ISS orbit intersection calculations Stephen [illegible] 17/12/18

$$1 = a^2c^2x^2 + b^2c^2x^2 + (a^2+b^2)x^2 + 2Bbc^2x + 2bB(a^2+b^2)x + c^2B^2 + b$$

$$0 = x^2(a^2c^2 + b^2c^2 + a^2 + b^2) + x(2Bbc^2 + 2bB(a^2+b^2)) + c^2B^2 + b^2B^2 - 1$$

$$0 = x^2(c^2(a^2+b^2) + (a^2+b^2)) + x(2Bb(a^2+b^2+c^2)) + c^2B^2 + b^2B^2 - 1$$

$$0 = x^2((a^2+b^2)(c+1)) + x(2Bb(a^2+b^2+c^2)) + c^2B^2 + b^2B^2 - 1$$

$$x = \frac{-2Bb(a^2+b^2+c^2) \pm \sqrt{(2Bb(a^2+b^2+c^2))^2 - 4((a^2+b^2)(c+1))(c^2B^2+b^2B^2-1)}}{2(a^2+b^2(c+1))}$$

$a^2 + b^2 + c^2 = 1$

$$\therefore x = \frac{-2Bb \pm \sqrt{(2Bb)^2 - 4(a^2+b^2(c+1))(c^2B^2+b^2B^2-1)}}{2(a^2+b^2(c+1))}$$

Test $B = .75 = d$

$a = -.43$

$b = -.75$

$c = .5$

CAS:

$$x = \frac{\pm a\sqrt{a^2+b^2-B^2} \times c}{(a^2+b^2)} - \frac{Bb}{a^2+b^2}$$

$$y = \frac{a \times B}{a^2+b^2} \pm \frac{\sqrt{a^2+b^2-B^2} \times b \times c}{a^2+b^2}$$

$$z = \pm\sqrt{a^2+b^2-B^2}$$

Pos

$$B = \cos(Rot) \times \sqrt{a^2+b^2}$$

ISS Orbit Gnomic Projection 17/12/17

$P_{rd} = \frac{P_{hw}}{\tan\theta}$

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

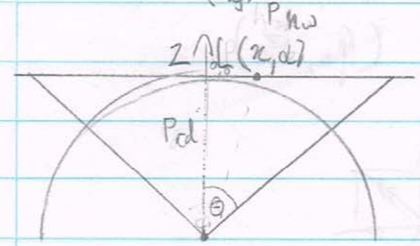


image polar pixel — pixel centered @ centre of view

$L(x, \alpha) \rightarrow L'\left(\arctan\left(\frac{x}{P_{rd}}\right), \alpha\right)$

spherical polar coordinates

$\varphi = \arctan\left(\frac{x}{P_{rd}}\right)$

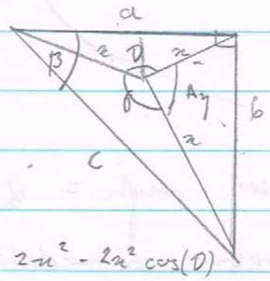
$\vec{L'} = (x, y, z)$

$\vec{L'} = \cos\alpha \sin\varphi \, \hat{i}' + \sin\alpha \sin\varphi \, \hat{j}' + \cos\varphi \, \hat{k}'$

Zenith $= \hat{k} = (0, 0)$

$D = 90 - Alt$ $\quad$ Im cent $= (D, A_z)$

Zenith ref:



$a^2 = 2x^2 - 2x^2\cos(D)$

$a^2 = 2x^2(1 - \cos(D))$

$b^2 = 2x^2(1 - \cos(A_z))$

$\beta = \arctan\left(\frac{\sqrt{2x^2(1-\cos(A_z))}}{\sqrt{2x^2(1-\cos(D))}}\right)$

$\beta = \arctan\left(\frac{1-\cos(A_z)}{1-\cos(D)}\right)$

$c^2 = a^2 + b^2$

$c^2 = 2x^2(1-\cos(D)) + 2x^2(1-\cos(A_z))$

$c^2 = 2x^2(1 - \cos(D) + 1 - \cos(A_z))$

$c^2 = 2x^2(2 - \cos(D) - \cos(A_z))$

$2x^2(2 - \cos D - \cos A_z) = (2x^2)(1 - \cos\gamma)$

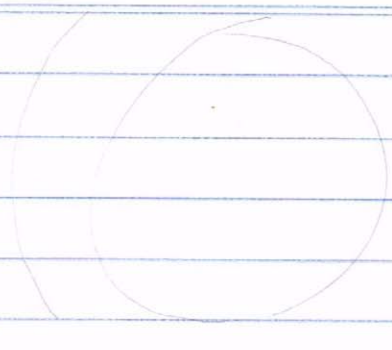
$2 - \cos D - \cos A_z = 1 - \cos\gamma$

