance traveled over time

School	Capacity	% Minority	1992	1993	1994	1995	1996	1997
1	698	48.0	552	580	602	621	628	647
2	592	12.4	587	581	592	571	566	569
3	1018	10.3	987	1000	1009	1008	1018	1009
4	726	20.1	726	713	706	718	704	720
5	606	11.6	545	555	554	550	535	521
6	619	47.3	512	536	539	552	552	562
7	686	50.0	492	504	531	548	568	593
8	593	44.9	577	586	591	593	586	590
9	527	37.9	472	490	503	497	511	522
Σ	6065	28.9	5450	5545	5627	5658	5668	5733
Average Distance from Home to School: 0.91					Number of Students Moved: 2174			

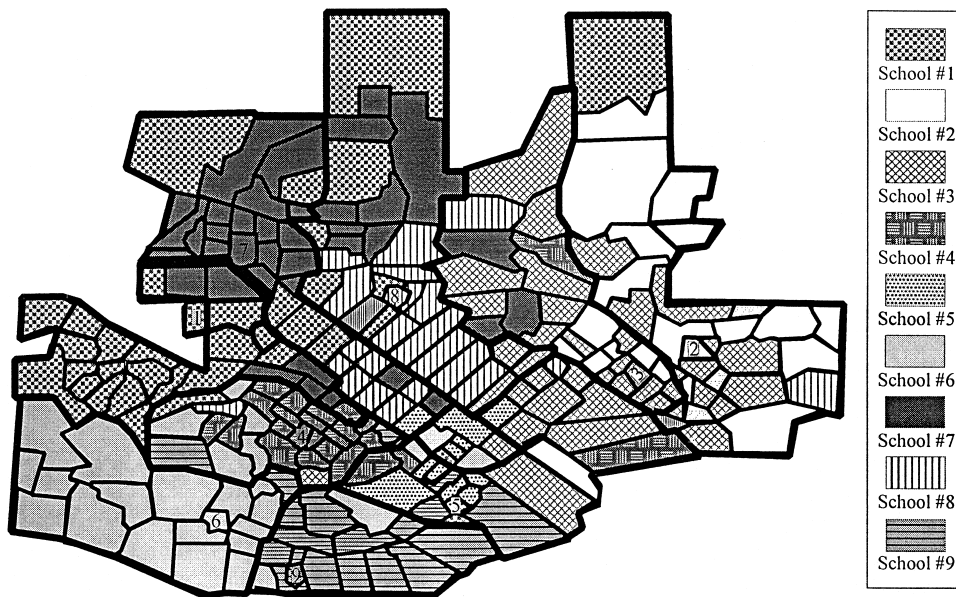


Fig. 3. Minimizing distance traveled over time.

possible to move even fewer students (20) if there was no constraint on future capacity, the difference (43) should be preferable to having to redistrict in each of the following 5 years.

As noted earlier, the final alternative involves a multiobjective SBS with 50% weights on both minimal distance and minimal student movement. It generated minor improvement in average distance traveled with no capacity violations over the five-year planning period (see Table 5). The optimal solution moved approximately 5% of the students in the district, but resulted in more compact and less fragmented and uneven attendance area boundaries than under the third alternative (see Figs. 4 and 5).

Table 4
Maximizing student stability

School	Capacity	% Minority	1992	1993	1994	1995	1996	1997
1	698	35.8	637	657	674	689	683	688
2	592	18.8	573	563	581	580	576	569
3	1018	10.3	779	797	795	771	783	804
4	726	23.9	698	687	689	703	697	724
5	606	11.3	557	578	597	606	600	573
6	619	40.3	586	604	602	605	605	619
7	686	47.3	567	593	618	637	666	686
8	593	50.1	561	560	552	554	537	547
9	527	29.3	492	507	519	513	521	523
Σ	6065	28.8	5450	5545	5627	5658	5668	5733
Average Distance from Home to School: 1.08					Number of Students Moved: 63			

Table 5

Multiobjective solution with equal weights on reassignments: 50% minimizing student moves/50% minimizing distance

School	Capacity	% Minority	1992	1993	1994	1995	1996	1997
1	698	35.7	630	647	660	673	662	668
2	592	16.5	557	551	571	570	566	559
3	1018	10.1	1008	1118	1006	978	991	988
4	726	23.9	698	687	689	703	697	724
5	606	10.5	553	578	597	606	601	580
6	619	40.3	586	604	602	605	605	618
7	686	53.4	499	532	576	601	638	663
8	593	48.0	590	592	586	593	574	588
9	527	43.5	329	337	342	332	338	350
Σ	6065	28.9	5450	5545	5627	5658	5668	5733

Average Distance from Home to School: 0.97 Number of Students Moved: 317

8. Summary of solutions

It is easy to see that the solution generated by the classical GdiP model is very disruptive, as many students are reassigned in order to minimize average distance. Further, such a solution does not age well, as many capacity constraints are violated in subsequent years. This clearly suggests the need for multi-year assignment models. Unfortunately, in a multi-year GdiP, many moves are required (2725) in order to maintain capacity and ethnic balance constraints and, at

