Machine Learning 410

Lesson 4

Convolutional Neural Networks and Feature Learning

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Reminders

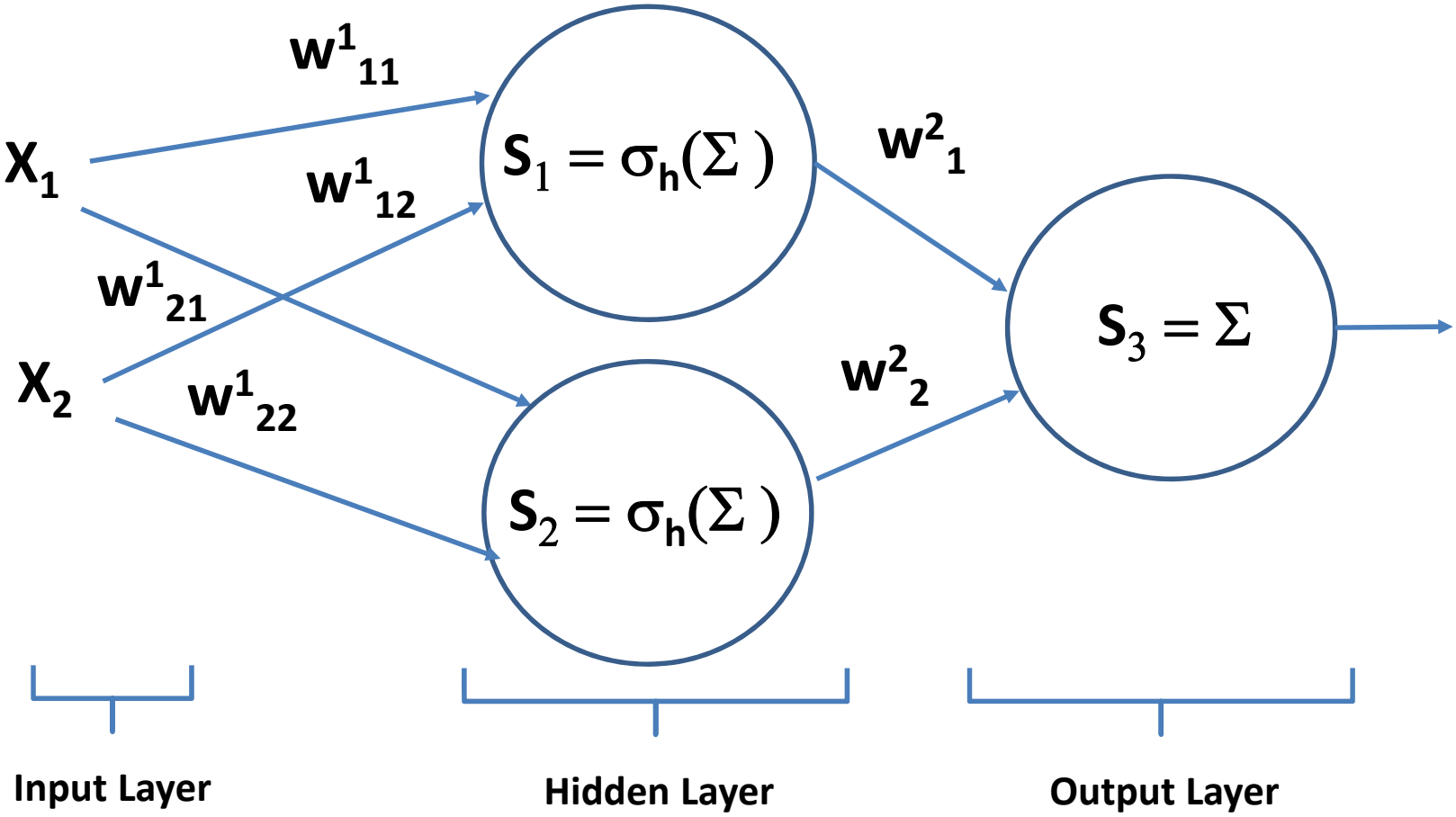
- Discussion – get an easy 5 points!
  - Discussion 3 closes tonight, Oct 17. Don't miss this!
  - Discussion 4 closes Thursday, Oct 14.
  - Discussion 5 closes Thursday, Oct 31.
- Homework – **Updated homework is in Canvas**
  - Homework 2 due Oct 20
  - Homework 3 due Oct 27
  - Homework 4 due Nov 3
- Materials in course Github repository
  - Notebook with reading
  - Slides

Review

- Forward propagation and inference
- Back propagation and gradients
- Loss functions
- Regularization and the bias-variance trade-off
- Optimization with stochastic gradient descent

Forward Propagation and Inference

Forward propagation achieved by **tensor multiplication**



The Backpropagation Algorithm

To **learn model weight tensor** we must **minimize the loss function** using the **gradient:**

$$W_{t+1} = W_t + \alpha \nabla_W J(W_t)$$

Where**:**

W_t = the tensor of weights or model parameters at step t

$J(W)$ = loss function given the weights

$\nabla_W J(W)$ = gradient of J with respect to the weights W

α = step size or learning rate

Example: Computing a Gradient

Goal is to compute the gradient:

The loss function is:

$$J(W) = -\frac{1}{2}\sum_{l=1}^{n}(y_l - S_{3,l})^2$$

$$\frac{\partial J(W)}{\partial W} = \begin{bmatrix} \frac{\partial J(W)}{\partial W_{11}^2} \\ \frac{\partial J(W)}{\partial W_{12}^2} \\ \frac{\partial J(W)}{\partial W_{21}^2} \\ \frac{\partial J(W)}{\partial W_{22}^2} \\ \frac{\partial J(W)}{\partial W_{1}^1} \\ \frac{\partial J(W)}{\partial W_{2}^1} \end{bmatrix}$$

Loss Functions for Training Neural Networks

Given: $\mathbb{D}_{KL}(P \parallel Q) = \mathbb{H}(P) + \mathbb{H}(P, Q)$

The term $\mathbb{H}(P)$ is constant

So, we only need the **cross entropy** term:

$$\mathbb{H}(P, Q) = -\sum_{i=1}^{n} p(x_i)\, ln_b\, q(x_i)$$

The Bias-Variance Trade-Off

- High capacity models fit training data well
  - Exhibit high variance
  - Do not generalize well; exhibit **brittle behavior**
  - $\text{Error}_{\text{training}} << \text{Error}_{\text{test}}$
- Low capacity models have high bias
  - Generalize well
  - Do not fit data well
- Regularization adds bias
  - Strong regularization adds significant bias
  - Weak regularization leads to high variance

The Bias-Variance Trade-Off

- How can we understand the bias-variance trade-off?
- We start with the error:

$$\Delta y = E\left[Y - \hat{f}(X)\right]$$

Where:

$Y =$ the label vector.

$X =$ the feature matrix.

$\hat{f}(x) =$ the trained model.

The Bias-Variance Trade-Off

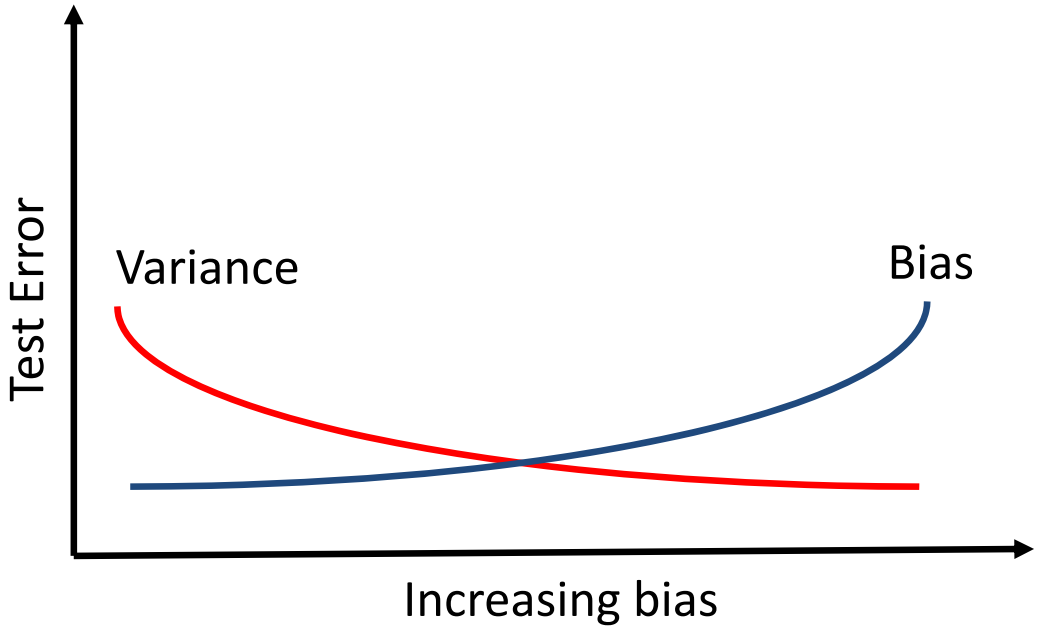
- We can expand the error term

$$\Delta x = \left(E[\hat{f}(X)] - \hat{f}(X)\right)^2 + E\left[(\hat{f}(X) - E[\hat{f}(X)])^2\right] + \sigma^2$$

$$\Delta x = Bias^2 + Variance + Irreducible\ Error$$

- Increasing bias decreases variance
- Notice that even if the bias and variance are 0 there is still irreducible error

The Bias-Variance Trade-Off



l2 Regularization

- One way to limit the size of the model parameters is to constrain the **l2** or **Euclidian norm**:

$$||W||^2 = \left(w_1^2 + w_2^2 + \ldots + w_n^2\right)^{\frac{1}{2}} = \left(\sum_{i=1}^{n} w_i^2\right)^{\frac{1}{2}}$$

- The regularized loss function is then:

$$J(W) = J_{MLE}(W) + \lambda||W||^2$$

- Where λ is the regularization hyperparameter
  - Large λ increases bias but reduces variance
  - Small λ decreases bias and increases variance

