Wave-optical lensing of gravitational and electromagnetic waves via Picard-Lefschetz theory

Arthur Suvorov
Melbourne Uni (Group Meeting); 2/5/22

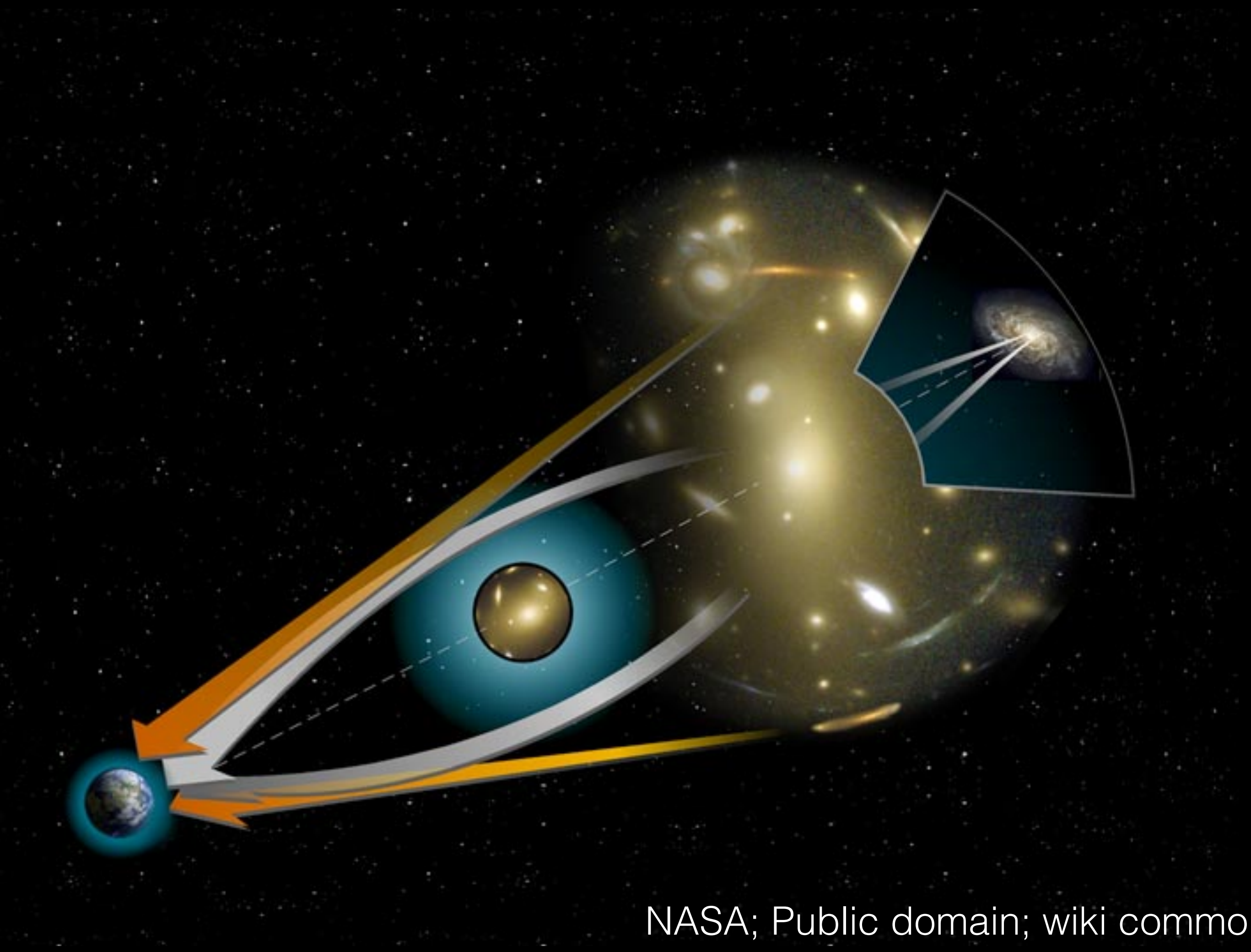
Image credit: Hubble telescope

Basic lensing introduction

- Space is not totally empty. Radiation that reaches us from distant sources often interacts with intervening material, refracting the signal and possibly amplifying it. Can also have multi-image formation.

- The amplification factor describes the increase of the area of the image (at constant surface brightness).

- Large amplification factors can be achieved depending on the nature of foreground objects; galaxies, for example, allow for large amplifications.



NASA; Public domain; wiki commons.

Basic lensing introduction. II

- Very exciting recent discoveries!

- The most distant (and thus earliest) detected star, WHL0137-LS (aka, Eärendil), was detectable due to gravitational lensing caused by the foreground galaxy cluster WHL0137-08. Simulations of the lensing effect suggest that Earendel's brightness was magnified between one thousand and forty thousand times [Welch++, Nature 603, 2022]

- First astrometric detection of an isolated, stellar-mass (~7 solars) BH lensing source star OGLE-2011-BLG-0462 [Sahu++, arXiv 2201.13296, 2022]

- Multi-image formation can make a source appear to be repeating when it isnt, for example repeat FRBs? (Chen++ 2017, ApJ 912 134).

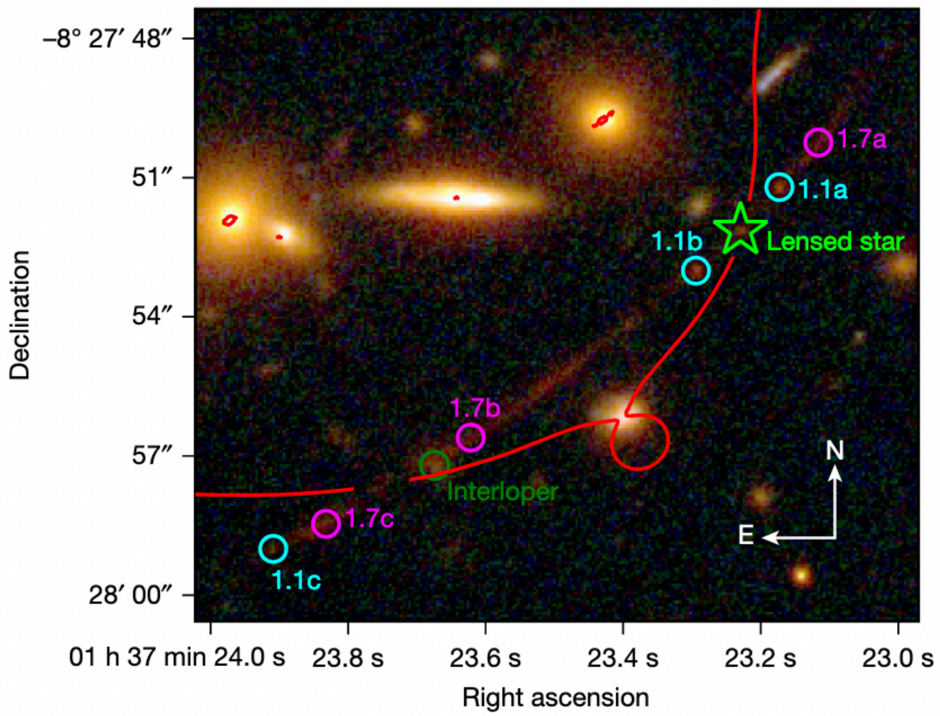


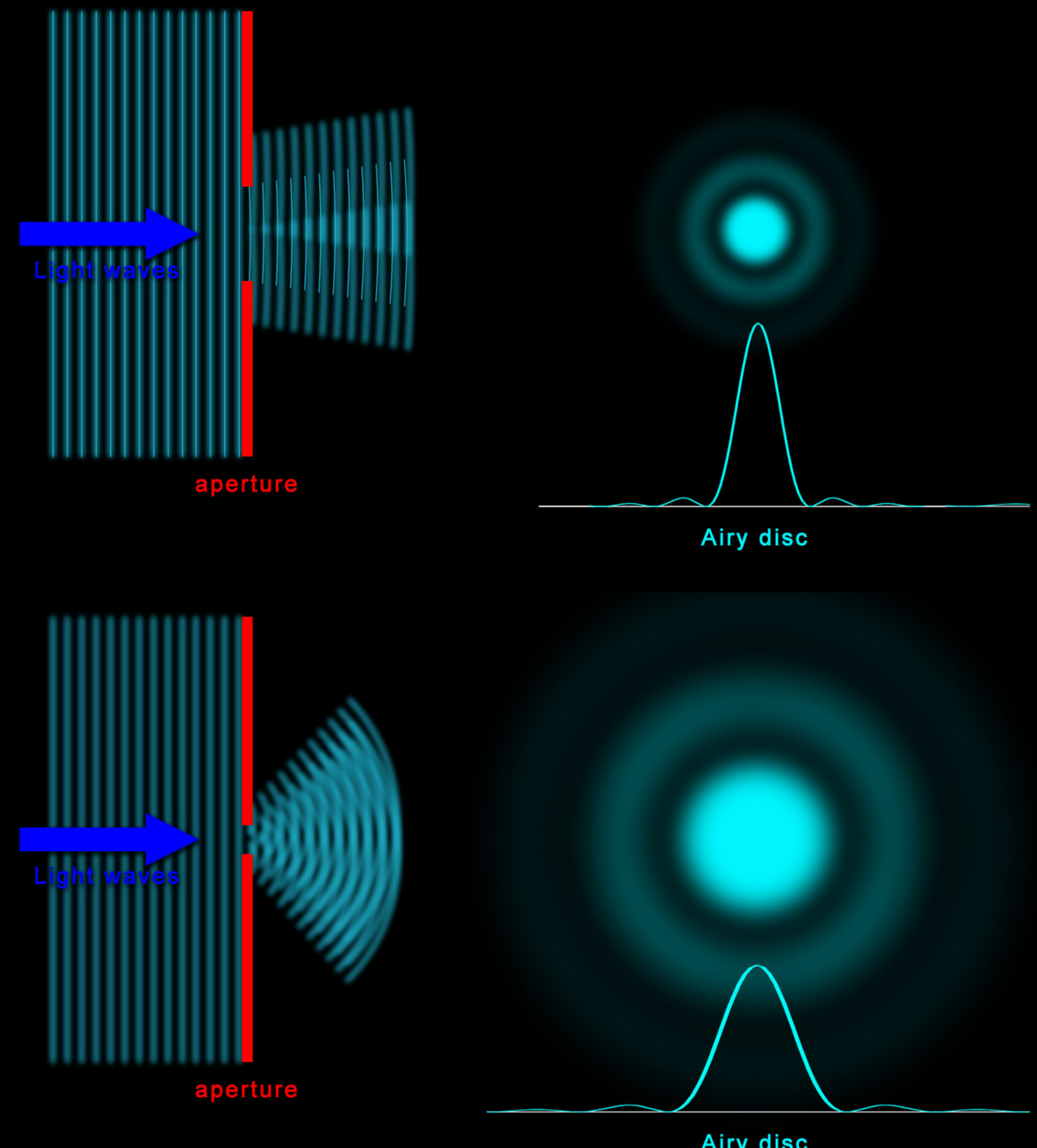
Fig. 1 | Labelled colour image of WHL0137-zD1. The Sunrise Arc at $z = 6.2$ is the longest lensed arc of a galaxy at $z > 6$, with an angular size on the sky exceeding 15 arcseconds. The arc is triply imaged and contains a total of seven star-forming clumps; the two systems used in lens modelling are circled, with system 1.1 in cyan and system 1.7 in magenta. The highly magnified star Earendel is labelled in green. The best-fit lensing cluster critical curve from the LTM model is shown in red, broken where it crosses the arc for clarity. The colour composite image shows the F435W filter image in blue, F606W + F814W in green, and the full WFC3/IR stack (F105W + F110W + F125W + F140W + F160W) in red.

