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ARTICLE: The Computational Complexity of Automated Redistricting: Is Automation the Answer?

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Text

[*81]

There is only one way to do reapportionment - feed into the computer all the factors except political registration.

- Ronald Reagan [1]

The rapid advances in computer technology and education during the last two decades make it relatively simple to draw contiguous districts of equal population [and] at the same time to further whatever secondary goals the State has.

- Justice William Brennan [2]

I. ***Redistricting*** and Computers

Ronald Reagan and Justice Brennan have both suggested that computers can remove the controversy and politics from ***redistricting***. [3] In fact proponents of ***automated*** ***redistricting*** claim that the "optimal" districting plan can be determined, given any set of specified values. The Supreme Court has expressed a similar sentiment by addressing such mechanical principles as contiguity and compactness in two recent ***redistricting*** cases, Shaw v. Reno [4] **[*82]** and Miller v. Johnson. [5]

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1 Tom Goff, Reinecke Denounces Court: Legislative Leaders Praise Action, L.A. Times, Jan. 19, 1972, at A24.

2 *Karcher v. Daggett, 462 U.S. 725, 733 (1983).*

3 See supra notes 2 & 3 and accompanying text.

4 *509 U.S. 630 (1993).* In Shaw, the Court stated that "district lines may be drawn, for example, to provide for compact districts of contiguous territory, or maintain the integrity of political subdivisions." *Id. at 646.*

5 *115 S. Ct. 2475 (1995).* In Miller, the Court maintained that a state could defeat a claim that a district has been gerrymandering on racial lines by using neutral considerations such as compactness and contiguity as a basis for ***redistricting***. See *id. at 2488.*

Will we soon be able to write out a function that captures the social value of a districting arrangement, plug this function into a computer, and wait for the "optimal" ***redistricting*** plan to emerge from our laser-printers? This rosy future is unlikely to be realized soon, if at all, because the three problems that face ***automated*** ***redistricting*** are unlikely to be solved. First, current methods of ***redistricting*** are flawed in that they consist primarily of trial and error. Second, ***redistricting*** problems are computationally complex, so they will not be solved with the use of faster computers. Third, ***automated*** ***redistricting*** cannot meaningfully capture the social worth of political districts.

Part II of this Article illustrates how current ***redistricting*** methods are not adequate for the purposes of ***automated*** ***redistricting***. Current ***automation*** techniques must resort to unproven guesswork in order to handle the size of real ***redistricting*** plans. Consequently, before ***automated*** ***redistricting*** produces trustworthy results, large gaps in the process must be filled.

Proponents of ***automation*** assume that despite current shortcomings, finding the optimal ***redistricting*** plan simply requires the development of faster computers. Parts III and IV will demonstrate that this assumption is false-in general, ***redistricting*** is a far more difficult mathematical problem than has been recognized. In fact, the ***redistricting*** problem is so computationally complex that it is unlikely that any mere increase in the speed of computers will solve it.

Even if these difficulties can be overcome, ***automated*** ***redistricting*** still faces a serious limitation: to use ***automated*** ***redistricting***, a function must be written which is rigid enough for computer processing but subtle enough to meaningfully capture the social worth of districts. Part V argues that such a function will necessarily have to ignore values that are based on the subtle patterns of community and representation which cannot be captured mechanically. **[*83]**

II. Current Research on ***Automated*** ***Redistricting***

Although the literature on ***automated*** ***redistricting*** is at least thirty-five years old, it has seen a recent resurgence. This research generally falls into two categories: the first addresses the merits of ***automated*** ***redistricting*** per se, and the second suggests methods we can use to create districts automatically. [6] This part briefly summarizes the prior research in each of these two categories.

A. Arguments for ***Automating*** the ***Redistricting*** Process

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[6] There is a third category for which authors typically suggest particular criteria for drawing optimal districts - much of the literature on geographical compactness falls into this category.

Some authors argue gerrymandering can be eliminated by drawing districts which are maximally compact. Cf. Richard G. Niemi et al., Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering, 52 J. Pol. 1155 (1990) (arguing that compactness should be defined by multiple measures such as geographic dispersion, district perimeter and population); H.P. Young, Measuring the Compactness of Legislative Districts, 8 Legis. Stud. Q. 105 (1988) (concluding that compactness is not a very useful or operational criterion for judging whether a districting plan is fair). See generally Curtis C. Harris Jr., A Scientific Method of Districting, 9 Behav. Sci. 219 (1964) (arguing that the solution to gerrymandering in districts with nearly equal population is to apply the precise rule for compactness); Henry F. Kaiser, An Objective Method for Establishing Legislative Districts, 10 Midwest J. Pol. Sci. 200 (1966) (explicating the mathematical notions of population equality and compactness); David D. Polsby & Robert D. Popper, The Third Criterion: Compactness as a Procedural Safeguard Against Partisan Gerrymandering, 9 Yale L. & Pol'y Rev. 301 (1991) (suggesting that those who define district boundaries must also be required to respect the constraint of compactness); Robert S. Stern, Political Gerrymandering: A Statutory Compactness Standard as an Antidote for Judicial Impotence, 41 U. Chi. L. Rev. 398 (1974) (maintaining that a defined standard of compactness would provide an effective and easily understandable criterion to guide apportionment design and judicial review); David Wells, Against Affirmative Gerrymandering, in Representation and ***Redistricting*** Issues 77 (1982) (asserting that politically and ethnically neutral criteria such as equality of population, compactness and contiguity would make gerrymandering impossible).

While these authors focus primarily on the criteria for evaluating districts, their core argument is the same as that examined above i.e., that ***redistricting*** can be performed best by automatically optimizing a pre-specified representation function.

In 1961, William Vickrey, in one of the earliest papers on this subject, proposed that districting be ***automated***, and that this **[*84]** ***automation*** process be based upon two specific values: population equality and geographical compactness. [7] Under his proposal political actors would be permitted to specify or add criteria to a goal function for ***redistricting***, but would not be permitted to submit specific ***redistricting*** plans. [8] With no further human input, plans would be created automatically from census blocks to meet the goal function. In essence then, ***automated redistricting*** is an attempt to push all decision-making to the beginning of the ***redistricting*** process.

Vickrey asserted that ***automated redistricting*** provides a simple and straightforward method of eliminating gerrymandering. [9] More recently, Michelle Browdy elaborated on Vickrey's arguments, creating what seems to be the best model for ***automated redistricting***. [10] Five main arguments are offered in the literature:

1) ***Automated redistricting*** creates a neutral and unbiased district map; [11]

2) ***Automated redistricting*** prevents manipulation by denying political actors the opportunity to choose district plans, while simultaneously producing districts that meet specified social goals; [12]

3) ***Automated redistricting*** promotes fair outcomes by forcing political debate over the general goals of ***redistricting***, and not over particular plans, where selfish interests are most likely to be manifest; [13] **[*85]**

4) ***Automated*** procedures provide a recognizably fair process by meeting any representational goals that are chosen by the political process. [14] Browdy argues that such procedural fairness will help to curtail legal challenges to district plans; [15] and

5) ***Automated redistricting*** eases judicial and public review because the goals and methods of the districting process are open to view and because the ***automation*** process creates a clear separation between the intent and effect of ***redistricting***. [16]

B.

Criticisms of ***Automated Redistricting***

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[7] See William Vickrey, On The Prevention of Gerrymandering, 76 Pol. Sci. Q. 105 (1961) (proposing the ***automation*** of ***redistricting*** based on population equality and geographical compactness). For a discussion of population equality and compactness, see Kaiser, supra note 6, at 201-05.

[8] See Vickrey, supra note 7, at 106-07.

[9] See id. at 106.

[10] See generally Michelle H. Browdy, Computer Models and Post-Bandemer ***Redistricting***, *99 Yale L.J. 1379 (1990)* (proposing a feasible computer program which can reapportion geographical districts and describing computer techniques that may solve policy problems).

[11] See Edward Forrest, Apportionment by Computer, 7 Amer. Behav. Sci. 23, 23 (1964).

[12] See Robert G. Torricelli & John I. Porter, Toward the 1980 Census: The Reapportionment of New Jersey's Congressional Districts, 7 Rutgers J. Computers, Tech. & L. 135, 141-43 (1979); Stern, supra note 6, at 421 n.68; Browdy, supra note 10, at 1386-87; Vickrey, supra note 7, at 106.

[13] See Vickrey, supra note 7, at 106.

[14] See Vickrey, supra note 7, at 110; Browdy, supra note 10, at 1385-86. Similarly, Polsby & Popper argue that an application of formal compactness standards would diminish the practical value of gerrymandering. See Polsby & Popper, supra note 6, at 332-34.

[15] See Browdy, supra note 10, at 1386-87.

[16] See id. at 1394; Samuel Issacharoff, Judging Politics: The Elusive Quest for Judicial Review of Political Fairness, *71 Tex. L. Rev. 1643, 1697 (1993)* (examining various means of checking on the ***redistricting*** and reapportionment process).

Authors have raised two central objections to ***automated*** ***redistricting***: (1) automatic ***redistricting*** should not be viewed as inherently objective, and (2) because it is up to legislators to select among ***automation*** plans, political bias is unavoidable. [17] John **[*86]** Appel argues that ***redistricting*** standards and processes embody political values and that ***automation*** of this process hides the fundamental conflict over values. [18] Robert Dixon points out that ***automated*** processes, even if based on nonpolitical criteria, may have politically significant results. [19] Recently, Arthur Anderson and William Dahlstrom have cautioned that political consequences of ***redistricting*** goals makes ***redistricting***, whether it is ***automated*** or not, inescapably political. [20]

The idea that ***automated*** ***redistricting*** is not inherently objective seems both correct and unavoidable. ***Automation*** is a process for obtaining a given set of ***redistricting*** goals. Neutrality, however, is a function of three factors the process selected, the goals themselves, and the effects of seeking to obtain those goals in a particular set of demographic and political circumstances. Since there is no general consensus over what objectively neutral goals are, or whether they exist at all, no amount of ***automation*** can make the ***redistricting*** process objectively neutral. [21]

This objection only relates to the claim that neutrality can be achieved through computer ***redistricting***. Many proponents do not claim that ***automated*** ***redistricting*** is objectively neutral, but instead explicitly acknowledge the political nature of ***redistricting*** **[*87]** goals. They propose that the ***automated*** process be used neutrally and effectively to meet goals previously generated by a political process. [22] For example, Robert Dixon, who concluded that ***automated*** ***redistricting*** could not be objectively neutral, nonetheless freely promoted its use in this context. Anderson and Dahlstrom, echoing Dixon, provide a second critique of ***automated*** ***redistricting*** by drawing attention to the legislative process used to select automatically generated plans. [23] They argue that the

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[17] Note that a number of papers argue against particular formal measurements, rather than against ***automated*** ***redistricting***. See generally Arend Lijphart, Comparative Perspectives on Fair Representation: The Plurality-Majority Rule, Geographical Districting, and Alternative Electoral Arrangements, in Representation and ***Redistricting*** Issues 143 (1989) (arguing that inconsistencies in the criteria used for ***redistricting*** lead to contradictory recommendations, plurality-majority rule and geographically drawn electoral districts); David H. Lowenstein and Jonathan Steinberg, The Quest for Legislative Districting in the Public Interest: Elusive or Illusory?, *33 UCLA L. Rev. 1 (1985)* (suggesting that no coherent public interest criteria exist for legislative districting independent of substantive conceptions of the public interest); David R. Mayhew, Congressional Representation: Theory and Practice in Drawing the Districts, in Reapportionment in the 1970s 249 (Nelson W. Polsby ed., 1971); Bruce E. Cain, The Reapportionment Puzzle 77 (1984) (concluding that a nonpartisan, noncontroversial reapportionment process is impossible because reapportionment is a political question).

[18] See John S. Appel, A Note Concerning Apportionment by Computer, 7 Amer. Behav. Sci. 36, 36 (1965) (concluding that computerized apportionment is possible as an adjunct to sets of values and may be helpful and inexpensive, so long as one follows public policy directives).

[19] See Robert Dixon, Democratic Representation: Reapportionment in Law and Politics 528 (1968) (asserting "it is a hard-core fact that every line drawn on a map, whether blindly or designedly, has partisan implications different from any other kind of line that could be drawn").

[20] See Arthur J. Anderson & William S. Dahlstorm, Technological Gerrymandering: How Computers Can Be Used in the ***Redistricting*** Process to Comply with Judicial Criteria, 22 The Urban Lawyer 59, 76 (1990) (arguing that ***automated*** ***redistricting*** is not apolitical, but ***automation*** will help create districts that comply with judicial standards and the requirements imposed by both federal and state constitutions).

[21] Much doubt has been expressed as to whether such goals exist. Furthermore, the fundamental conflicts between some of the more commonly proposed goals make such a consensus unlikely. For a discussion of these, see Cain, supra note 17, at 68-73; Dixon, supra note 19, at 528; Bernard Grofman, Criteria for Districting: A Social Science Perspective, *33 UCLA L. Rev. 77, 78 (1985);* Lijphart, supra note 17, at 147-52.

[22] See Vickrey, supra note 7, at 110; Issacharoff, supra note 16, at 1697; cf. Browdy, supra note 10, at 1389 (asserting that even if the procedure used to design the ***redistricting*** algorithm is carefully planned to facilitate public scrutiny, computers cannot transform ***redistricting*** into a "neutral" process).

legislature's willingness to accept plans generated by an ***automated*** process will be politically motivated, reintroducing political bias into the process. [24]

Although Anderson and Dahlstrom's criticism seems valid, there is a simple cure. Any attempt to reintroduce political bias can be prevented by mandating legislative acceptance of the results in advance. In general, however, researchers have largely underestimated the potential for political biases to become part of the ***automation*** process. [25]

C.

The Core Argument: ***Automation*** as a "Veil of Ignorance"

Among its proponents, ***automation*** essentially plays the role of a Rawlsian "veil of ignorance" which creates fairness by hiding each actor's position in the final outcome. [26] The "veil of ***automation***" attempts to hide the final outcome (i.e., ***redistricting*** plans) from those bargaining over the social contract (i.e., ***redistricting*** goals and procedures). Thus, advocates of the ***automation*** process claim to prevent manipulation by promoting a recognizably fair method that will, on average, promote fair outcomes.