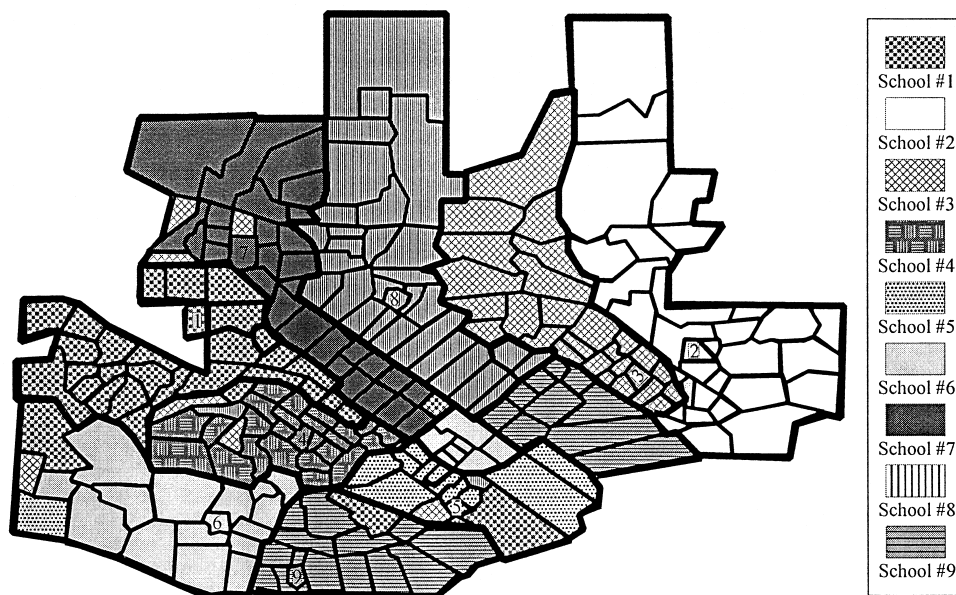


Fig. 4. Maximizing student stability.

the same time, minimize distance. Again, a solution involving wholesale reassignment would be politically unacceptable.

The third model alternative (SBS) addresses this issue of reassignment. Its solution moves only 63 students in order to remain feasible more than five years. Such a solution would be both politically expedient and publically acceptable. The final scenario demonstrates that reasonable alternatives exist in the tradeoff between minimizing distance and maximizing stability. Thus, with some sacrifice in stability, one may devise more compact boundaries that are likely to be even more publically acceptable. Such results clearly establish the value of the SBS model vs earlier GdiP formulations.

9. Conclusions

The problem of drawing boundaries for school assignments is often accomplished in a politically-charged atmosphere. We have presented results showing that past models may lead to a variety of problems. In particular, while making compact efficient school assignments, earlier models reassign large percentages of students to other schools. Further, such solutions do not “age” well. To counter this, we have proposed the SBS model. It generates school assignment patterns that are both stable over time and minimally disruptive in terms of boundaries. Such a model is thus likely to encourage the wider use of optimization techniques in school district planning practice.

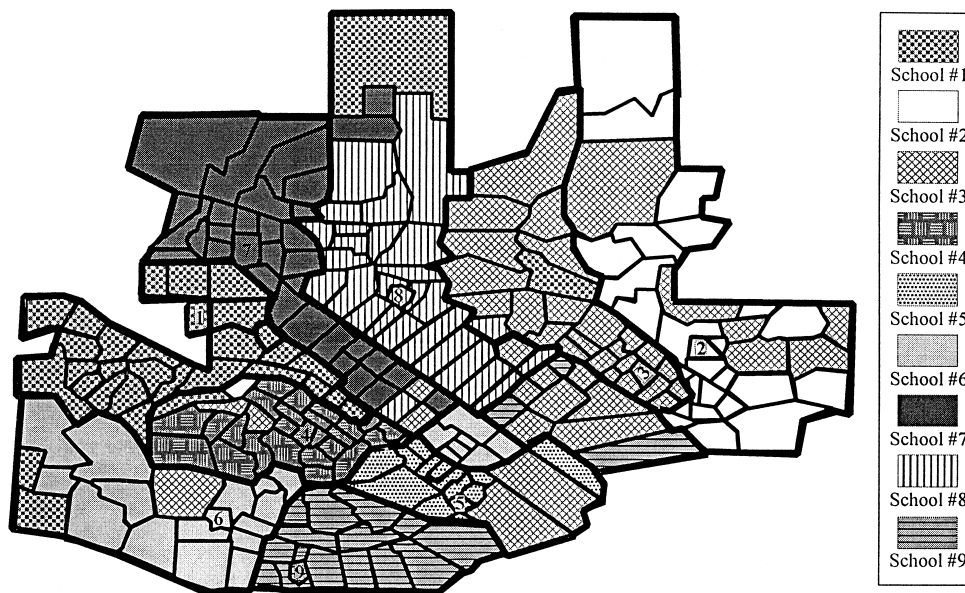


Fig. 5. Multiobjective SBS solution with equal weight on reassignments: 50% minimizing student moves/50% minimizing distance.

