



# Multicriteria spatial allocation of educational resources: an overview

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## Abstract

The multiple criteria decision-making (MCDM) problem involves a set of alternative allocation plans evaluated on the basis of multiple, conflicting and noncommensurate criteria by several interest groups. These are often characterized by unique preferences with respect to the relative importances of criteria against which the alternative plans are evaluated. It is argued that central to many spatial (geographical) decision making problems in the public sector is the search for consensus among various interest parties. We suggest that multicriteria decision analysis may be used successfully to develop alternative allocation plans in facilitating compromise among competing interests. In this regard, a variety of normative approaches developed over the past 20 years, together with an increasing interest in GIS (Geographic Information Systems)-based analysis, will likely lead to greater emphasis on interactive search procedures and interactive computer-based decision support system concepts. © 2000 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Through the 1960s and 1970s, optimization models were widely used in public sector planning. The type of planning problem to which such models were applied covered a broad spectrum and included decisions on the use of educational resources. Although the models themselves differ greatly in detail, a common feature exists: typically, the models were designed and implemented for purposes of identifying an optimal solution to a planning problem.

Among the many criticisms directed at the optimization approaches, perhaps the most

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disturbing and recurrent ones suggested that they (1) are incomprehensible to interested parties, (2) implicitly assume a social consensus, and (3) usually fail to include criteria or goals that decision-makers believe to be important. Insistence that rigorous optimization techniques will yield best solutions to educational resource allocation problems is predicated on the assumption that the solutions will be acceptable to the interested parties (e.g. administrators, taxpayers, parents, teachers, students). This assumption, however, fails to recognize the complexity of resource allocation decisions in the public sector. Such decisions are complex because of the variety of interest groups and the difficulty in measuring, assessing and evaluating the quality and quantity of impacts associated with alternative allocation patterns. Responsible decision-making demands that those in authority who make allocation decisions be accountable, and that the selection of any formal technique should contribute to this accountability by allowing the analysis to be scrutinized by appropriate interest groups.

Dissatisfaction with traditional optimization approaches has prompted a marked interest in multicriteria decision-making (MCDM) analysis during the last 15–20 years [24,81]. It is argued here that MCDM techniques can be used to develop alternative plans for allocating educational resources. In particular, we suggest that successful plan evaluation, implementation and monitoring can involve formal methods of the multicriteria variety. However, it is vital that such methods be used interactively by those who make decisions and are subsequently responsible for the outcomes. To this end, the concept of a computer-based interactive decision support system (DSS) can provide a framework for integrating MCDM techniques with Geographical Information System (GIS) capabilities. The main aim of this paper is thus to provide a critical overview of research on multicriteria analysis of the spatial (geographical) allocation of educational resources. In so doing, we attempt to bring together works from diverse areas of MCDM to complement and extend the surveys of Brown [13], Knutson et al. [49], and Thomas [87].

## 2. Criteria for allocating educational resources

The process of allocating educational resources involves two broad interrelated decision problems. First, are the decision problems concerned with the *spatial (re)organization of educational systems*. Typical examples include reorganization of the system by locating and/or closing educational facilities, reorganization of school districts by allocating users to facilities, determination of the boundaries of educational jurisdictions (e.g. local education authorities and school boards), linkage of primary and secondary school systems, and the school busing problem. The spatial organization of educational systems provides a framework for allocating resources. *Allocation of educational resources among jurisdictions and functional units* constitutes the second category of decision problem. This grouping is concerned with such issues as the spatial pattern of the tax base for educational service provision, and the spatial distribution of financial and human resources among geographical units (school districts/schools).

### 2.1. Spatial reorganization criteria

Spatial reorganization of educational systems involves issues of spatial accessibility. One can

identify two types of accessibility criteria: the first is based on spatial efficiency, and the second on equity [57]. Spatial efficiency is typically operationalized in terms of the minimization of total (or average) costs of travel (time or distance) from places of residence to places of education. Most earlier research in the spatial (re)organization of educational systems was concerned with this efficiency criterion [40,56,60,68,92].

There are several measures of equity available for evaluating spatial accessibility to educational services [36,68]. One can, for example, impose a standard such that the average (or maximum) travel time to schools not exceed a specified norm [65]. Alternatively, the concept of equity can be operationalized by means of a minimax criterion [6]. This would involve minimization of maximum costs (time or distance) of travel from place of residence to educational facility. The minimax criterion is consistent with Rawls' [71] principle of justice. According to this principle, educational services should be organized in a way that maximizes the accessibility of the most remotely situated individual within a school district. The equity concept can also be operationalized by minimizing variability of access to educational services [57,64]. Variability can be measured in terms of the standard deviation or variance of accessibility costs (time or distance) from place of residence to place of educational service [58,65,68].