Welch++ (2022); Nature

Long or short?

- The “size” of the lens relative to the radiation wavelength is critically important.

- In general, there is a characteristic length-scale associated with changes in the background spacetime curvature, which is related to the Schwarzschild radius of a relatively slow lens.

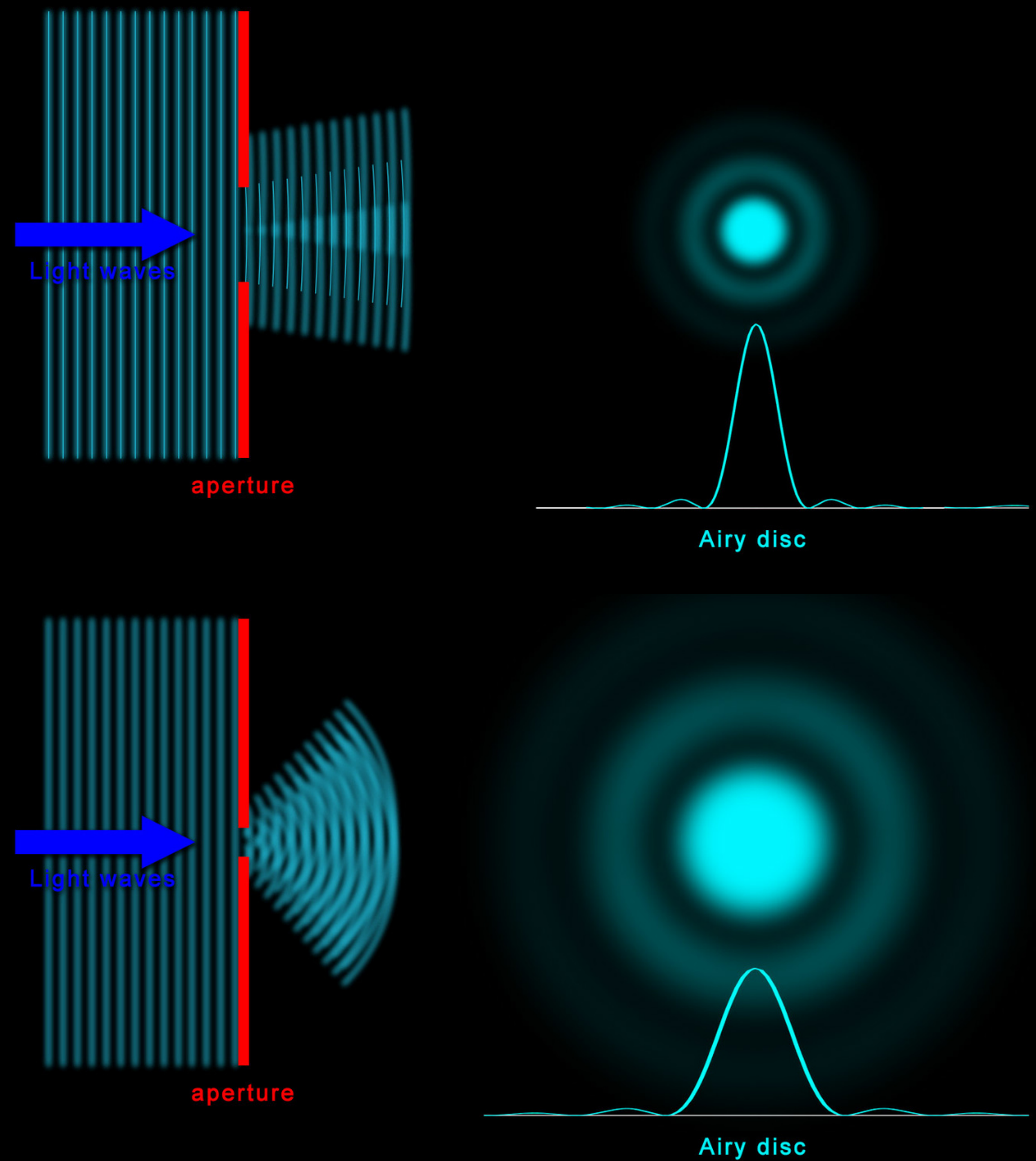


Credit: Nando Harmsen, 2021.

Long or short?

- If the radiation is short wavelength, then the entirety of a radiation packet is contained within that characteristic scale, and can be treated as a point particle traversing the spacetime geodesics. in a simple euclidean/Newtonian approximation of the spacetime, we then have a pythagorean setup.

- If, however, the wavelength is long, then there will be significant diffraction that takes place, and self-interference effects also become importan. More generally these regimes are not so clean cut, and some effects more important than others in each limit but often all important.



Credit: Nando Harmsen, 2021.

When is it really wave optics?

$$\Omega \approx 1.2 \times 10^5 (M/M_\odot)(\nu/\mathrm{GHz})$$

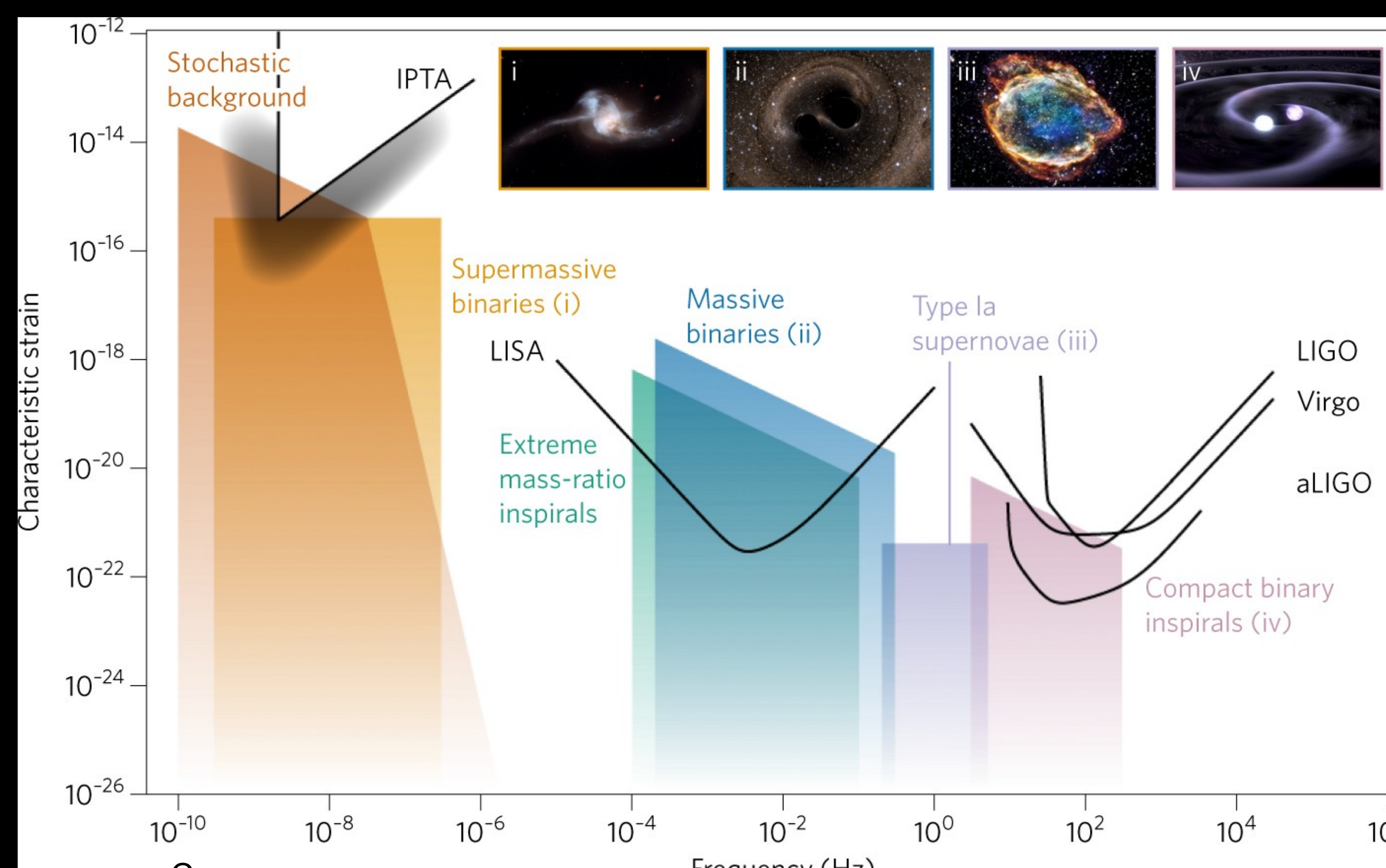
Geometric optics valid for Omega >>1

- In this talk we are interested in the long wavelength limit.
- For EM radiation this is likely uninteresting for gamma-rays, and so on. But for radio there can be planetary mass lenses, which has all sorts of interesting implications as concerns radio pulsars and FRBs.
- Here: GWs!