Eigenvalues and Regularization

l2 regularization with eigenvalue-eigenvector decomposition

- For the inverse $(A^T A + \alpha^2 \cdot I)^{-1}$ the eigenvalue matrix is:

$$\Lambda^{+}_{Tikhonov} = \begin{bmatrix} \frac{1}{\lambda_1 + \alpha^2} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\lambda_2 + \alpha^2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{\lambda_m + \alpha^2} \end{bmatrix}$$

- Even for an eigenvalue, λ, of 0, the biased inverse becomes $1/\alpha^2$
- Thus, the bias term, α^2, creates a 'ridge' of nonzero values on the diagonal ensuring the inverse exists and is stable.

l1 Regularization

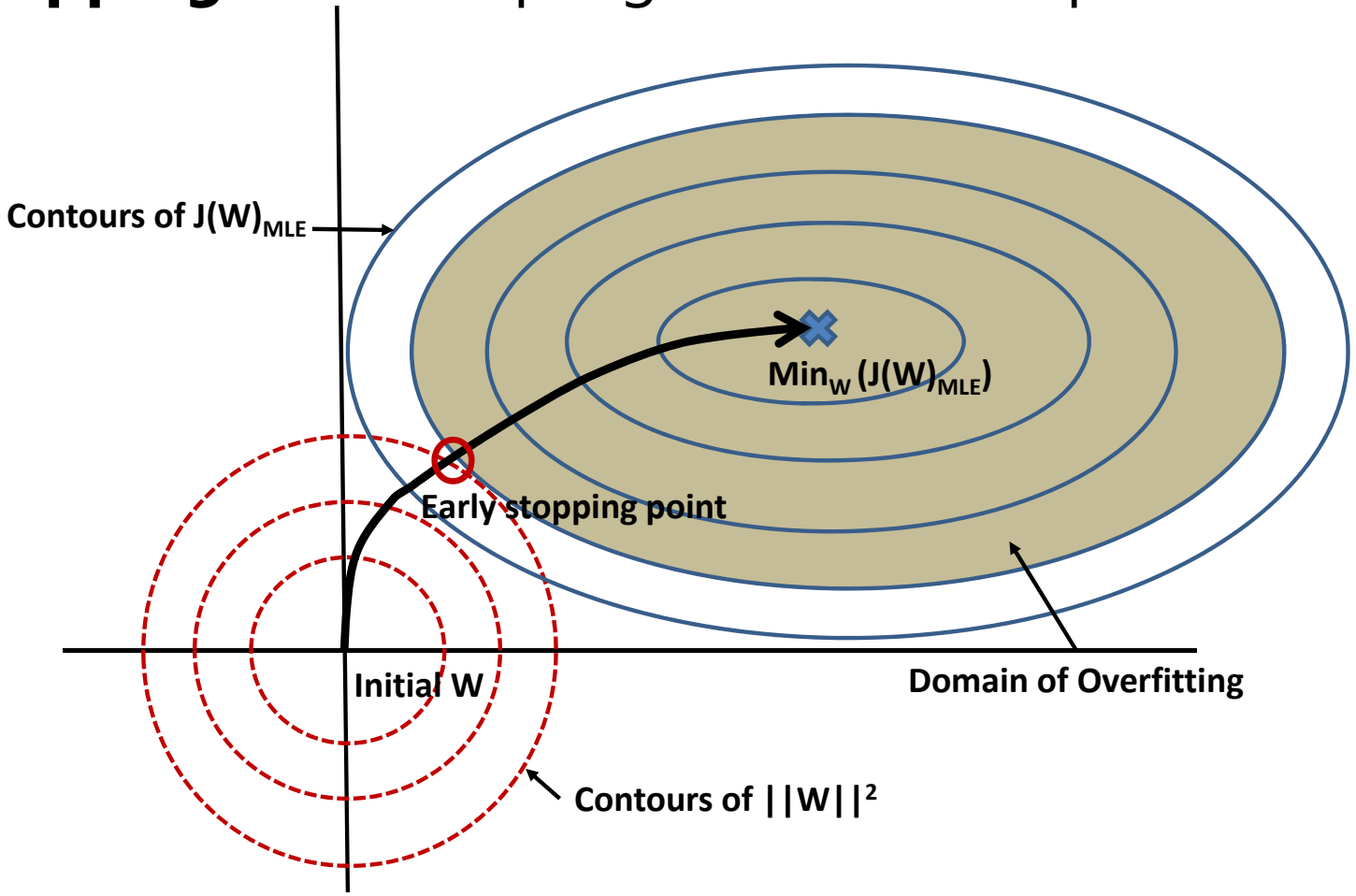
- Given the l1 norm of the weights, the loss function becomes:

$$J(W) = J_{MLE}(W) + \alpha ||W||^1$$

- Where α is the regularization hyperparameter
  - Large α increases bias but reduces variance
  - Small α decreases bias and increases variance
- The l1 constraint drives some weights to exactly 0
  - This behavior leads to the term **lasso regularization**

Early Stopping

Early stopping has a simple geometric interpretation



Dropout regularization

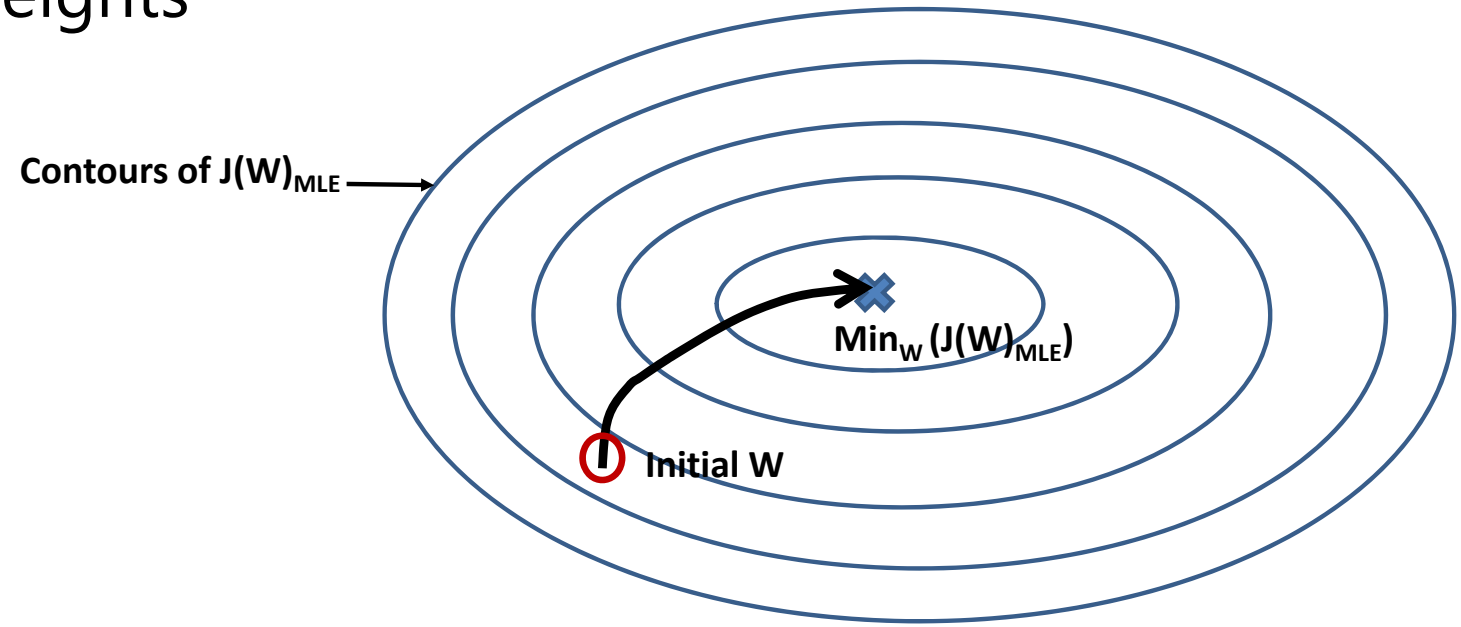
- **Dropout regularization** is a conceptually simple method unique to deep learning
  - At each step of the gradient decent some **fraction, p, of the weights are dropped-out** of each layer
  - The result is a series of models trained for each dropout sample
  - The final model is a **geometric mean** of the individual models
- Weight values are clipped in a small range as a further regularization
- For full details see the readable paper by Srivastava et. al., 2014 http://www.cs.toronto.edu/~rsalakhu/papers/srivastava14a.pdf

Batch Normalization

- In deep neural networks there is a high chance that units in a hidden layer have a **large range of output values**
  - Causes shifts in the covariance of the output values
  - Leads to difficulty computing the gradient
  - Slows convergence for optimizers
- A solution is to normalize the output of the hidden layers in the network as a batch
- This simple idea can be really effective
- For more details see Sergey and Szegedy, 2015: https://arxiv.org/pdf/1502.03167.pdf

Local Convergence of Gradient Descent

- Ideally, the loss function, J(W), is **convex** with respect to the weights



- Convex loss function has **one unique minimum**
- **Convergence** for convex loss function is **guaranteed**

Local Convergence of Gradient Descent

Expand loss function to understand convergence properties of gradient descent:

$$J(W^{(l+1)}) = J(W^{(l)}) + (W^{(l+1)} - W^{(i)})\vec{g} + \frac{1}{2}(W^{(l+1)} - W^{(i)})^T H(W^{(l+1)} - W^{(i)})$$

Where:

$W^{(l)}$ is the tensor of weights at step l

$\vec{g}$ is the gradient vector

H is the **Hessian** matrix.