Spatial accessibility may fail to account for economies of scale and operating efficiency in the provision of educational services. The accessibility criteria should thus be considered within the context of capacities (size of facilities) and number of facilities serving a given area/population. The more schools, the closer each potential user can be to the one serving him/her. On the other hand, with more locations, the fewer will be the number sharing each with less advantage taken of economies of scale. Clearly, these two criteria must be traded off in determining the number, size, and location of educational facilities in a given area [47,61,68,89]. The conflict between accessibility criteria and scale economies is associated with the preferences of different interest groups concerned with the spatial (re)organization of educational systems. Broadly speaking, school administrators tend to stress the operating efficiency of fewer facilities, while the public is more concerned with the issue of equity [22,89,90].

School closure programs provide examples of the conflict that arises between operating efficiency and accessibility [4,22,43]. The aim of closure programs is to rationalize educational service provision in response to falling school enrollments associated with declining birth rates or outmigration from inner city areas [1,11,25,70,88]. A school closure in a community diminishes the accessibility to educational services for the local residents. On the other hand, the closure decision may increase the operational efficiency of the educational system. Consequently, school closures create conflict between administrators, elected officials, and the public. This is particularly true for closures affecting the most disadvantaged inner city areas [4,11,25].

## *2.2. Subdivision of relevant geographic space criteria*

Subdivision of geographic space within a country (region, province, state) into several jurisdictional and functional units provides a basis for allocating educational resources among local authorities, school boards, or school districts. These spatial units may vary in terms of their tax bases and educational expenditures, as well as socioeconomic and demographic

compositions. The differentiations of geographic space may result in variability of educational need as well as the quantity and quality of services provided by different educational units to meet that need. This raises the issues of equality, equity, and efficiency in the allocation of financial and human resources.

The simplest measure of equality in allocating educational resources is the student/teacher ratio and/or the expenditure per student [33,41,59]. Equality is achieved when the number of students per teacher and/or the amount of expenditure per student are the same for all schools across a school district. This allocation criterion is based on the human and financial inputs to educational systems. Equal allocation of educational resources may, however, produce inequalities in the quantity and quality of the outputs of education, measured in terms of educational achievement [33,40].

Equality of outputs may require inequality in the inputs, such as financial and human resources, as well as in the general characteristics of the schools and school districts. The equity criterion thus relates the outputs of the educational process to its inputs. It is concerned with how to allocate resources among school districts characterized by different levels of educational achievement [59,80]. Further complicating application of the equity criterion, at least in the North American context, is the fact that the inputs should be related to the spatial pattern of taxation. The concept of fiscal equity, however, can be used to relate the tax base and the allocation of resources among school districts. This concept requires that the same tax revenues per student be generated, irrespective of spatial variation in the tax base. In other words, fiscal equity is achieved if all school districts, in a given area, that choose to levy a particular tax rate, have the same level of expenditure per pupil [41].

The criterion of economic efficiency is based on the concepts of cost and production functions [5,8,27,33]. Efficiency in allocating educational resources can be evaluated in two ways: (i) maximize possible outputs achieved given the level of inputs (resources) available; and (ii) minimize the cost of a given level of outputs by an optimal combination of inputs [52]. The first is known as technical efficiency; the latter as allocative. Both concepts imply that educational resources are used to their fullest. The underlying assumption is that outputs of the educational process are optimized [27].

From the above arguments, we conclude that there are three major sets of factors involved in evaluating the allocation of educational resources: spatial accessibility, equity (along with the related concept of equality), and efficiency. Broadly speaking, these criteria are conflicting. Thus, the efficiency criterion usually conflicts with that of equity or equality [66,80]. Maximum efficiency of resource allocation is often achievable only at the expense of equity, and vice versa. Further, this efficiency–equity conflict may be reinforced by spatial accessibility considerations. As noted earlier, the accessibility criterion may fail to account for economic efficiency in the allocation of resources. On the other hand, if economic efficiency (e.g. scale economies) is maximized, then equity in spatial accessibility must be sacrificed.

As previously noted, when dealing with the allocation of educational resources, several interest groups must be considered [22,69]. These groups (including administrators, taxpayers, parents, teachers, and students) may have different preferences (importances) with respect to the evaluating criteria. For example, the elected school boards and school administrators are often concerned with maximizing efficiency of service provision. Teaching staff are generally most concerned with equity across the system (student/teacher ratios; expenditures/teacher).