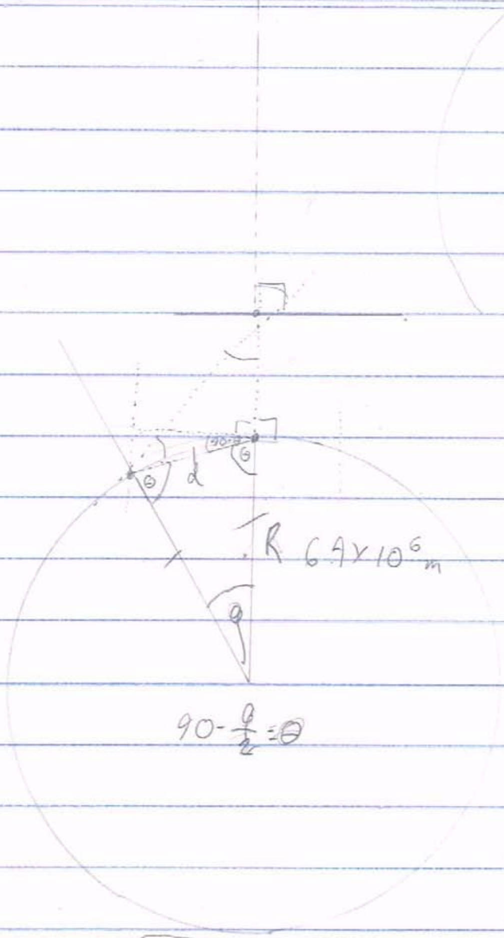
$1 - \cos D - \cos A_z = -\cos\gamma$

$\cos D + \cos A_z - 1 = \cos\gamma$

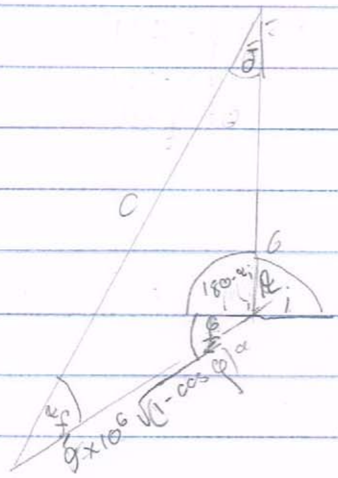
$\gamma = \arccos(\cos D + \cos A_z - 1)$

188 orbit calculation





$90 - \frac{\varphi}{2} = \theta$



$180 = \alpha_f + (180 - \alpha_i) + \delta$

$\cancel{180} = \alpha_f + \cancel{180} - \alpha_i + \delta$

$0 = \alpha_f - \alpha_i + \delta$

$\alpha_i - \alpha_f = \delta$

$\delta \approx 45^\circ$

$\frac{\sin 45^\circ}{a} = \frac{\sin \alpha_f}{b}$

$\frac{a \sin \alpha_f}{s} = b$

$d = \sqrt{a^2 + b^2 - 2ab\cos C}$

$d = \sqrt{2(6.4\times10^6)^2 - 2(6.4\times10^6)^2 \cos\varphi}$

$d = \sqrt{2(6.4\times10^6)^2(1-\cos\varphi)}$

$d = \sqrt{2} \times 6.4\times10^6 \sqrt{1-\cos\varphi}$

$180 - (90 - \frac{\varphi}{2})$

$180 - 90 + \frac{\varphi}{2} = \theta$

$90 + \frac{\varphi}{2}$

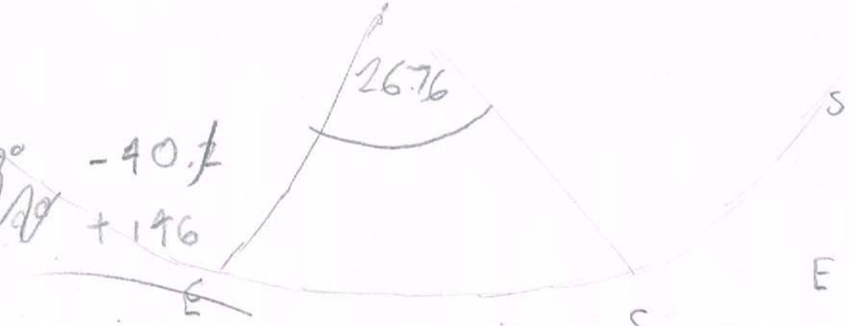
Pos Start 2016/5/20 18:40:35 AEST Alt: +52° Az: +154° RA: 13h 09m DEC: -67° 05'

Pos End 2016/5/20 18:41:05 AEST Alt: +48° Az: +114° RA: 13h 53m DEC: -42° 33'

Δ RA: 44m

Δ DEC: 24° 31'