Local Convergence of Gradient Descent

- How can you understand the Hessian matrix?

$$H\left(f(\vec{x})\right) = \begin{bmatrix} \frac{\partial^2 f(\vec{x})}{\partial x_1^2} & \frac{\partial^2 f(\vec{x})}{\partial x_2 \partial x_1} & \cdots & \frac{\partial^2 f(\vec{x})}{\partial x_n \partial x_1} \\ \frac{\partial^2 f(\vec{x})}{\partial x_1 \partial x_2} & \frac{\partial^2 f(\vec{x})}{\partial x_2^2} & \cdots & \frac{\partial^2 f(\vec{x})}{\partial x_1 \partial x_n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial^2 f(\vec{x})}{\partial x_1 \partial x_n} & \frac{\partial^2 f(\vec{x})}{\partial x_2 \partial x_n} & \cdots & \frac{\partial^2 f(\vec{x})}{\partial x_n^2} \end{bmatrix}$$

- The Hessian is the matrix of all derivatives of the gradient
- Properties of the Hessian determine convergence rate

Local Convergence of Gradient Descent

- Real-world loss functions are **typically not convex**
- There can be multiple minimums and maximums; a **multi-modal loss function**
- Finding the **globally optimal solution** is hard!
- The minimum reached by an optimizer depends on the **starting value of W**
- In practice, we are happy with a **good local solution**, if not, the globally optimal solution
- Regularization can ‘smooth’ the loss function, at least locally
- First order optimization found to perform as well, or better, than second order

The Nature of Gradients

- Some key properties of the Hessian matrix:
  - The Hessian is symmetric since $\frac{\partial f(\vec{x})}{\partial x_1 \partial x_2} = \frac{\partial f(\vec{x})}{\partial x_2 \partial x_1}$
  - For a convex loss function the Hessian has all **positive eigenvalues**; it is **positive definite**
  - At a maximum point the Hessian has all **negative eigenvalues**; it is **negative definite**
  - The Hessian has **some positive and some negative eigenvalues** at a **saddle point**
  - Saddle points are problematic since direction of descent to the minimum is unclear
  - If Hessian has some **very small eigenvalues**, the gradient is low and **convergence will be slow**

The Nature of Gradients

- For quadratic optimization, the rate of convergence is determined by the **condition number** of the Hessian:

$$\kappa(H) = \frac{|\lambda_{max}(H)|}{|\lambda_{min}(H)|}$$

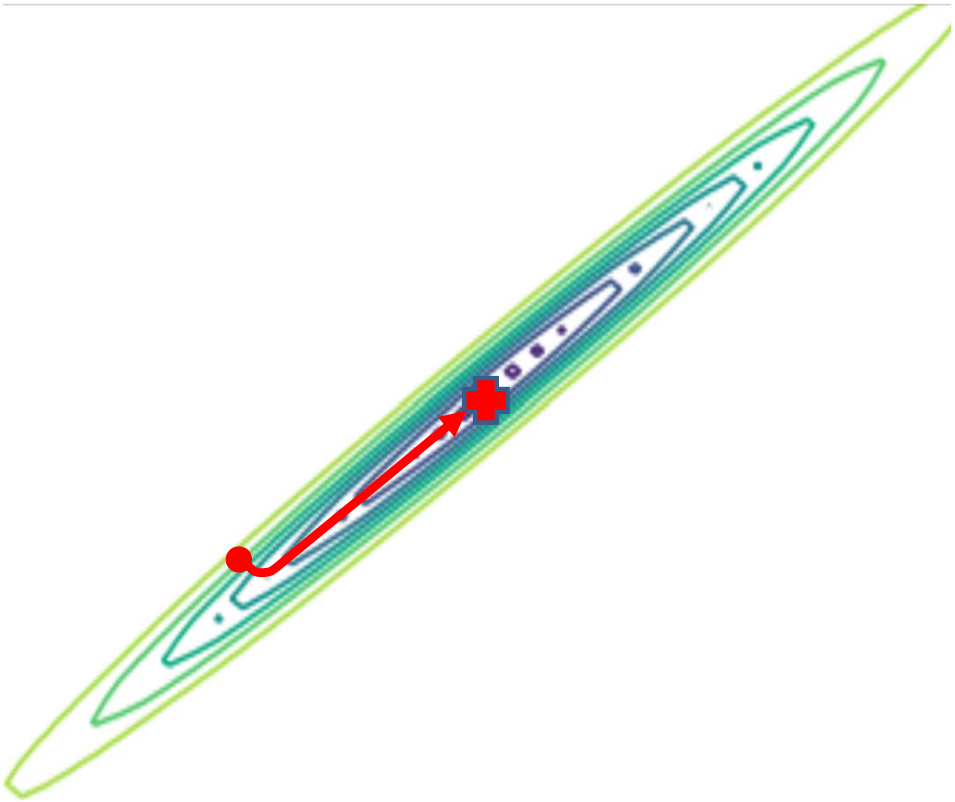
- Where:

$|\lambda_{max}(H)|$ is the absolute value of the largest eigenvalue of H

$|\lambda_{min}(H)|$ is the absolute value of the smallest eigenvalue of H

- If the condition number is close to 1.0, the Hessian is **well conditioned** and convergence will be fast
- If the condition number is large, the Hessian is **ill-conditioned** and convergence will be slow; gradient is flat in some dimensions

Convex vs. Non-Convex Optimization

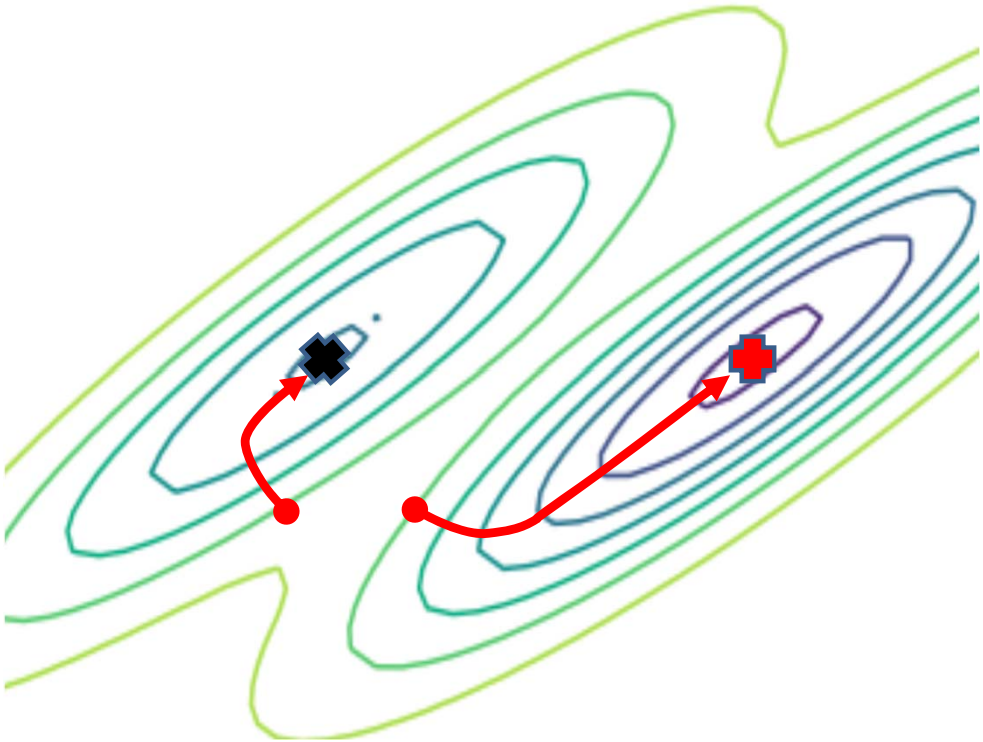


Convex loss
Poorly conditioned

Gradient descent is well-behaved with **convex loss function**

- Only 1 **global minimum**
- From any starting point the gradient leads to global minimum
- Hessian is always **positive definite**

Convex vs. Non-Convex Optimization

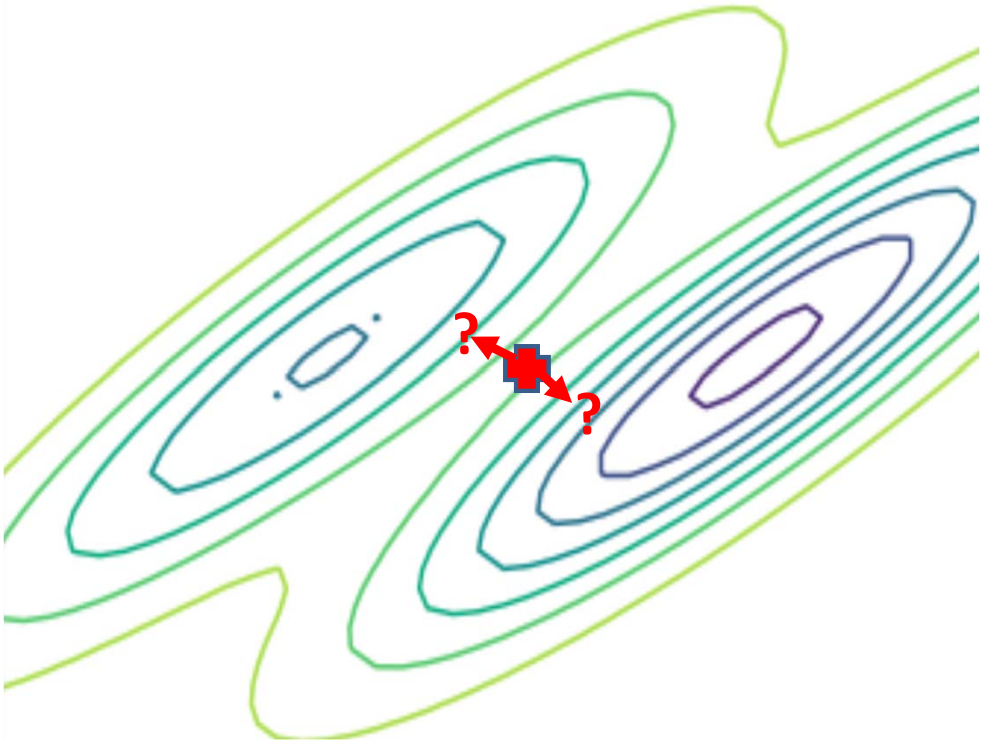


Nonconvex loss

Gradient descent can be problematic with **nonconvex loss function**

- There is a **global minimum**
- Possibly many **local minimums**
- Minimum found with gradient descent depends on starting point
- A good minimum may be good enough
- Hessian positive definite at any minimum, but globally who knows??