Diffraction for:

EM: ~planets

GW: ~stars



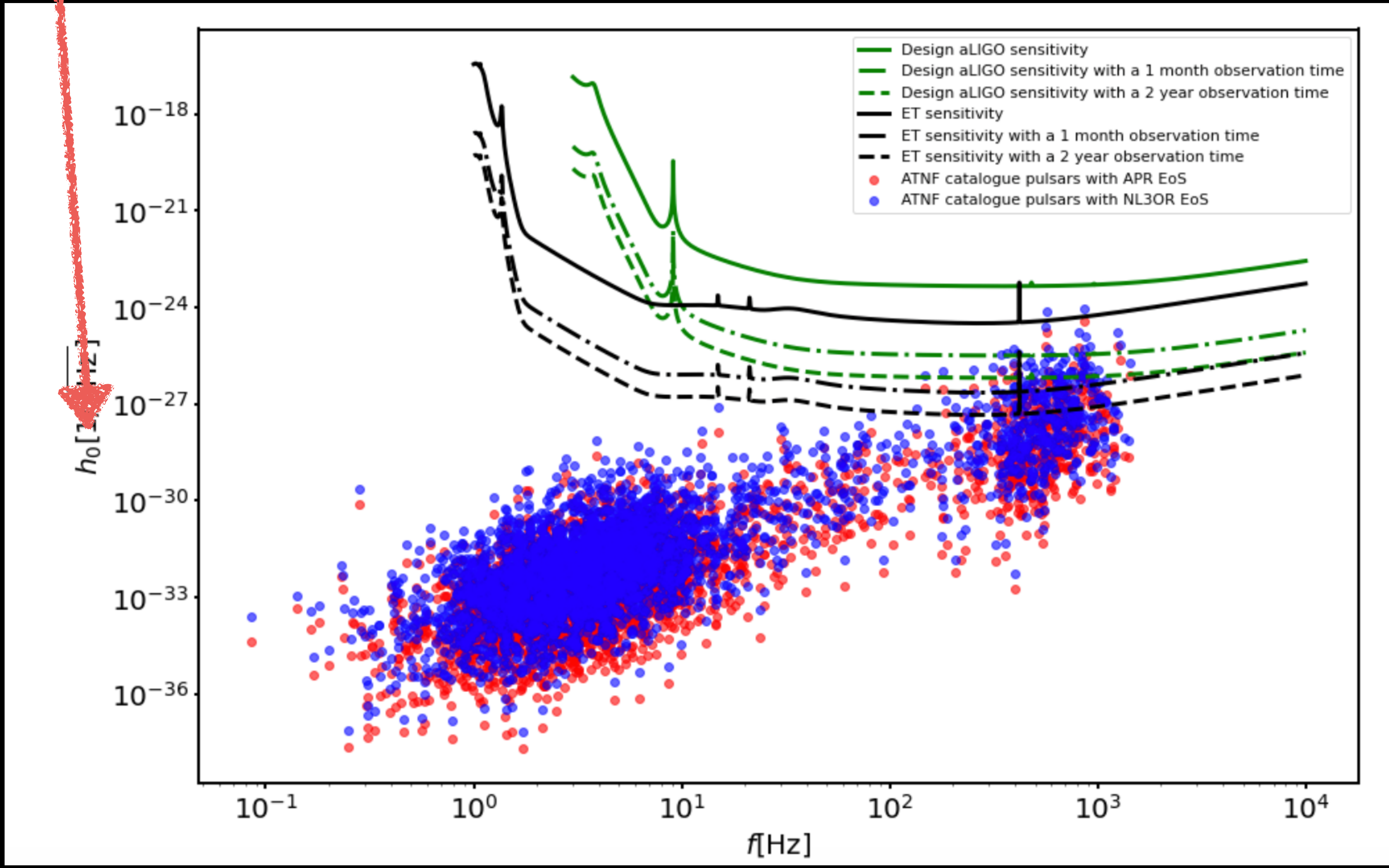
GWPlotter; Moustakas 2017

Continuous GWs from neutron stars

- For rapidly rotating neutron stars, several mechanisms can organically (but also inorganically, e.g., accretion) induce large momentum or energy fluxes within the stellar interior.
- It is hoped that these waves will be detected in the near future;
- However, owing to their faintness, it will likely require long periods of persistent monitoring

$$h_0 \approx 1.4 \times 10^{-28} \left(\frac{Q_{22}}{10^{36}\ \mathrm{g\ cm}^2} \right) \times \left(\frac{\nu_\star}{500\ \mathrm{Hz}} \right)^2 \left(\frac{10\ \mathrm{kpc}}{D_{OS}} \right),$$

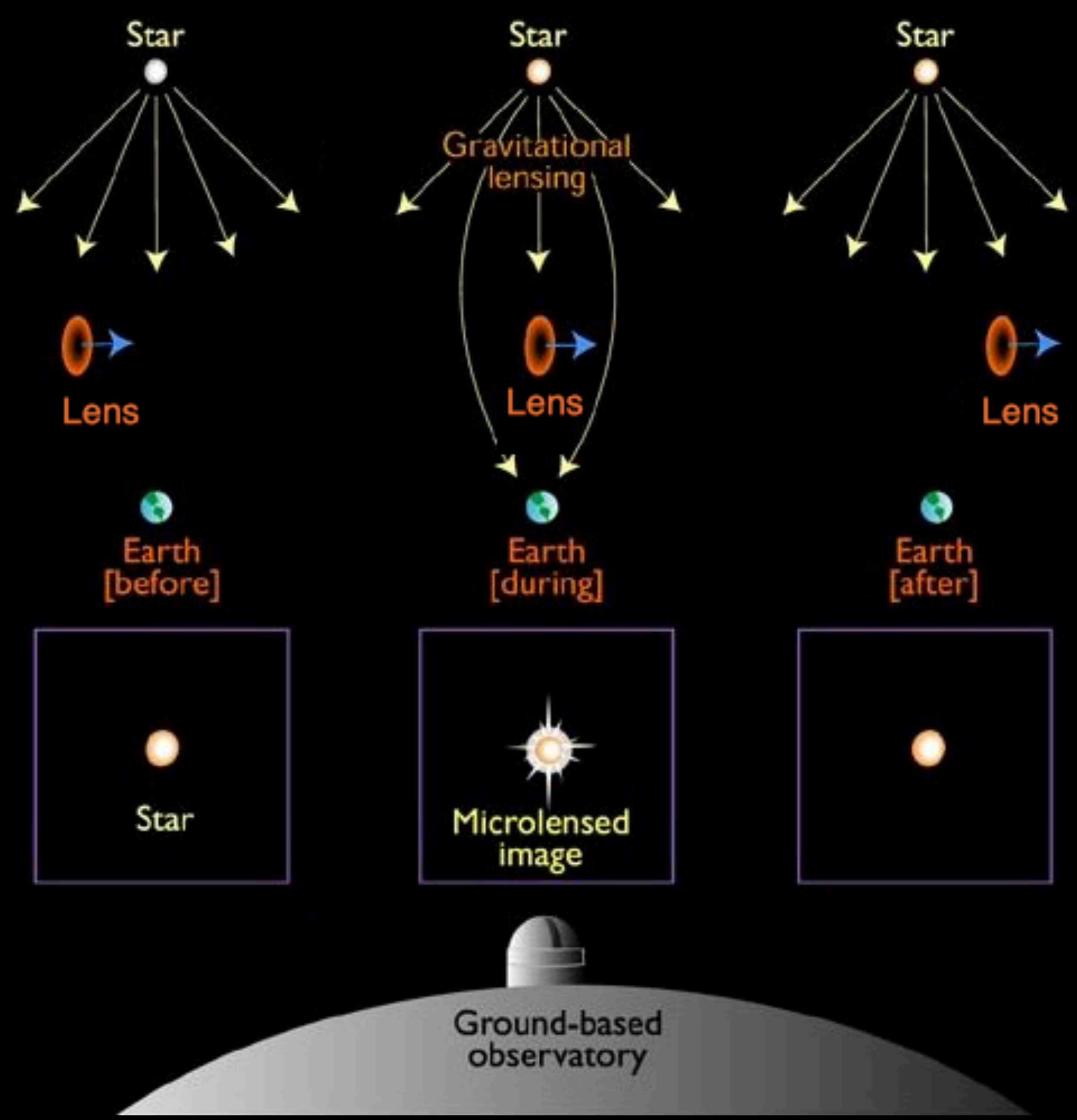
$$Q_{22} \approx 7 \times 10^{35} \left(\frac{B_\star}{10^{13}\ \mathrm{G}} \right)^2 \left(\frac{R_\star}{10^6\ \mathrm{cm}} \right)^6 \times \left(\frac{1.4 M_\odot}{M_\star} \right) \mathrm{g\ cm}^2,$$



Magnetic deformation estimates; Soldateschi & Bucciantini 2021

Long-term observations

- Hard to detect continuous GWs in general; pulsars, especially of the millisecond variety, are however rather stable! -> Narrow-band search.
- Any individual microlensing event is however unlikely, but when you take into account that a year or so of observation may be required by narrow-band searches, this implies actually that the probability is non-negligible (especially for stars with large supernova kicks).
- We could potentially misintrepret source behaviour, or miss a detection, by not accounting for this.



Modified from ESA/Hubble illustration [https://esahubble.org]

Physical optics

- Relevant equations can be derived in a number of ways: by considering a GW perturbation in the Lorentz gauge
- Or: the GWs/light take all possible paths to reach the observer, weighted by some probability (Nakamura+Deguchi 1999)

$$ds^2 = g^{(L)}_{\mu\nu} dx^\mu dx^\nu = -(1+2U)\, dt^2 + (1-2U)\, d\boldsymbol{x}^2$$

Perturbation

$$0 = \nabla_\alpha \nabla^\alpha h_{\mu\nu} + 2R^{(L)}_{\alpha\mu\beta\nu} h^{\alpha\beta} + \mathcal{O}(h^2)$$

Keeping terms to linear order in U (for consistency)

$$0 = \left(\nabla^2 + \omega^2\right) \phi - 4\omega^2 U \phi$$

Kirchhoff theorem — Feynman path integral

$$\phi(\boldsymbol{x}) = \phi^{(0)}(\boldsymbol{x}) - \frac{\omega^2}{\pi} \int d^3\boldsymbol{x}' \frac{e^{i\omega|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} U(\boldsymbol{x}') \phi^{(0)}(\boldsymbol{x}')$$

Integral over the aperture; which continues to infinity because the gravitational potential never totally vanishes

Physical optics

- We look at here a 2D projection, that it is to assume that the lenses are relatively far away from either the source or the observer; collect them all on a plane and collapse the z dimension.

- Maybe inappropriate, for example, for a binary and the sun (see Marchant++ 2021).