Taxpayers, on the other hand, are primarily interested in the efficient distribution of expenditures, while parents and students may emphasize the importance of equity in access to educational services [14,22].

It is against this background that MCDM methods can be used to analyse the spatial allocation of educational resources. We thus argue that such approaches can accommodate the multidimensional and conflicting nature of the spatial (re)organization of educational systems, including the allocation of educational resources.

### 3. The multicriteria decision-making (MCDM) problem

The MCDM problem may be represented as:

$$\text{maximize } \mathbf{z}(\bar{\mathbf{x}}) = (z_1(\bar{\mathbf{x}}), z_2(\bar{\mathbf{x}}), \dots, z_k(\bar{\mathbf{x}})) \quad (1)$$

$$\text{subject to: } \bar{\mathbf{x}} \in \mathbf{X} = \{\bar{\mathbf{x}} \mid \bar{\mathbf{x}} \in \mathbb{R}^n, g_c(\bar{\mathbf{x}}) \leq 0, c = 1, 2, \dots, t\} \quad (2)$$

where  $\bar{\mathbf{x}}$  is an  $n$ -dimensional vector of decision variables,  $z_1, z_2, \dots, z_k$  are  $k$  distinct criteria functions of decision vector  $\bar{\mathbf{x}}$ ,  $g_1, g_2, \dots, g_t$  are inequality constraints, and  $\mathbf{X}$  is the feasible set of constrained decisions. Depending on the nature of the decision-making problem, the vector of decision variables can represent locational and/or allocational variables. The former are associated with the specific location of an educational facility or an area (e.g. school district). The latter variables typically represent allocation of resources to educational units (school boards, local education authorities, school districts, etc.), or the allocation of users (students) to educational facilities (schools).

If reorganization of an educational system requires location and/or closure decisions, then a spatial pattern of educational facilities can be represented by a binary (logical) vector  $\bar{\mathbf{x}} = (x_1, \dots, x_n)$ , where  $x_j = 1$  if a facility is located (closed) at a site or within an area  $j$  ( $j = 1, 2, \dots, n$ ), and  $x_j = 0$  otherwise.

For certain classes of spatial organization problems, a vector of allocation variables is required. The allocation decision can be expressed in terms of a binary variable,  $x_{ij}^*$  ( $x_{ij}^* = 1$ , if some quantity is allocated from  $i$  to  $j$ , and  $x_{ij}^* = 0$ , otherwise); or, in the case when the allocated quantity can be split among two or more sites/areas, an integer or continuous variable. Then,  $x_{ij}^{**}$  is a portion of the quantity allocated from  $i$  to  $j$ , ( $x_{ij}^{**} \geq 0$ ). In a typical spatial reorganization problem, the vector  $\bar{\mathbf{x}}$  is thus an aggregate of two types of decision variables:  $\bar{\mathbf{x}} = (\bar{\mathbf{x}}', \bar{\mathbf{x}}'')$ , where  $\bar{\mathbf{x}}'$  is a binary vector of locational decisions, and  $\bar{\mathbf{x}}'' = \bar{\mathbf{x}}^*$  or  $\bar{\mathbf{x}}^{**}$ , which is an integer (0–1 or general integer) or continuous vector of allocation decisions. For example, closure of a school may be represented by a 0–1 variable, the allocation of students to schools can be represented by a general integer variable, while the allocation of financial resources is usually represented by a continuous variable.

#### 4. Multicriteria decision-making (MCDM) methods

##### 4.1. Pareto-optimality analysis

Due to the generally conflicting nature of objectives in allocating educational resources, an optimal solution that simultaneously maximizes all relevant criteria is normally not achievable. For example, maximization of economic efficiency may very well conflict with the minimization of equal distribution of resources. This situation will yield several solutions to the MCDM problem in question. These are referred to as efficient, nondominated, and noninferior or Pareto-optimal. Decision vector  $\bar{x}$  is said to be Pareto-optimal if there does not exist a  $\bar{y}$  such that for  $z_v(\bar{y}) \geq z_v(\bar{x})$  for  $v = 1, 2, \dots, k$ , and  $z_v(\bar{y}) > z_v(\bar{x})$  for at least one  $v$ . Accordingly, the function  $z(\bar{x})$  is a Pareto-optimal criterion vector if  $\bar{x}$  is a Pareto-optimal decision vector.