Convex vs. Non-Convex Optimization



Nonconvex loss

Gradient descent can be problematic with **nonconvex loss function**

- Can get stuck at **saddle point!**
- Gradient is ambiguous at saddle point
- Hessian is not positive definite

Batch Gradient Descent

- Recall the basic gradient descent equation:

$$W_{t+1} = W_t + \alpha \nabla_W J(W_t)$$

- Where:

W_t = the tensor of weights or model parameters at step t

$\nabla_W J(W)$ = gradient of J with respect to the weights W

α = step size or learning rate

- The gradients of the multi-layer NN are computed using the chain rule

Batch Gradient Descent

- Can we use the gradient descent equation directly?
- Yes, we can
- Iterate the weight tensor relation until a **stopping criteria** or **error tolerance** is reached:

$$||W^{t+1} - W^{t}|| < \text{tolerance}$$

- But;
  - Must compute the gradient for all weights at one time as a **batch** at each step
  - Does not scale if there are a large number of weights

Stochastic Gradient Descent

- We need a more scalable way to apply gradient descent
- **Stochastic gradient descent** is just such a method
- The weight tensor update for stochastic gradient descent follows this relationship:

$$W_{t+1} = W_t + \alpha \, E_{\hat{p}data}\left[\nabla_W J(W_t)\right]$$

Where:

$\hat{p}data$ is the Bernoulli sampled mini-batch

$E_{\hat{p}data}[]$ is the expected value of the gradient given the Bernoulli sample

Stochastic Gradient Descent

- Stochastic gradient descent is known to converge well in practice
- Empirically, using mini-batch samples provide a better exploration of the loss function space
  - Can help solution escape from small local gradient problems
  - Sampling is dependent on mini-batch size

Stochastic Gradient Descent

- Stochastic gradient descent algorithm

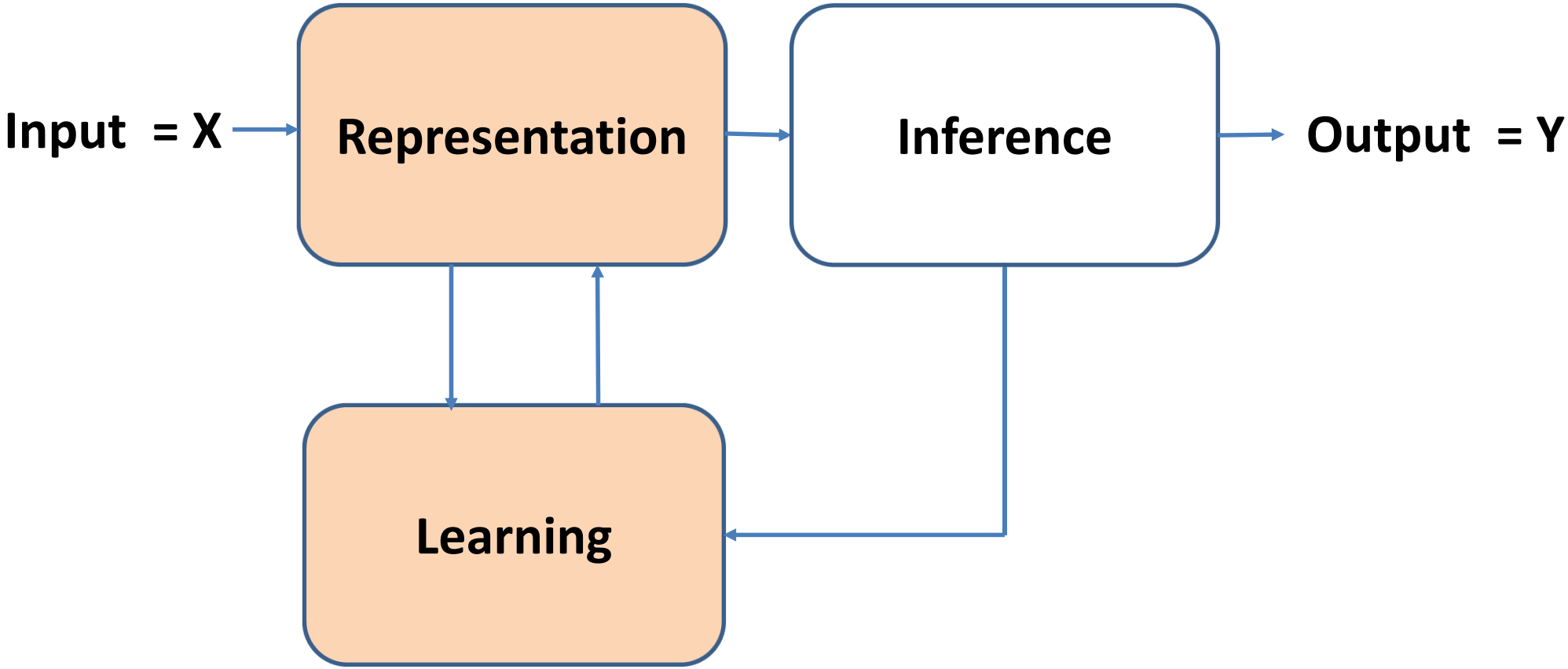
```
Random_sort(cases)
while(grad > stopping_criteria):
    mini-batch = sample_next_n(cases)
    grad = compute_expected_grad(mini_batch)
    weights = update_weights(weights, grad)
```

- Notice that the addition rounds repeat the samples
  - In practice this does not create much bias
  - For large samples this may not happen

Pitfalls in Training Deep Neural Networks

- Fully connected neural networks are **complex function approximators**
  - **Very high dimensional space**
  - **Hard to visualize or understand**
  - Need **regularization to ensure generalization**
  - Gradient descent can find a **good local solution**
- But, training **data may not cover the solution space**
  - **'Holes' in space covered by training** data lead to unexpected results!
  - **More data may not help** if samples have same bias
  - **Biased samples lead to poor generalization**
  - **Regularization can smooth over holes in solution space** and help optimization, at least locally – but **don't expect magic**

Convolutional Neural Networks



Convolutional Neural Networks

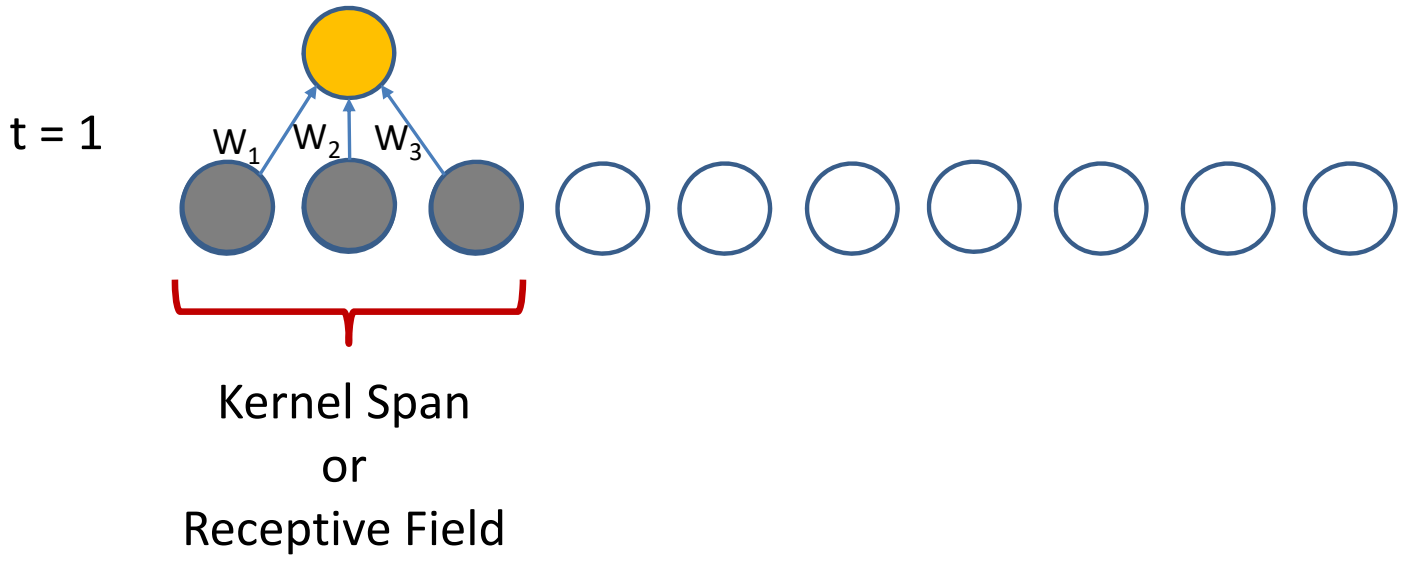
- **CNN**s are used to **learn complex feature maps**
  - **Invariant** to translation and distortion of features
  - **Reduce the dimensionality** of input tensors
  - **Share weights** and are relatively easy to train
- CNNs have a long history
  - LeCun et. al. (1998) first employed CNNs for automatic check handling
  - Era of general use started when Krizhevsky et. al. (2012) won an ImageNet object recognition competition
  - Now commonly used for image, speech and text problems

1-D Convolution

- 1-D CNNs are a simple, but useful example
  - Time series data
  - Text data
  - Speech
- Convolution kernel is moved along the input tensor
  - Kernel has a small span compared to dimension of input tensor
  - At each step a weighted output value is computed