In this vein, Vickrey emphasizes that in order for ***automation*** to be successful at promoting fairness, it must be sufficiently unpre- **[*88]** dictable. [27] It should not be possible for political actors to deduce the results of the ***redistricting*** goals over which they bargain. [28] Without unpredictablity the choice of objective functions collapses into a choice of individual plans, and the incentive to gerrymander remains unameliorated by the ***automation*** process. If actors can predict the plans that will result from specific values, the veil of ***automation*** could be pierced. [29]

While proponents of ***automated*** ***redistricting*** recognize the need for unpredictability, they do not always consider the danger from unpredictable results. An ***automated*** process for creating districts in accordance with agreed upon values must predictably achieve (or at least approach) the goals that were agreed upon in the bargaining stage, or otherwise lose legitimacy.

The ***automated*** ***redistricting*** process must maintain a delicate balance. To prevent manipulation while maintaining fairness, the ***automated*** process must predictably implement the ***redistricting*** goals agreed upon in the bargaining process, but it must be unpredictable in every other dimension of interest to the bargaining agents. These requirements are difficult to satisfy when bargaining agents seek specific, hand-tailored gerrymanders. Such requirements become even more difficult to meet when bargaining agents represent interest groups or political parties unconcerned with particular incumbents, because such agents are interested in far more general properties of ***redistricting*** plans. Can ***automated*** ***redistricting*** methods reliably produce plans that exclusively embody any specific set of ***redistricting*** goals?

D. Current Methods Are Inadequate for ***Automated*** ***Redistricting***

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[23] See Anderson & Dahlstorm, supra note 20, at 76-77.

[24] See id.

[25] For a detailed discussion of political biases in the ***automation*** process see infra Part IV.

[26] See John Rawls, A Theory of Justice 136-42 (1971).

[27] Unpredictable" more accurately describes what Vickrey calls "random." Vickrey maintains that the exact results from using a particular value function should not be obvious to political actors. In order to achieve this goal it is not necessary that the process be random. For example, if the process is sufficiently chaotic, it may not be random but it may still be unpredictable. See Vickrey, supra note 7, at 106.

[28] See id.

[29] See id.

Initially, many researchers expressed optimism about the ease of achieving ***redistricting*** goals through ***automation***. [30] Vickrey **[*89]** best summarized this spirit in 1961:

Elimination of gerrymandering would seem to require the establishment of an automatic and impersonal procedure for carrying out a ***redistricting***. It appears to be not all difficult to devise rules for doing this which will produce results not markedly inferior to those which would be arrived at by a genuinely disinterested commission. [31]

While this initial optimism has dulled since it has been recognized that purely ***automated*** ***redistricting*** techniques remain generally unsatisfactory, [32] many authors still assume that ***automation*** of the ***redistricting*** process is within reach. [33]

Vickrey sketched a method of ***automated*** ***redistricting***, but did not give a precise plan for implementing this method. [34] Much of the subsequent scholarship assumed the benefits of ***automated*** ***redistricting*** and focused primarily on providing criteria and methods to use in such ***automation***. For example, John Liittschwager discussed the extension and application of Vickrey's method to Iowa's ***redistricting*** process. [35] James Weaver and Sidney Hess, as well as Stuart Nagel, have developed similar methods and/or measures for drawing districts in accordance

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[30] See generally Stuart S. Nagel, Simplified Bipartisan Computer ***Redistricting***, 17 Stan. L. Rev. 863 (1965) (describing a computer program which can reapportion a legislature or populace who represent geographical districts and exploring programming techniques to solve policy problems); James B. Weaver & Sidney W. Hess, A Procedure for Nonpartisan Districting: Development of Computer Techniques, 72 Yale L.J. 288 (1963) (proposing the utilization of available computer programming techniques to identify a certain number of districts within a certain area, by grouping together smaller areas according to specific principles of representation); Toricelli & Porter, supra note 12, at 141; Vickrey, supra note 7, at 110.

[31] Vickrey, supra note 7, at 110.

[32] See Charles H. Backstrom, Problems of Implementing ***Redistricting***, in Representation and ***Redistricting*** Issues 43, 51 (Bernard Grofman et al;. eds.,1982) (stating that "hopefully, there is no one who thinks the computer is a black box with magical problem-solving capacity of its own").

[33] See Anderson & Dahlstorm, supra note 20, at 77; Browdy, supra note 10, at 1386-89.

[34] See Vickrey, supra note 7, at 110.

[35] See John M. Liittschwager, The Iowa ***Redistricting*** System, 219 Annals N.Y. Acad. Sci. 221 (1973) (illustrating that in the Iowa ***redistricting*** system "the implementative understanding of the judicial and constitutional intent has been, both manually and by computer, considerably more evolutionary than revolutionary").

with principles of **[*90]** population equality and geographical compactness. [36] Recently, Browdy proposed that the method of simulated annealing may be generally applicable to the problem of drawing optimal Districts. [37]

Different methods have been used or suggested for finding optimal districts. These techniques can be categorized as: (i) exact and (ii) heuristic methods. The next two sections will address the methods used to search for optimal districts. **[*91]**

1. Limitations of Exact Methods

Exact methods systematically examine all legal districts, either explicitly or implicitly. "Explicit enumeration," or "brute force" search methods literally evaluate every district. More sophisticated methods such as "implicit-enumeration," "branch-and-bound," or "branch-and-cut" techniques exclude classes of solutions which are inferably sub-optimal without an explicit examination. Finding the optimal districts with these methods is merely a matter of sorting a list of district scores. These methods have been used by several authors to approach very small ***redistricting*** problems. [38]

Exact methods have two major shortcomings. First, no exact method has been developed that will solve ***redistricting*** problems for a reasonably sized plan, and, as shown in Part IV, the mathematical structure of the

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[36] See generally Weaver & Hess, supra note 30 (proposing a mathematical procedure for creating districts that promote population equality and more contiguous and compact districts); Nagel, supra note 30 (proposing a program where the relative weight given to population equality, contiguity, compactness and political balance of power can be adjusted by a user). For a review of early attempts at ***automated*** ***redistricting***, see generally G. Gudgin & P. J. Taylor, Seats, Votes, and the Spatial Organization of Elections 121-61 (1979) (providing basic algorithms for ***redistricting***, application of the algorithms, and guidelines for choosing optimal solutions) and L. Papayanopoulos, Quantitative Principles Underlying Apportionment Methods, 219 Annals N.Y. Acad. Sci. 181 (1973) (constructing a simple apportionment model and examining methods to create equal population districts).

Note that federal courts no longer require compactness in federal ***redistricting***. See Grofman, supra note 21, at 84-85. Federal courts have never required compactness for state legislative districting. See id. Courts have frequently mentioned the desirability of compactness. See id. Twenty-five state constitutions have compactness provisions. See id.

[37] See Michelle H. Browdy, Simulated Annealing: An Improved Computer Model for Political ***Redistricting***, 8 Yale L. & Pol'y Rev. 163, 172-75 (1990). Annealing in the scientific community describes the method by which hot metal is cooled. See id. at 172. Just as annealing allows molecules to arrange themselves slowly into stable, low-energy configurations, simulated annealing allows computer scientists to utilize this slow cooling principal to locate the optimal solution for a computer problem. See id. Simulated annealing in computer science can be summarized as follows:

If one starts with any random configuration of a system (for example, think of the current ***redistricting*** plan for a state as the initial configuration) and then tries to change random pieces of the system (for example, moving groups [enumeration districts] between legislative districts), always keeping changes that improve the system (for example, making districts closer in equality of population) and occasionally keeping changes which harm the system (for example, making districts less equal in population), then eventually one should reach an optimal or nearly optimal configuration of the system (a plan that meets the objective function, such as the most compact plan).

See id. at 172-73.

[38] See Gudgin & Taylor, supra note 36, at 143-46; R.S. Garfinkel & G.L. Nemhauser, Optimal Political Districting by Implicit Enumeration Techniques, 16 Mgmt. Sci. B-495, B-500 to B-502 (1970) (exploring methods of reaching optimal ***redistricting*** plans); Michael A. Jenkins & John W. Shepherd, The Amalgamation of High School Districts in Detroit A Computer Evaluation of Alternative Strategies in the Political Control of Decentralization: Summary, in Computer Science Association of Canada 160, 171-78 (1970). A close examination of the algorithms discussed by the authors reveals that in order to make enumeration complete in a feasible amount of time, "short-cuts" are used where some sub-classes of partitions are assumed to be unreasonable, and are disregarded without examination and without proof of sub-optimality. Restrictions such as "exclusion distance" in Garfinkel & Nemhauser or limiting examination to "amalgamations" in Shepherd & Jenkins must be formally classified as heuristic rather than exhaustive.

redistricting problem makes it unlikely that such a method will be developed in the future. Second, even if an exact method were developed, such a method would reveal the precise plan corresponding to a particular set of ***redistricting*** goals. As a result, exact methods would have completely predictable results, violating the ideal requirements of an ***automated*** ***redistricting*** method.

2. Limitations on Heuristic Methods

Heuristic procedures use a variety of methods to structure the search for high-valued ***redistricting*** plans. By definition, heuristic algorithms provide no guarantee of convergence to the optimal **[*92]** district plan in a finite amount of time. At best, they are merely good guesses.

All general ***redistricting*** heuristics are based upon making iterative improvements to a proposed ***redistricting*** plan. [39] The single most popular ***redistricting*** heuristic seems to be hill climbing and its variants, however, some researchers add more sophisticated features such as neighborhood search techniques. [40]

Hill climbing methods work by making small improvements on a potential solution until a local optimum is reached. Hill climbing often starts with the current district plan or with a randomly generated plan, and it makes improvements through repeatedly trading census blocks between districts. [41] Another variant of this method selects arbitrary census tracts to form the nuclei of each district, and then repeatedly adds tracts that most improve [42] the current district until each district is fully populated. [43] **[*93]**

Neighborhood search methods are similar to hill climbing techniques in that they seek to improve potential solutions by examining the value of "nearby" solutions. Unlike hill climbing algorithms, neighborhood search methods employ sophisticated techniques in an attempt to avoid becoming stuck at local optima. [44]

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[39] In the field of computer science, heuristic algorithms are divided into two categories: iterative improvement and divide-and-conquer methods. These two general broad categories include variants such as simulated annealing and genetic algorithms. See infra Part IV.

[40] In addition to general ***redistricting*** methods there are a number of special purpose methods. For example, an iterative graphical remapping has been used to generate districts of equal population. See W.R. Tobler, A Continuous Transformation Useful for Districting, 219 Annals N.Y. Acad. Sci. 215, 215-19 (1973). This process gradually distorts geographical maps to create new maps where population is equivalent to area, facilitating the creation of districts with equal population. See id.

[41] Usually, improvements are made sequentially for each district, but if the number of census tracts is small, several trades may be examined simultaneously. See generally Jack Moshman & Elaine M. Kokiko, A ***Redistricting*** Algorithm Applied to Geographic Reorganization of Circuit Courts, 219 Annals N.Y. Acad. Sci. 236, 241 (1973) (arguing that "by commencing with a manually derived starting point … the algorithm continually attempted to improve the solution"); Nagel, supra note 30, at 864 (observing that the program will move one unit to an adjoining district or by trading units between districts).

[42] Often this is simply the tract that is closest to the selected district center (in whatever metric used). The Weaver and Hess algorithm uses linear-programming techniques to select population units to add to the district center. See Weaver & Hess, supra note 30, at 292 (asserting that this technique would favor areas of high and low population densities).

[43] See Vickrey, supra note 7, at 106 (urging that the process should be as far removed as possible from human contact); Weaver & Hess, supra note 30, at 307-08 (pointing out that the program will reduce the number of choices that need to be made); Liittschwager, supra note 35, at 228-33 (emphasizing that data regarding population regions is inserted and checked by programs which create a report); Lawrence D. Bodin, A Districting Experiment with a Clustering Algorithm, 219 Annals N.Y. Acad. Sci. 209, 209 (1973) (arguing that the algorithm is not discriminating enough to produce one final partition because it produces several results and the best has to be accepted); Rose Institute of State and Local Government, Reapportionment and Computers in California ***Redistricting*** (1980); P.J. Taylor, Some Implications of the Spatial Organizations of Elections, 60 Transactions, Institute of British Geographers 121 (1973).

[44] One particular type of neighborhood search algorithm is simulated annealing. See Browdy, supra note 37, at 172-75 (expressing an optimistic perspective on simulated annealing's potential for success in ***redistricting***). Simulated annealing and other neighborhood search techniques will be discussed in greater detail infra Part IV.

The main difficulty with all heuristics is that they are informed guessing procedures. [45] This is not necessarily bad - when faced with a difficult problem, guessing may frequently give rise to an adequate solution cheaply. Using guessing procedures to decide political questions, however, requires a showing that the guesses are unbiased and are likely to produce positive solutions. [46] None of the non-exhaustive methods referred to in this Article have **[*94]** been proved to approximately optimal [47] or unbiased, [48] and this author has been unable to find any such methods mentioned in the literature.

III. ***Automated*** ***Redistricting*** May be Intractable

Automated ***redistricting*** has been unsuccessful not only because of current techniques but because of inherent ***complexities*** in the structure of the ***redistricting*** problem. Advocates of ***automated*** ***redistricting*** suggest that developing "optimal" ***redistricting*** plans simply requires formulating the ***redistricting*** problem in mathematical terms. Regardless of the formulation, however, the ***re-*** **[*95]** ***districting*** problem is practically impossible to solve exactly it is computationally intractible.

A. ***Redistricting*** is a Large Mathematical Problem

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[45] Compare Papayanopoulos, supra note 36, at 188 (advocating experimentation with heuristic methods until "a precise, highly efficient solution method" is developed). Papayanopoulos acknowledges the imperfection of heuristic methods: "Some may work under certain circumstances and not others. Some may be faster but less accurate than others, and so on." Id.