LST: 299°

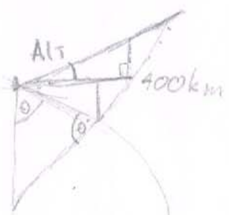


Pos Start Lat ~~-69°~~ -40.~~1~~

Long ~~160°~~ +146

+30s Lat = -39°

Long = +148



S $x = 0.9099$

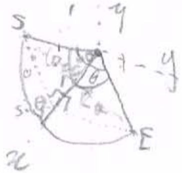
$y = 0.414$

$z = 0$

E $x = 0.981$

$y = .190$

$z = 0$



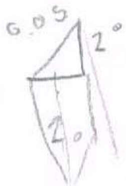
$c^2 = a^2 + b^2 - 2ab\cos C$

$\frac{c^2 - a^2 + b^2}{2ab} = \cos C$

$H = \sqrt{(.08)^2 + (.914)^2 - (.190)^2}$

$H = 0.46$

$\theta = 26.76$

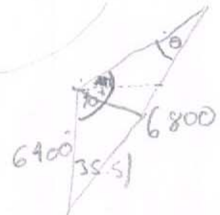


ω

$a = m s^{-2}$ 1.14

$a = \omega^2 r$ 2.2

$v = \omega r$



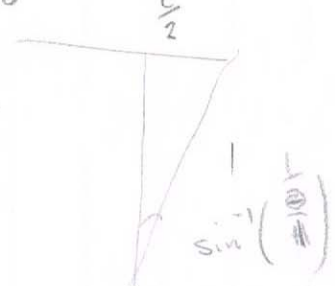
$2.82° / 30$

$\omega = 0.0016$

$T = \frac{2\pi}{\omega}$

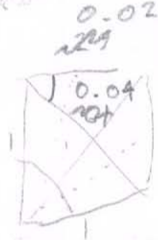
$T = 3818 s$

$T = 1.06h$

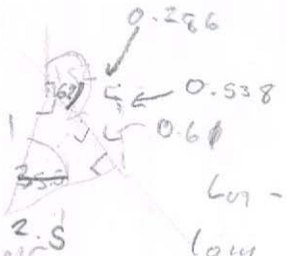


$\sin^{-1}\left(\frac{L}{2}\right)$

0.0204

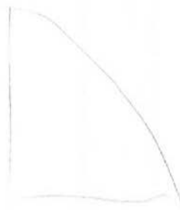


0.385



Lat -3°

Long 16



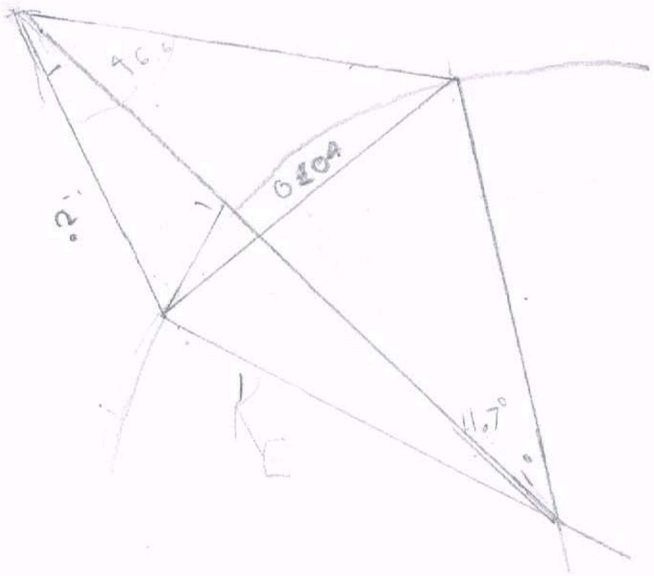
$\frac{v^2}{r} = \frac{GM}{r^2} = \sqrt{\frac{GM}{r}}$