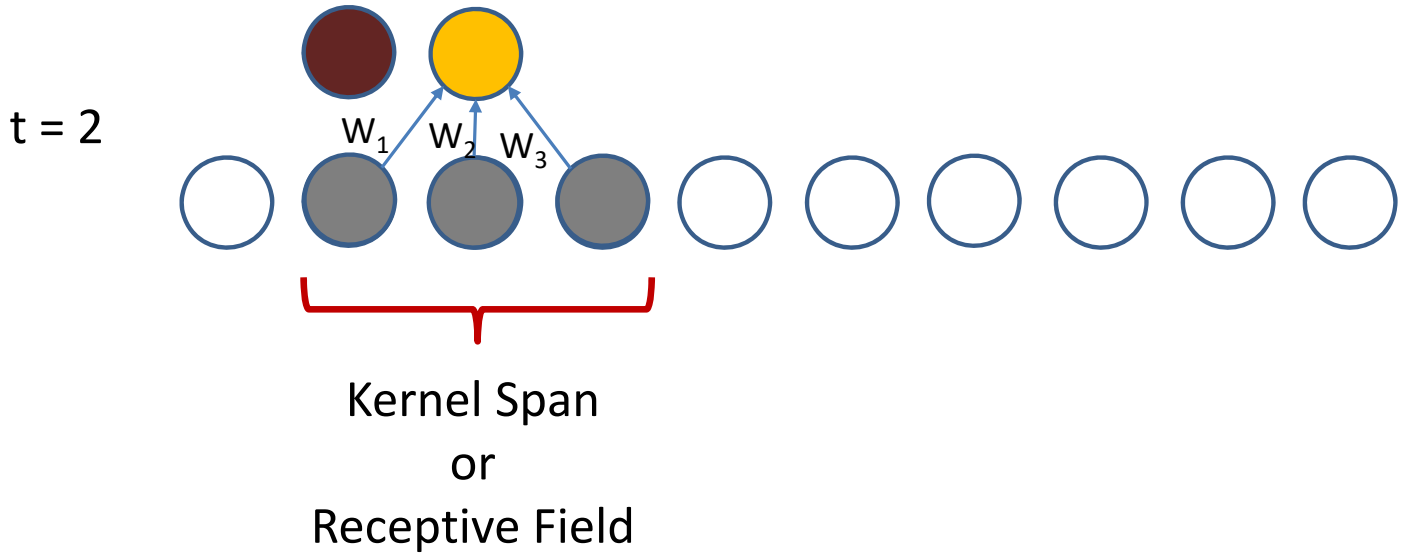
1-D Convolution

1-D CNNs are a simple, but useful example



1-D Convolution

1-D CNNs are a simple, but useful example



1-D Convolution

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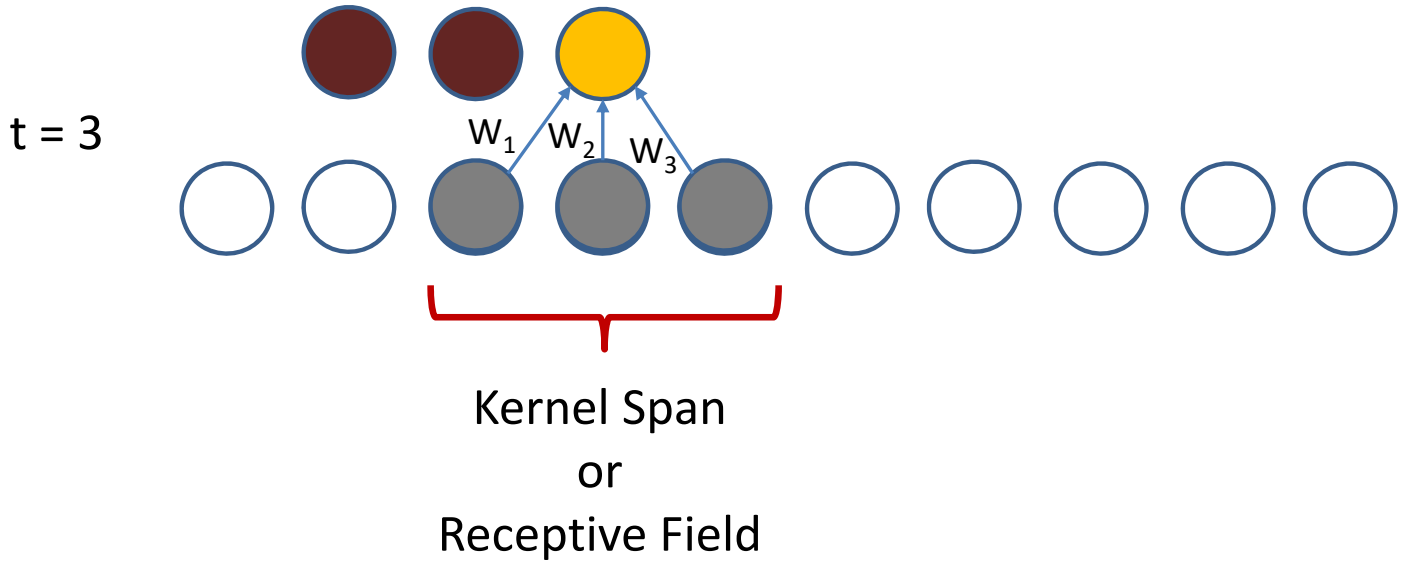
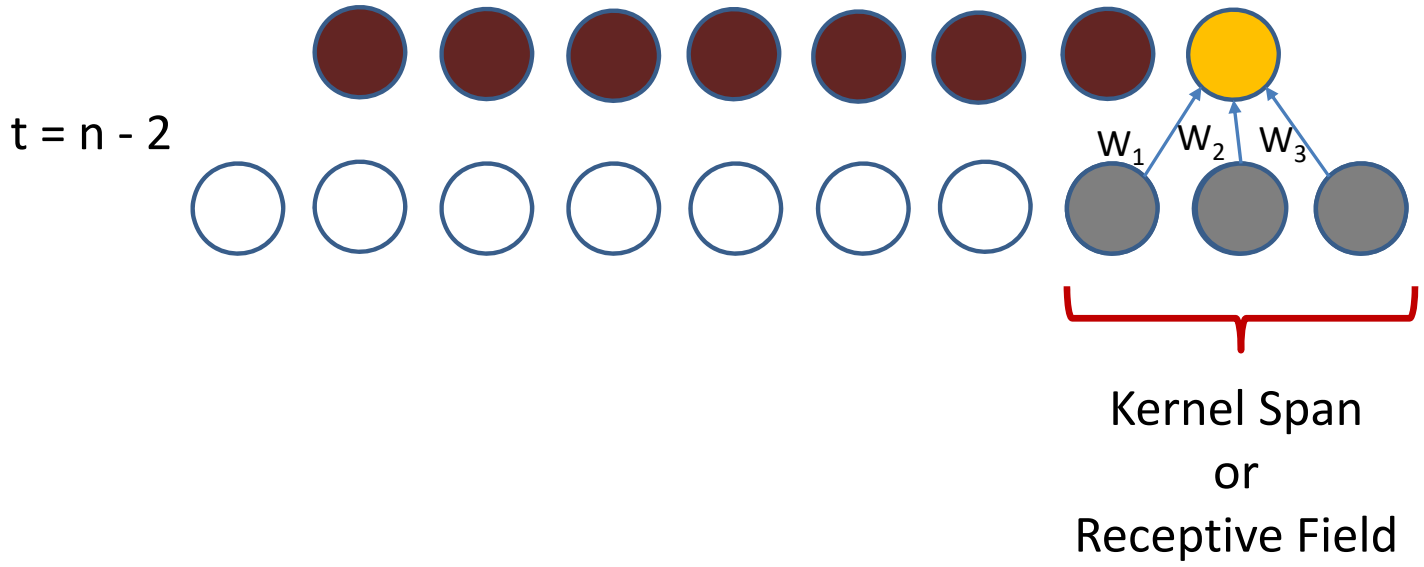


1-D Convolution

1-D CNNs are a simple, but useful example



1-D Convolution

Mathematically, express 1-d convolution as a weighted sum over a set of discrete kernel values:

$$s(t) = (x * k)(t) = \sum_{\tau=t-a}^{t+a} x(t)k(t-\tau)$$

Where:

$s(t)$ is the output of the convolution operation at time t

x is the series of values,

k is the convolution kernel

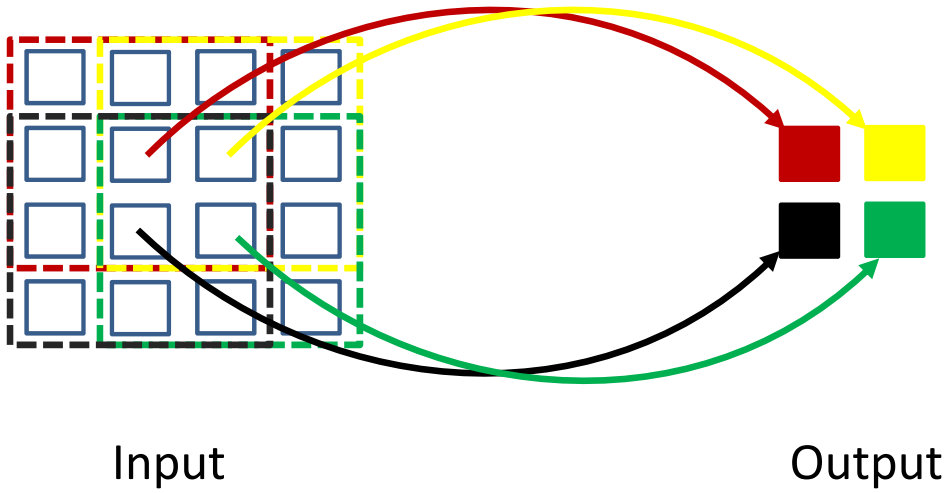
$*$ is the convolution operator

τ is the time difference of the convolution operator.