All Pareto-optimal solutions represent alternative patterns of educational resources for which no improvement in any individual evaluation criterion is possible without sacrificing one or more of the other criteria. A Pareto-optimal allocation of resources among educational areas thus exists if it is not possible to reallocate resources to improve the well-being of one area without making at least one other area worse off.

While, in the real world, it is normally difficult to achieve a Pareto-optimal allocation of resources among educational areas, we may still be interested in using the concept to judge among arbitrary non-optimal allocations of resources. In this regard, Lerman [51] and Phipps and Anglin [69] provide examples of applications of the Pareto-optimality concept in evaluating public school closure decisions. The authors identify conditions under which the decision to close a school, given a set of potential candidates for closing, is Pareto-optimal. The objectives here include the cost savings from closing the school and the aggregate additional transportation costs for the displaced students travelling to their next-nearest schools.

Resources are allocated according to Pareto-optimality if a school board closes “a school as soon as the savings from closure exceed the additional costs for the displaced students and for the school board” [69, p. 340]. According to these authors, in analyzing five public school closures in Saskatoon using Pareto-optimality, the school board did not act rationally. Rather, they subsidized declining schools until they had enrollments fall to 150 students. This was done in preference to closing them at 400 students, the point their study suggested would be most economically efficient. On the other hand, Lerman [51] was concerned with whether “the postclosing utility level” exceeds that which existed before closing. In this case, the author was not seeking an optimal closing level, but, rather, a comparison of utility levels before and after closing.

##### 4.2. Data envelopment analysis (DEA)

The aim of DEA is to measure how efficiently a decision making unit (DMU) uses the resources available to generate a set of outputs [18,19]. In the current case, a DMU is referred to as a spatial unit such as a school board, local education authority, or school district. The hypothesis underlying DEA is that multiple, incommensurate inputs in a given spatial unit

generate multiple, incommensurate outputs in that unit [7]. The combination of inputs may vary from one unit to the other, and may generate spatial variation of outputs. Efficiency of a DMU is defined in terms of the Pareto-optimality concept as follows: (1) if none of the outputs in a spatial unit can be increased without increasing one or more of its inputs, or without decreasing some of its other outputs; or (2) if none of the inputs in a spatial unit can be decreased without decreasing some of its outputs, or without increasing some of its other inputs. Otherwise, the spatial unit is characterized as 100% efficient [18]. This definition allows us to measure the level of efficiency achieved in a given unit in relation to the levels of efficiency attained elsewhere under similar circumstances [67]. Specifically, the efficiency of a spatial unit,  $q$ , is measured by the following mathematical programming model:

$$\text{maximize } z(w_r, w_s) = \left( \sum_{r=1}^v w_r y_{rq} / \sum_{s=1}^n w_s x_{sq} \right) \quad (3)$$

$$\text{subject to: } \left( \sum_{r=1}^v w_r y_{rj} / \sum_{s=1}^u w_s x_{sj} \right) \leq 1, \text{ for } j = 1, 2, \dots, q, n \quad w_r, w_s \geq 0, \quad (4)$$

$$\text{for } r = 1, 2, \dots, v; s = 1, 2, \dots, u$$

where  $y_{rq}$  is the amount of output  $r$  ( $r = 1, 2, \dots, v$ ) generated by unit  $q$ ;  $x_{sq}$  is the quantity of input  $s$  ( $s = 1, 2, \dots, u$ ) used in unit  $q$ ; and  $w_r$  and  $w_s$  are the weights for output  $r$  and input  $s$ , respectively. The problem is to find those weights  $w_r$  and  $w_s$  that maximize the efficiency ratio of the spatial unit  $q$ , subject to the constraint that no spatial unit has an efficiency ratio exceeding unity. The value of the objective function  $z(w_r, w_s)$  ranges from 0 to 1, where  $z(w_r, w_s) = 1$  indicates 100% efficiency.

DEA techniques have been used to analyse the efficiency of educational units both in the US and in the UK. For example, Bessent et al. [10] used this approach to examine the efficiency of 241 school districts in Houston (see also [8,9]). Two measures of academic achievement were employed as outputs, along with 13 measures of inputs related to socioeconomic conditions, pupil inputs, school organizational climate, and classroom instructional processes [8]. The relationship between inputs and both school size and student performance for a sample of Maine elementary schools was analyzed by Deller and Rudnicki [27] using DEA. Similar work, with minor variations to inputs and outputs, can be found in Chalos and Cherian [17], Desai and Schinnar [29], Garrett [39] and Ruggiero [72,73]. The state of North Carolina has also used DEA to encourage operational efficiency and expenditure reduction in student busing. As a result, in a competitive atmosphere, local school districts have become rather innovative in their handling of school transportation [78].