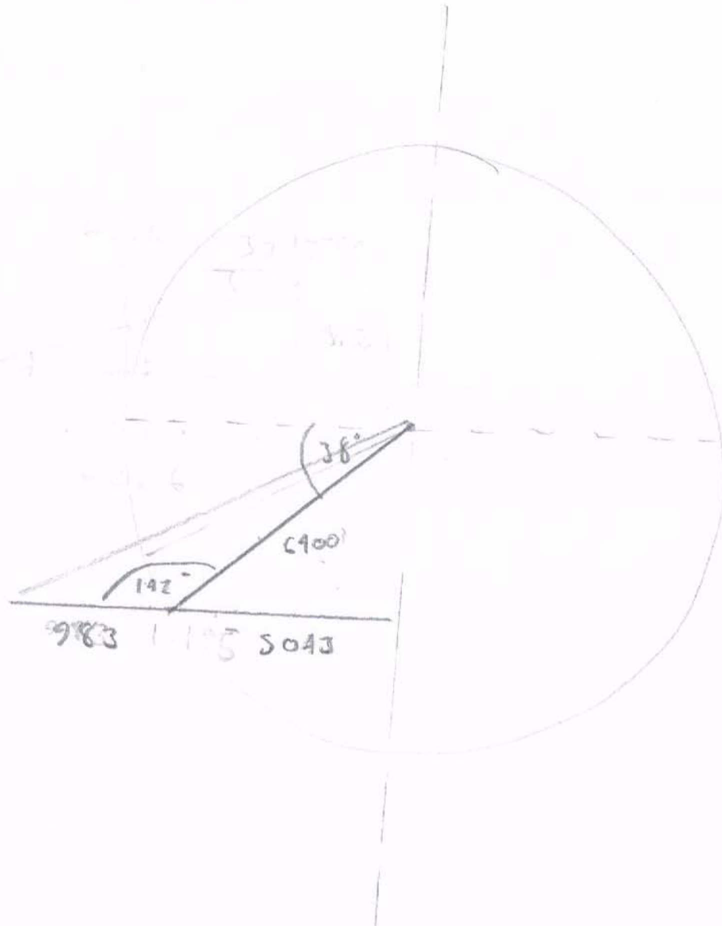
$T^2 = \frac{4\pi^2 a^3}{GM}$

$U = \frac{1}{2}mv^2 - \frac{mMG}{R} = -\frac{mMG}{2a}$

$v = \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}} = 11 \frac{km}{s}$

ISS orbit [illegible] Height Trigonometry [illegible] 17/12/26





$c^2 = a^2 + b^2 - 2ab\cos\theta$

$c^2 = 1.117$

$c = 1.057\,R_E$

$H = 0.057\,R_E$

ISS orbit measurement — Stephen Corsgrove — 17/12/20

Determining periods

$\frac{1}{83352.7}$ $n = \frac{1}{\tau}$ $n\tau = 83352.7$ $\frac{1}{}$

$\frac{1}{m} n_1$

~~xxxx~~

$n\tau = 83352.7$

$\frac{1}{m} n_1 = \frac{1}{\tau}$

$\frac{1}{m} n_2 = \frac{1}{2}$

$+ \frac{1}{m} n_1 + \frac{1}{m} n_2 = \frac{2}{\tau}$

$\tau = \frac{833527}{n_0}$ $n_0 \in \mathbb{N}$

$\tau = \frac{88902.7}{n_1}$ $n_1 \in \mathbb{N}$

~~833527~~

$n_4 = \cancel{\frac{833527}{n_0}}$

$\frac{\sin}{x} = \frac{\sin}{x}$

$\frac{833527}{n_0} = \frac{88902.7}{n_1}$

n_3

n_0

$\frac{n_1}{n_0} = \frac{88902.7}{833527}$

$\frac{n_1}{n_2} = a$ $\frac{n_2}{n_3} = b$ $\frac{n_3}{n_1} = c$

$a + \delta = \frac{R_E(\gamma + 2)}{R_E + h}$ $\frac{a+\delta}{R_E} = \frac{\gamma + \zeta}{R_E + h}$

$\frac{\gamma + \zeta}{(1+h)}$ $1 = ax$

$ax + by = c$

$an_1 + bn_2 = c$

$n_1 + n_2 = \frac{2}{\tau}$

~~$n_1 + n_2$~~

$an_1 + bn_2 = \frac{2}{\tau}$

$an_1 + cn_3 = \frac{2}{\tau}$

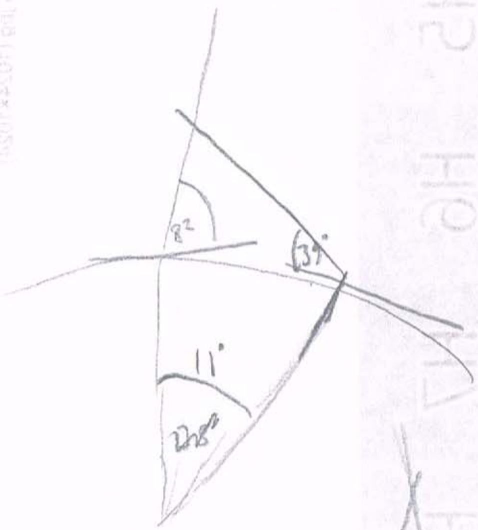
$+ \frac{\alpha}{R_E} = \frac{\beta}{R_E + h}$

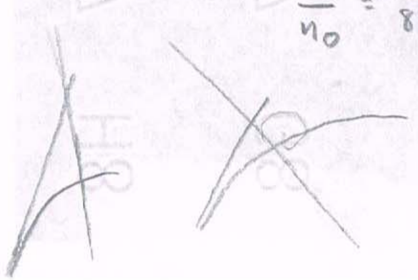
$+ \frac{\delta}{R_E} = \frac{\zeta}{R_E + h}$

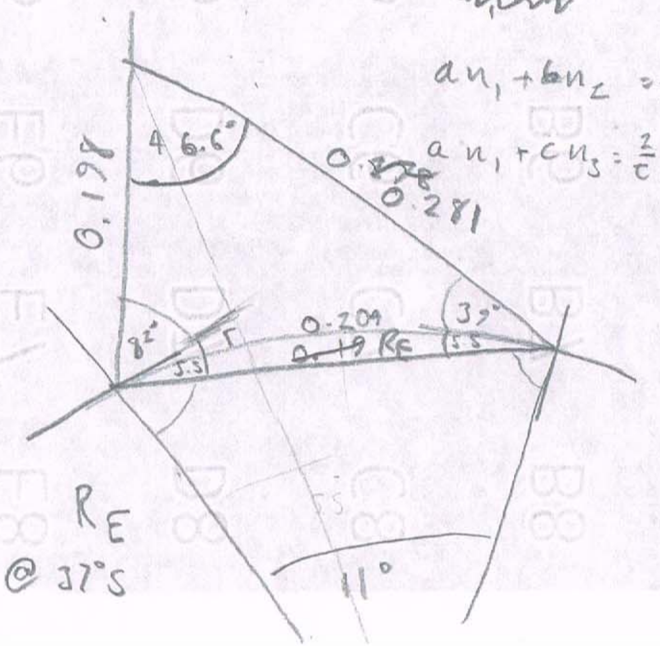
$\frac{R_E}{\alpha} = \frac{x}{\beta} = \frac{R_E + h}{\gamma}$