$a = \frac{1}{2}(kernel\_span + 1)$, for an odd length kernel

2-D Convolution

- 4 x 4 input tensor
- 3 x 3 convolution operator
- 2 x 2 output tensor



2-D Convolution

- Mathematically, we express 2-d convolution as a weighted sum over a discrete rectangular kernel:

$$S(i,j) = (I * K)(i,j) = \sum_{m=i-a}^{i+a} \sum_{n=j-a}^{j+a} I(i,j)K(i-m, j-n)$$

- Where S, I and K are now tensors

2-D Convolution

- The image and kernel tensors are **commutative** in the convolution relationship
- This allows an operation known as **kernel flipping** with the following alternative result:

$$s(i,j) = (I * K)(i,j) = \sum_{m=i-a}^{i+a} \sum_{n=j-a}^{j+a} I(i-m, j-n)K(i,j)$$

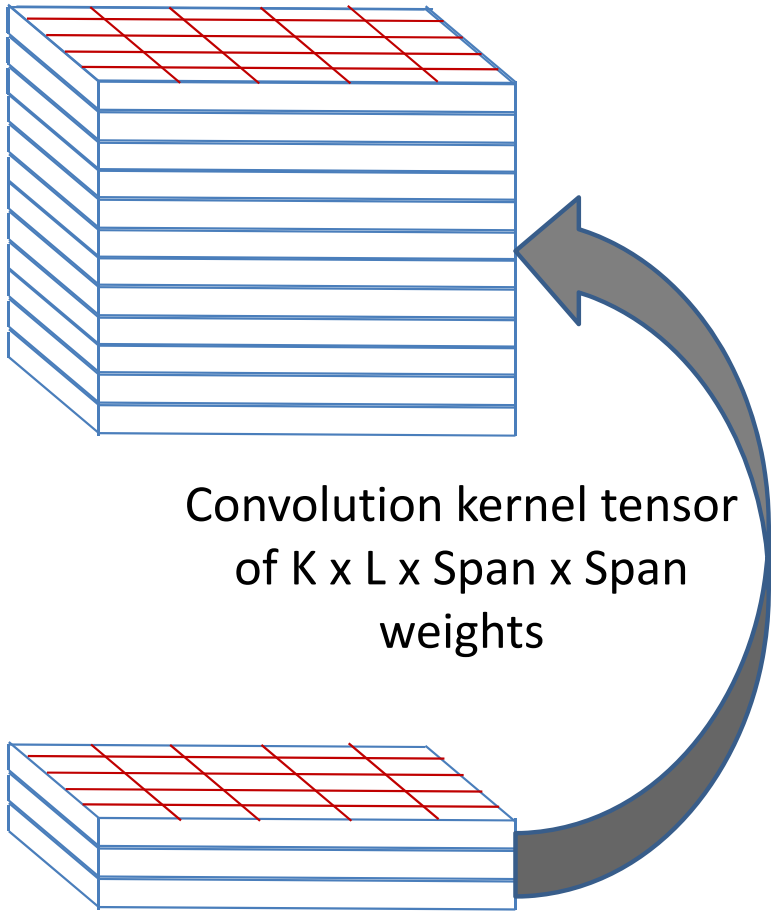
Convolution in Higher Dimensions

- Tensor notation allows easy extension to higher dimensions
  - Input tensor has **multiple input channels**
  - 3-D for color image
  - 4-D for video
- Create **multiple feature maps**
  - Convolution kernel tensor has **multiple output channels**
  - Each output channel is a different feature map
  - Feature in a channel might be vertical lines, horizontal lines, corners, etc

Convolution in Higher Dimensions

Tensor of K output channels

Each channel is a part of **feature map**



Convolution kernel tensor of K x L x Span x Span weights

L input channels
i x j dimensions

Convolution in Higher Dimensions

A multi-dimensional convolution relationship can be written:

$$Z_{i,j,k} = (V * Z)(i,j,k,l) = \sum_{l} \sum_{m=-a}^{a} \sum_{n=-a}^{a} V_{i,j,l} \cdot K_{i-m,j-n,k,l}$$

Where: i, j are the spatial dimenstions

l is the index of the input channel

k is the index of the output channel

$K_{i,j,k,l}$ is the kernel connecting the lth channel of the input to the kth channel of the output for pixel offsets i and j

$V_{i,j,l}$ is the i, j input pixel offsets from channel l of the input,

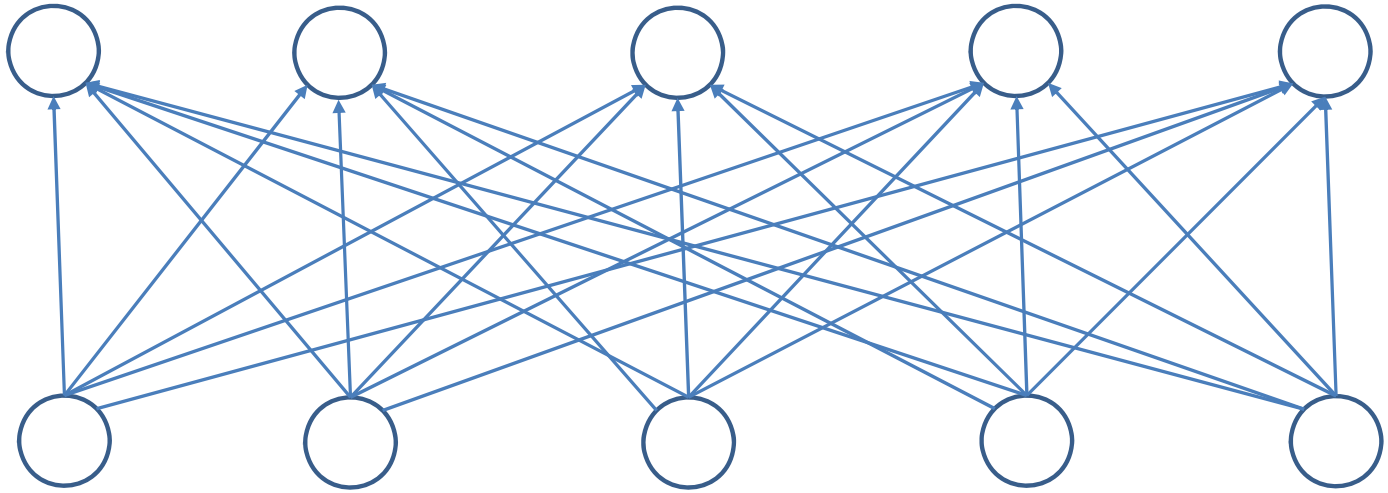
$a = \frac{1}{2}(kernel\_span + 1)$, for an odd length kernel

Parameter Sharing

- The weights of the convolutional kernel must be **learned**
- Each weight of a fully connected network must be learned independently
- CNNs are efficient to train
- CNNs use **parameter sharing**
  - **Statistical strength** from more samples per weight
  - Reduced variance of parameter estimates
- Weights are learned using backpropagation and gradient descent methods
- Also called **tied weights** or **sparce interaction**

Parameter Sharing

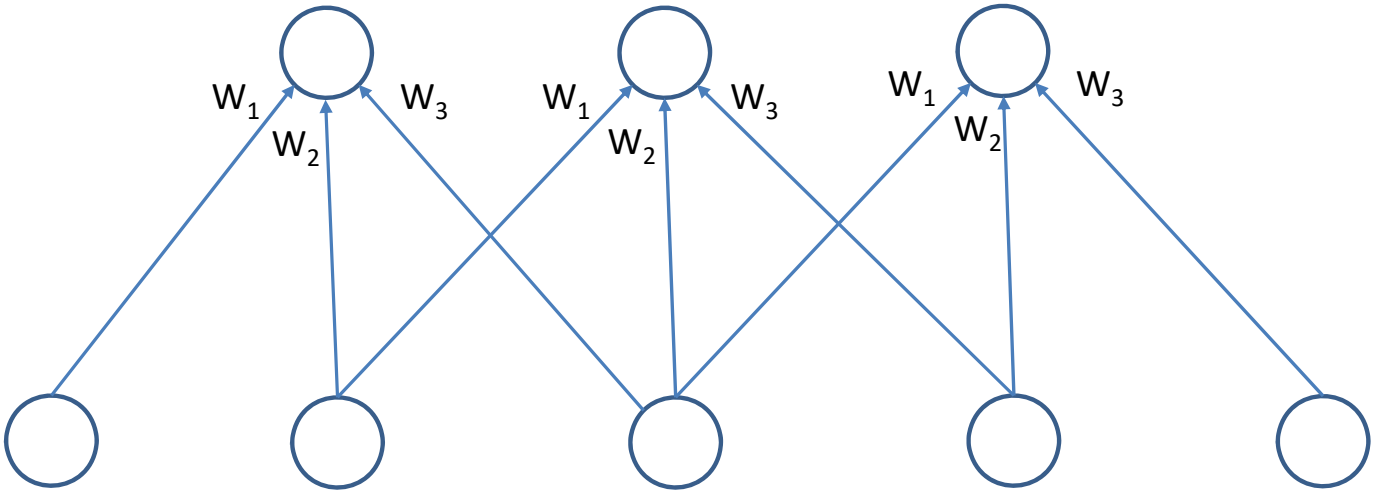
Weights of fully connected network are independent



e.g. requires $5^2 = 25$ weights

Parameter Sharing

Weights are shared in CNN



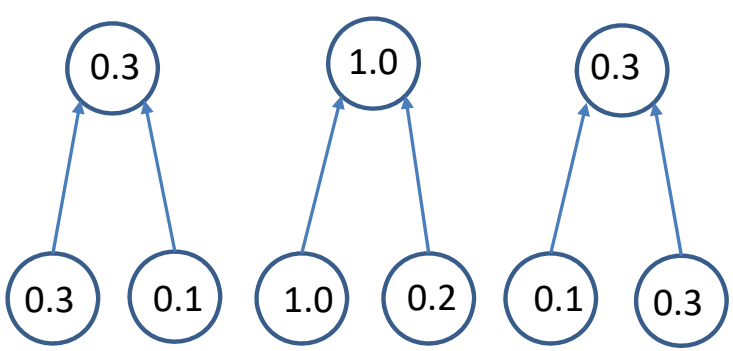
Requires 3 weights

Pooling and Invariance

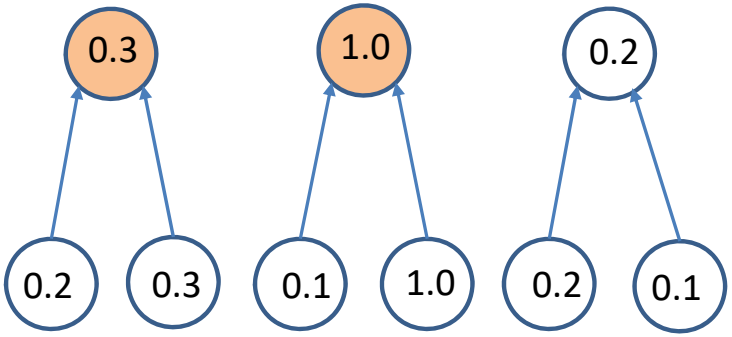
- Convolution provides reduced dimensionality of input tensor
- How can we obtain greater reduction in dimensionality?
- **Pooling** of convolution kernel output values reduces dimensionality
  - e.g. 2x2 operator pools the 4 values into 1
- How to pool?
  - Average?
  - **Max pooling**; simple and highly effective