There are also several studies that employed DEA to measure the efficiency of Local Education Authorities (LEAs) in England and Wales. Thus, Jesson et al. [46] analyzed LEA efficiency by taking two measures of exam performance as outputs, and total expenditure per pupil and three socioeconomic background variables as inputs. More recently, Bates [5] employed DEA in relating measures of academic attainment to the use of resources and to socioeconomic variables for the 96 LEAs of England. Further, Norman and Stoker [67] used

DEA to examine the efficiency of the 132 secondary schools in the Inner London Education Authority. Similar studies have been conducted more recently by Boussofiane et al. [12] and Thanassoulis [86].

These DEA applications have all proved helpful to educational planners and administrators. This is the case since DEA examines the degree to which inefficient educational units deviate from an efficient one while it measures the extent to which inputs are underutilized [8,67].

The major practical disadvantage of DEA is its high informational requirements. Further, it is often not clear which variables should be used in measuring the inputs and outputs [91]. It can be argued that the results would be different if different sets of variables were involved. Furthermore, the technique may point to perfect efficiency of an educational unit even if the unit is inefficient in the Pareto sense. This may be revealed when the solution to the mathematical programming problem for units with efficiency scores of 1.0 is also characterized by slacks in the input or output constraints.

#### 4.3. Parametric programming methods

The spatial allocation of educational resources has been traditionally modelled using mathematical programming methods that involve a single-criterion objective function [23,42,45,56,60,62,82,92]. In these studies, the problem is formulated as a linear or mixed-integer linear programming model that minimizes the total travel distance (cost or time) subject to a set of constraints imposed on the number of facilities, their capacities, gender- and racial-balance considerations, number of students to be allocated, etc.

It should be emphasized that various modeling techniques and solution strategies within the framework of single-criterion mathematical programming can be used to tackle multicriteria problems (see Cohon [24] for an overview). The best-known of these are the weighting and constraint parametric linear programming models. Both transform the multicriteria model (1)–(2) into a single-criterion form. First, they generate one efficient solution. Then, by parametric variation of the single-criterion problem, the complete set, or a subset of efficient solutions, is generated. The basic difference between these two methods lies in how they make the transformation from a multiple to a single-criterion problem.

The weighting method involves assigning a weight,  $w_v$  ( $v = 1, 2, \dots, k$ ), to each of the criterion functions  $z_v$ . The multiple criteria function (1) can then be converted into a single-criterion form through the linear combination of criteria with corresponding weights. The problem (1)–(2) is thus transformed to the following form: minimize  $\{\mathbf{w}_1 \mathbf{z}_1(\bar{\mathbf{x}}) + \mathbf{w}_2 \mathbf{z}_2(\bar{\mathbf{x}}) + \dots + \mathbf{w}_k \mathbf{z}_k(\bar{\mathbf{x}})\}$ , subject to:  $\bar{\mathbf{x}} \in \mathbf{X}$ , and  $w_v > 0$ , for  $v = 1, 2, \dots, k$ . The problem can then be solved by standard linear programming methods.

Alternatively, the constraint method can be used to convert problem (1)–(2) to one which is single-criterion. This is done by optimizing only one of the criterion functions while all others are transformed into inequality constraints. The MCDM problem (1)–(2) is thus changed to the following single-criterion problem: minimize  $\{\mathbf{z}_1(\bar{\mathbf{x}})\}$  subject to:  $\bar{\mathbf{x}} \in \mathbf{X}$ , and  $\mathbf{z}_p(\bar{\mathbf{x}}) \geq \epsilon_p$ , for  $p = 1, 2, \dots, l-1, l+1, \dots, k$ ; where  $\epsilon_p$  is the minimum allowable level for the  $p$ th criterion function. The set of efficient solutions can then be generated by solving the single-criterion problem with parametric variation of the weights (in the weighted method), or by parametric variation of the  $\epsilon_p$  (in the constraint method).



There are several published applications of parametric programming to spatial allocation of educational resources [21,30,77]. Diamond and Wright [30], for example, proposed a multicriteria model for school districting and consolidation. The model includes criteria based on accessibility to schools, safety, school utilization, and student dislocation. Central to the Diamond–Wright model is a combination of districting and closure considerations along with the equalized utilization of open schools. Church and Murray [21] demonstrated that the Diamond–Wright model has an unintended built-in bias. Thus, the model may seek to close smaller schools as this lowers average utilization rates even if that results in reduced accessibility for students. Subsequently, Church and Murray reformulated the Diamond–Wright model to overcome its inherent bias.

