

# TRINITY CONSISTENCY

## A Formal Logical Consistency Model of the Nicene Doctrine of the Trinity

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### Abstract

The Christian doctrine of the Trinity, as articulated in the Nicene-Constantinopolitan Creed, affirms that the Father, the Son, and the Holy Spirit are three distinct Persons sharing one divine essence. Critics often claim this doctrine is inherently contradictory, citing the perceived impossibility of being simultaneously “three” and “one.” This paper presents a formal first-order logic model of the Nicene formulation, implemented in the Z3 theorem prover. The axioms describing the Trinity are shown to be logically satisfiable within classical logic. Attempts to encode major historical heresies—Modalism, Tritheism, and Subordinationism—produce logical contradictions under this system. The result is a reproducible, machine-verifiable proof of internal coherence, reinforcing the doctrine’s logical viability and offering a clear, accessible framework for evaluating its internal consistency.

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### 1. Introduction

The Trinity is central to historic Christianity and remains a focal point for philosophical and interreligious debate. While analogies and philosophical reasoning have defended its coherence, such approaches can be vulnerable to subjective interpretation. By formalizing the doctrine in a theorem-proving environment, this study assesses its internal consistency without reliance on rhetoric or interpretive bias. The Nicene view is encoded in classical first-order logic and evaluated for satisfiability using the Z3 SMT solver, a high-performance automated reasoning tool.

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### 2. Methodology

#### 2.1 Logical Framework

This study uses first-order logic with equality under classical rules. The Z3 solver checks whether the axioms are satisfiable (SAT) or unsatisfiable (UNSAT), indicating logical consistency or contradiction, respectively.

#### 2.2 Entities and Sorts

- **Person:** {Father, Son, Spirit} — Nicene Creed (381 AD).
- **Essence:** One constant E — Nicene Creed, “of one substance with the Father” (*homoousios*).
- **Attr:** Essential attributes such as omnipotence and eternity — Gregory of Nazianzus, *Oration 31.14*.
- **Will:** The will of the divine nature — John 10:30; John 6:38.

## 2.3 Relations and Functions

- `Shares(Person, Essence)` — each Person fully possesses the one essence.
- `Has(Essence, Attr)` / `HasP(Person, Attr)` — attributes of the essence belong equally to all Persons.
- `Begets(Person, Person)` — eternal generation of the Son.
- `Proceeds(Person, Person)` — procession of the Spirit.
- `will_of(Person)` — unity of will across Persons.

## 2.4 Axioms (Core Nicene Formulation)

1. **Unique Essence:** `ForAll(x, x == E)` — one divine *ousia*.
2. **Consubstantiality:** Each Person shares `E` fully.
3. **Attribute Sharing:** All divine attributes are possessed by all Persons.
4. **Relations of Origin:** Father begets Son; Son does not beget Father; Spirit proceeds from Father.
5. **Unity of Will:** All Persons share the same will.
6. **Distinctness:** The three Persons are pairwise distinct.

## 2.5 Anti-Heresy Tests

- **Modalism:** Father = Son — violates distinctness.
- **Tritheism:** Introduces a second essence — violates monotheism.
- **Subordinationism:** Denies an essential attribute to the Son — violates attribute sharing.

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## 3. Results

Test Case	Result	Interpretation
Core Nicene Axioms	SAT	Doctrine is internally consistent
Modalism (Father = Son)	UNSAT	Contradiction detected
Tritheism (Second essence)	UNSAT	Contradiction detected
Subordinationism (Son lacks essential attribute)	UNSAT	Contradiction detected

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## 4. Discussion

The SAT result for the Nicene axioms confirms that, as formally specified, the doctrine is logically consistent under classical logic. The UNSAT results for heretical formulations demonstrate that such deviations inherently contradict the Nicene structure. While this does not establish metaphysical truth, it confirms that the doctrine's claims do not entail a logical contradiction.

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## 5. Limitations

- Dependent on the accuracy of the formalization.
- Assumes classical first-order logic.
- Avoids metaphysical commitments, operating solely within symbolic formalism.

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## 6. Conclusion

Encoding the Nicene doctrine of the Trinity into a satisfiable first-order logic model shows it can be expressed coherently. While not proving its truth, this counters the claim that it is inherently self-contradictory. Automated theorem proving offers a repeatable, verifiable method for others to test, critique, and refine the model.

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## References

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