Pooling and Invariance

Max pooling provides **invariance** to small shifts of the input tensor:



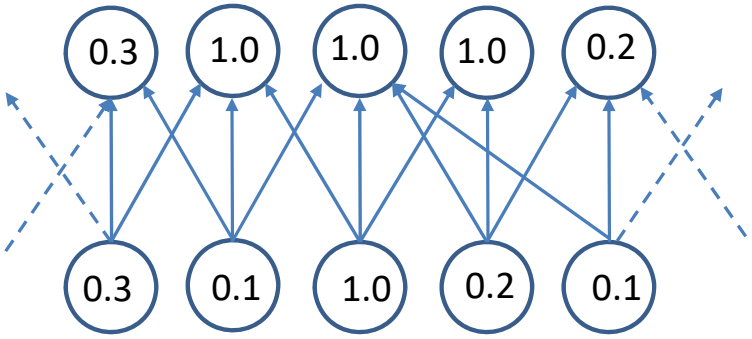
Original



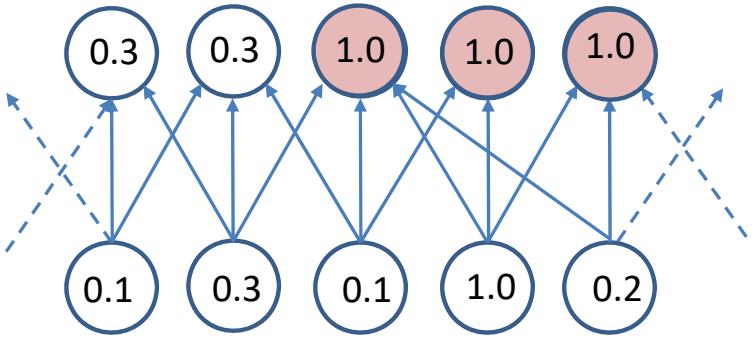
Shift Right by One

Pooling and Invariance

Max pooling provides **invariance** to small shifts of the input tensor:



Original



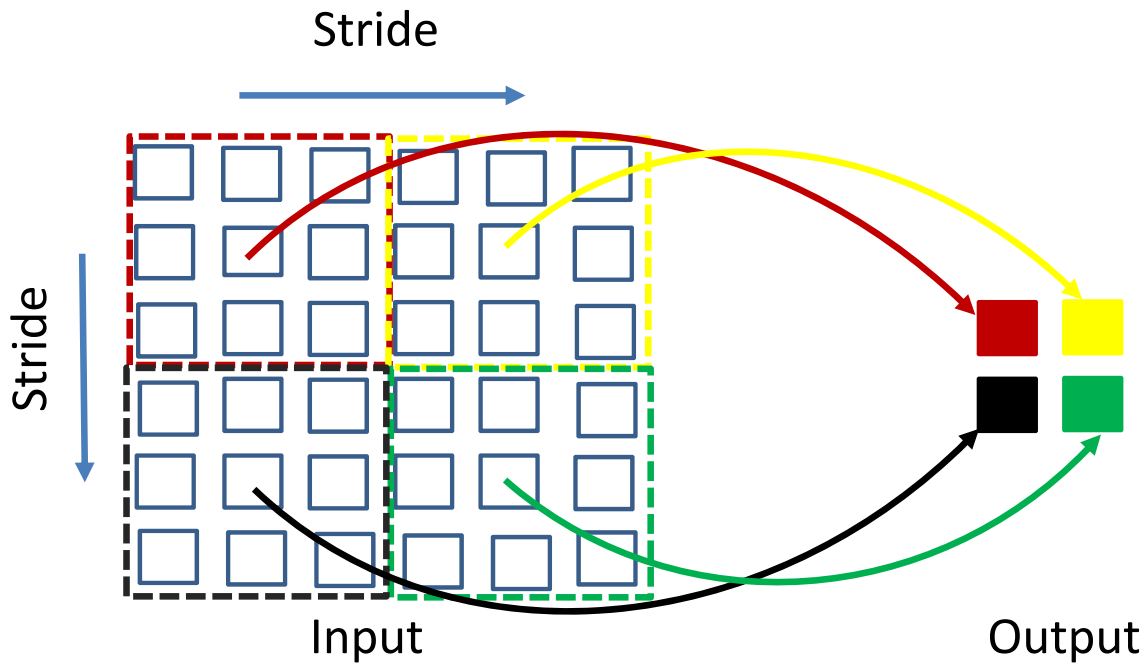
Shift Right by One

Stride and Tiling

- Do we always move the convolution kernel by 1 data value?
- **No!**
  - May not need the full resolution of the input
- We can choose a **stride > 1** for the convolution operator
  - Convolution kernel moved by stride at each step
- Stride > 1 **down-samples** the data
  - Reduces dimensionality
- Stride < 1 up-samples data
  - Useful to create tensor of required dimension

Stride and Tiling

Tiling is a special case when stride = span:



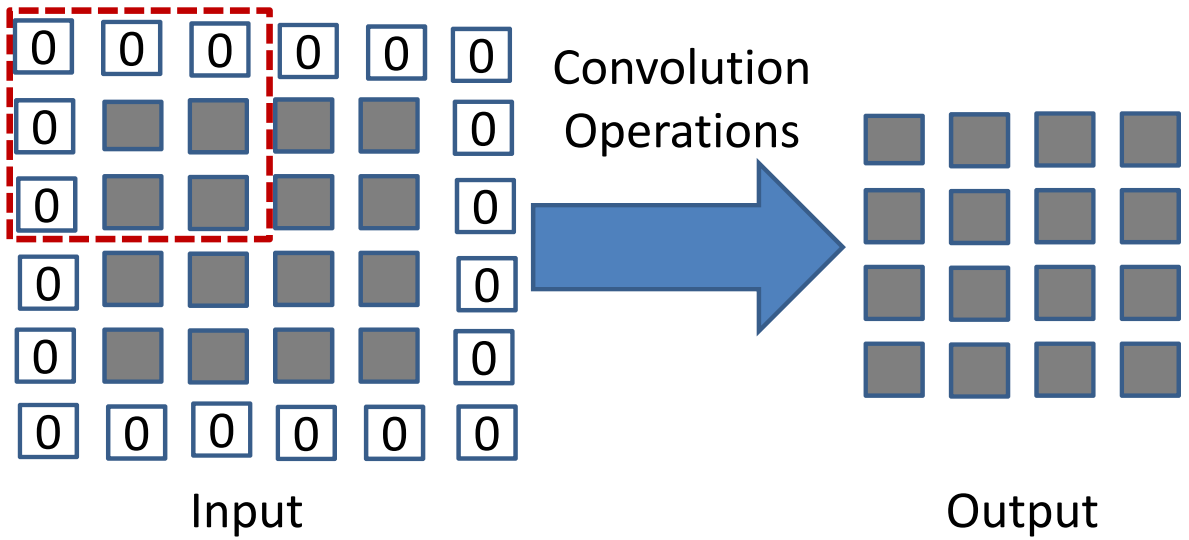
Stride >1 reduces dimensionality

Padding for Convolution

- How can we best deal with **edges of the input** tensor when performing convolution?
- A **valid convolution** confines the kernel to the input tensor dimensions
  - For odd kernel shape, output dimension is (span + 1)/2 less than input dimension
  - After many convolution layers, dimension is reduced further
- We can **zero pad** the input tensor
  - Dimensionality is maintained