$\frac{R_E}{\delta} = \frac{y}{\varepsilon} = \frac{R_E + h}{\zeta}$

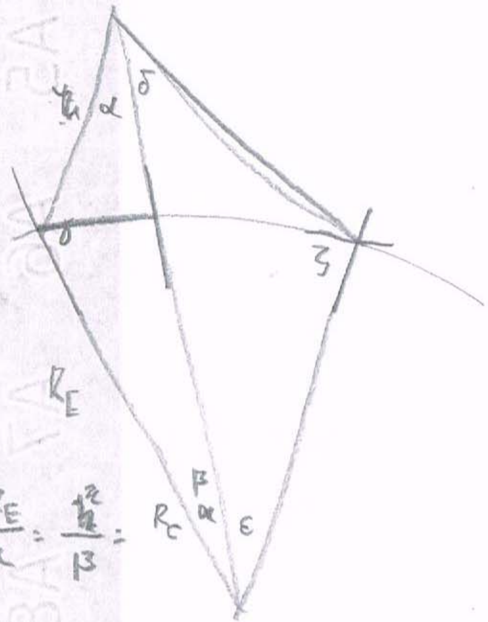
$\frac{\alpha}{R_E} \frac{x}{\beta} = \frac{R_E + h}{\gamma}$



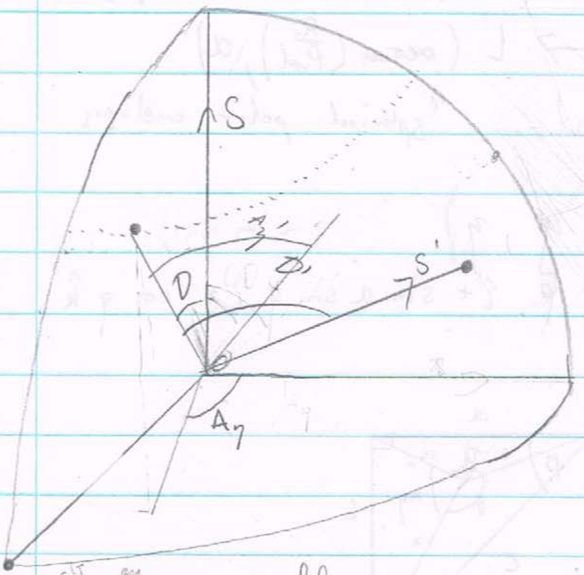




R_E @ 37°S

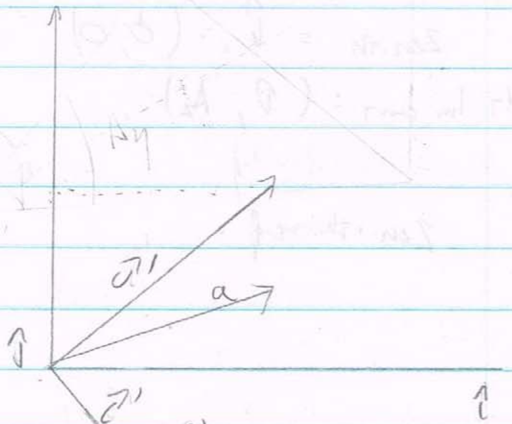


$\frac{R_E}{\alpha} = \frac{h^2}{\beta} = R_E$



S in S $(0,0)$

S' in S (φ_s, θ_s)



$S(\overset{alt}{90}, \overset{az}{90})$

$s(\overset{alt}{90}, \overset{az}{0})$

offset angle = α

measure counterclockwise from x axis.

Z = [illegible] coordinate system

$L'_c = (1, \varphi, \theta) = \cos\theta \sin\varphi\, \hat i + \sin\theta \sin\varphi\, \hat j + \cos\varphi\, \hat k = \hat k'$

$\hat k' \times \hat k$

$\hat k \times \hat k' =$ Angle $\quad \cos\alpha = \dfrac{\text{Angle} \cdot \hat i'}{|\text{Angle}||\hat i'|}$

$\hat k' \times \hat i' = \hat j'$

Expansion

→ Angle $= A_x \hat i + A_y \hat j + A_\eta \hat k$

$\therefore \cos\alpha = \dfrac{i_x A_x i + i_y A_y j + i_\eta A_\eta k}{\sqrt{A_x^2 + A_y^2 + A_\eta^2}\sqrt{i_x^2 + i_y^2 + i_\eta^2}}$