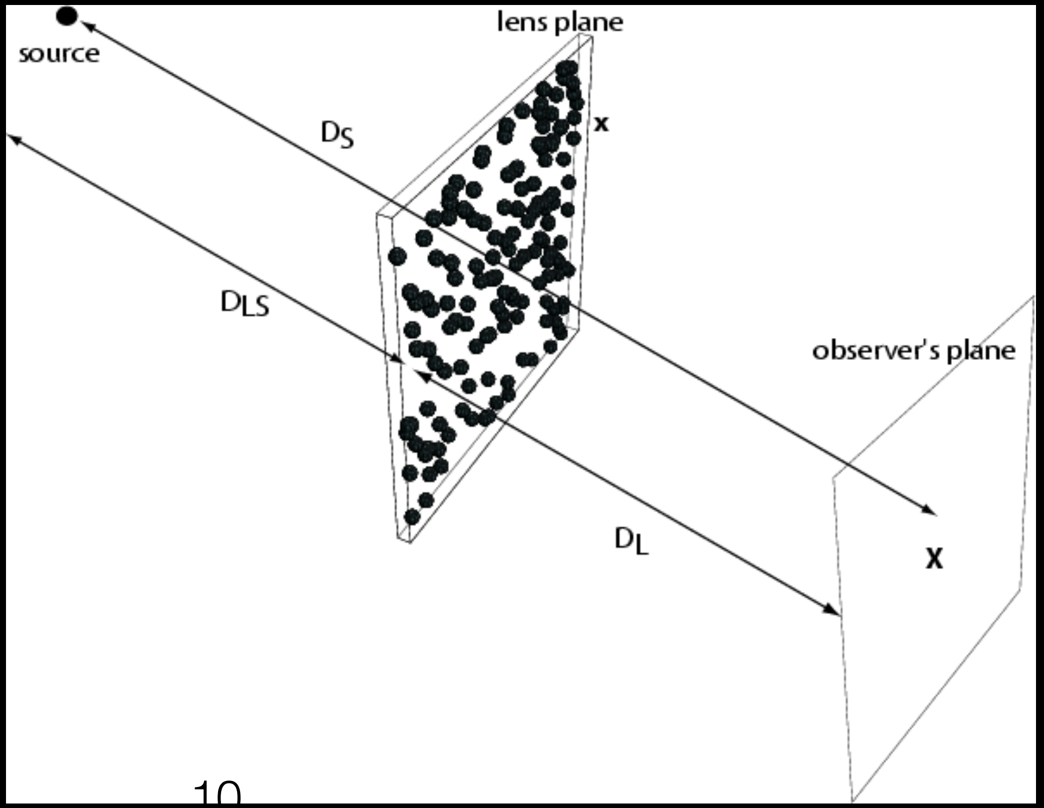
$$\phi(\boldsymbol{x}) = \phi^{(0)}(\boldsymbol{x}) - \frac{\omega^2}{\pi} \int d^3\boldsymbol{x}' \, \frac{e^{i\omega|\boldsymbol{x}-\boldsymbol{x}'|}}{|\boldsymbol{x}-\boldsymbol{x}'|} U(\boldsymbol{x}')\phi^{(0)}(\boldsymbol{x}')$$

2D Projection

$$F(\boldsymbol{x}_s) = \frac{w}{2\pi i} \int_{\mathbb{R}^2} d^2\boldsymbol{x} \exp\left(iw \left[\frac{|\boldsymbol{x}-\boldsymbol{x}_s|^2}{2} - \sum_{j \leq N} \left(\frac{M_j}{M_L}\right) \log\sqrt{(x-x_j)^2 + (y-y_j)^2} \right] \right)$$

Pythagorean contribution

Shapiro delay



Macquardt (2004), A&A 422, 761

Number crunch time!

- Eventually, we want to evaluate integrals that look like this:

$$\int e^{i\phi}dx \sim \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdye^{i\nu t_d(x,y,x_s,y_s)}$$

How?

- Numerical methods tend to fail, because:

#1: Euler's formula: integrand is like cos + isin; does not decay fast (cutoffs also fail).

#2: Conditionally convergent; certain methods involving coordinate transforms fail.

#3: Singularities at microlens positions (due to our approx.)

Picard-Lefschetz fundamental idea

- General strategy: complexity coordinates such that the oscillatory (imaginary) part of the integral is constant

- In 1D: we have a phase and complexity x->z = x+iy

$$e^{i\phi(z)} = e^{i\phi(x+iy)} = e^{h(x,y)+iH(x,y)} = \overbrace{e^{h(x,y)}}^{\text{Well behaved}} \underbrace{e^{iH(x,y)}}_{\text{Oscillatory}}$$

If we manufacture a contour y = y(x) such that the function H is constant, then an integral along this curve is going to be well-behaved and evaluable with standard techniques

Refs: Witten 2010; Feldbrugge++2019;2020

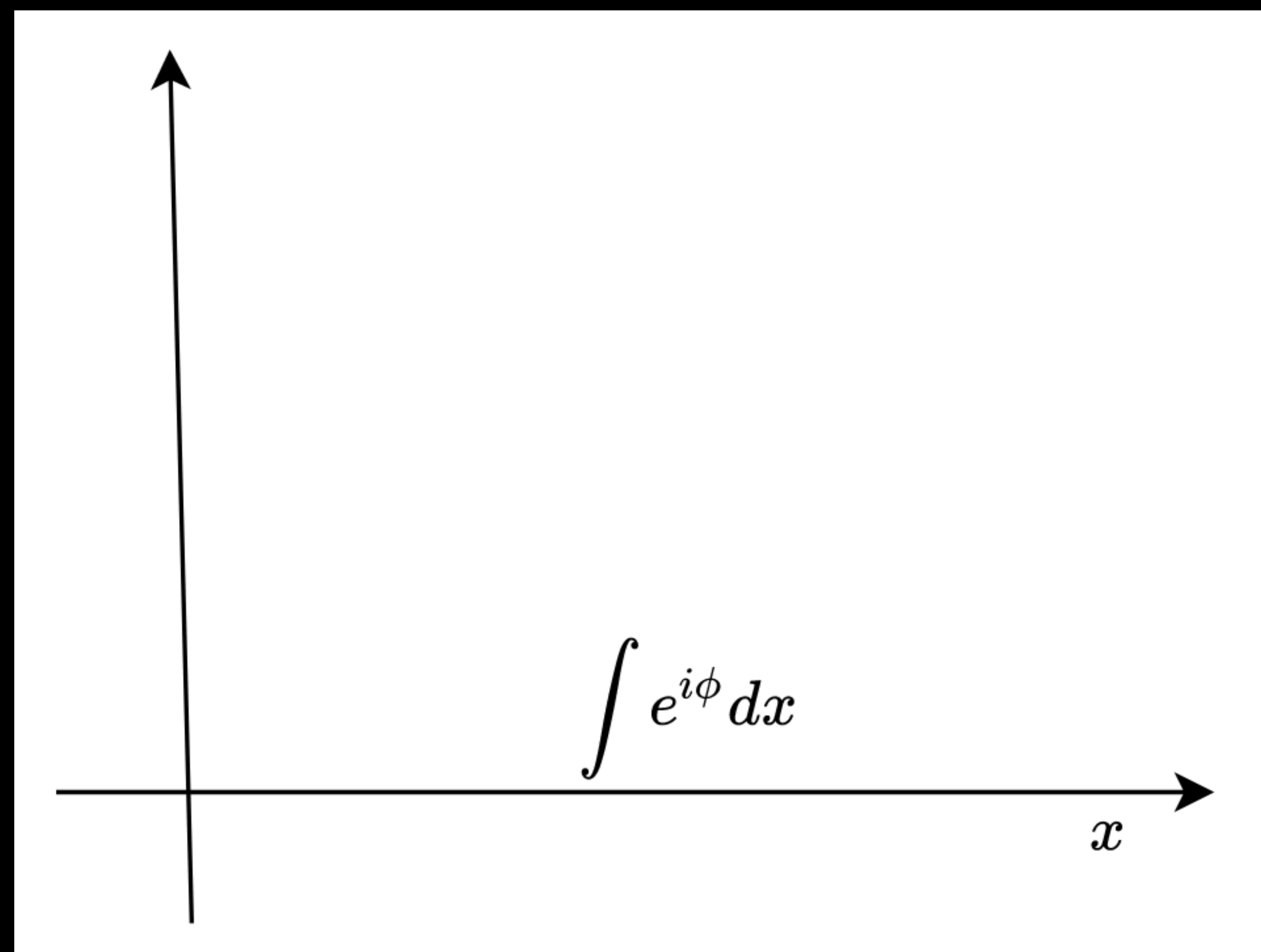
Promotion to the complex plane

- Eventually, we want to evaluate integrals that look like this:

$$\int e^{i\phi} dx \sim \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy e^{i\nu t_d(x,y,x_s,y_s)}$$

