A Formal Logical Consistency Model of the Nicene Doctrine of the Trinity

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Abstract

The Christian doctrine of the Trinity, as articulated in the Nicene-Constantinopolitan Creed, affirms that the Father, the Son, and the Holy Spirit are three distinct persons who share one divine essence. Critics often assert that this doctrine is inherently contradictory, citing the apparent impossibility of being simultaneously 'three' and 'one'. In this paper, we present a formal first-order logic model of the Nicene formulation implemented in the Z3 theorem prover. We demonstrate that the axioms describing the Trinity are logically satisfiable within classical logic and that attempts to encode major historical heresies—Modalism, Tritheism, and Subordinationism—produce logical contradictions under this system. The result is a reproducible, machine-verifiable proof of internal coherence.

1. Introduction

The doctrine of the Trinity is foundational to historic Christianity yet remains a focal point for philosophical and interreligious debate. While theological discourse has historically defended its coherence through analogy and philosophical reasoning, such arguments remain susceptible to subjective interpretation. By formalizing the doctrine in a theorem-proving environment, we can evaluate its internal consistency without reliance on rhetoric or human interpretive bias. This paper encodes the Nicene-orthodox view of the Trinity in classical first-order logic and tests for satisfiability using the Z3 SMT (Satisfiability Modulo Theories) solver.

2. Methodology

2.1 Logical Framework
We employ first-order logic with equality under classical logic rules. The Z3 solver is used to evaluate satisfiability (SAT) and unsatisfiability (UNSAT) of the axioms.

2.2 Entities and Sorts
- Person: Enumerated type containing {Father, Son, Spirit}. - Essence: Declared sort with constant E representing the one divine essence. - Attr: Declared sort for essential attributes (e.g., omnipotence, eternity). - Will: Declared sort for the will of the divine nature.

2.3 Relations and Functions
- Shares(Person, Essence) — Person shares the divine essence. - Has(Essence, Attr) — Essence possesses an attribute. - HasP(Person, Attr) — Person possesses an attribute. - Begets(Person, Person) — Personal relation of generation. - Proceeds(Person, Person) — Personal relation of procession. - will_of(Person) — Maps a Person to their will.

2.4 Axioms (Core Nicene Formulation)
1. Unique Essence: All essences are identical to E. 2. Consubstantiality: Each Person shares E. 3. Attribute Sharing: All attributes of E are had by all Persons. 4. Relations of Origin: - Father begets Son. - Son does not beget Father. - Spirit proceeds from Father (no Filioque). 5. Unity of Will: All Persons share will_of_E. 6. Distinctness: Father, Son, and Spirit are pairwise distinct.

2.5 Anti-Heresy Tests
- Modalism: Assert Father = Son. - Tritheism: Introduce a second essence E2 ≠ E. - Subordinationism: Assert the Son lacks an essential attribute of E.

3. Results

Test Case	Result	Interpretation
Core Nicene Axioms	SAT	Doctrine is internally consistent
Modalism (Father = Son)	UNSAT	Contradiction detected
Tritheism (Second essence)	UNSAT	Contradiction detected
Subordinationism (Son lacks essential attribute)	UNSAT	Contradiction detected

4. Discussion

The solver's SAT result for the Nicene axioms demonstrates that the doctrine, as formally specified, is logically consistent under classical logic. The UNSAT results for heretical formulations indicate that these deviations inherently violate the Nicene structure. Importantly, this result does not establish the truth of the Trinity in a metaphysical sense. Instead, it provides a reproducible formal model showing that the doctrine's claims do not entail a contradiction. This addresses a common objection that the Trinity is 'illogical' by providing a machine-verifiable counterexample.

5. Limitations

- The model's conclusions are only as sound as its formalization. If key theological nuances are omitted or misrepresented, the result may not fully capture the doctrine. - Classical first-order logic is assumed; results may differ under alternative logical systems. - The model avoids metaphysical commitments and operates strictly within symbolic formalism.

6. Conclusion

By encoding the Nicene doctrine of the Trinity into a satisfiable first-order logic model, we have demonstrated that it can be expressed in a way that is internally coherent. This does not prove its truth but removes the assertion that it is inherently self-contradictory. The use of automated theorem proving provides a repeatable method for others to test, critique, and refine the formalization.

References

1. Nicene-Constantinopolitan Creed, 381 AD. 2. De Moura, L. & Bjørner, N. (2008). Z3: An Efficient SMT Solver. TACAS. 3. Leftow, B. (2004). A Latin Trinity. Faith and Philosophy, 21(3). 4. Tuggy, D. (2013). The Unfinished Business of Trinitarian Theorizing.