VI cons $\hat{j}' = \hat{k}' \times \hat{i}'$

III Local Cartesian = $x\hat{i}' + y\hat{j}' + z\hat{k}'$

substitute in $\hat{i}' = \ldots \hat{i} + \ldots \hat{j} + \ldots \hat{k}$ for $\hat{i}'\ \hat{j}'\ \hat{k}'$

$$\hat{i}' = a\hat{i} + b\hat{j} + c\hat{k}$$
$$\hat{j}' = d\hat{i}\quad e\hat{j}\quad f\hat{k}$$
$$\hat{k}' = h\hat{i}\quad i\hat{j}\quad j\hat{k}$$

Global cartesian = $x(a\hat{i} + b\hat{j} + c\hat{k}) + y(d\hat{i} + e\hat{j} + f\hat{k}) + z(h\hat{i} + i\hat{j} + j\hat{k})$

$$= xa + yd + zh\,\hat{i} + xb + ye + zi\,\hat{j} + xc + yf + zj\,\hat{k}$$

which is also the Matrix produces

$$\begin{bmatrix} a & d & h \\ b & e & i \\ c & f & j \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

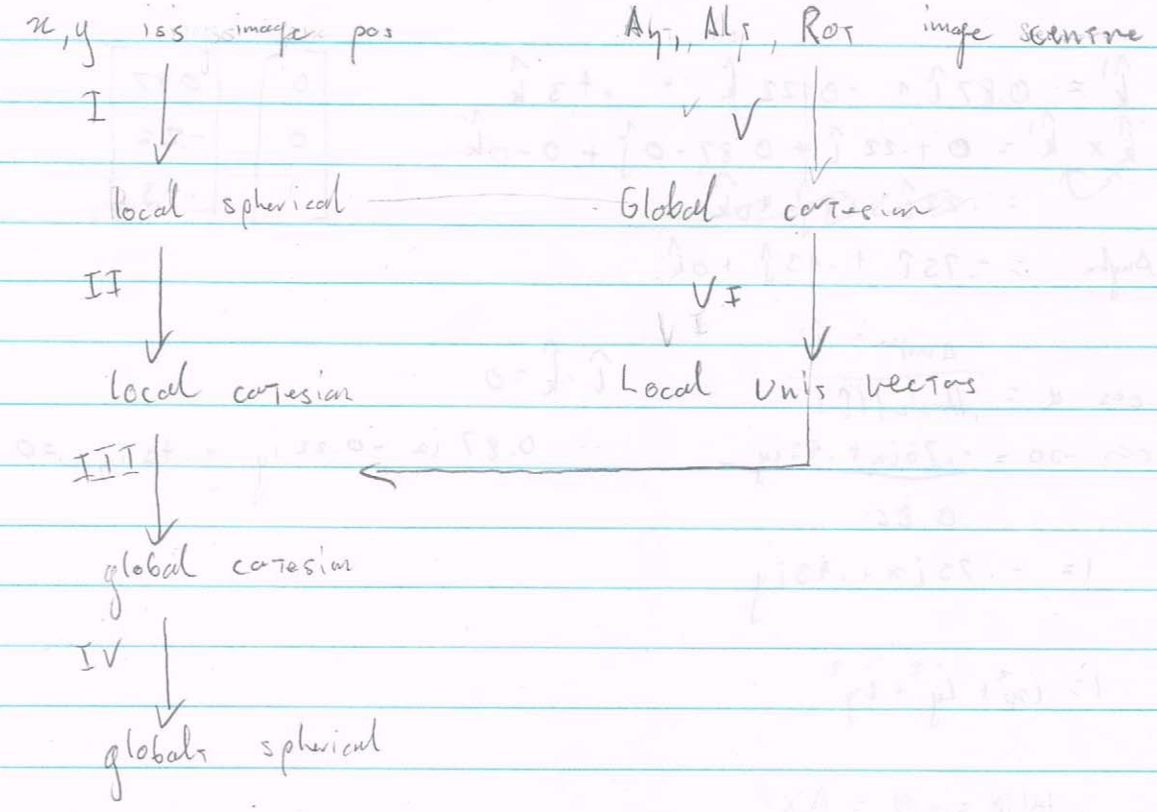
IV Global cartesian (x, y, z)

Global spherical (r, θ, φ)

$$r = 1 \quad \sqrt{x^2 + y^2 + z^2}$$
$$\theta = A(x, y) \quad \begin{cases} \tan^{-1}\left(\frac{y}{x}\right), & 0 \le x \le y \\ \tan^{-1}\left(\frac{y}{x}\right), & y < 0 \le x \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi, & x < 0 \le y \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi, & x \le y \le 0 \end{cases}$$
$$\varphi = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}(z)$$

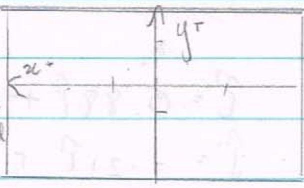
Flow diagram ISS orbit measurement | Stephen Parsons 17/12/18

flow



I I: ISS pos data (x, y) form (pixels)