[46] See Browdy, supra note 37, at 168. Browdy notes that a computer generated ***redistricting*** plan must be defensible in order to demonstrate that the ***automated*** process is more rational than the present system. Two standards are suggested as useful in defending a computer generated plan: the model could produce the least changed plan which meets the particular ***redistricting*** goal, or, the algorithm's solution could attain optimality. Browdy asserts that optimality is the strongest possible defense but concedes that "it is not clear that … any ***redistricting*** solution is optimal." Id.

Papayanopoulos concurs that heuristic methods provide no guarantee that an optimal result will be achieved; rather, success depends upon consistency in producing a "reasonably good solution." Papayanopoulos, supra note 36, at 188. A "reasonably good solution" is defined as "one that comes close to being optimal or one that the designer considers acceptable." Id. at 188 n.*. The problem of bias is discussed infra note 49 and accompanying text.

[47] Optimality can be described either as local or global. See generally Browdy, supra note 37, at 169-70 (providing a simple analogy to clearly demonstrate the distinction between local and global optimality). A globally optimal solution provides the unique and best solution to the problem. See id. at 169.

Heuristic methods are proficient at generating alternative plans, see infra note 60 for the definition of a "plan," comprised of ***redistricting*** goals set at various local optima. See Papayanopoulos, supra note 36, at 187; Weaver & Hess, supra note 30, at 303-04.

Manual and computerized random selections have both been suggested as methods of choosing a final plan from computer-generated alternatives. See Weaver & Hess, supra note 30, at 304 (arguing that manual selections can supplement "a collection of plans"); Browdy, supra note 37, at 170-71 n.21 (citing Herbert Simon's notion of "satisficing" to propose that good decisions, which are a function of technological constraints, are sufficient where optimization is not possible or feasible). Each of these suggestions, however, raises important concerns. Permitting legislators to manually select from a group of plans defeats the purpose of ***automated*** ***redistricting*** to insure against bias or gerrymandering by denying the politician the ultimate choice of a plan. See, e.g., Vickrey, supra note 7, at 106. Alternatively, where a computer process randomly selects the final plan, defensibility or justification of the final result persists as problematic. See Browdy, supra note 37, at 170-71.

[48] Bias can be introduced into the process at two distinct phases. One was mentioned in a previous footnote, see supra note 47, that manually choosing a final plan provides authorities with an unchecked opportunity to pursue illegitimate objectives. In addition, the efficiency and "success" of heuristic methods depend strongly on the initial input chosen. See Browdy, supra note 37, at 170 n.20; Papayanopoulos, supra note 36, at 185 ("The manner in which these criteria are taken into account strongly affects the outcome"). See generally James B. Weaver, Fair and Equal Districts: A How-To-Do-It Manual on Computer Use 40-45 (1970) (emphasizing the significance of the choice of ***redistricting*** goals and the importance of obtaining authoritative consent as to each; to discern why a method can implement only some criteria might be politically problematic).

Because the size of the solution set can be enormous, the ***redistricting*** problem poses special difficulties. [49] It will be impossible to attack the problem by a brute force search through all possible districting arrangements. [50]

The ***redistricting*** problem can be characterized mathematically in a number of different ways. [51] One way is to think of it as a set partitioning problem. [52] More particularly, ***redistricting*** can be characterized as a combinatorial optimization problem. [53] Imagine that census blocks are indivisible, and that complete information about voting and demographic information exists for every census block in the state. [54] The ***redistricting*** problem is to partition [55] the **[*96]** entire set of units into districts such that a value function [56] is maximized. [57] This partitioning

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[49] There has been some success in ***redistricting*** experiments that implement exhaustive methods. See, e.g., Garfinkel & Nemhauser, supra note 38, at B-503 to -06. These are "laboratory successes," however, as researchers candidly admit that "the necessary computations become too cumbersome even for a computer" when more than fifty census enumeration districts ("EDs") are produced. See Papayanopoulos, supra note 36, at 188. To appreciate the precise degree of this limitation, note that the small state of Delaware had fifty-four EDs as determined by the 1960 Census. See Weaver & Hess, supra note 30, at 306.

[50] See Weaver & Hess, supra note 30, at 302-04 (proposing the "brute force" districting technique). While this technique may be a viable option in the context of a small-scale experiment, it is clearly unworkable when the size of the solution set becomes unmanageably large. Quite aside from the impracticability of manual selection from alternative plans, the brute force search method reintroduces the "human element" and the potential for political bias into the ***redistricting*** process. See supra note 49.

[51] See generally Papayanopoulos, supra note 36, at 188.

[52] The characterization "set partitioning" will be used extensively in the Article. There are other plausible characterizations including graph partition, polygonal dissection and integer programming. See infra Appendix 1. Since all of these characteristics are computationally equivalent, see infra Part IV, the results in this section are independent of which characterization is chosen.

[53] ***Redistricting*** usually is characterized as a combinatorial optimization problem. See Browdy, supra note 37, at 171; Papayanopoulos, supra note 36, at 188. See generally Gudgin & Taylor, supra note 37, at 140-43.

[54] Commentators also use the term "enumeration district" (ED) to represent the smallest indivisible population unit. See, e.g., Browdy, supra note 37, at 166; Papayanopoulos, supra note 36, at 185; Weaver & Hess, supra note 30, at 300. The ED is the smallest unit of population count provided by the United States Census. See id.

[55] A partition divides a set into component groups which are exhaustive and exclusive. More formally:

For any set x = [x<1>, x<2>, ..., x<n>], a partition is defined as

a set of sets Y = [y<1>, y<2>, ..., y<k>,] such that

[SEE EQUATION IN ORIGINAL]

[56] The terms "value function" and "objective function" are interchangeable; each describes the measure of a desirable characteristic or "goal." See Browdy, supra note 37, at 170. Measures of a district's population equality, or of contiguity, or of both combined are examples of value functions. Obviously, maximizing these measures is desirable.

[57] More formally:

Given

(1) a set of census blocks x

(2) the set of all partitions of x, Y

(3) a value function on partitions, V(y)

The optimal district plan is

[SEE EQUATION IN ORIGINAL]

problem may be complicated by the addition of a set of constraints on districts. [58] These constraints may limit the set of legal plans. [59] **[*97]**

Formally , the total number of distinct plans [60] that can be created using n population blocks to draw r districts is characterized by the function: [61]

[SEE EQUATION IN ORIGINAL]

Even assuming that each district is composed of exactly k population blocks [62] (hence r = [su'n']/<k>) the number of possible plans remains a rapidly growing function:

[SEE EQUATION IN ORIGINAL]

The magnitude of this problem is seldom fully recognized. For even a small number of census tracts and districts, the number of possible districting arrangements is enormous. Table One illustrates the number of possible plans that could be used to divide a small hypothetical state into two districts: [63]

[SEE TABLE IN ORIGINAL]

[*98]

The number of districts, r, is also an important factor in determining how many plans are possible. The number of possible plans will increase in r up to a point, and then decrease:

[SEE TABLE IN ORIGINAL]

An exhaustive search for optimality will be impractical for all but the coarsest population units and extreme number of districts per census block. Only by making a large population unit (such as a county) indivisible can exhaustive

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[58] One or more ***redistricting*** goals which are combined with a single partitioning objective become "constraints" in the problem; these act as additional criteria or requirements which the computer must satisfy to produce a solution. See Papayanopoulos, supra note 36, at 185-88.

[59] The districting scheme which results from these processes is called a "plan." See Garfinkel & Nemhauser, supra note 38, at B-495. The more criteria or constraints introduced into the problem, the fewer the solutions or plans that can satisfy them. See Papayanopoulos, supra note 36, at 187. Indeed, "it may well happen that no solution meeting all the conditions exists." Id. When represented formally, a constraint not only introduces an additional criterion for the computer to evaluate, it may also inhibit satisfaction of the original objective.

Some techniques, such as simulated annealing, provide for absorption of the constraint into the value function, thereby creating an unconstrained problem. This phenomenon is called relaxation: it provides a means of solving a constrained problem as an unconstrained one. See Browdy, supra note 37, at 173-75. Note that simulated annealing is a heuristic method, however, see supra note 37, and entirely distinct from partitioning which is exhaustive, see supra note 55.

[60] This number reflects districts that are distinct and ignores the numbering order of districts. Merely renumbering the districts without changing the composition of at least one does not result in a different plan.

[61] S is known as a "Stirling Number of the Second Kind." For a helpful introduction to this concept, see generally Shimon Even, Algorithmic Combinatorics 58-61 (1973).

[62] Which is not correct, but closer to the real situation than the formula above.

[63] As evidenced, the size of the ***redistricting*** problem grows rapidly as a function of the number of population units being used. In fact, this table understates the size of the problem, because it assumes that all districts have identical numbers of blocks.

search manageability be achieved. [64] If districting is to be performed using accurate, fine-grained population units such as census tracts or blocks, the number of plans generated make exhaustive searches burdensome. [65]

Confronted with a prohibitive number of plans, several proponents of ***automated*** ***redistricting*** have suggested that non- **[*99]** exhaustive procedures be employed. [66] Concededly, exhaustive searches are not the only means to generate optimal districts. Any method for finding optimal districts, however, is likely to be computationally difficult, and therefore impractical for all but the "smallest" ***redistricting*** problems.

B. Candidate Value Functions

The difficulty of solving the ***redistricting*** problem will depend upon which particular value function and constraints are utilized. [67] While virtually all political values are subject to debate, a general consensus does exist for a number of ***redistricting*** criteria. Five of the least contested ***redistricting*** objectives are listed below:

1) Population equality: [68] this criterion is satisfied when there is no substantial deviation in population between districts; **[*100]**

2) Contiguity: [69] a district is contiguous if it is possible to reach every point in the district from another point without crossing the district boundary;

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[64] Either the quality of solutions will suffer, or the likelihood of any solution at all will diminish. Cf. Papayanopoulos, supra note 36, at 187 (arguing that using more criteria will result in fewer solutions, if any at all). Essentially, the fewer population units, the greater the success in achieving optimality. See, e.g., Garfinkel & Nemhauser, supra note 38, at B-506. Achieving this optimality will necessitate that fewer ***redistricting*** goals can be realized. Cf. Browdy, supra note 37, at 1386 (noting that ideal ***automated*** ***redistricting*** models will produce one ***redistricting*** plan); Papayanopoulos, supra note 36, at 187 (asserting that if ***redistricting*** goals consider too many ethnic and economic distinctions, the resulting districts may not satisfy other important considerations).

[65] Consider, for example, the state of California, where approximately twenty thousand census block-groups (and a much greater number of blocks) must be assigned to fifty-two districts. See infra note 129. If it started at the creation of the universe, a computer that could examine a million districts a second still would not be finished.

[66] See Stuart Nagel, Computers and the Law and Politics of ***Redistricting***, 5 Polity 77 (1972); Papayanopoulos, supra note 36, at 188.

[67] See, e.g., Browdy, supra note 37, at 177.

[68] This is probably the most important and universally accepted of all ***redistricting*** criteria. Most districting plans that have been held unconstitutional have failed because of the disproportionality in population between one or more of the districts. See Garfinkel & Nemhauser, supra note 38, at B-495. The United States Supreme Court regards population equality as indispensible to fair districting, but applies a significantly more rigorous standard in the context of congressional districting as compared to state or local. See Grofman, supra note 21, at 80.

While there are other significant and legitimate ***redistricting*** goals, population equality assumes a clearly elevated position among them. See Grofman, supra note 21, at 87; see also id. at 82 (citing *Kirkpatrick v. Preisler, 394 U.S. 526 (1969)* to illustrate the Court's reluctance to accept that a competing goal such as avoiding fragmentation of local economic or social interests could justify even minimal population disparities among districts). But see id. at 82 & n.24 (implying that other ***redistricting*** criteria become more significant when applied consistently and when the population inequality is minimal).

Population equality is one way to insure equal voting power between districts. For a clear and instructive explanation of population equality, weighted voting, and multimember districting as alternative means of maintaining equal voting power among districts see generally Papayanopoulos, supra note 36, at 182-85.

The Supreme Court's "one-man-one-vote" rule requires that districts be substantially equal in population. See Grofman, supra note 21, at 80 (citing *Reynolds v. Sims, 377 U.S. 533, 558 (1964)).*

[69] See, e.g., Garfinkel & Nemhauser, supra note 38, at B-496; Grofman, supra note 21, at 84. Thirty-seven states require that districts be contiguous. See id.

3) Compactness: [70] a district is compact if its shape is geometrically regular;

4) Creating fair electoral contests: there are many characteristics that could be attributed to "a fair contest." The most common ones include maximal competitiveness, [71] neutrality, [72] and a constant swing ratio [73] for each party; and

5) Representational goals: these goals seek to insure that all social sects have a political voice in an election; examples include the protection of communities of interest [74] and non-dilution of minority representation. [75] **[*101]**

Once these ***redistricting*** goals have been decided upon by a political process, ***automation*** can merely implement them in a neutral way. [76] Formalization or ***automation*** cannot make these goals "objective." [77] The politics and controversy surrounding ***redistricting*** cannot be remedied completely by ***automation***.

C. What is a Computationally "Hard" Problem?

For any of the aforementioned value functions, or combinations thereof, the problem of finding an optimal districting plan is computationally complex; any attempt will most likely be thwarted by the size and ***complexity*** of the

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[70] See Garfinkel & Nemhauser, supra note 38, at B-496. A district is geographically compact if it is circular or square in shape rather than long and narrow. See id. Although compactness has never been a federal requirement for state legislative districting, see Grofman, supra note 21, at 85, it is desirable nonetheless, because it reduces the possibility of gerrymandering, see Garfinkel & Nemhauser, supra note 38, at B-496. Although twenty-five states require it, compactness is defined by only two states. See Grofman, supra note 21, at 85. Another type of compactness less frequently considered is population compactness. See Garfinkel & Nemhauser, supra note 38, at B-496.

