Q1

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$L_1 = \{ a^n b^m c^h \mid n, m, h \in Z^+, 2h = n - 3m \}$
$L_2 = \{ a^n b^m c^h \mid n, m, h \in Z^+, 2h \geq n - 3m \}$

Prove L_1 is ~~regular~~ with pumping lemma
$L_1 = \{ a^n b^m c^h \mid n, m, h \in Z^+, 2h = n - 3m \}$

$p = 2o + 3l$

$w = a^p b^l c^o \mid 2o = p - 3l, \; w \in L_1, \; |w| > p$

$y = a^k, \; p \geq k \geq 1, \; |xy| \leq p, \; |y| \geq 1$

$xz = a^{p-k} b^l c^o, \; p - k < 2o + 3l$

$\Rightarrow xz \notin L_1$

$\therefore L_1$ is non regular

Theorem 1.1 (2.4.1) *Let L be a regular language. Then*

- *there is an integer $n \geq 1$ such that*
- *any string $w \in L$ with $|w| \geq n$ can be written as $w = xyz$ where x, y, and z are strings and $y \neq \epsilon$, $|xy| \leq n$ and*
- *$x(y^k)z \in L$ for all $k \geq 0$.*

Proof: Because L is regular, there is a finite automaton M such that L is the language recognized by M.

- Let n be the number of states of M.
- Let $a_1, a_2, \ldots, a_n$ be the first n symbols of w. Let q_0 be the start state of M, and let q_i be the state M is in after reading the symbols $a_1, a_2, \ldots, a_i$.
- The sequence $q_0, q_1, q_2, \ldots, q_n$ of states has $n+1$ elements, but there are only n states in M.
- Thus there must be i and j with $i \neq j$ such that $q_i = q_j$.
- Let x be $a_1, a_2, \ldots, a_i$, let y be $a_{i+1}, a_{i+2}, \ldots, a_j$, and let z be the rest of w, that is, $xyz = w$.
- Then x causes M to go from q_0 to q_i, y causes M to go from q_i to q_j, which is equal to q_i, and z causes M to go from q_j to some accepting state r of M.
- Then y^k causes M to go from q_i to q_i for any k.
- Thus for any k, the string $x(y^k)z$ is accepted, because x causes M to go from q_0 to q_i, y^i causes M to go from q_i to q_i, and z causes M to go from q_i to the accepting state r of M.

- Therefore the string $x(y^k)z$ is in L.