Fig 1: ISS pos data with superimposed axis



Local spherical (r, φ, θ)

$r = 1$

$$\theta = A(x,y) \begin{cases} \tan^{-1}\left(\frac{y}{x}\right), & 0 \le x \le y \\ \tan^{-1}\left(\frac{y}{x}\right), & y \le 0 \le x \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi, & x < 0 \le y \\ \tan^{-1}\left(\frac{y}{x}\right) + \pi, & x \le y \le 0 \end{cases}$$

$$\varphi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{(Phw/\tan\alpha)}\right)$$

where:

Phw is half the width (pixels) of the image &

α is the angular field of view of half the width of the image

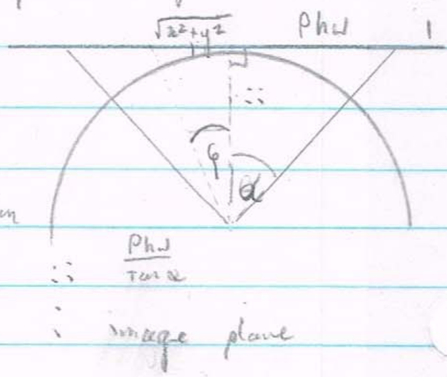


Fig 2: Pos to local spherical conversion diagram

Iss orbit measurement Test Mars Stephen Cygnus 17/12/17

$\hat{k}' = 0.87\hat{i} + -0.22\hat{k} + .43\hat{k}$

$\hat{k} \times \hat{k}' = 0 + .22\hat{i} + 0.87 - 0\hat{j} + 0 - 0\hat{k}$

$= \cancel{.22\hat{i}\ .88\hat{j}} + 0\hat{k}$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0.87 \\ -.22 \\ .43 \end{bmatrix}$$

Angle $= -.75\hat{i} + .43\hat{j} + 0\hat{k}$

$\cos\alpha = \dfrac{\text{Angle} \cdot \hat{i}'}{|\text{Angle}||\hat{i}'|}$ $\qquad \hat{i} \cdot \hat{k} = 0$

$\cos -30 = \dfrac{-.75 i_x + .43 i_y}{0.86}$ $\qquad \therefore\ 0.87 i_x - 0.22 i_y + .43 i_y = 0$

$1 = -.75 i_x + .43 i_y$

$1 = i_x^2 + i_y^2 + i_y^2$

$|A||B| \sin\theta = A \times B$

$\sin\theta = \dfrac{A \times B}{|A||B|}$

$\hat{i}' = 0.88\hat{i} + 0.22\hat{j} + 0.43\hat{k}$

$\hat{j}' = -.21\hat{i} + .62\hat{j} + 0.75\hat{k}$

$\hat{k}' = -.43\hat{i} - .75\hat{j} + .5\hat{k}$

$k' = \hat{i}' + \hat{j}' + \hat{k}'$

$k =$

Flow diagram ISS orbit measurement 2 Stephen Cossens 17/12/18

II Local spherical : $(1, \varphi, \theta)$

Local cartesian : $x\hat{i}' + y\hat{j}' + z\hat{k}'$ where $\hat{i}', \hat{j}'$ & $\hat{k}'$ are the local unit vectors.

$x = \cos\theta \sin\varphi$
$y = \sin\theta \sin\varphi$
$z = \cos\varphi$

V Image centre $(r=1, A_y, \pi - Al_t, R_{ot})$

In cartesian global : $\hat{k}' = \cos A_y \sin(\pi - Al_t)\hat{i} + \sin A_y \sin(\pi - A_y)\hat{j} + \cos(\pi - Al_t)\hat{k}$

VI $\hat{k} \times \hat{k}' = \text{Ang}$

$\hat{i}' = x\hat{i} + y\hat{j} + z\hat{k}$

$$(x, y, z) : \begin{cases} \cos(R_{ot}) = \frac{\text{Ang} \times \hat{i}'}{|\text{Ang}|} \\ 1 = x^2 + y^2 + z^2 \\ 0 = \hat{k}' \cdot \hat{i}' \end{cases}$$

Ang is in the same plane as $\hat{i}'$ the angle it and $\hat{i}'$ is the rotation of the image so

$\cos(R_{ot}) = \frac{\text{Ang} \cdot \hat{i}'}{|\text{Ang}||\hat{i}'|}$

$|\hat{i}'| = 1 \therefore \sqrt{x^2 + y^2 + z^2} = 1$

$\hat{i}' \cdot \hat{k}' = 0$

With this known solving the following system of equations will give $\hat{i}'$

The +ve z solution is accepted for a clockwise rotation, the -ve z solution is accepted for a counter clockwise rotation.