[71] Maximal competitiveness is simply the maximization of the number of close elections.

[72] Neutrality proscribes electoral biases in the award of seats to a particular political party for a certain percentage of the vote. See Grofman, supra note 21, at 150. Two states require what might loosely be described as an electoral response function. See Del. Const. of 1897, art. II, 2A; Haw. Const. art. IV, 6. These constitutional directives are generally worded to the effect that the districting authority, "shall district to not unduly favor any person or political party or faction."

[73] The term swing ratio indicates "the percentage point change in a party's seat share obtained for each additional percentage point increment in its vote share above fifty percent." Grofman, supra note 21, at 151.

[74] Five states explicitly mention and extend protection to communities of interest in their ***redistricting*** statutes. See *Idaho Code 72-1506(2)* (1996); Tenn. Code Ann. 7-21-492 (1995); *Wash. Rev. Code Ann. 44.05.090(2)(a)* (West 1995); *W. Va. Code 1-2-1(c)(5)* (1995); *Wis. Stat. Ann. 4.001(3)* (West 1995).

[75] Section five of the Voting Rights Act of 1965, *42 U.S.C. 1971* (1994), was intended to protect minorities from the denial or abridgment of voting rights. See Grofman, supra note 21, at 95. In cases brought under this section, the state had the burden of proving that its districting plan was not intended to have, nor would have the effect of diluting minorities' voting rights. See id.

[76] See Browdy, supra note 37, at 167-68. Browdy advances the argument that legislators and politicians would be accountable for their choice of objectives since these would be available to the courts and the public. See id. at 167. This system of checks and balances is desirable and compelling, but other commentators assume a more skeptical position. See, e.g., Papayanopoulos, supra note 36, at 182-83 ("We may draw an analogy between legislative representation and some hypothetical game of skill. The model we are constructing corresponds to the rules set down by the designer of the game. It contains nothing concerning the players' abilities or their ethical adherence to the rules. After all reapportionment is about fair rules, not fair play.").

[77] Cf. Weaver, supra note 48, at 40.

redistricting problem. Some formal definitions from ***computational*** ***complexity*** theory will provide structure for this argument. [78]

The two theories of theoretical computer science are ***computational*** ***complexity*** theory [79] and the related field of computability **[*102]** theory. [80] These disciplines are devoted to analyzing the difficulty of solving specified discrete problems using computers. [81]

Researchers in computer science and in operations research use ***computational*** ***complexity*** theory extensively when analyzing problems. [82] While this type of analysis has been adopted only recently by political scientists, ***computational*** tractability is becoming recognized as a prerequisite for practical electoral rules. [83]

The definition of a problem is basic to ***computational*** ***complexity*** theory. [84] A problem is a general question to be answered. [85] In the case of ***redistricting***, the problem is to find the districting plan that maximizes the value function; formally, the problem is finding the optimal partition. [86] The term ***redistricting*** sub-problem will be used to distinguish the case where a particular value function has been pre-specified, [87] rather than where the value function **[*103]** itself is a parameter. [88] A problem possesses several parameters, [89] or free variables. [90] For

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[78] ***Computational*** ***complexity*** theory studies the inherent difficulty of certain mathematical problems and seeks optimal means for their solutions. See J.F. Traub & H. Wozniakowski, Perspectives on Information-Based ***Complexity***, 26 Bull. of the Am. Mathematical Soc'y 29, 29 (1992). ***Complexity*** theory is mainly concerned with the constraints upon the computation of functions; which functions are capable of computation; the amount of time needed for computation; and the amount of memory necessary to perform the computation. See D. Deutsch, Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer, 400 Proc. of the Royal Soc'y of London 97, 114 (1985). For a more lengthy and formal characterization, see generally Christos H. Papadimitriou, ***Computational*** ***Complexity*** (1994).

[79] ***Computational*** ***complexity*** is otherwise known as structural ***complexity***. See Ronald V. Book, Revitalizations of the P[fc?]NP and Other Problems: Developments in Structural ***Complexity*** Theory, 36 SIAM Rev. 157, 159-60 (1994).

[80] See Jerry S. Kelly, Social Choice and ***Computational*** ***Complexity***, 17 J. Mathematical Econ. 1, 1-2 (1988).

[81] See generally Book, supra note 79, at 157 (reviewing recent developments in the field); Traub & Wozniakowski, supra note 78, at 29 (detailing the intricacies of the peripherally relevant "information-based ***complexity***" field).

[82] See, e.g., Papadimitriou, supra note 78, at 3-13.

[83] One researcher, for example, has analyzed the ***complexity*** of a number of voting rules, and has established some conditions for computable electoral rules. See Kelly, supra note 80, at 3-8. Others have analyzed the ***complexity*** of manipulating elections, arguing that while almost all electoral rules are theoretically open to manipulation, some rules may be impervious to manipulation as a practical matter because of the ***complexity*** of the calculations that would be required. See J.J. Bartholdi III, et al., The ***Computational*** Difficulty of Manipulating an Election, 6 Social Choice and Welfare 227 (1989); J.J. Bartholdi III, et al., How Hard is it to Control an Election? 16 Mathematical & Computer Modeling 27 (1992).

[84] See Papadimitriou, supra note 78, at 3-5.

[85] See id. at 3.

[86] ***Redistricting*** can be characterized as a general problem where the value function itself is unspecified ex ante: for any of the formulations in the Appendix simply substitute a general value function V* for the specific one chosen. Showing that this general problem is hard is simple (substitute any "hard" combinatorial optimization for V*) but uninteresting because the value function has not been limited to realistic ***redistricting*** goals. In this section, it is demonstrated that the ***redistricting*** problem remains hard even for common ***redistricting*** goals. Even for the specific value functions discussed in this Article, there are many possible characterizations of the problem; fortunately, the choice of characterization will not affect the results discussed herein. See infra note 104.

[87] See supra note 56 (defining value function). Finding the arrangement of Iowa's 1980 census tracts that maximize population equality is an illustration of a ***redistricting*** sub-problem.

any ***redistricting*** sub-problem, the parameters consist of the population units from which one draws the plan, and the vector of values assigned to those population units. An instance of a problem is created by assigning values to all parameters. [91]

The second set of terms refers to solutions to the preceding problems. An algorithm is a general set of instructions in a formal computer language that, when executed, solves a specified problem. [92] An algorithm solves a problem if, and only if, it can be applied to any instance of that problem and is guaranteed to produce an exact solution to that instance. [93]

The final set of terms refers to properties of problems and their solutions. A problem is said to be computable [94] if and only if **[*104]** there exists [95] an algorithm which solves the problem. [96] For computable problems, the time-***complexity*** ("***complexity***") of an algorithm is defined to be a function that represents the number of the instructions that algorithm must execute to reach a solution. [97] The ***complexity*** of an algorithm is expressed in terms of the size of the problem, roughly equivalent to the number of input parameters. [98] The size of ***redistricting*** is simply

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[88] This method of district partitioning makes the ***redistricting*** problem somewhat more manageable because in the absence of a pre-specified value function, any "hard value" function may be substituted into the general problem to show that this problem is "hard."

[89] A parameter is a numeric descriptive measure of a population. See William Mendenhall & Terry Sincich, A Second Course in Business Statistics: Regression Analysis 20 (4th ed. 1993).

[90] Free, or independent, variables and their associated coefficients comprise the function through which resolution of a problem, or dependent variable, is sought. See id. at 84.

[91] An instance is a mathematical object to which questions are presented and answers are expected. See Papadimitriou, supra note 78, at 3. The specific kind of question asked characterizes the problem. See id. Problems capable of a deterministic solution, "yes' or "no' for example, are called decision problems. See id. Considering only decision problems, rather than problems requiring many different answers, is the much simpler alternative in ***complexity*** theory. See id.

[92] See, e.g., Browdy, supra note 37, at 165.

[93] To solve a problem is usually defined as obtaining any one output that has a specified property. See David Deutsch & Richard Jozsa, Rapid Solution of Problems by Quantum Computation, 439 Proc. Royal Soc'y London 553, 554 (1992). For example, an algorithm solves a population equality ***redistricting*** problem only if it is guaranteed to find a population equality maximizing solution for any set of census tracts.

[94] An expression is computable when it is calculable by finite means. Alan M. Turing, On Computable Numbers, With an Application to the Entscheidunsgproblem, 42 Proc. London Mathematical Soc'y 230, 230 (1937).

[95] The term "exists' is used in the formal mathematical sense, implying that it is not necessary to know which algorithm solves a problem to show that such an algorithm must exist. See Kelly, supra note 80, at 3-4 (distinguishing between computable and non-computable problems in the context of social choice rules).

[96] Turing demonstrated that there are non-computable problems for which solutions exist. See Turing, supra note 94, at 230-49.

[97] This definition assumes a serial (single processor) computation model, but the results are not altered if parallel-processing is used: the sum of the time needed by a set of parallel-processors to solve a problem can be no less than the total required in the serial model.

[98] The time ***complexity*** of an algorithm is conventionally denoted as O(f (n)) where n is the size of the problem. Papadimitriou, supra note 78, at 5-7. The number of steps necessary to solve an algorithm of O(n) ***complexity*** is a linear function of the number of inputs. See id. at 6.

the number of population units that are used as input. An algorithm is said to take polynomial time [99] if its time-***complexity*** function is a polynomial, and is said to take exponential time. [100]

A problem is computationally tractable if there exists an algorithm which solves the problem and is of polynomial ***complexity*** for all instances. [101] Conversely, a problem is computationally in- **[*105]** tractable if the (provably) optimal algorithm for solving the problem cannot solve all instances in polynomial time. [102]

This characterization of problem difficulty has two main strengths: (1) it is independent of any particular computer hardware design technology; and, (2) it classifies the difficulty of the problems themselves, not of particular methods used to solve these problems.

First, results under this characterization are implementation independent. Different computer languages (and encoding schemes for the parameters) may alter the time ***complexity*** of an algorithm, but no "reasonable" [103] language will convert a polynomial **[*106]** algorithm to an exponential algorithm. While a more powerful computer

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[99] Polynomial time is used to describe an algorithm whose "running time is a polynomial in the length of the input." Stephen A. Vavasis, Nonlinear Optimization: ***Complexity*** Issues 22 (1991). Polynomial time algorithms are considered efficient. See id. Such polynomial rates of growth are acceptable time requirements. See Papadimitriou, supra note 78, at 6. Nonpolynomial rates are the causes of concern; if the rates persist and the algorithms devised fail to optimally solve the problem in polynomial time, the problem becomes intractable and is not amenable to a practically efficient solution. See id.

[100] Any polynomial will grow at a markedly slower rate than any exponential. See Papadimitriou, supra note 78, at 6. In addition to analyzing the time required to solve a problem, analogous tractability criteria for the space requirements of a problem can be formulated. See id. at 29. Problems that require exponential space will also require exponential time. See generally id. at 6, 45, 147-51, 182, 491.

[101] See Book, supra note 79, at 159-60; Papadimitriou, supra note 78, at 182-84.

[102] See Book, supra note 79, at 160; Papadimitriou, supra note 78, at 45. Computationally intractable problems are also referred to as "computationally complex" or "computationally hard."

Although ***complexity*** has been defined in terms of time, it may be useful to think of it as a measure of cost as well. If time is costly, and if there are no exponential economies of scale associated with time, computationally intractable problems will be prohibitively expensive, since the cost to solve such problems will also grow at an exponential rate. Cf. Papadimitriou, supra note 78, at 360-62 (arguing that "using too many processors in our basic algorithm eventually translates to high parallel time when we apply our algorithm to large instances on a machine with a fixed number of processors"). Obviously, the time-costs of ***redistricting*** are unlikely to exhibit exponential economies of scale. If at all, wasted time will likely exhibit constant or negative economies of scale. If ***redistricting*** takes too long, it will disrupt elections.

[103] All known physically constructable computer architectures are "reasonable" in this sense, and it is believed that all possible computers based on classical physical principles preserve this property. See generally Papadimitriou, supra note 78, 359-67. There is some debate, however, over whether this implementation independence applies to hypothetical computers designed to utilize unexplored properties of quantum physics.

One researcher asserts that under the "many-universes" interpretation of quantum theory, one could design a device to exploit an infinite number of alternative universes for parallel calculation. See Deutsch, supra note 78, at 97-99. Under this controversial interpretation of quantum theory, devices may be built which would be able to compute some problems in polynomial time, even though these problems are only computable in exponential time on all convential computers. See Deutsch & Jozsa, supra note 93, at 556-58.

Another researcher makes a somewhat different argument that currently unresolved areas of quantum physics may provide fundamentally different ways of solving problems than those represented by the Turing model. See Roger Penrose, The Emperor's New Mind: Concerning Computers, Minds and the Laws of Physics 398-404 (1989). This argument asserts not only that quantum physics introduces mechanisms for problem solving which are different from those employed by today's computer, but that the human brain actually employs such mechanisms. See id. at 405-18; see also Roger Penrose, Shadows of the Mind: A Search for the Missing Science of Consciousness 348-88 (1994).

may be able to perform each atomic operation more quickly, it will not alter the time ***complexity*** function of the problem. Intractable problems cannot be made tractable through improvements in hardware technology. [104]

Second, results under this characterization apply to the problem, not to a particular method used to solve this problem. Problems which are shown to be difficult under this characterization are difficult for any computer method. Since it is the problem itself that requires exponential time, these problems cannot be made tractable through advances in software or algorithmic design.

This characterization is also subject to several important limitations. These limitations have caused its use as an absolute measure of problem difficulty to be criticized justifiably. [105] A brief summary of these limitations follow.

First, the distinction between tractable and intractable problems is most important for instances of large size, i.e. where the exponential factors in the time requirements of these problems become dominant. Consider the following two problems: Problem "A" is computationally intractable and takes O(1.1<prime><prime>) steps to solve; Problem "B" is computationally tractable and takes O(<n>[su'14']) steps. Although the time needed to solve problem A will eventually become much greater than the time required for problem B, for problem sizes less than one thousand, problem A can actually be solved much more quickly.