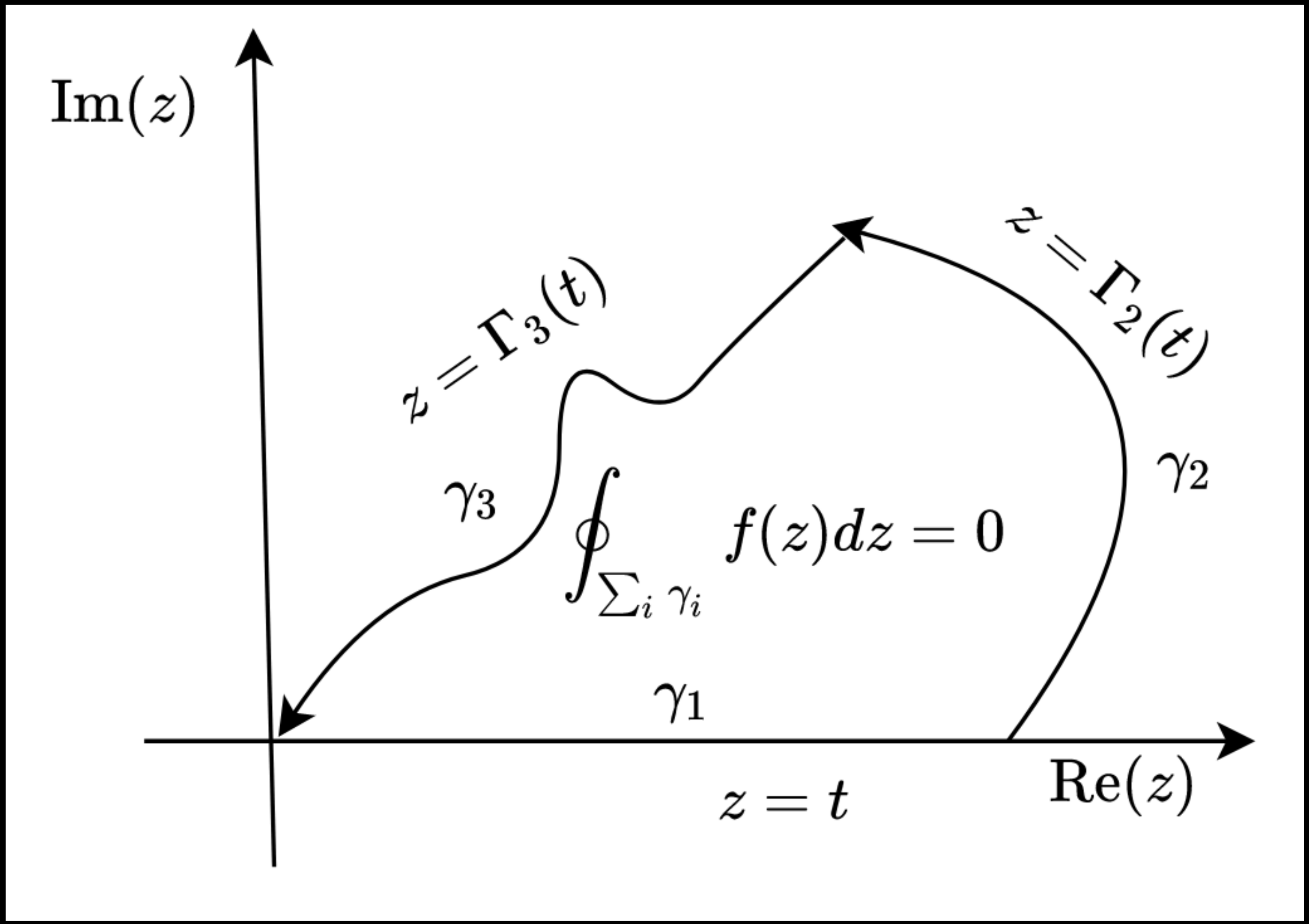
- The oscillatory integral can be thought of a line integral over the reals (trivially)

- Cauchy's theorem can then be used by deforming the "contour" into the complex plane



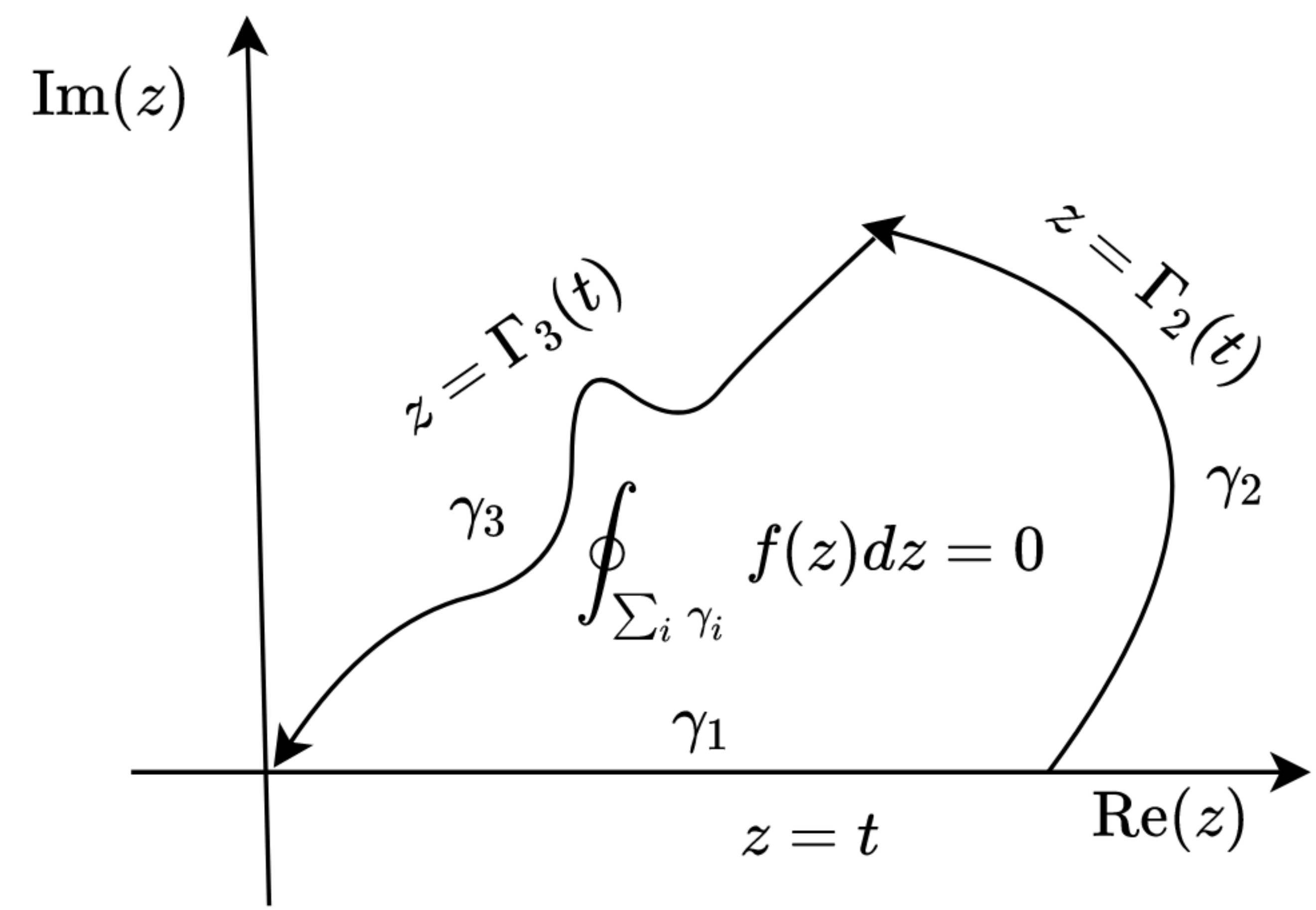
Promotion to the complex plane

- Cauchy's theorem tells us that the integral, provided the function is holomorphic (complex analytic), is zero



Promotion to the complex plane

- Cauchy's theorem tells us that the integral, provided the function is holomorphic (complex analytic), is zero
- Since the sum is zero, we can evaluate the real integral without actually evaluating it, but evaluating two other integrals instead



$$\overset{\text{Cauchy}}{\Longrightarrow}$$

$$\int e^{i\phi} dx = -\int_{\gamma_2} e^{i\phi[\Gamma_2(t)]}\dot{\Gamma}_2(t)dt - \int_{\gamma_3} e^{i\phi[\Gamma_3(t)]}\dot{\Gamma}_3(t)dt$$

Tends to vanish by Jordan's lemma

Fresnel integral

Trying this on the Fresnel integral we all know and love leads to

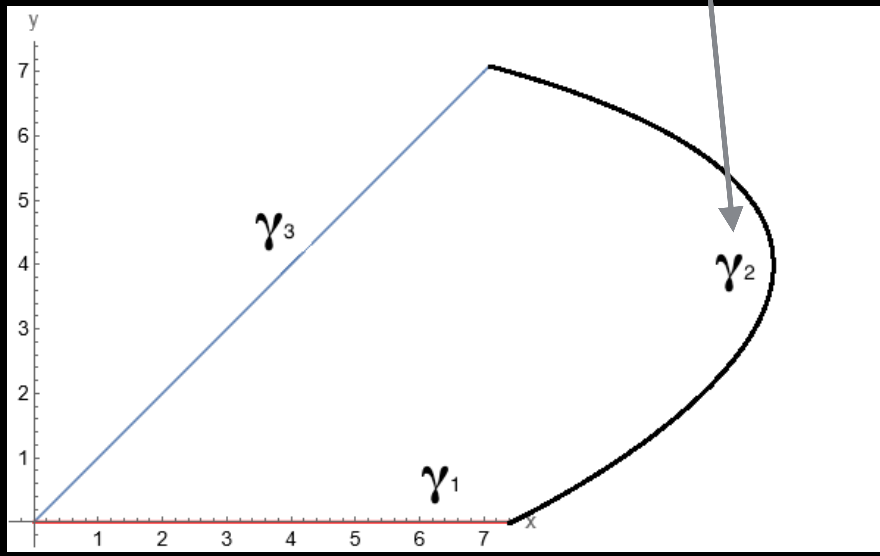
$$\phi = x^2 \quad \Longrightarrow \quad i\phi(z) \mapsto -2xy + \underbrace{i(x^2 - y^2)}_{y = \pm x}$$

The solution of constant imaginary component for x>0 is precisely the line y=x, which is the line of argument π/4

$$\phi = x^2; x \mapsto e^{i\pi/4}t$$

$$\Longrightarrow \int_0^\infty e^{ix^2}\,dx = e^{i\pi/4}\int_0^\infty e^{-t^2}\,dt$$

Vanishes by Jordan Lemma



In some more generality

- The basic tool that we begin is the “Morse flow equation” with some generic affine parameter [(x(t),y(t)], which is a geometric flow on the complex plane that leads to steepest descent

Morse flow equation

$$\frac{d\gamma^i}{dt} = -g^{ij}\frac{\partial h}{\partial \gamma^j}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -2g^{i\bar{j}}\partial_i h \bar{\partial}_{\bar{j}} h \, (=: -|\nabla h|^2) \leq 0.$$

Witten (2010)

In some more generality

- The basic tool that we begin is the “Morse flow equation” with some generic affine parameter [(x(t),y(t)], which is a geometric flow on the complex plane that leads to steepest descent
- The main perk of this is that the imaginary component is precisely constant along this flow
- The Lefschetz thimble is the multi-dimensional surface traced by the flow

Morse flow equation

$$\frac{d\gamma^i}{dt} = -g^{ij}\frac{\partial h}{\partial \gamma^j}$$

Witten (2010)