References

- [1] Armstrong MP, Lolonis P, Honey R. A spatial decision support system for school redistricting. *URISA Journal* 1993;5(1):40–51.
- [2] Belford PC, Ratliff HD. A network-flow model for racially balancing schools. *Ops Res* 1972;20(3):619–28.
- [3] Bovet J. Simple heuristics for the school assignment problem. *J Opl Res Soc* 1982;33:695–703.
- [4] Brown PJB. Trade-off and compromise in secondary school reorganization schemes: a goal programming approach. *Pap Reg Sci Ass* 1987;61:115–30.
- [5] Bruno JE, Anderson PW. Analytical methods for planning educational facilities in an era of declining enrollments. *Socio-Econ Plann Sci* 1982;16(3):121–31.
- [6] California Department of Education, Office of School Facility Planning, personal communication, October 17, 1996.
- [7] Church RL, Murray AT. Modeling school utilization and consolidation. *J Urban Plann Dev* 1993;119(1):23–38.
- [8] Church RL, Schoepfle OB. The choice alternative to school assignment. *Envir & Plann B* 1993;20:447–57.
- [9] Clarke S, Surkis J. An operations research approach to the desegregation of school systems. *Socio-Econ Plann Sci* 1968;1:259–72.
- [10] Diamond JT, Wright JR. Multiobjective analysis of public school consolidation. *J Urban Plann Dev* 1987;113(1):1–18.
- [11] Ferland JA, Guénette G. Decision support system for the school districting problem. *Ops Res* 1990;38(1):15–21.
- [12] Franklin AD, Koenigsburg E. Computed school assignments in a large district. *Ops Res* 1978;21(2):413–26.
- [13] Garrison W. Spatial structure of the economy: II. *Ann AAG* 1959;49(4):471–82.
- [14] Greenleaf NE, Harrison TP. A mathematical programming approach to elementary school facility decisions. *Socio-Econ Plann Sci* 1987;21(6):395–401.
- [15] Heckman LB, Taylor HM. School rezoning to achieve a racial balance: a linear programming approach. *Socio-Econ Plann Sci* 1969;3(2):127–33.
- [16] Henig M, Gerchak Y. Dynamic capacity planning of public schools in changing urban communities. *Socio-Econ Plann Sci* 1986;20(5):319–24.
- [17] Holloway CA, Wehrung DA, Zeitlin MP, Nelson RT. An interactive procedure for the school boundary problem with declining enrollments. *Ops Res* 1975;23(2):191–206.
- [18] Jennergren PL, Obel B. A study in the use of linear programming for school planning in Odense. *J Opl Res Soc* 1980;31(9):791–9.
- [19] Knutson DL, Marquis LM, Ricchute DM, Saunders GM. A goal programming model for achieving racial balance in public schools. *Socio-Econ Plann Sci* 1980;14:109–16.
- [20] Lemberg DS. 10 Year enrollment projections. Santa Barbara School District. Santa Barbara, CA 1992.
- [21] Lemberg DS, Church RL. Feasible alternatives generation in collaborative spatial decision making. In: *GIS/LIS '96 Proceedings, GIS/LIS '96*, Denver, CO, November, 1996, 1996.
- [22] Lemberg DS, Smith E. Geographic information systems (GIS) and school facilities planning. *CASBO Journal* 1989;54(1):19–22.
- [23] Liggett RS. The application of an implicit enumeration algorithm to the school desegregation problem. *Man Sci* 1983;20(1):159–68.
- [24] Maxfield DW. Spatial planning of school districts. *Ann AAG* 1972;62(4):580–90.
- [25] McDaniel RD. Case study of the use of the transportation algorithm for school districting under federal integration guidelines. *Socio-Econ Plann Sci* 1975;9(6):271–2.
- [26] McKeown P, Workman B. A study in using linear programming to assign students to schools. *Interfaces* 1976;6(4):96–101.
- [27] Ploughman T, Darnton W, Heusser W. An assignment program to establish school attendance area boundaries and forecast construction needs. *Socio-Econ Plann Sci* 1968;1:243–58.
- [28] Schoepfle OB. The modeling of schools under declining and fluctuating enrollments. PhD Dissertation, Department of Geography, University of California at Santa Barbara 1988.
- [29] Schoepfle OB, Church RL. A fast, network-based hybrid heuristic for the assignment of students to schools. *J Opl Res Soc* 1989;40(11):1029–40.

- [30] Schoepfle OB, Church RL. A new network representation of a “classic” school districting problem. *Socio-Econ Plann Sci* 1991;25(3):189–97.
- [31] Sutcliffe C, Board J, Cheshire P. Goal programming and allocating children to secondary schools in Reading. *J Ops Res Soc* 1984;35(8):719–30.
- [32] Trifon R, Livnat A. The spatial allocation of schools over time in cities. *Reg and Urban Econ* 1973;2(4):387–400.
- [33] Woodall M, Cromley RG, Semple RK, Green MB. The elimination of racially identifiable schools. *Prof Geogr* 1980;32(4):412–20.
- [34] Yeates M. Hinterland delimitation: a distance minimizing approach. *Prof Geogr* 1963;15(6):7–10.