Second, this characterization requires that problems be solved exactly. Some problems that are computationally difficult to solve may be approximated much more quickly. If the approxi- **[*107]** mation reached is, provably or empirically, close enough to the optimal solution to the problem, for practical purposes, the exact best solution need not be achieved. [106]

Third, this characterization requires that the algorithms always reach a correct solution for every problem instance, a requirement that makes ***computational*** ***complexity*** a function of the worst-case problem instance. Since the analysis is based on the worst-case, the ***complexity*** of the problem on average may be overstated. Furthermore, since the solution-algorithm is permitted neither to make errors nor to give up on a problem, some algorithms that are probabilistic will be dropped from the analysis. While such algorithms do not formally "solve" a computationally hard problem, they may be quite useful if their rates of error and of failure are sufficiently low.

The three caveats above offer possible escape routes around computationally intractable problems. As will be established in Part V, the requirements of ***automated*** ***redistricting*** procedures make these avenues unlikely to be fruitful.

D. ***Redistricting*** is a Computationally Hard Problem

Redistricting is deeply connected to mathematical partitioning problems. Many researchers in computer science have examined partition problems and have reached some conclusions about their ***computational*** ***complexity***. The ***redistricting*** problem in general, and even many simpler ***redistricting*** sub-problems, are likely to be intractable.

Proving that a problem is intractable is difficult; researchers have been unable to determine whether most problems are tractable. [107] There are, however, a number of large classes of problems that computer scientists believe to be intractable. [108] The oldest of **[*108]** these is called the class of NP-complete problems. [109]

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[104] The time required to solve "intractable" problems, by the present definition, grows exponentially with increases in the size of the problem. This exponential growth quickly overwhelms even significant increases in computer speed. See Michael R. Garey & David S. Johnson, Computers and Intractability: A Guide to NP-Completeness, 6-11 (1983).

[105] S.E. Page, Two Measures of Difficulty, 929 Social Science Working Paper, Cal. Inst. of Tech. (1994).

[106] Heuristic methods yield such approximations. See Papadimitriou, supra note 78, at 299. Heuristics are "quick and dirty" algorithms that return feasible solutions that are not necessarily optimal. See id.

[107] See generally id. at 1-13.

Search for a proof of the intractability [110] of NP-complete problems has been one of the most famous open problems in computer science for over two decades. While there is no proof of intractability, no polynomial algorithms have been found to solve any of these problems. Because of the breadth of the class of problems, it is widely believed that no such algorithms exist. [111]

The NP-hard class is a superset containing the NP-complete class. This class is potentially harder to solve than NP-complete problems, because if any NP-complete problem is intractable, then all NP-hard problems are intractable. However, the reverse is not true. [112] Figure 1 below illustrates the probable relationship **[*109]** between the NP-complete, NP-hard and tractable classes of problems:

[SEE FIGURE IN ORIGINAL]

The most common ***redistricting*** sub-problems, such as finding the optimal set of compact districts, are NP-complete or NP-hard. [113] Table Three summarizes these formal results:

[SEE TABLE IN ORIGINAL]

[*110]

While reference to a number of particular characterizations as- **[*111]** sists in explanation, it is important to realize that none of these results are dependent on the use of a particular characterization. [122] Since all of these

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[108] Whether an NP-complete problem, see infra note 110, is intractable is an unresolved issue. See Book, supra note 79, at 157. The inquiry into the tractability of the problem is:

Whether there is an NP-complete problem that is not in the class P, where P is the class of decision problems solvable deterministically in polynomial time. If some NP-complete problem is not in the class P, then no NP-complete problem is in P; and if some NP-complete problem is in the class P, then every NP-complete problem is in P and P=NP. Thus the question of P=?NP is the question of whether or not the apparently intractable NP-complete problems are in fact tractable.

Id.

[109] Cook defined the first set of NP-complete problems which is now recognized as belonging to a large set consisting of hundreds of problems in many fields. See S.A. Cook, The ***Complexity*** of Theorem-Proving Procedures, in New York Assoc. for Computing Machinery 152 (1971). Richard Karp is known for characterizing polynomial-time reducibility as the most important property of NP-complete problems. See Richard M. Karp, Reducibilty Among Cominatorial Problems, in ***Complexity*** of Computer Computations 91-93 (Raymond E. Miller & James W. Thatcher eds. 1972); see also Book, supra note 79, at 157-58. Polynomial reductions are defined so as to preserve space ***complexity*** characteristics as well. See Papadimitriou, supra note 78, at 141, 177. Furthermore, there is a deep equivalency among all "complete problems": each problem can be transformed to any other by a simple functional mapping. See id. at 165-72.

[110] Only problems solvable in polynomial time are considered computationally tractable. See Book, supra note 79, at 159. Book furthered the notions of polynomial-time computations and polynomial time computable reductions that demonstrate how some problems can be solved through reductive processes to other problems. See id. Book also highlighted the importance of the class NP of decision problems that are solvable nondeterministically in polynomial time. See id.

[111] The class of NP-complete problems is not the only class that is believed to be intractable. See generally Book, supra note 79, at 157-74. For present purposes, however, only the NP-complete class and the related NP-hard classes of problems are considered.

[112] Any NP-hard problem can be shown to be NP-complete for at least some instances, but not necessarily for all instances; the problem becomes "hard' because almost every instance is difficult to compute. See Book, supra note 79, at 171-73.

[113] See infra App. 1. This finding contradicts other researchers' predictions that the time required for an experimental integer programming technique should be approximately linear in population units. See Garfinkel & Nemhauser, supra note 38, at B-506.

[122] Reformulating the ***redistricting*** problem in other ways can be useful if it suggests natural restrictions that can be put on the problem in order to simplify it. Restrictions that are natural in one context, such as planarity or graphs in the graph-partitioning

characterizations are in the set of NP-complete problems, they are all formally equivalent. Any algorithm that solves one problem can be simply reformulated to solve any other without a change in ***complexity*** class. The advantage of multiple characterizations is that each characterization may suggest various limitations that can be put on the problem making the problem easier to solve or approximate. [123]

These ***complexity*** results apply to the general ***redistricting*** problem as well as to the sub-problem illustrated. Since ***complexity*** results describe worst case properties, these results demonstrate that the ***redistricting*** problem, in its most general form, is at least as difficult as any NP-complete problem. [124] A further implication of these results is that finding an optimal district under any combination of these "hard" goals is also difficult. ***Automated*** optimal ***redistricting*** should not be expected to be tractable for an arbitrary choice of population units, number of districts, and an arbitrary value function. [125] **[*112]**

The ***redistricting*** goals listed in Table 3 are commonly used, straightforward, and relatively simple. For example, of all the ***redistricting*** goals specified in the literature, population equality is probably the most straightforward to quantify, and the easiest to evaluate. [126] Yet even the task of drawing a district plan to minimize population deviation alone is computationally hard. This does not demonstrate that the ***redistricting*** problem is hopeless. Part V explores the avenues available for managing intractable problems and the promise of these avenues for ***redistricting***.

IV. Attempts to Escape the Intractability of ***Automated*** ***Redistricting***

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problem, may not be obvious when the problem is formulated as an integer partition problem because these restrictions require torturous definitions in this context.

[123] For example, the independent set graphing problem is NP-Complete for general graphs, but can be solved in polynomial time for certain types of graphs, such as circle graphs. See Garey & Johnson, supra note 104, at 195.

[124] Each of these ***redistricting*** sub-problems is an instance of the ***redistricting*** problem. If some instances of the sub-problems require exponential time, then the general problem as well must also require this amount of time. Thus, the ***redistricting*** problem has a time ***complexity*** at least as large as its sub-problems. It has not shown that these sub-problems represent the worst cases of the ***redistricting*** problem. Therefore, some instances of the general ***redistricting*** problem may be worse than NP-complete, or even unsolvable.

[125] Some object to the claim that ***automated*** ***redistricting*** is formally intractable, since political actors are able to gerrymander so well. If political actors can gerrymander optimally, then so should computers. This argument is based on three false assumptions:

Claim One: Humans already perform (near) optimal gerrymandering. There is little evidence for this claim. In fact there is much evidence to the contrary, including many examples of attempted gerrymanders that have had far from the intended results.

Claim Two: ***Automated*** ***redistricting*** is no more difficult than gerrymandering. Again, the available evidence seems to point toward the opposite conclusion. ***Automated*** ***redistricting*** is significantly more complicated than gerrymandering in three important respects. First, gerrymandering usually involves the maximization of one simple goal. Optimal ***redistricting*** may involve many simultaneous, complicated, and conflicting goals. Next, gerrymandering is often limited to relatively small modifications of an existing plan. ***Automated*** ***redistricting*** processes must examine a much wider range of possible plans. Finally, gerrymanders need not be optimal to be politically effective. The social value function being optimized by an ***automated*** process may be much more sensitive to suboptimality.

Claim Three: If humans can perform a (mathematical) task well, computers can (at least theoretically) perform the same task as well. The lack of success in the field of artificial intelligence is evidence to the contrary. See Hubert L. Dreyfus, What Computers Still Can't Do 96-97 (1992) (reviewing the successes and failures in this field and arguing against claim three). Even advocates of artificial intelligence recognize that such tasks as recognizing human social and political relationships, and applying "common sense" are among the hardest problems for computers. See William A. Taylor, What Every Engineer Should Know Now About Artificial Intelligence 38 (1988)

[126] Even this goal can be defined in a number of ways depending on how the inequality between two districts is measured. For instance, one might focus on the maximum differences between the largest and smallest districts, or their ratios, or their average differences.

Part IV demonstrates that the ***redistricting*** problem is NP-complete and briefly reviews several caveats to the definition of intractability. [127] The section will explore whether these caveats **[*113]** allow ***automated redistricting*** to escape intractability.

NP-completeness is a limited form of intractability. [128] There are a number of possible avenues for dealing with these types of problems.

Not all variations of ***redistricting*** sub-problems are intractable. If the ***redistricting*** values are restricted sufficiently, then a tractable way of finding optimal districts becomes possible. Similarly, if the set of inputs in the problem are restricted enough, the problem may be quite tractable.

Some practical approaches to general ***redistricting*** problems also remain plausible. If the problem cannot be made tractable through restricting the values or data, a probabilistic method might solve the problem most of the time, or a deterministic algorithm might work well in most cases. Alternatively, a method that quickly finds an approximately-optimal solution may be developed. Nevertheless, caution should be exercised. Although **[*114]** these escape routes are open in theory, each presents technical and political difficulties. This section will explore these possible escape routes, and will discuss technical problems and political ramifications attendant to each.

A. Solving NP-Complete Problems: An Example

Despite the fact that all NP-complete problems are reducible to each other, [129] in a practical sense they are not all equally hard to solve. [130] Some problems can be restricted easily, approximated closely, or answered

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[127] Recall that a problem is tractable only if a polynomial-time algorithm exists which is guaranteed to exactly solve all instances of this problem. This is an extremely demanding problem.

[128] NP-complete problems certainly do not encompass all intractable problems, nor do they comprise the "worst" class of intractable problems. See Papadimitrou, supra note 78, at 182 (explaining that when a problem is NP-complete, it is established that it is not P and therefore, that there is at least some likelihood that the problem can be solved in polynomial time). A problem may be intractable for a number of different reasons. Three are identified and discussed in order to illustrate varying levels of problem difficulty.

First, it may be that the solution is unmanageably large or otherwise unmanageable. For example, consider the problem: "Enumerate the set of all possible district plans." The solution for this type of problem is untenable for all but the most minuscule instances. This type of problem is provably intractable. Even if eventually obtained, solutions to these kinds of problems usually have minimal utility.

Second, it may be that the solution is manageable, but difficult to verify. Problems of this type are esoteric and too awkward to describe. See generally Mihir Bellare & Shafi Goldwasser, The ***Complexity*** of Decision Versus Search, 23 SIAM J. Computing 97 (1994) for one demonstration that proving membership in a solution set can be harder than deciding it.

Third, it may be that finding the solution is difficult but, once found, it can be easily verified and put to use. See Papadimitriou, supra note 78, at 183-84 (providing a proof for and concurring that this type of problem is "perhaps the most useful"). An example of this type of problem: "Does there exist a district plan where the maximum population difference between districts is less that B; if so, specify that plan." As long as computation of the value function f is tractable, finding a satisfactory plan may be difficult; once found however, it is easy to verify that the value of the plan exceeds B. Accordingly, these problems present the least difficult form of intractability.

[129] See Garey & Johnson, supra note 104, at 13 (referencing Stephen Cook's work in "The ***Complexity*** of Theorem Proving Procedures"). A reduction is an algorithm that solves problem A by transforming any instance of A to an equivalent instance of the previously solved problem B. For further elaboration on the concept of reduction, see generally Papadimitriou, supra note 78, at 12.

[130] See Garey & Johnson, supra note 104, at 8-9; see also Papadimitriou, supra note 78, at 6 n. + (discussing the simplex method, which is known to be exponential in the worst case, but to perform consistently well in practice); cf. id. at 6-7 (discussing the controversy surrounding the functional utility of using the "polynomial paradigm" at all since "there are efficient computations that are not polynomial, and polynomial computations that are not efficient in practice").

probabilistically. [131] A simple example illustrates the practical difference between two formally intractable problems:

A wealthy art-collector has died, leaving an estate of unique and valuable items. The executor must divide the estate between its inheritors according to three rules: (1) all of the items must be given away (sale and division of proceeds is not permitted); (2) each item must go to a single inheritor; and (3) the subjective equality of the division must be maximized (i.e., the value that Inheritor A subjectively assigns to his share must be as close as possible to the value that Inheritor B subjectively assigns to her share, etc.).

Formally this is an NP-complete problem, but practically it might not be very "hard" to solve. All of the inheritors might be antique dealers who know current market prices and therefore share common values for each object. Perhaps there are only two inheritors, and no item is "priceless." In such cases, an optimal solution can be derived in polynomial time using dynamic pro- **[*115]** gramming. [132]

Suppose, however, that each inheritor assigns a different value to each object. For example, Inheritor A attaches great sentimental value to the sofa and end-table combination, whereas Inheritor B hates the end-table but has always coveted the sofa and matching wall-hanging. Although the initial problem is structurally the same, this latter scenario presents practical difficulties that were not presented by the former.