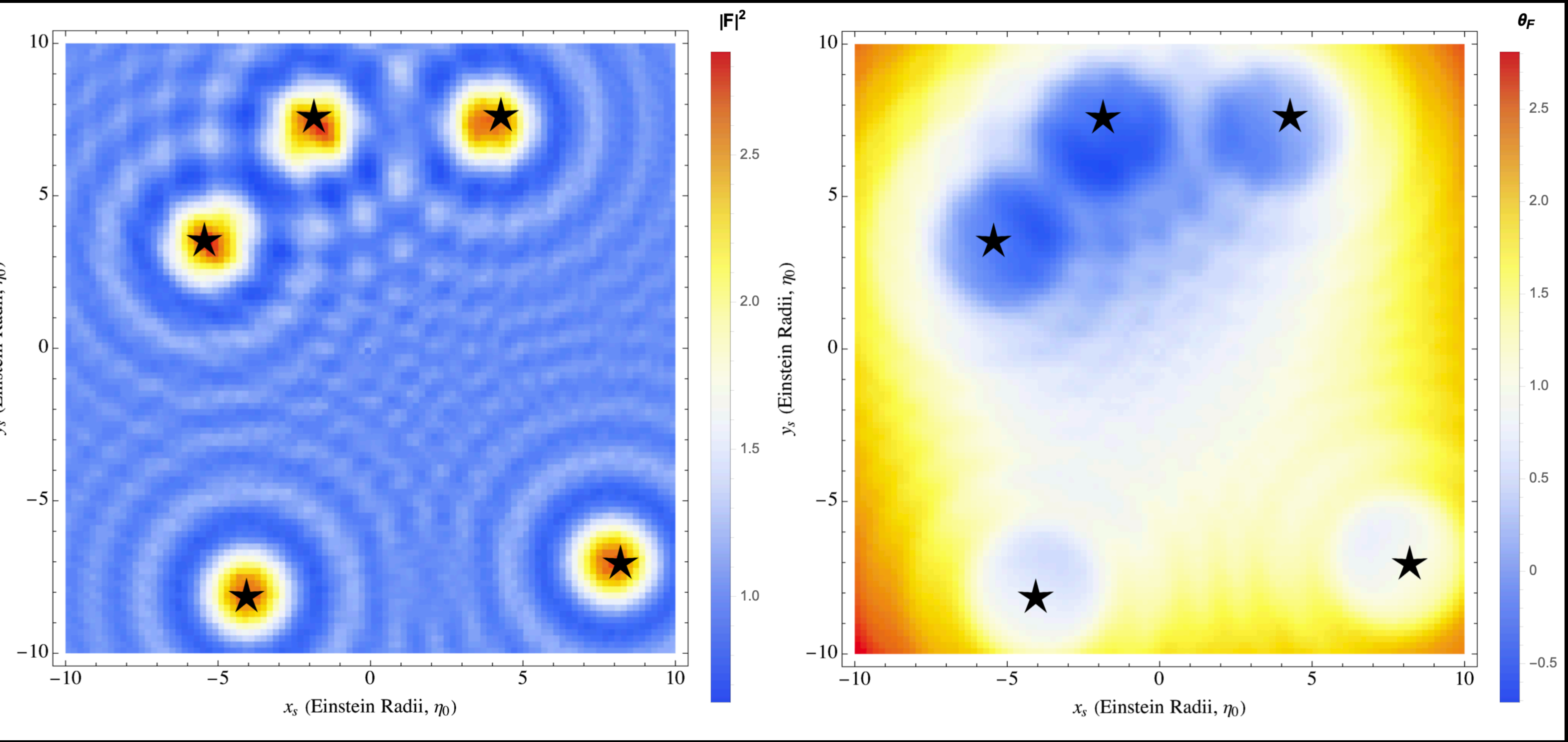
$$\frac{\mathrm{d}h}{\mathrm{d}t} = -2g^{i\bar{j}}\partial_i h \bar{\partial}_{\bar{j}} h \, (=: -|\nabla h|^2) \leq 0.$$

$$\Longrightarrow \quad \begin{aligned} \frac{dH}{dt} &= \frac{d\gamma^i}{dt}\frac{\partial H}{\partial x^i} \\ &= -g^{ij}\frac{\partial h}{\partial \gamma^j}\frac{\partial H}{\partial x^i} \\ &= 0 \end{aligned}$$

Cauchy-Riemann:

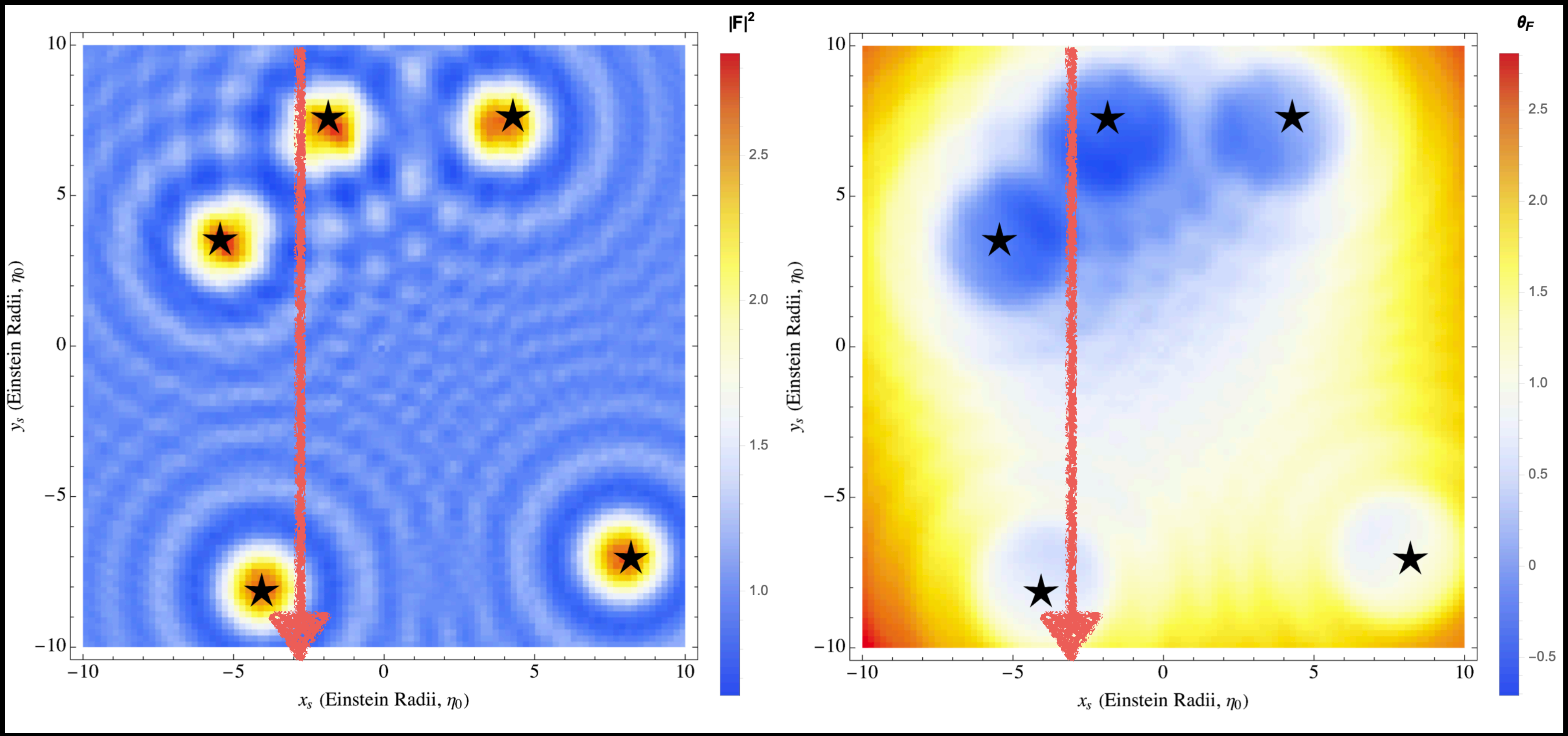
$(\nabla h \cdot \nabla H = 0)$

A simple worked, example

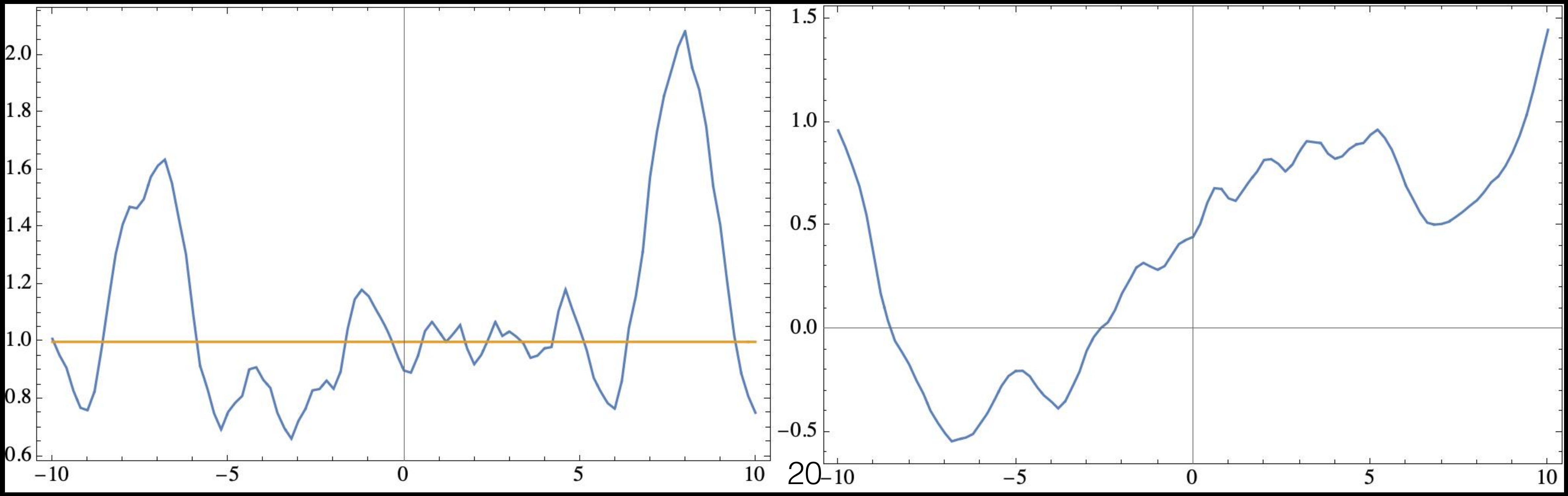


Here: five micro lenses of mass ~8 solars each, with a millisecond pulsar source (400 Hz GW frequency).

Relative motion



Imagine the source is moving relative to the lens over time



Parameter estimation?