The most important feature of these techniques is an ability to eliminate inferior spatial allocation patterns, thus saving both time and money in searching for the best (most preferred) solution. An additional advantage of parametric programming is its ability to provide school administrators with a range of possible decision outcomes and the corresponding tradeoffs. For example, Silverstein [79] and Schoepfle and Church [77] demonstrated the use of parametric programming in seeking the spatial reorganization of educational systems. The major disadvantage of parametric programming is the size of the set of Pareto-optimal solutions, which is usually large for real world resource allocation problems. Identifying the best allocation pattern among a set of Pareto-optimal solutions thus requires explicit or implicit accounting of the decision-maker's preferences. This line of thinking leads us to the class of preference-based MCDM techniques that includes goal programming.

#### 4.4. Goal programming methods

Goal programming (GP) is an extension of linear programming designed to solve MCDM problems [24]. It requires that the decision-maker specify his/her preferences with respect to evaluation criteria in the form of aspiration levels or goals. The criterion functions (1) are thus transformed into goals:

$$\mathbf{z}_v(\bar{\mathbf{x}}) + \mathbf{d}_v^- - \mathbf{d}_v^+ = \mathbf{a}_v, \text{ for } v = 1, 2, \dots, k; \quad (5)$$

$$\mathbf{d}_v^- \geq 0; \mathbf{d}_v^+ \geq 0; \text{ and } \mathbf{d}_v^- - \mathbf{d}_v^+ = 0 \quad (6)$$

where  $\mathbf{a}_v$  is the aspiration level for the  $v$ th criterion, and  $\mathbf{d}_v^-$ , and  $\mathbf{d}_v^+$  are negative and positive goal deviations, respectively; i.e. nonnegative state variables that measure deviations of the  $v$ th criterion function from the corresponding aspiration level.

An optimal GP solution is that which minimizes the deviations from the aspiration levels. Various measures of multidimensional deviations are introduced and expressed as achievement functions. Accordingly, a range of GP forms has been proposed. In the context of spatial allocation of educational resources, one can distinguish two approaches to GP: weighted GP and preemptive or lexicographic GP.

The weighted GP method incorporates a decision-maker's preferences with respect to over- and under-achievement of goals by assigning relative weights to positive and negative deviations. The weighted GP model can thus be written as:

$$\text{minimize } \mathbf{z}(\bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+) = \sum_{v=1}^k (\mathbf{w}_v^- \mathbf{d}_v^- + \mathbf{w}_v^+ \mathbf{d}_v^+) \quad (7)$$

subject to: (2), (5), and (6), where  $\mathbf{w}_v^-$  and  $\mathbf{w}_v^+$  are weights assigned to the negative and positive goal deviations, respectively.

In lexicographic GP, the achievement functions are prioritized in the strict sense that attainment of  $a_1$  far outweighs in importance attainment of  $a_2$ , which far outweighs  $a_3$ , etc. This corresponds to the situation where  $\mathbf{w}_1^- \gg \mathbf{w}_2^- \gg \mathbf{w}_3^- \dots$  and  $\mathbf{w}_1^+ \gg \mathbf{w}_2^+ \gg \mathbf{w}_3^+ \dots$ ; i.e. goal one is infinitely more important than goal two, goal two is infinitely more important than goal three, etc.

Goal programming methods have been applied to several real-life and hypothetical problems involving resource allocation and reorganization within educational systems. Lee and Moore [50], for example, used a lexicographic GP model to analyse the school busing problem. Their model considers both equity and efficiency criteria for allocating students to schools, including equal educational opportunity for all children in the district, racial balance, total transportation costs, transportation time, and utilization of school capacity. The model is analyzed for different goal rankings within a hypothetical busing problem. Knutson et al. [49] and Saunders [75] also analyzed the school busing problem via GP. These two studies differ from the Lee–Moore analysis in that they use a weighted GP model to solve a (hypothetical) problem involving three goals and a set of conventional constraints. The goals were to minimize the deviations from the average racial balance, the deviation from maximum total busing distance, and the deviation from school capacities.

While school busing and racial desegregation have been the focus of much research in the US, in the UK, GP methods have been applied primarily to the problem of declining enrollments [13,83,84]. Sutcliffe and associates thus used a weighted GP to analyse the problem of catchment area reorganization for 17 secondary schools in the Greater Reading area. The problem involved six educational goals; specifically, the average racial balance, the average reading-age retarded proportion, total road distance traveled, total difficulty of travel, average capacity utilization, and average gender proportions. Deviations from these objectives were minimized subject to a set of conventional constraints imposed on the number of students allocated to each secondary school and the number of allocations; i.e. each student must be allocated to a school exactly once.