Solution: where $B = \cos(R_{ot}) \times \sqrt{a^2+b^2}$

$$x = \frac{\pm ac\sqrt{a^2+b^2 - (\cos(R) \times \sqrt{a^2+b^2})^2}}{a^2+b^2} - \frac{\cos(R)\sqrt{a^2+b^2} \times b}{a^2+b^2}$$

$$y = \frac{\pm b \times c\sqrt{a^2+b^2 - (\cos(R) \times \sqrt{a^2+b^2})^2}}{a^2+b^2} + \frac{a \times \cos(R)\sqrt{a^2+b^2}}{a^2+b^2}$$

$$z = \pm\sqrt{a^2+b^2 - (\cos(R)\sqrt{a^2+b^2})^2}$$

185 Orbit maneuver Vector calculations

$$\hat{i}' = x\hat{i} + y\hat{j} + z\hat{k} \quad , \quad (x,y,z): \begin{cases} \cos(Rot) = \dfrac{Ang \times \hat{i}'}{|Ang|} \\ 1 = x^2 + y^2 + z^2 \\ 0 = \hat{i}' \cdot \hat{k}' \end{cases}$$

let $\cos(Rot): R$

$Ang = \hat{k} \times \hat{k}'$ $\qquad \hat{k}' = a\hat{i} + b\hat{j} + c\hat{k}$

$Ang = -b\hat{i} + a\hat{j}$

$$R = \frac{Ang \cdot \hat{i}'}{|Ang|}$$

$$1 = x^2 + y^2 + z^2$$

$$0 = \hat{i}' \cdot \hat{k}'$$

$$R = \frac{-bx + ay}{\sqrt{a^2+b^2}} \qquad 1 = x^2 + y^2 + z^2 \qquad 0 = ax + by + cz$$

$R \times \sqrt{a^2+b^2} = -bx + ay$ $\qquad cz = -ax - by$

$R \times \sqrt{a^2+b^2} + bx = ay$ $\qquad z = \dfrac{-ax-by}{c}$

$\dfrac{R \times \sqrt{a^2+b^2} + bx}{a} = y$ $\qquad z = \dfrac{-ax - b\left(\frac{B+bx}{a}\right)}{c}$

let $R \times \sqrt{a^2+b^2} = B$

$$\frac{B+bx}{a} = y$$

$ax + b\left(\dfrac{B+bx}{a}\right)$ $\qquad 1 = x^2 + \left(\dfrac{B+bx}{a}\right)^2 + \left(\dfrac{-ax - b\left(\frac{B+bx}{a}\right)}{c}\right)^2$

$ax + \dfrac{bB}{a} + \dfrac{b^2 x}{a}$ $\qquad 1 = x^2 + \dfrac{b^2x^2 + 2Bbx + B^2}{a^2} + \dfrac{\left(ax - b\left(\frac{B+bx}{a}\right)\right)^2}{c^2}$

$a^2x + bB + b^2x$

$\dfrac{\left(x(a^2+b^2) + bB\right)^2}{a^2}$ $\qquad 1 = x^2 + \dfrac{b^2x^2 + 2Bbx + B^2}{a^2} + \dfrac{\left(ax + b\left(\frac{B+bx}{a}\right)\right)^2}{c^2}$

$$\frac{(a^2+b^2)x^2}{a^2} + \frac{2bB(a^2+b^2)x}{a^2} + \frac{b^2B^2}{a^2}$$

$$a^2 = x^2 + \frac{b^2x^2 + 2Bbx + B^2}{a^2} + \frac{(a^2+b^2)x^2 + 2bB(a^2+b^2)x + b^2B^2}{c^2 a^2}$$

$$1 = a^2x^2 + b^2x^2 + 2Bbx + B^2 + \frac{(a^2+b^2)x^2 + 2bB(a^2+b^2)x + b^2B^2}{c^2}$$

$$1 = a^2\cancel{c^2}x^2 + b^2\cancel{c^2}x^2 + \cancel{2Bbc^2x} + c^2B^2 + \cancel{(a^2+b^2)x^2} + \cancel{2bB(a^2+b^2)x} + b^2B^2$$

$\tau = 7s$ $\quad e \approx 0$

$\varphi \in (5.2, 5.4)$

~~$r \in (1.05, 1.50)$~~

$r \in (1.05\cancel{4}^{2}, 1.05\cancel{5}^{8})$

$j \in (1.053, 1.062)$

$\varphi = 5.289$

$r = 1.0552$

$j = 1.05533 50656$

$k = \frac{j}{r}$

$\frac{29}{30}$

0

30 29 31

60 58 62

90 93

120

$\therefore \quad \ldots \quad ?$

$e^2 = 1 - \frac{b}{a}\left(\frac{r}{j}\right)^2$

$e = \sqrt{1 - \left(\frac{1.0552}{1.05533 50656}\right)}$

~~$e = 0.015298$~~

$e = 0.003898\ 695.$

$inc = 5\cancel{8}.61°$

$actual = 51.63$

Nodal precession $4.95° / day$

$V = 7.3\ km s^{-1}$

$h_p = 400 km$

ξ

$A = \cancel{7.43951 \times 10^{-4} m^2}$ $\quad 0.00\cancel{0}42626\ m^2$

$\alpha = 0.43115°$

$A_T = \pi ab$

$\alpha = 1.0628 0287723 \times 1.062794$

$A = 3.54855\ m^2$

$\tau = 9\cancel{4}\ min$

$T = 94\ min$