- At the simplest level, much of the information concerning the oscillations can be "killed off" in just considering the RMS strain; h -> |F| h

- It may be important in rare cases that the effects of lensing, which could ostensibly imply large variations in the quadrupole moment over the observational window, be accounted for

$$\frac{\mathrm{SNR}(F \neq 1)}{\mathrm{SNR}(F = 1)} \approx \sqrt{\frac{\int_0^{T_{\mathrm{obs}}} |F(t)|^2 \sin^2(f_{\mathrm{GW}} t) dt}{\int_0^{T_{\mathrm{obs}}} \sin^2(f_{\mathrm{GW}} t) dt}} = 1.66.$$

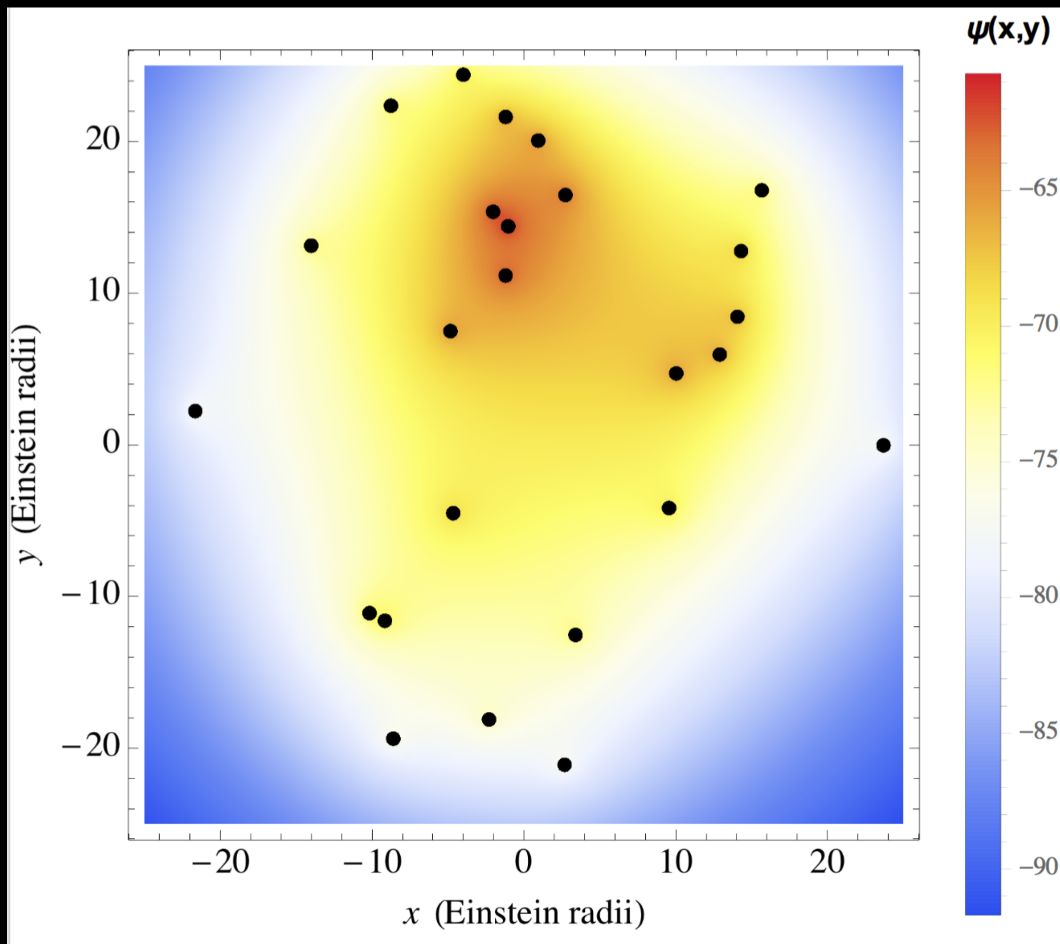
- One can introduce the Fisher matrix using the same inner product, which can be used to investigate parameter errors;

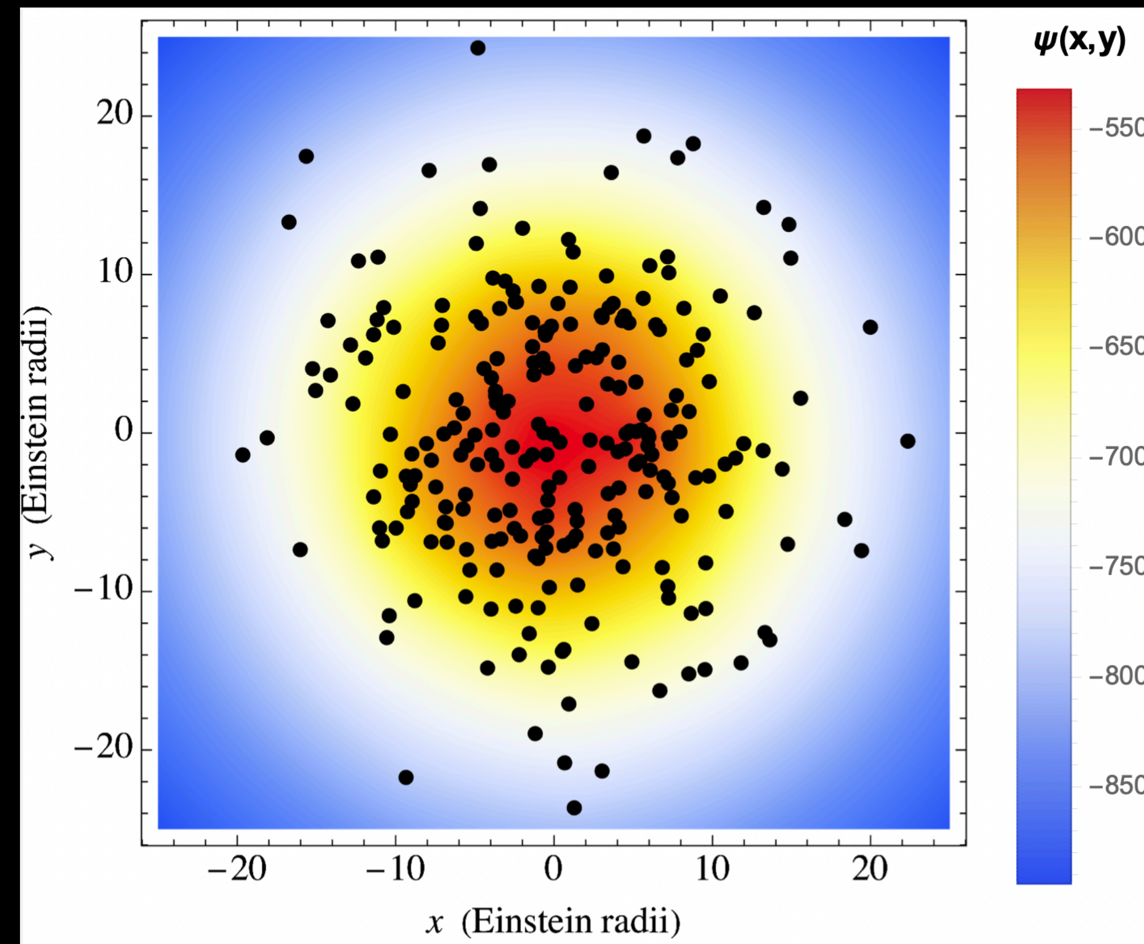
$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right)$$

2

Jaranowski++ (1998)