B. Problem Size and ***Computational*** ***Complexity***

Even problems which are computationally hard may be solved easily for sufficiently small cases. [133] The ***redistricting*** problem is large enough however, that reliance upon this principle of tractability is impractical. [134]

The number of possible solutions to a problem grows both as a function of the number of population blocks that are used, and to a more limited extent, as a function of the number of districts being drawn. [135] The time necessary to solve ***redistricting*** sub-problems grows exponentially as a factor of both districts and population blocks. [136]

The number of districts per state is reasonably small, typically ranging from one to approximately fifty. [137] In contrast, the num- **[*116]** ber of population blocks is quite large, generally ranging from several thousand to approximately one hundred thousand. [138]

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[131] See Papadimitriou, supra note 78, at 183.

[132] If there are more than two inheritors but all of the objects are of approximately equal value, then minimizing the inequality between shares is not difficult. Even if the antiques differ greatly in value, it is possible to find a relatively equal division. Approximability results for this partition problem are unavailable, but the following algorithm is quite successful for bin packing: "Order all of the objects from least to most valuable, then assign each object to each inheritor in turn until the list is exhausted."

[133] See supra note 50; cf. Papadimitriou, supra note 78, at 183 (advancing that any sufficiently generalized problem will become NP-complete or worse).

[134] Papadimitriou glorifies the utility of the NP-complete problem in the context of various practically important and natural ***computational*** problems. See Papadimitriou, supra note 78, at 182. While portraying NP-completeness as beneficial for mathematical abstractions of "actual, physical objects or real-life plans that will ultimately be constructed or implemented," Papadimitriou properly qualifies his statement: "In most applications, the certificates are not astronomically large, in terms of the input data." Id.

[135] See supra Part III.

[136] See infra App. 1.

[137] See generally Congressional Quarterly Inc., Congressional Quarterly's Guide to 1990 Congressional ***Redistricting***: Part 2, at 410-20 (1993) (providing a ***redistricting*** summary for a majority of the states and including the number of districts in each). California has fifty-two districts, the most of any state listed, see id. at 411; Maine, Nevada, New Hampshire, and Rhode Island each have two, the least among the states included in the summary, see id. at 414-16, 419.

Under these conditions, the time required to solve the ***redistricting*** problems can be only approximated. Because of the large number of solutions that must be searched however, it is reasonable to expect the exponential time-requirements of the algorithm to be dominant, even if the exponential growth is relatively small. [139]

C. Restricting the ***Redistricting*** Problem

It is well understood that any problem, when sufficiently generalized, becomes computationally hard, and conversely, any sufficiently restricted problem becomes trivial. [140] Because of this common-sense principle, it may be asked whether natural restrictions [141] placed upon the ***redistricting*** problem could make it tractable. [142]

The ***redistricting*** problem is amenable to three basic types of restrictions. First, consideration can be restricted to particular ***re-*** **[*117]** ***districting*** goals. Second, the population units or data fed into the ***automated*** ***redistricting*** process can be restricted. Third, results can be restricted through the elimination of undesirable or illegal plans.

Each of these restriction possibilities will be discussed separately. In practice, however, all types of restrictions often are combined to simplify problems and avoid ***computational*** ***complexity***. Although combining multiple restrictions might remedy the practical difficulties of finding a plan, political and normative problems persist.

1. Restrictions on Value Functions

The computer's task becomes easier if the number and types of goal functions to be optimized are restricted. Such restrictions could provide for the use of specially tailored optimization algorithms. Restrictions also could eliminate exponential time requirements, allowing for the utilization of more general optimization algorithms. Goal functions which are "easy" to optimize must exist, but their use in ***redistricting*** present two difficulties.

The first difficulty is a purely practical one: the present ***redistricting*** goals do not lend themselves to easy computation. To optimize even the simplest and most universally-accepted value functions presents a computationally hard problem. [143] Moreover, any (simply weighted) combination of these functions, with each other or with any other function, also will be computationally hard.

The second difficulty is normative and political. Proponents argue that ***automated*** ***redistricting*** allows for the neutral implementation of goals which have been derived from political processes. When restriction of these goals becomes technically necessary, however, the separation of goal choice and implementation is violated. Thus, it may be normatively and politically unacceptable to allow the limitations of a computerized process to restrict, ex ante, the types of representational goals that society is allowed to pursue. **[*118]**

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[138] The U.S. Census supplies data for ***redistricting*** at the block level and at higher levels of aggregation resulting in up to approximately 580,000 units. See Bureau of the Census, U.S. Dept. of Commerce, 1990 Census of Population and Housing, Population and Housing Characteristics for Census Tracts and Block Numbering Areas, Sec. 1 of 3, 15 (1993). Much ***redistricting*** is performed using larger units such as block groups, which number from 692 in Alaska to 21521 in California. See id.

[139] See generally Papadimitriou, supra note 78, at 10-11 (explaining that time or space requirements can make a problem intractable because either could be exponential). An exponential growth factor as low as 1.001n is likely to make such large problems intractable.

[140] See id. at 183.

[141] The term "restriction" refers to limitations placed upon the goals and inputs to be considered by the ***redistricting*** algorithm. These restrictions are distinct from constraints on districting. Cf. supra note 59 (defining "constraint" and explaining its ***computational*** significance).

[142] See generally Papadimitriou, supra note 78, at 183-84 (arguing that restrictions on NP-complete problems can be accomplished by "rewriting any instance so that "undesirable features' go away.").

[143] See supra Part III.

2. Restrictions on Input

Theoretically, intractability can be avoided by restricting input. In the context of ***automated*** ***redistricting***, the basic inputs are defined in terms of vector-valued population blocks. Restrictions can be placed on population blocks directly or indirectly.

If it is possible to predict which cases create problems for ***redistricting*** algorithms, then it is possible to directly reduce the ***computational*** ***complexity*** of a problem by eliminating input which gives rise to its worst cases. For example, if the only goal is to create district plans to minimize population inequalities, then the problem can be simplified by restricting all population blocks to be equal in size. [144] While not completely trivializing the problem, this eliminates the possibility of cases which might cause the algorithm to take exponential time to complete. [145]

Input restrictions can be used indirectly for more complicated problems. In order to reduce the number of solutions that an optimization algorithm needs to consider, input restrictions often are combined with restrictions on ***redistricting*** goals. "Branch and bound" and "cutting planes methods" are variant applications of this general principle. [146] These methods usually are very sensitive to the choice of goal functions. The input set must be restricted to guarantee that the branch and bound proce- **[*119]** dure itself does not take exponential time. [147]

Although the ***automated*** ***redistricting*** process can be made easier through restricting inputs, many useful restrictions may not be practical or possible. The irregularity of population blocks is the chief cause of the worst case behavior of ***redistricting*** programs. Given the irregular boundaries and non-uniform population distribution in many states, most of the worst-case characteristics are inherently exogenous. For example, geographical regularities can help to draw compact districts. If a state were a perfect square with people uniformly distributed across it, then drawing equally-sized, compact districts would be simple. Drawing districts as equally-sized rectangles would even be possible. Because, however, the vast majority of the states have jagged or irregular boundaries, it is impossible to draw such regular districts, and finding the best set of districts is quite difficult.

Furthermore, input restriction can eliminate worst-case behavior only when value functions are similarly restricted. Restrictions that simplify the satisfaction of one goal may exacerbate the difficulties involved in satisfying other

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[144] An alternate strategy for drawing equally-populated districts is to arbitrarily split population blocks. This method is among the most popular in the political arena. For present purposes however, "population unit" is assumed to reflect the finest grained unit. Splitting population blocks, therefore, must be regarded as an approximation method. See generally supra Section 3.C. (discussing approximation methods and their limitations).

[145] Similarly, for a problem restricted as such, any heuristic method will quickly find an optimal solution. See generally supra Part II (introducing and briefly discussing heuristic methods).

[146] See E. Balas & P. Toth, Branch and Bound Methods, in The Traveling Salesman Problem 361 (E. L. Lawler et al. eds., 1985); Gerhard Reinelt, The Traveling Salesman: ***Computational*** Solutions for TSP Applications 202-03 (1991). Enumerative methods, such as branch and bound and implicit enumeration, solve optimization problems by performing a repetitive process whereby progressively more relaxed problems and more viable solutions evolve. See Balas & Toth, supra, at 362. The procedure ends either when a feasible solution is derived or when no better solution is possible. See id. The best solution found is considered a global optimum. See id.

[147] Many kinds of "relaxations" can be applied to the ***redistricting*** problem. For example, if census blocks are assumed to be infinitely divisible, then the ***redistricting*** problem could be reformulated as an integer-linear program that is simple to solve.

Properly setting the bounds of the problem is the most challenging. Knowledge about the goal function, as well as mathematical intuition and cleverness, are required to successfully relax a problem.

goals. In addition, few restrictions are demonstrably useful across a variety of value-functions. [148] As a result, such restrictions may not be politically or normatively justifiable. **[*120]**

3. Restrictions on Plans

Restricting the types of districts or plans might be more useful than attempting to restrict goals or data. This form of restriction would simplify optimization by eliminating plans which do not meet particular goals. [149] Most political planners, however, would consider such restrictions unrealistic and overly restrictive.

Although mathematically convenient, plan restrictions implicate restrictions on value functions or inputs which could be politically and normatively problematic. For instance, if the restriction requires that plans be contiguous, then any value functions that allow contiguity to be weighed against other goals will be excluded. This equivalence demonstrates how reformulating limitations as plan-restrictions inescapably restricts value functions and input. Plan restrictions may be valuable for formulating issues clearly, but they also may be open to abuse by political manipulators. What might appear to be a legitimate plan restriction, potentially could be a deliberate restriction of a value function.

D. Can Sub-Optimal ***Redistricting*** Help?

Recall the demanding requirements attendant to theoretical ***computational*** ***complexity***. Political solutions may not require the precision that theory demands. Perhaps, if requirements are relaxed, easy and practical methods of ***redistricting*** may become possible. This section explores whether plans that are optimal "most of the time," or are "close enough" approximations, might present practical ***redistricting*** options.

1. Optimal ***Redistricting*** - Most of the Time

Potentially, two alternative methods could provide optimal ***redistricting*** "most" of the time. The first method would demand optimal results for most plans. The second would yield close-to-optimal results for all plans.

Theory requires that algorithms be guaranteed to generate a **[*121]** correct solution to a problem; [150] relaxation methods abate this requirement to allow occasional errors. [151] Hypothetically, relaxation presents the possibility of "solving" some NP-complete problems. Current research suggests, however, that NP-complete problems are not susceptible to this type of solution. [152]

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[148] There is a notable exception: problems sometimes can be solved in polynomial time by restricting the maximum value assigned to any one population block. NP-complete problems that are demonstrably solvable under this restriction are called "pseudopolynomial." Those that remain intractable under this restriction are termed "strongly NP-complete".

A number of ***redistricting*** sub-problems are "pseudopolynomial" when further restricted to draw only two districts. That is, it is possible to compute the optimal plan in polynomial time for many, but not all, sub-problems if (1) only two plans are to be drawn; and (2) an upper bound is set on the value of each population block. For drawing more than two districts, these same problems are strongly NP-complete. See infra App. A for more details.

[149] Hypothesize, for example, that the only goal is maximizing one form of compactness. It becomes easier to find "optimal districts" if those plans that are not contiguous rectangles are eliminated.

[150] See Garey & Johnson, supra note 104, at 4.

[151] See id. at 3-4. Expanding the solutions to algorithms which produce the result with a probability [epsilon] does not bound the magnitude of the error, rather the probability of its occurrence.

[152] An algorithm is "boundedly probabilistic polynomial" (BPP) if it computes a "solution" to all instances of a problem in polynomial time, and all "solutions" have a probability greater than 50% of being correct. Because the probability of error on each execution of the algorithm is independent, BPP algorithms can be executed repeatedly to obtain any desired probability of correctness. It is hypothesized that no NP-complete problem is a member of the class BPP. See Papadimitriou, supra note 78, at 269.

As long as a case exists for which solution requires exponential time, the problem is computationally intractable by definition. Reliance upon probabilistic methods might not be necessary, however, if algorithms solve the ***redistricting*** problem with certainty either on average or in practice. [153]

This possibility presents an empirical inquiry; specifically, how well do real methods work on real data? [154] Previous attempts to generate optimal districting plans suggest that this problem is quite difficult "in practice." As of yet, no procedure is both demonstrably optimal and generally effective on data sets large enough to be useful for political ***redistricting***. [155]

The question persists whether ***redistricting*** sub-problems can be solved in practice. In most cases, analysis of average-case ***complexity*** requires the specification of a particular distribution func- **[*122]** tion. Average-case ***complexity*** results specific to ***redistricting*** sub-problems currently are unavailable for any distributions. [156] For the class of simple computable distributions, however, the average-case ***complexity*** of all NP-complete problems is exponential. [157] For at least some distributions, therefore, average-case ***complexity*** cannot avoid intractability.

In sum, the prospects are dim that a tractable procedure for optimal ***redistricting*** will be discovered. Realistically, ***automated*** procedures almost certainly will result in sub-optimal ***redistricting***. Accordingly, it is necessary to discuss both the practical and political implications surrounding approximation methods.

2. Guaranteed Approximations

Computational ***complexity*** measures the difficulty of obtaining optimal solutions, but it says little about the difficulty of approximation. While equivalent for optimal solutions, NP-complete problems are not equivalent for approximation. Some problems are much easier to approximate than others.

For some NP-complete optimization problems, there are methods that will generate, in polynomial time, a solution that is guaranteed to be within a specified percentage of the optimal value. [158] Methods that yield arbitrarily close

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[153] Concerning its behavior upon problems drawn from a theoretical, mathematically-defined probability distribution, an algorithm might be said to work well on average; evaluated against actual empirical data, an algorithm would be deemed to work well in practice.

