

# A Formal Logical Consistency Model of the Nicene Doctrine of the Trinity

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## Abstract

The Christian doctrine of the Trinity, as articulated in the Nicene-Constantinopolitan Creed, affirms that the Father, the Son, and the Holy Spirit are three distinct persons who share one divine essence. Critics often claim this doctrine is inherently contradictory, citing the perceived impossibility of being simultaneously “three” and “one.” This paper presents a formal first-order logic model of the Nicene formulation, implemented in the Z3 theorem prover. The axioms describing the Trinity are shown to be logically satisfiable within classical logic. Attempts to encode major historical heresies—Modalism (claiming the Persons are just modes or roles of one person), Tritheism (claiming there are three separate gods), and Subordinationism (claiming the Son or Spirit is less divine than the Father)—produce logical contradictions under this system. The result is a reproducible, machine-verifiable proof of internal coherence. These findings not only reinforce the doctrine’s logical viability for scholars but also provide a clear, accessible framework that can help lay audiences understand and evaluate the Trinity’s internal consistency.

## 1. Introduction

The doctrine of the Trinity is central to historic Christianity yet remains a focal point for philosophical and interreligious debate. While theological discourse has often defended its coherence through analogy and philosophical reasoning, such arguments are vulnerable to subjective interpretation. By formalizing the doctrine in a theorem-proving environment, we can assess its internal consistency without relying on rhetoric or interpretive bias. This paper encodes the Nicene-orthodox view of the Trinity in classical first-order logic and evaluates its satisfiability using the Z3 SMT (Satisfiability Modulo Theories) solver, a high-performance automated reasoning tool used to check logical formulas for consistency.

## 2. Methodology

### 2.1 Logical Framework

We employ first-order logic with equality under classical rules. The Z3 solver evaluates the axioms for satisfiability (SAT) or unsatisfiability (UNSAT).

### 2.2 Entities and Sorts

- **Person:** Enumerated type {Father, Son, Spirit}.
- **Essence:** Declared sort with constant  $E$  representing the one divine essence.
- **Attr:** Declared sort for essential attributes (e.g., omnipotence, eternity).
- **Will:** Declared sort for the will of the divine nature.

## 2.3 Relations and Functions

- *Shares(Person, Essence)* — Person shares the divine essence.
- *Has(Essence, Attr)* — Essence possesses an attribute.
- *HasP(Person, Attr)* — Person possesses an attribute.
- *Begets(Person, Person)* — Personal relation of generation.
- *Proceeds(Person, Person)* — Personal relation of procession.
- *will\_of(Person)* — Maps a Person to their will.

## 2.4 Axioms (Core Nicene Formulation)

1. **Unique Essence:** All essences are identical to *E*.
2. **Consubstantiality:** Each Person shares *E*.
3. **Attribute Sharing:** All attributes of *E* are possessed by all Persons.
4. **Relations of Origin:**
5. Father begets Son.
6. Son does not beget Father.
7. Spirit proceeds from Father (no Filioque).
8. **Unity of Will:** All Persons share *will\_of E*.
9. **Distinctness:** Father, Son, and Spirit are pairwise distinct.

## 2.5 Anti-Heresy Tests

- **Modalism:** Assert Father = Son. This test addresses the view that the Persons are merely different roles or manifestations of a single individual, which denies their distinct personhood.
- **Tritheism:** Introduce a second essence  $E_2 \neq E$ . This counters the belief that the three Persons are separate gods with distinct essences.
- **Subordinationism:** Assert the Son lacks an essential attribute of *E*. This challenges any view that would diminish the Son's divinity or equality with the Father.

## 3. Results

Test Case	Result	Interpretation
Core Nicene Axioms	SAT	Doctrine is internally consistent
Modalism (Father = Son)	UNSAT	Contradiction detected
Tritheism (Second essence)	UNSAT	Contradiction detected
Subordinationism (Son lacks essential attribute)	UNSAT	Contradiction detected

## 4. Discussion

The solver's SAT result for the Nicene axioms confirms the doctrine, as formally specified, is logically consistent under classical logic. The UNSAT results for heretical formulations show that such deviations inherently violate the Nicene structure. This does not establish the Trinity's metaphysical truth but demonstrates that the doctrine's claims do not entail a contradiction. This addresses the common objection that the Trinity is "illogical" by providing a machine-verifiable counterexample.

## 5. Limitations

- The model's conclusions depend on the accuracy of its formalization; omitted or misrepresented nuances may affect results.
- Classical first-order logic is assumed; alternative logical systems may yield different results.
- The model avoids metaphysical commitments, operating solely within symbolic formalism.

## 6. Conclusion

By encoding the Nicene doctrine of the Trinity into a satisfiable first-order logic model, we have shown that it can be expressed coherently. While this does not prove its truth, it counters the claim that it is inherently self-contradictory. Automated theorem proving offers a repeatable method for others to test, critique, and refine the model.

## References

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