The UK studies examined several alternative strategies for school system reorganization by establishing a set of plausible weights. A similar approach was applied by Brown [13] to analyze a range of strategies for reorganizing the secondary school system in Liverpool. This study suggests several goals in addition to those in the Greater Reading model; e.g. the number of children required to travel more than a specified distance, the average proportion of children in each secondary school in particular “social groups”, and unit operating costs of school facilities.

Goal programming methods are viewed by many as quite flexible. This flexibility allows for the consideration of a wide range of conflicting, noncommensurate criteria and for analyzing alternative spatial allocation strategies [83]. GP also offers a high degree of computational efficiency as it stays within the efficient linear programming computational environment.

There are, however, several technical and conceptual problems when using GP methods.

First, an important computational drawback arises from a strong tendency to generate inefficient solutions. For many analysts, this “inefficiency” problem seriously limits the utility of GP as a tool for tackling MCDM problems [24].

Secondly, GP requires the decision-maker to specify fairly detailed a priori information about his/her aspiration levels, preemptive priorities, and the importance of goals as weights. One can expect that in a complex educational resource allocation problem, the decision maker will find it difficult to provide such precise information. In the Greater Reading study, Sutcliffe and Board [83] reported that it was relatively easy to specify the aspired goal values, but the task of establishing a set of goal weights was much more difficult. This is particularly true when the allocation problem involves multiple decision makers [37]. These difficulties are further aggravated when the goals are unrelated to each other. To avoid such problems, Brown [13] has suggested an application of the Analytic Hierarchy Process, developed by Saaty [74], to estimate the goal weights for the Liverpool study. Although this approach increases the practical usefulness of GP methods, it only partially solves the problems associated with determining goal weights.

Consider, now, a third issue. Although it can be argued that lexicographic GP solves the problem of preference weighting, it is not, in general, superior to the weighted method. Since the lexicographic approach assumes that a higher priority goal is of overriding importance with respect to a goal of the next lower priority, tradeoffs between goals are not allowed. Consequently, an allocation plan that performs best on the highest priority will always be identified as the best, irrespective of (1) its performance on other criteria, or (2) how well other plans perform on these criteria. We should note that this property of the preemptive approach seriously limits its applicability to problems of spatial allocation of educational resources [83,84].

Finally, the problems associated with a priori information can be overcome, at least partially, by an interactive approach. To this end, it should be noted that the aspiration levels, preemptive priorities and goal weights can be changed during the analysis if a GP model is used as a basis of some interactive decision support approach. This leads us to the concept of the multicriteria-decision support system (MC-DSS), discussed below.

## **5. MC-DSS and GIS**

A decision support system (DSS) can be defined as an interactive computer-based system designed to support a decision-maker in a complex environment [48]. Spatial Decision Support Systems (SDSS) are different from the ordinary DSS in that they integrate GIS and model base management system (MBMS) capabilities. The common feature of GIS systems is their focus on the capture, storage, manipulation, analysis and display of geographically referenced data. Importantly, they only implicitly assume a support of spatial decision-making through analytical modeling operations [28]. An integration of GIS with multicriteria decision models can thus significantly increase the utility of GIS as a decision support system [16,31,32,35].

The geographically referenced data base, and multicriteria decision model base systems can be considered as major elements of a multicriteria-spatial decision support system (MC-SDSS). Such a system can support a variety of decision-making styles in various decision situations. It

thus allows for both integrated data analysis and educational system modeling, while taking multiple criteria and decision-maker preferences into consideration.

There have been several applications of GIS-based DSSs to school districting and school busing problems [3,15,17,34,38, 44,76,85]. An early SDSS system, still used, is IBM's *Geodata Analysis and Display System (GADS)*. It has been applied to a variety of spatial planning problems including the reorganization of school districts [15]. GADS is an interactive system enabling users to solve the problem of allocating students to schools. It employs a districting algorithm and presents solutions as maps.

Similar DSS systems applied to the school districting problem have been developed by Szekely et al. [85] and Ferland and Guenette [38]. The analytic and cartographic display capabilities of these systems are, however, rather limited as the systems are based on a districting algorithm and a simple display of school district boundaries. On the other hand, the Integrated Transportation Optimization and Planning System (ITOPS) has significantly greater analytic and display capabilities [34]. ITOPS' capabilities include: school district optimization techniques; procedures for identifying the most efficient busing pattern and for allocating vehicles; forecasting methods for analyzing changes in the spatial distribution of school populations; and a variety of ARC/INFO-based cartographic display capabilities (ARC/INFO is specific GIS software developed by the Environmental Systems Research Institute).