[154] Compare the conceptual distinction identified in the above text with the hypothetical antique dealer/executor problem discussed supra Part IV.A. Recall that the empirical nature of the objects and inheritors influenced the practical difficulty of problem solution. The same principle operates in the context of ***redistricting***, yet inevitably the empirical data seems to offer no relief from practical ***computational*** difficulty.

[155] See Gudgin & Taylor, supra note 36, at 143-47 (discussing some early attempts at ***redistricting***); see generally supra Part II.

[156] It is not obvious what mathematical function can be used to accurately describe the distribution of values over population blocks. But see Gudgin & Taylor, supra note 36, at 31-53.

[157] See Ming Li & Paul M. B. Vitanyi, Average Case ***Complexity*** Under the Universal Distribution Equals Worst-Case ***Complexity***, 42 Info. Processing Letters 145, 146, 149 (1992).

[158] See Papadimitriou, supra note 78, at 299-300. Measurements of approximation based on the value of the solutions produced are distinct from those based on the percentage of solutions which are excluded. For example, hand-drawn districts are likely to be valued in the top 99% of a set of all possible solutions, merely because there are so many possible districts with little value. These hand-drawn districts may be significantly lower in value, however, than the optimal district.

"solutions" in polynomial time are known as fully polynomial approximations. [159] No fully **[*123]** polynomial approximations exist for many of the ***redistricting*** sub-problems. [160]

Although the ***redistricting*** problem does not allow arbitrarily close approximations, this fact has not excluded the possibility of approximating a solution within some fixed percentage of the optimum. No such guaranteed approximation procedure for a ***redistricting*** sub-problem has yet been demonstrated, but the question remains open. [161]

3. Making an Educated Guess

Indisputably, heuristic methods are the most common approach to ***automated*** ***redistricting***. [162] Although considered useful for problem solving, heuristics do not guarantee optimality (or approximate optimality). Heuristic methods can be used, however, to reduce the set of possible plans.

Several heuristics have proven useful in solving partition problems:

Simulated annealing: this method's behavior is analogized to the slow cooling of hot metals, in order to allow molecules to arrange themselves into a stable, low-energy [163] configuration. [164] **[*124]** One variation of this technique has been useful on a number of set-partition problems. [165]

Genetic algorithms: these use processes that are analogous to crossover, mutation, and evolutionary selection. [166] Variants of genetic algorithms have generated solutions to selected graph-partitioning problems. [167]

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[159] Fully polynomial approximations are possible by combining (1) time requirements bounded by a polynomial function with (2) an approximation method, known as an epsilon approximation, that will yield only solutions within a minimum deviation from the optimum. See, e.g., id. at 301-02. More specifically, epsilon approximation methods guarantee that the ratio of the value of the approximal solution to the value of the true optimum is no less than 1- [epsilon] . See id. at 302.

[160] See id. at 307 (demonstrating that no fully polynomial approximations exist for problems which are strongly NP-complete); see also infra App. 1 (illustrating that a number of ***redistricting*** sub-problems are at least strongly NP-complete).

[161] It seems probable that effective guaranteed approximation limits are obtainable for the problem of minimizing population inequalities between districts.

[162] See generally supra Part II (noting that all ***automated*** ***redistricting*** procedures implemented thus far have relied upon heuristic methods).

[163] The "energy" component in the annealing process is analogous to an algorithm's objective function. See Browdy, supra note 37, at 173. As minimizing the energy configuration is the goal of annealing when hot metal is cooled, so minimizing the objective function is the goal of simulated annealing when optimization is sought. See id. at 173 n.30 (exemplifying the "energy" of one system as "Energy = Compactness + Contiguity + Population Equality" and providing the mathematical expression of such a system).

[164] Simulated annealing works by starting with a random configuration of a system and then changing random pieces of that system. See id. The process retains changes that improve the system and occasionally keeps changes that harm the system until an optimal or near optimal configuration is reached. See id. at 172 & n.28.

[165] See V. Zissimopoulos, et al., On the Approximation of NP-Complete Problems by Using the Boltzmann Machine Method: The Cases of Some Covering and Packing Problems, 40 IEEE Transactions on Computers 1413, 1417-18 (1991) (summarizing the effectiveness of this technique on problems including Maximum Independent Set, Set Partitioning, Clique, Minimum Vertex Cover, Minimum Set Cover, and Maximum Set Packing).

[166] The "natural evolution" metaphor aptly conceptualizes the process by which genetic algorithms operate. Genetic algorithms work with a population of solutions which undergoes "adaptation." See R. Chandrasekharam et al., Genetic Algorithm for Node Partitioning Problem and Applications in VLSI Design, 140 Computers and Digital Techniques 255, 255 (1993). By the simulated evolution process of a genetic algorithm, the candidate solutions progressively improve as they retain the better characteristics of multiple solutions of earlier generations. See id. For definitions and illustrations of "crossover" and "mutation," see generally id. at 256-57.

Divide-and-conquer methods: these divide a set of solutions into subsets depending on the solutions' qualitative value; the lowest-valued subsets are then eliminated, and the process is repeated until only the "optimal" solution (or subset of solutions) remains. [168] Set-partitioning algorithms can be used in conjunction with this method, in order to exact the optimal subset divisions. [169]

Branch-and-bound methods: these solve a problem by repetitively relaxing the bounds on the problem such that the solution **[*125]** subsets derived therefrom steadily increase in value. [170] This technique has been used to transform set-partition problems into integer-programming problems. [171]

Encouraging test case results imply that these heuristics may be useful for ***redistricting***. The size of the experimental problems, however, have been confined to approximately ten districts and one-hundred population units. Obviously, this input is much smaller than that necessitated by the typical ***redistricting*** problem. Moreover, many of these methods have performed poorly for the common sub-problem of drawing compact districts according to the standard definition of compactness. [172] Therefore, further experimentation with heuristics is necessary.

4. Political Implications of Sub-Optimal ***Redistricting***

While sub-optimal ***redistricting*** methods offer the only practical means of implementing ***automated*** ***redistricting***, these methods have a number of disturbing normative and political implications. Proponents must, therefore, be cautious before arguing that ***automated*** ***redistricting*** presumptively remedies political manipulation of ***redistricting***. [173] These advocates are well advised to heed potential infirmities inherent in the current methods, and the difficulty of judicial review of these methods for constitutional legitimacy.

First, because heuristics currently present the only feasible method for ***automated*** ***redistricting***, there is no guarantee that a **[*126]** solution is close to optimal. In practice, the effectiveness of a heuristic process can be measured through experimentation and simulation, but such analyses will depend upon the value functions used. If the value functions prove unreliable, then ***automated*** ***redistricting*** methods lose much of their normative appeal.

Second, guarantees of performance notwithstanding, heuristics and approximation techniques are selected from a large body of solutions, and the selection may be biased. Assuming arguendo that it is possible to select methods which arrive at solutions in the top 99.99% of all possible district plans, these methods might still yield a solution that has a value significantly lower than other solutions in the set because the pool of possible solutions is staggeringly large. Furthermore, heuristic methods can be biased toward certain classes of solutions.

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[167] The performance of genetic algorithms has been rated for test scheduling problems, see id. at 257-58, input encoding problems, see id. at 258-59, and programmable logic arrays, see id. at 259. The ratings indicate that genetic algorithms yield more "acceptable results than those obtained by the heuristic methods currently in vogue." Id.

[168] See, e.g., Robert Sedgewick, Algorithms 48-51 (1983).

[169] See, e.g., John Hershberger & Subhashi Suri, Finding Tailored Partitions, 12 J. Algorithms 431, 432 (1991) (suggesting that a developed clustering technique capable of dividing a polygon into two optimally compact sub-regions in polynomial time, could be "used as a heuristic to obtain an arbitrary partitioning by recursive invocation").

[170] See Balas & Toth, supra note 138, at 362.

[171] See Elia El-Darzi & Gautam Mitra, Solution of Set-Covering and Set-Partitioning Problems Using Assignment Relaxations, 43 J. Operational Res. Soc'y 483, 483-85 (1992). Branch-and-bound techniques also have transformed set-partition problems into Langrangean (among other assignment) relaxation heuristics. See id. at 487.

[172] See Micah Altman, The Consistency and Effectiveness of Mandatory District Compactness Rules 33-43 (Apr. 6, 1995) (unpublished manuscript, on file with the Rutgers Computer and Technology Law Journal) (evaluating the consistency of seven typical measures of compactness).

[173] Cf. Issacharoff, supra note 16, at 1703 ("If reviewing courts use the absence of a verifiable computer algorithm or some other clear ex ante articulation of the bases for reapportionment decisions as presumptive evidence of constitutional infirmity, the logic of adjudication may … propel the states toward the use of verifiable criteria for reapportionment" (emphasis added)).

Third, another form of bias can enter the process, because heuristic methods are sensitive to initial conditions. [174] Most ***redistricting*** heuristics are path-dependent, therefore the starting conditions influence the types of plans generated by these methods. [175] If all near-optimal district plans are similar in type, then the starting conditions introduce no bias into the process. If dissimilar however, the choice of starting conditions provides an implicit opportunity to manipulate the outcome.

Fourth, heuristic methods are susceptible to manipulation. The Ohio State University (OSU) method provides one illustration of a method that both embodies a particular bias and can be manipulated for a variety of goals. The OSU method creates one central district from which other districts radiate. [176] For its 1980 **[*127]** ***redistricting*** plan, Arizona exploited the principle of the OSU method to perfect a racial gerrymander, as shown in Figure 2: [177]

[SEE FIGURE IN ORIGINAL]

Finally, judicial review of this process is inherently complicated. Proponents argue that courts could scrutinize whether a method is biased, and whether it is narrowly tailored to pursue legitimate goals. The quality of a plan and the presence of manipulation becomes an intensely technical question, dependent upon both the choice of value function and the behavior of the method. For example, it is unrealistic to expect a judge to possess the technical competence to determine if a ***redistricting*** goal, such as the creation of minority opportunity districts, has been properly weighted by the ***redistricting*** algorithm. The reality is that the political groups, equipped with large computing resources and capable of hiring experts in the field of computer science, are in a much better position to discover which combinations of methods and functions are advantageous to private aims. ***Automation***, therefore, may enhance the incidence of ***redistricting*** manipula- **[*128]** tion, because its technical sophistication could preclude effective judicial review. [178]

If manipulation cannot be eliminated by ***automation***, perhaps then, all parties should be afforded the opportunity to choose plans. Why not make computer resources available, let interested parties submit plans, and then select the plan with the highest value?

This scheme has three shortcomings. First, incentives remain for strategic manipulation at the value-choice stage. ***Automation*** cannot be used as a "veil of ignorance" to force participants to bargain fairly. Second, this scheme discourages compromise. Because the plan with the highest value acts as a "trump" and is implemented in toto, there is a strong incentive for secrecy. If an opponent knows the details of a proposed plan, he can determine whether his plan requires improvement. If so, he may be able to use iterative improvements to submit a slightly higher-valued plan that better suits his goals. Third, since ***redistricting*** problems are complicated, and ***automated redistricting*** problems are even more so, well-funded, private interests are likely to maintain a significant advantage in their ability to use computing resources even if such resources are made widely available.

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[174] See, e.g., Papayanopoulos, supra note 36, at 185 (acknowledging, as a drawback to heuristic methods, that different starting points can produce different sequences and different sets of districts); see also supra note 48.

[175] See generally supra Part II (describing the Weaver-Hess procedure and the Nagel method, both of which are sensitive to initial conditions).

[176] See Myron Q. Hale, Computer Methods of Districting, in Reapportioning Legislatures 96, 108-10 (Howard D. Hamilton ed., 1966); see also Figure 2a. This method will always draw the same type of plan, even though the plans' particulars will vary.

[177] See Figure 2. The final, court-approved plan is based upon the same general lines. Compare Figure 2 with Congressional Quarterly, Inc., supra note 137, at 8 and Larry Light, New Arizona Districts: A Quandary for Udall, 40 Cong. Q. Wkly Rep. 161, 162 (1982). To maximize the clarity of the comparison, Figure 2 illustrates only Arizona's state boundary and district lines.

[178] Judicial review of such convoluted and technical processes would be significantly more difficult than judicial review of the individual districting plans. Of course, scrutinizing the plan is always a viable option, whether it is hand-drawn or the product of ***automation***. The risk arises where courts presumptively assume that a plan is legitimate, merely because it is the product of an ***automated*** ***redistricting*** process. Cf. supra note 150 and accompanying text. Judicial "verification" of ***redistricting*** algorithms and ***automated*** methods is impractical and unrealistic. Accordingly, assigning presumptive weight to the mere presence of an ***automated*** scheme is ill-advised.

V. ***Automating*** the ***Redistricting*** Process Limits Conflicts with Representational Goals

Apart from any technical difficulties attendant to finding optimal districts, it is necessary to formalize the notion of an "optimal district." An attempt at formalization gives rise to two political and normative problems. First, the technical details of formalization may be used to manipulate the ***redistricting*** process; how goals are defined determines what type of plan will result. Sec- **[*129]** ond, formalization necessarily precludes consideration of a number of important representational goals.

A. Limit on the Formalization of ***Redistricting*** Goals

The first step to ***automating*** the ***redistricting*** process is to quantify the goals. The extent to which a plan satisfies, or does not satisfy, each of the goals must be represented by a number. [179] For example, population inequality might be quantified by subtracting the population of the largest from the population of the smallest districts.

Not all goals are easily quantifiable. Particularly problematic are representational goals, because these do not refer to a directly measurable aspect of a district. Unlike population or shape, which are objectively quantifiable, representational goals are defined by their efficiency at achieving political fairness. [180] Representational goals, such as protection of communities of interest and non-dilution of minority representation, are especially difficult to describe formally.