Lensing in globular clusters

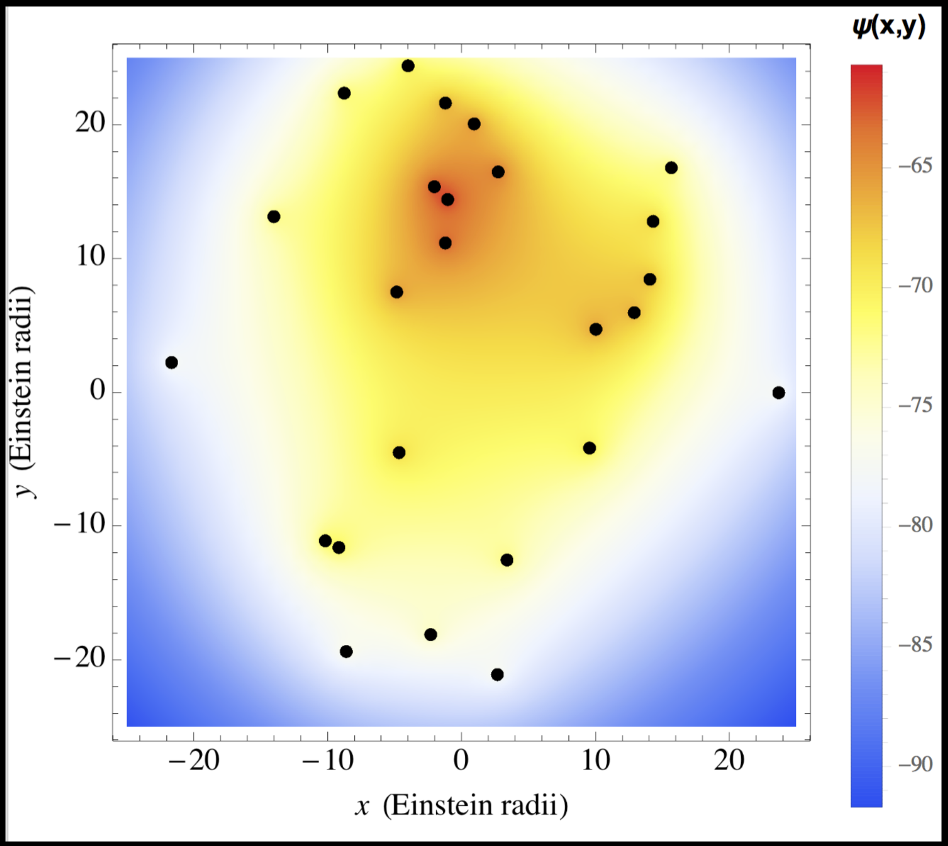




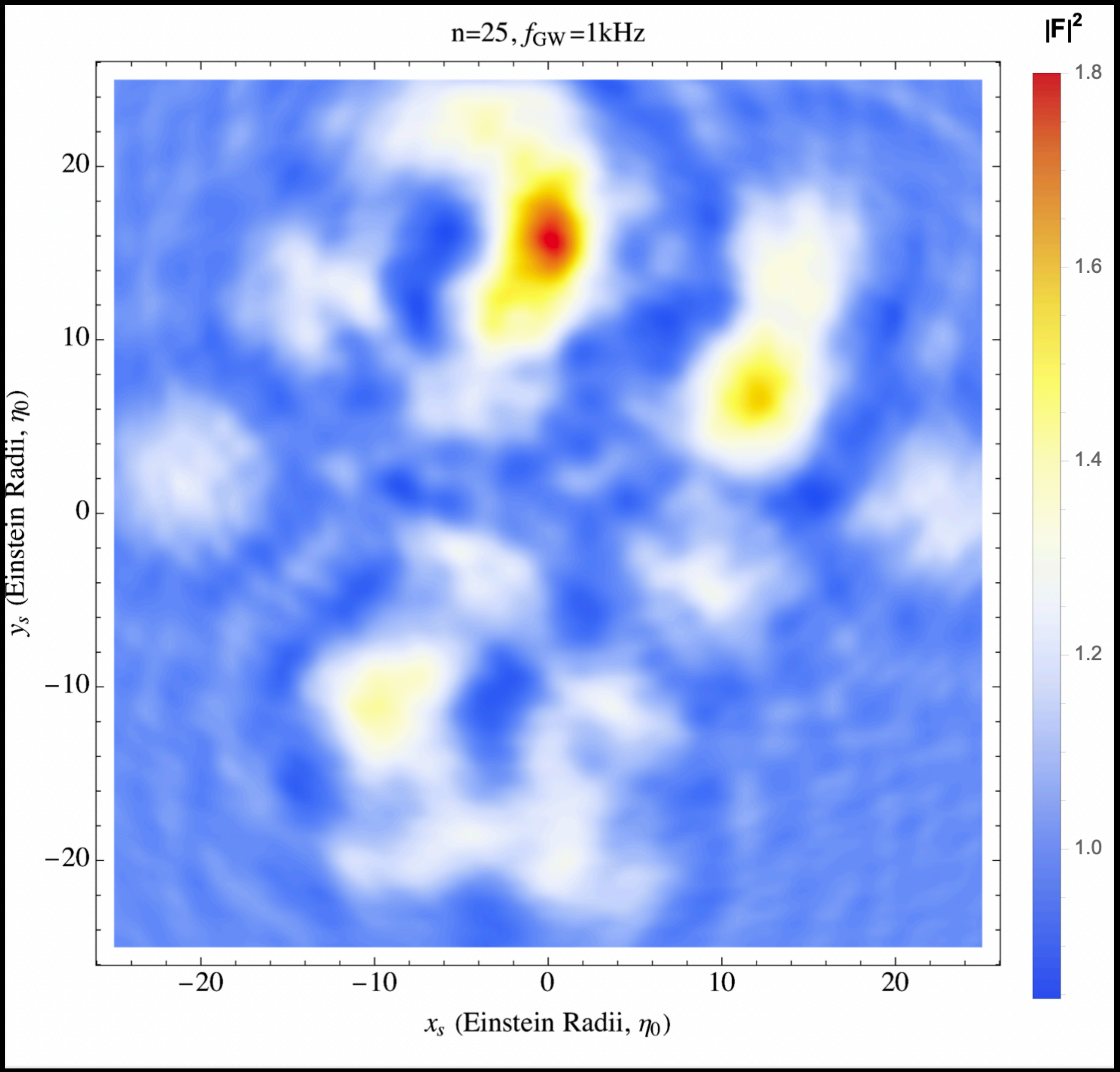
Suppose that the line of sight between the deformed neutron star and Earth intersects with a star cluster.

For example, the globular cluster Tuc 47 lies ~4 kpc from Earth, contains ~10^5 stars and at least 27 millisecond pulsars. (Freire++2017; Ridolfi++2021)

More complicated!



Depending on the relative motion of the pulsar through the plane, different 1D patterns emerge for the fluctuations in h(t)



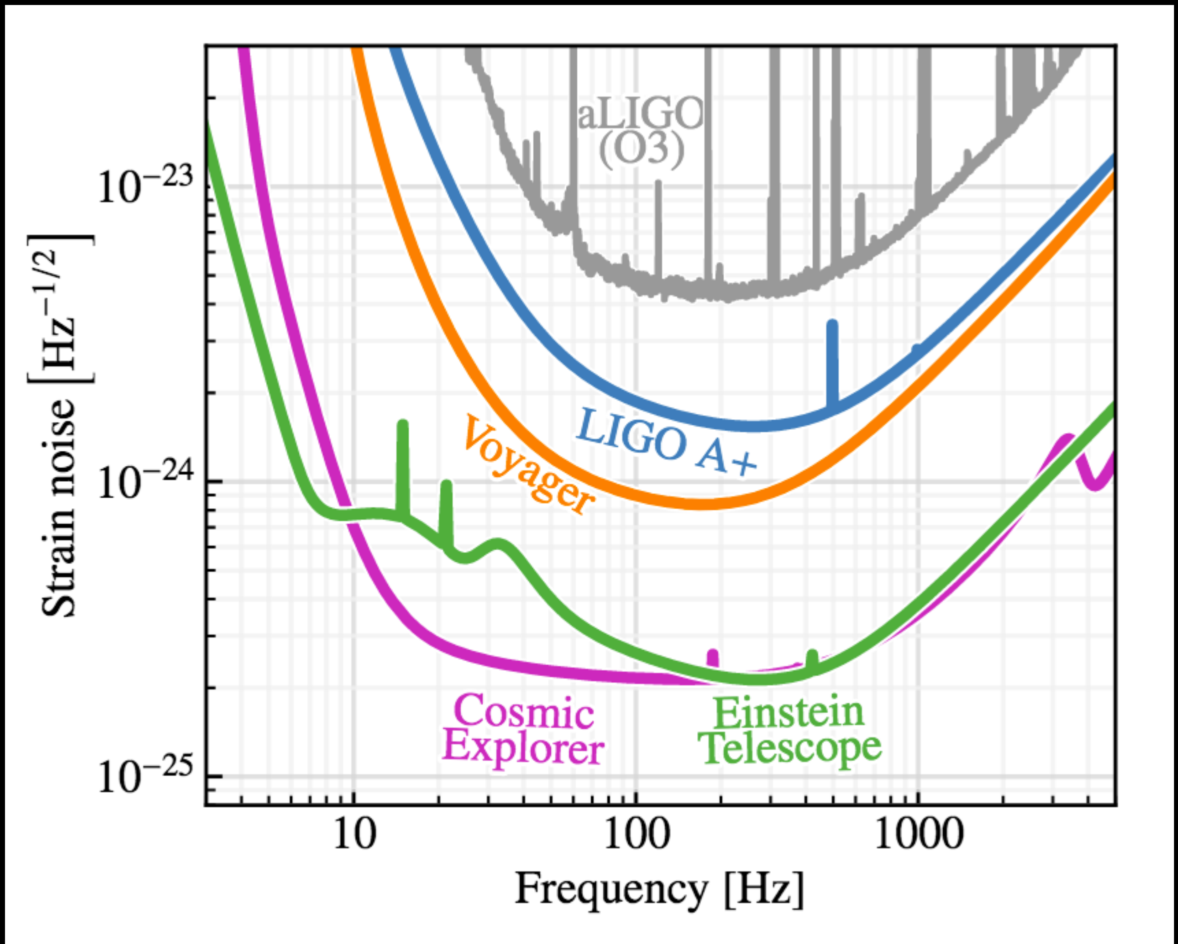
Suvorov (2022; ApJ)

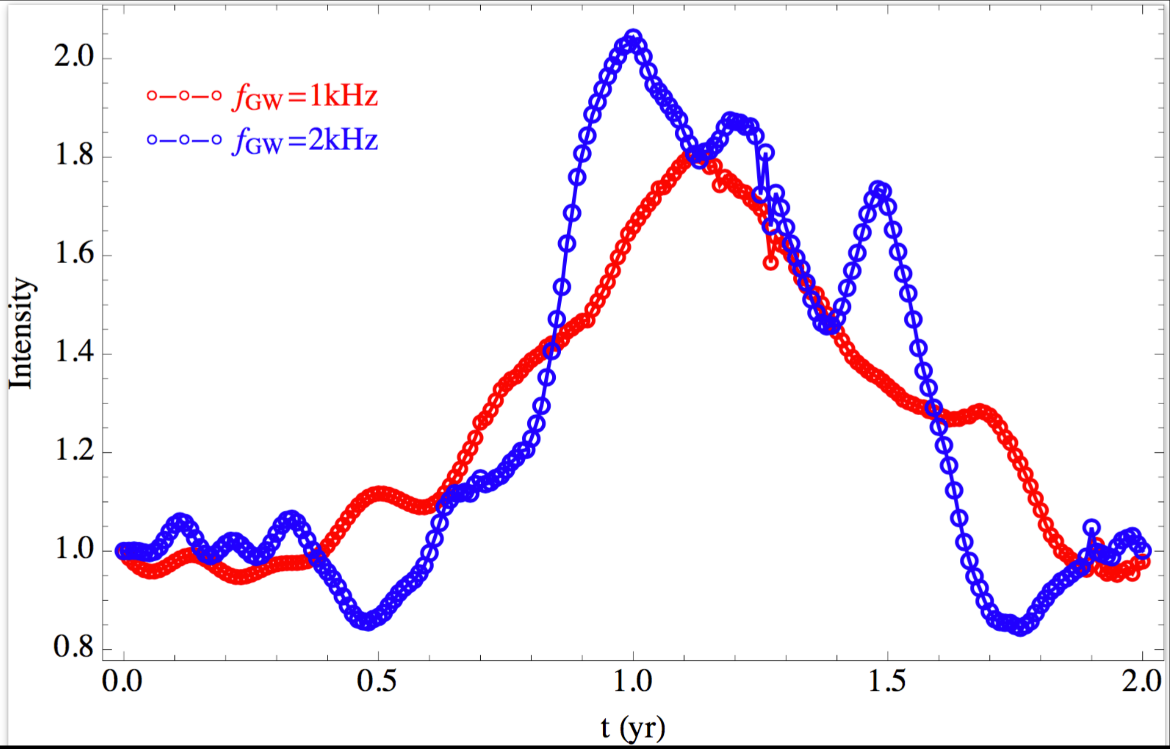
The intensity tends to cluster in the centre-of-mass for the individual lumps.

Interference effects are also there (though a bit hard to see)

Summary

- Gravitational wave propagation, when encountering astrophysical objects, will tend to be lensed.
- Importantly: the lensing will occur in the diffractive regime for stars, since the GW wavelength exceeds the Schwarzschild radius of the lens.
- The calculation in this case is hard because of the oscillatory nature of the Fresnell-Kirchhoff diffraction integral.
- Using some ideas from Picard-Lefschetz theory, we make progress (ask if you want tools!)
- Given a microlens distribution, can consider how the lensed waveform influences parameter inference.
- Future: plasma lenses, higher n, Fisher analysis





Modulation to h(t) from sparse cluster; Suvorov (ApJ; 2022)