In a related study, Schoenhaus et al. [76] developed an ARC/INFO-based system for spatial allocation of educational resources. Specifically, the system integrates school district boundaries and taxpayer address records to aid in more difficult and effective allocation of resources. This integration allows one to assign income wealth to school districts based on district codes reported by residents on their income tax returns. Similarly, Armstrong et al. [3] and Honey et al. [44] implemented an SDSS for the spatial reorganization of an educational system by combining the display capabilities of a commercially available mapping package with a microcomputer-based location-allocation analysis system. The system has been used in reorganizing the delivery system for special education services. The cartographic display techniques supported by this system are reported in Armstrong et al. [2].

Cortez et al. [26] used SDSS for the spatial organization of educational systems by applying an "open" architecture approach. They thus developed an ARC/INFO-based DSS for school bus routing and redistricting. Alternative bus transportation and redistricting scenarios were generated and evaluated on the basis of criteria such as: average cost of transportation, travel time, and the number of students transported by school, program and route. The system is based on a client/server architecture. This "open" approach to system design significantly enhances the utility of GIS for educational resource allocation since different users (area bus supervisors, student transportation supervisor, school superintendent, etc.) can access the system from a variety of locations through local and wide area networks.

Despite these considerations, several criticisms of current approaches to SDSS in educational resource allocation can be made. Perhaps the most disturbing is the implicit assumption of a social consensus and the failure to include criteria or goals that decision-makers, school administrators, and the public believe to be important. Indeed, most commercially available GIS systems provide rather limited analytic capabilities in support of the spatial resource allocation decisions required by educational interest groups [54]. This is due, in part, to the fact that the systems do not consider the process and context of the allocation.

MC-SDSS offers a framework for structuring resource allocation problems, to place them into a broader social and economic context, and to involve the interest groups and the decision-makers responsible for plan implementation. Such an approach changes the role of GIS in the planning process, shifting the focus from outcome-oriented to process-oriented, and from efficient to effective aspects of educational resource allocation. To this end, it is important that MC-SDSS be able to support the analysis of alternative solutions to such problems both in decision (geographical) and in criterion outcome spaces [20,55].

The display capabilities of GIS systems typically allow the user to visualize alternative spatial patterns in decision space [2]; i.e. once the problem is solved by multicriteria techniques, the results (decision variables) can be displayed using a mapping package. Most available GIS systems are not able to solve multicriteria decision problems in criterion space. An application of GIS for tackling such problems thus requires substantial user involvement in order to link the analytic components of multicriteria techniques with cartographic display techniques.

In this regard, there is the possibility of bias in the perception of alternative solutions in decision and criterion spaces. Aspects of alternative solutions missed when visualizing decision space may become apparent when viewed in criterion space [81]. This is important in the school districting and school busing problems. The spatial patterns that seem to be “insignificantly” different when viewed in decision space, might vary “significantly” in criterion space and vice versa [20,55]. A solution that seems to be most preferred in criterion space might thus be recognized as inferior when viewed in decision space. For example, a solution to the districting problem may be most preferred in terms of total transportation costs and maximum distance traveled, while another solution may be most preferred in terms of location-allocation patterns. The most effective way to deal with such problems is to present alternative solutions in several different formats in both decision and criterion spaces [81]. Graphic script can be applied for the sequenced visualization of alternative solutions [63], incorporating a variety of techniques useful in composing sequences of dynamic maps, graphs, tables and text blocks. Such an approach to integrating MC-SDSS components can thus increase flexibility in analyzing and solving the educational resource allocation problem [53].

## **6. Conclusions**

The preceding discussion provides a survey and critique of the very extensive literature on the spatial allocation of educational resources. The need for multicriteria analysis is shown to result from often conflicting and noncommensurate objectives arising in problems of educational resource allocation. A variety of normative approaches developed over the last 20 years, together with an increasing interest in GIS-based analysis, will likely lead to greater emphasis on both interactive search procedures and interactive computer-based decision support systems. We thus believe that educational resource allocation problems can be analyzed more efficiently and effectively within an interactive MC-SDSS framework. This is especially true when the MC-SDSS concept includes visualization techniques. The ability to present alternative resource allocation plans in various formats both in criterion and decision spaces can greatly benefit those attempting to understand why one solution is preferred over

another. This, in turn, can enhance the confidence of those charged with allocating scarce educational resources.

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