The Court has recognized a wide variety of valid goals for ***redistricting*** in a number of cases. In Karcher v. Daggett, [181] the Court's opinion stated that "any number of consistently applied legislative policies" might be important enough to justify deviations from absolute population equality, including compactness, protection of municipal boundaries, preserving the core of previous districts, and avoiding contests between incumbents. [182] In Davis v. Bandemer, [183] the Court determined that "vote dilution" is a relevant and justiciable factor in ***redistricting***. [184] Most recently, **[*130]** in Miller v. Johnson, [185] the Court chastised the defendants for ignoring "traditional" principles including contiguity, compactness, race neutrality and "respect for political subdivisions or

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179 In theory this number could represent either an ordinal or a cardinal ranking. All of the ***automated*** procedures developed to date have used cardinal rankings.

180 See generally *Gaffney v. Cummings, 412 U.S. 735, 753 (1973)* ("The very essence of districting is to produce a different a more "politically fair' result than would be reached with elections at large, in which the winning party would take 100% of the legislative seats.").

181 *462 U.S. 725 (1983).*

182 *Id. at 740.*

183 *478 U.S. 109 (1986).*

184 *Id. at 143.*

185 *115 S. Ct. 2475 (1995).*

communities defined by actual shared interests." [186] It would be difficult to program a computer to evaluate and balance all of these factors.

To evaluate the representational goals of a ***redistricting*** plan, it is necessary to recognize natural geographic, social, and historical patterns and to apply knowledge about political and social relationships. [187] Computers, however, are least able to recognize these patterns because they are dynamic and not easily reduced to **[*131]** formulae. [188] In fact, humans perform better than any computer algorithms in just these sorts of natural recognition tasks. [189] Evaluation of any one of the circumstantial factors discussed above is beyond the reach of current computer technology, and it is unlikely that the "totality of the circumstances" [190] can be adequately formalized in the foreseeable future. [191] A state legislature would be abdicating its responsibility if it allowed such goals to be abandoned in favor of ***automation***.

Even goals that may seem to be simple, easily quantified, and widely accepted may be subject to several different, unequal quantifications. For example, Niemi lists twenty-four quantifications for the goal of "compactness," most of which will differ in the values they assign to districts. [192] The controversy over apportionment [193] methods in the

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[186] *Id. at 2481.* The ***complexity*** and subtlety of representational goals are illustrated by the Court's review of multi-member districts in *White v. Regester, 412 U.S. 755, 765 (1975).* In White, the Court recognized that the "totality of the circumstances" may be considered in the evaluating ***redistricting*** plans. *412 U.S. at 769-70.* Affirming the district court's decision invalidating two multi-member districts in Texas, the Court reiterated several important circumstantial factors that the lower court considered, including the racial motivation for a plan, the history of discrimination and voting patterns in the state, and the number of districts in the state where the majority of voters belonged to an affected class. See *id. at 765-69;* see also S. Rep. No. 47, 97th Cong., 2d Sess. 28-29 (1982), reprinted in 1982 U.S.C.C.A.N. 177, 206-07. After listing seven typical factors that establish grounds for ***redistricting*** under the Voting Rights Act, the Senate report then states:

While these enumerated factors will often be the most relevant ones, in some cases other factors will be indicative of the alleged dilutions …

…

The cases demonstrate, and the Committee intends that there is no requirement that any particular number of factors be proved, or that a majority of them point one way or another.

Id. at 29, 1982 U.S.C.C.A.N at 207 (emphasis added). Although the Senate Committee intended these factors to be used only to determine whether multi-member, at-large districts were discriminatory, and neither the court nor the Senate report specified that these factors should be used to draw district lines, the factors show how complex the concept of fair representation can be.

[187] Even a seemingly simple concept like compactness is difficult to formalize in some of its forms. See Bernard Grofman, Would Vince Lombardi Have Been Right if He Had Said, "When it Comes to ***Redistricting***, Race Isn't Everything, It's the Only Thing," *14 Cardozo L. Rev. 1237, 1261-63 (1993)* (arguing that instead of applying purely formal compactness criteria to determine district geography, legislatures should apply the principle of "cognizability," which is defined as the ability to describe, in common sense language, the geographic characteristics of a district).

[188] See Dreyfus, supra note 117, at 97.

[189] See id.

[190] Despite the Supreme Court's formulation of a three-pronged test for challenging at-large elections under the Voting Rights Act, see *Thornburg v. Gingles, 478 U.S. 30, 50-51 (1986),* the "totality of circumstances" approach is still applied to single-member districts. See *Johnson v. DeGrandy, 114 S. Ct. 2647, 2657 (1994);* *Voinovich v. Quilter, 113 S. Ct. 1149, 1156 (1993); Growe v. Emison, 113 S. Ct. 1075, 1084-85 (1993).*

[191] See generally Pamela S. Karlan, The Rights to Vote: Some Pessimism about Formalism, *71 Tex. L. Rev. 1705 (1993)* (raising further arguments concerning the limited ability of formalism to capture representational goals).

[192] See Niemi, supra note 6, at 1161-62.

[193] Apportionment is the allocation of representatives based on population. See Michel L. Balinski & H. Peyton Young, Fair Representation 5 (1982).

United States is another example of how even seemingly small differences in the formalization of an informal standard may be of great practical importance. [194]

Furthermore, ***redistricting*** typically involves assigning weights **[*132]** to multiple social goals. [195] The number of possible weights and resulting weighting functions is practically infinite. Even should overall ***redistricting*** goals and their formulations be agreed upon, the choice of weights may still be subject to contention. The ***complexity*** of weighting different goals should not be underestimated. It is unreasonable to expect that a dynamic and subtle weighing of social values can be captured by a simple fixed set of linear weights. A study by Pankaj Sheth and Sidney Hess, in which political scientists were asked to give fixed linear weights to two popular ***redistricting*** goals, provides an example of the dramatic differences in weighting that can occur even in a relatively homogenous group deciding among extremely restricted options. [196] **[*133]**

B. Political Implications of ***Redistricting*** Formalization

There are few limits on formalization of ***redistricting*** goals per se. Nevertheless, there are limits that cannot be ignored on the interpretation of a particular formalization as "objectively neutral." [197] Some proponents of ***automated*** ***redistricting*** do not claim that ***automation*** makes ***redistricting*** neutral, but instead specify that the decisions involved in choosing and formalizing ***redistricting*** goals must be made by a political process. [198] Even when the goals and formalizations are chosen by a political process, however, this type of formalization will have its disadvantages.

Seemingly simple characterizations of formal criteria, such as geographical compactness, can have counterintuitive implications in specific cases. [199] Given this fact, and the multiplicity of possible formalizations that can be

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[194] See generally id. (discussing the politics of choosing an apportionment method); see also *United States Dep't of Commerce v. Montana, 503 U.S. 442, 449-56 (1992)* (presenting a historic overview of apportionment and its legal significance).

[195] Many state constitutions contain multiple criteria for evaluating districts. See Grofman, supra note 21, at 85-87, 126, 165. An extreme example of what an ***automated*** procedure might have to accomplish is illustrated by the Hawaii Constitution, which requires the following:

1. No district shall extend beyond the boundaries of any basic island unit.

2. No district shall be so drawn as to unduly favor a person or political faction.

3. Except in the case of districts encompassing more than one island, districts shall be contiguous.

4. Insofar as practicable, districts shall be compact.

5. Where possible, district lines shall follow permanent and easily recognized features, such as streets, streams and clear geographical features, and, when practicable, shall coincide with census tract boundaries.

6. Where practicable, representative districts shall be wholly included within senatorial districts.

7. Not more than four members shall be elected from any district.

8. Where practicable, submergence of an area in a larger district wherein substantially different socio-economic interests predominate shall be avoided.

Haw. Const. art. IV, 6.

[196] Pankaj J. Sheth & Sidney W. Hess, Multiple Criteria in Political ***Redistricting***: Development of Relative Values, 2 Rutgers J. Computers & L. 44, 67 (1971) ("Conflicting results lead us to conclude that there is no consensus among the political scientists on the relative use of [population equality and compactness] criteria.").

[197] See Browdy, supra note 10, at 1389.

[198] See Vickrey, supra note 7, at 106; see also Browdy, supra note 10, at 1381; Issacharoff, supra note 16, at 1693-94.

[199] See Young, supra note 6, at 106-12; see also Altman, supra note 172, at 18-24 (providing examples of nonintuitive districts resulting from seemingly simple compactness measurements).

developed even for such seemingly intuitive goals as "geographic compactness," ***automated*** ***redistricting*** will have three serious defects.

First, ***automation*** will shift the political debate from ***redistricting*** goals to technical details. ***Automation*** therefore seems likely to engender extensive and arcane battles over the mathematical details of goal functions. "There surely is a tension between the language of optimization … and [the language of] creative problem solving, which implicitly is understood to be the proper language for looking at the real decision process." [200] ***Redistricting*** should be concerned fundamentally with principles of representation and of communities of interest; however, these are questions that resist quantification. Because ***automated*** ***redistricting*** requires such quantification, it leads decision-makers away from these principles and towards standards that are more formal but less deep.

Second, requiring that all ***redistricting*** criteria be formalized will underemphasize social values that are difficult to reduce to **[*134]** formulae. Representational goals such as preserving communities of interest or preventing minority vote dilution are particularly difficult to quantify, because they succinctly condense a host of dynamically changeable, interacting social factors and application of political philosophy. Current sophistication in computer programming is simply insufficient to achieve these goals. [201]

Third, proponents of ***automated*** ***redistricting*** claim that it eases judicial and public review by making political purposes clear. Instead, formal characterizations often mask, rather than clarify, political purposes. In cases where formal criteria have counterintuitive implications, debate over particular district plans may illuminate political motives much more clearly than debate over the technical characterizations of ***redistricting*** goals. [202]

VI. Conclusion

Computers are eminently useful tools for many purposes, including ***redistricting***. Unfortunately, these tools cannot eliminate political and social problems. ***Automated*** ***redistricting*** is neither as simple nor as objective as its proponents have claimed.

This Article has demonstrated that the general ***redistricting*** problem and common sub-problems belong to a class of problems that is widely believed to be computationally intractable. Consequently, practical methods for generating districts automatically will almost certainly be based on heuristics, procedures which make guesses at solutions. The current proposals for implementing ***automated*** ***redistricting*** make exactly such guesses. This guesswork is disturbing because researchers offer little in the way of empirical data or theoretical analysis to show that these methods are successful in implementing ***redistricting*** goals. **[*135]**

Even if heuristics are developed with better empirical and theoretical support, or better alternatives altogether arise, political and normative difficulties remain in formulating every districting goal as a computer algorithm. Representational goals, which are based upon the analysis of social and political patterns, cannot be quantified at the current state of computer technology. Many simpler goals, such as geographic compactness, can be easily quantified, but are open to a plethora of conflicting quantifications.

If districts are created in an open political process, perhaps districts need only meet minimum standards, such as population equality, in order to provide a "fair playing field"; the political process may determine the rest. Circumventing the political process and substituting a computer program requires confidence that the computer

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[200] Richard R. Nelson, The Moon and the Ghetto 33 (1977).

[201] See Lani Guinier, Eracing Democracy: The Voting Rights Cases, *108 Harv. L. Rev. 109, 121-24 (1994)* (arguing that the Voting Rights Act and minority vote dilution cannot be understood outside of the context of political theories of representation).

[202] Since the consequences of specific formulations are more uncertain than the consequences of plans in general, one might expect that risk averse bargainers would prefer to bargain over plans than to bargain over goals, ceteris paribus. The ceteris paribus assumption is a strong one, however, and a formal game model of the bargaining process is needed to prove this supposition in a realistic context.

cannot only satisfy the minimum standards, but that it can affirmatively meet representational goals. There is a meaningful difference between the two.

Proponents claim that ***automated*** ***redistricting*** promotes fairness and illuminates the ***redistricting*** process. They assume that social goals for ***redistricting*** are easily characterized. In fact, as this Article argues, even the most seemingly straightforward ***redistricting*** goals may be subject to many conflicting and confusing technical characterizations. More subtle social goals, especially those goals which are explicitly representational, may be given short shrift or have to be disregarded altogether to accommodate the ***automation*** process. Thus, because the ***automation*** process benefits nonrepresentational goals over representational ones, it cannot adequately serve a system of representation.

Some proponents also claim that ***automated*** ***redistricting*** can operate neutrally by merely implementing social goals chosen in a previous political debate. In fact, representational values are difficult or impossible to adequately formalize using current techniques, and no feasible methods for finding optimal plans for arbitrarily specified value functions have been discovered. Heuristic methods further complicate this picture because they intertwine choice of values with the process of creating districts.

Proponents claim that ***automated*** ***redistricting*** eases judicial review. In fact, the technical ***complexities*** of characterizing goals formally, and of finding plans to serve these goals, is a barrier to **[*136]** judicial and public review and to participation in the districting process.

Finally, proponents have claimed that ***redistricting*** will reduce political manipulation. In fact, ***automated*** ***redistricting*** may serve only to change the types of political manipulation that occur. ***Automation*** may reduce political manipulation that uses ***redistricting*** plans for the purpose of benefiting particular individuals. It may promote, however, political manipulation using formal characterization of goals and choice of ***automation*** methods so as to advantage particular political groups. It may disadvantage particular politicians, but strengthen partisanship.

Practical computer methods capable of guaranteeing optimal districts do not exist, and probably never will. In the real world, ***automated*** ***redistricting*** proceeds through educated guesses at solutions and crude attempts to describe representational goals with mathematical formulas. As a consequence, ***automation*** may not eliminate the opportunity to manipulate politically, but instead shift that opportunity toward those groups that have access to the most extensive computing facilities and expertise. At the same time, ***automation*** shrouds the manipulation in the illusion of neutrality and behind a cloud of technical details. Even if computing resources are equal, districts can be politically influenced by the choice of how to characterize values and how to arrive at specific plans.

Proponents hold up ***automation*** as a veil of ignorance that can be used to promote fairness and prevent manipulation in the ***redistricting*** process. This veil may be pierced, however, by well-funded groups that have ample opportunity to manipulate how goals are characterized and implemented.

[SEE APPENDIX IN ORIGINAL]