Padding for Convolution

- Example: 4x4 input tensor with 3x3 kernel

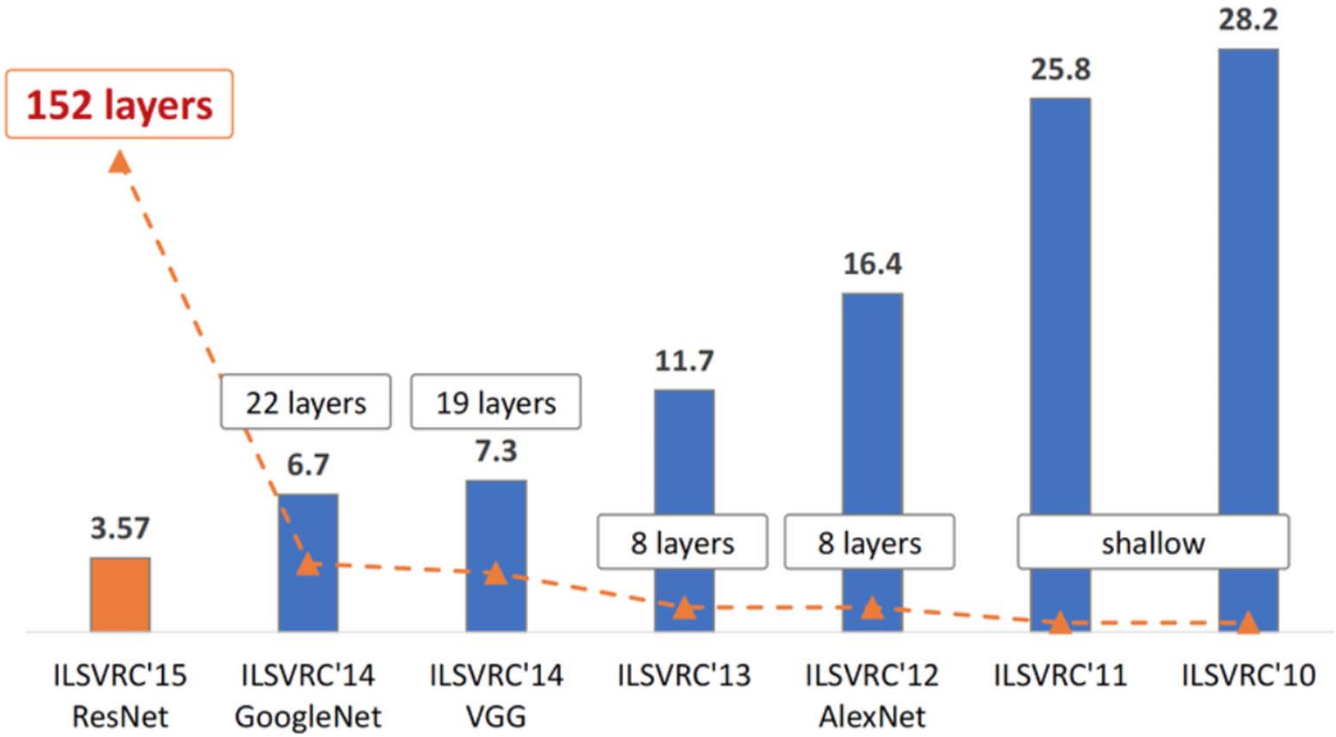


- Dimension of output tensor = dimension of input tensor
- 0 values have little effect with max pooling

Deep and Multi-Scale Architectures

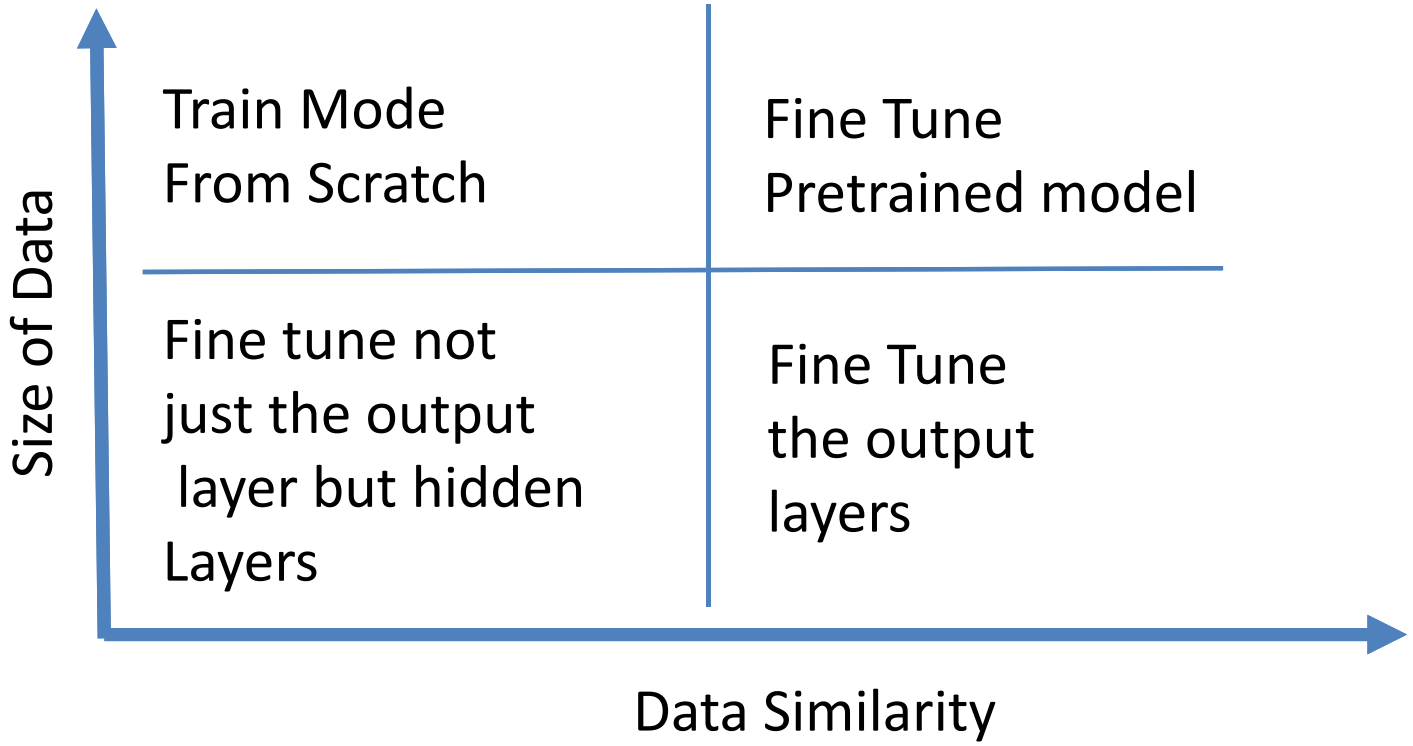
- Deep architectures create large and complex feature maps
- Feature maps have a higher number of channels
- Trained on very large benchmark datasets
- Classification accuracy found to improve with depth

Deep and Multi-Scale Architectures



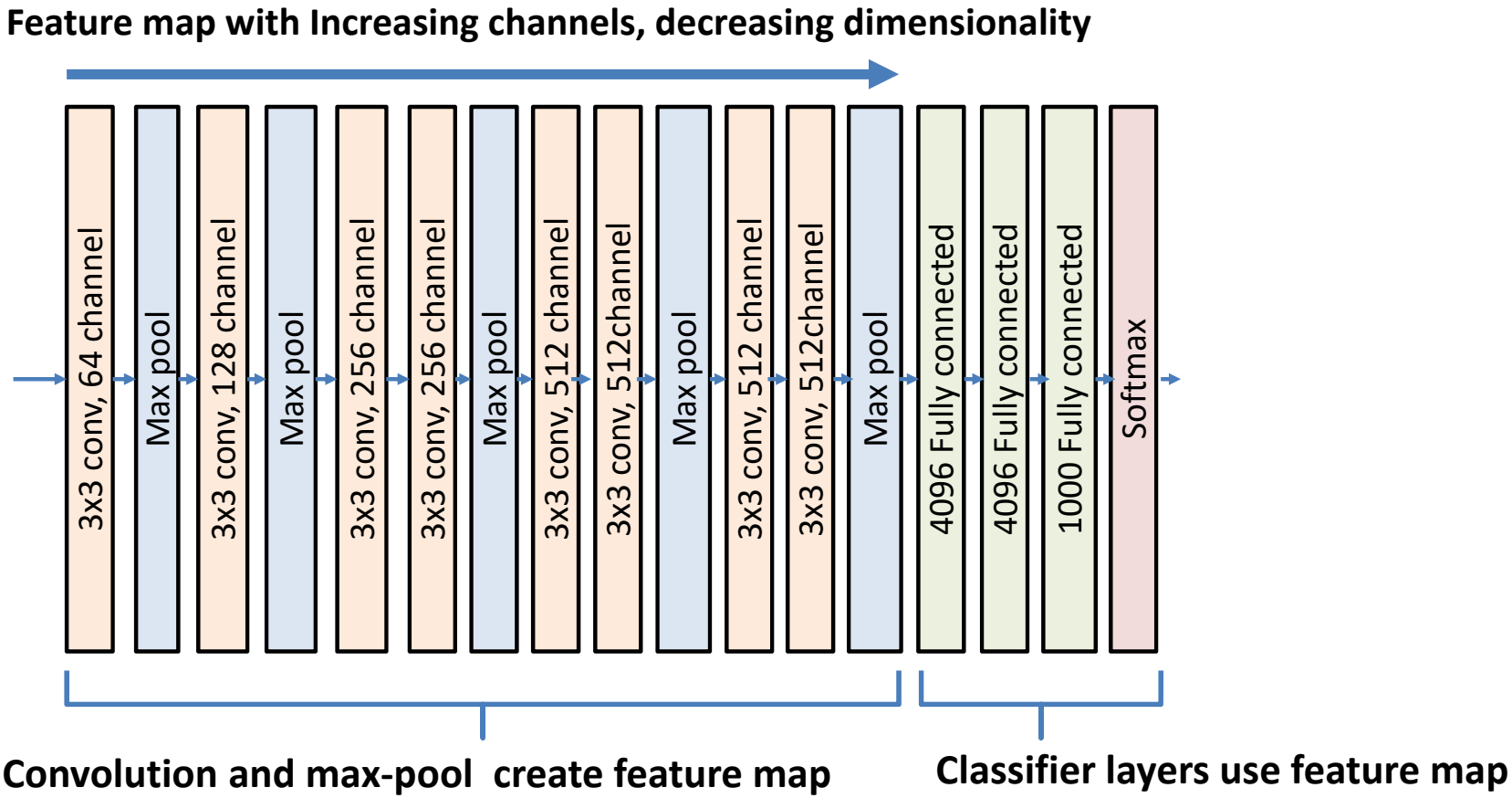
Transfer Learning

- **Transfer learning** exploits learning of a model trained with similar data
- Transfer learning can greatly speed learning for specific solutions



Deep and Multi-Scale Architectures

VGG11 - Simanyan and Zdisserman, 2015



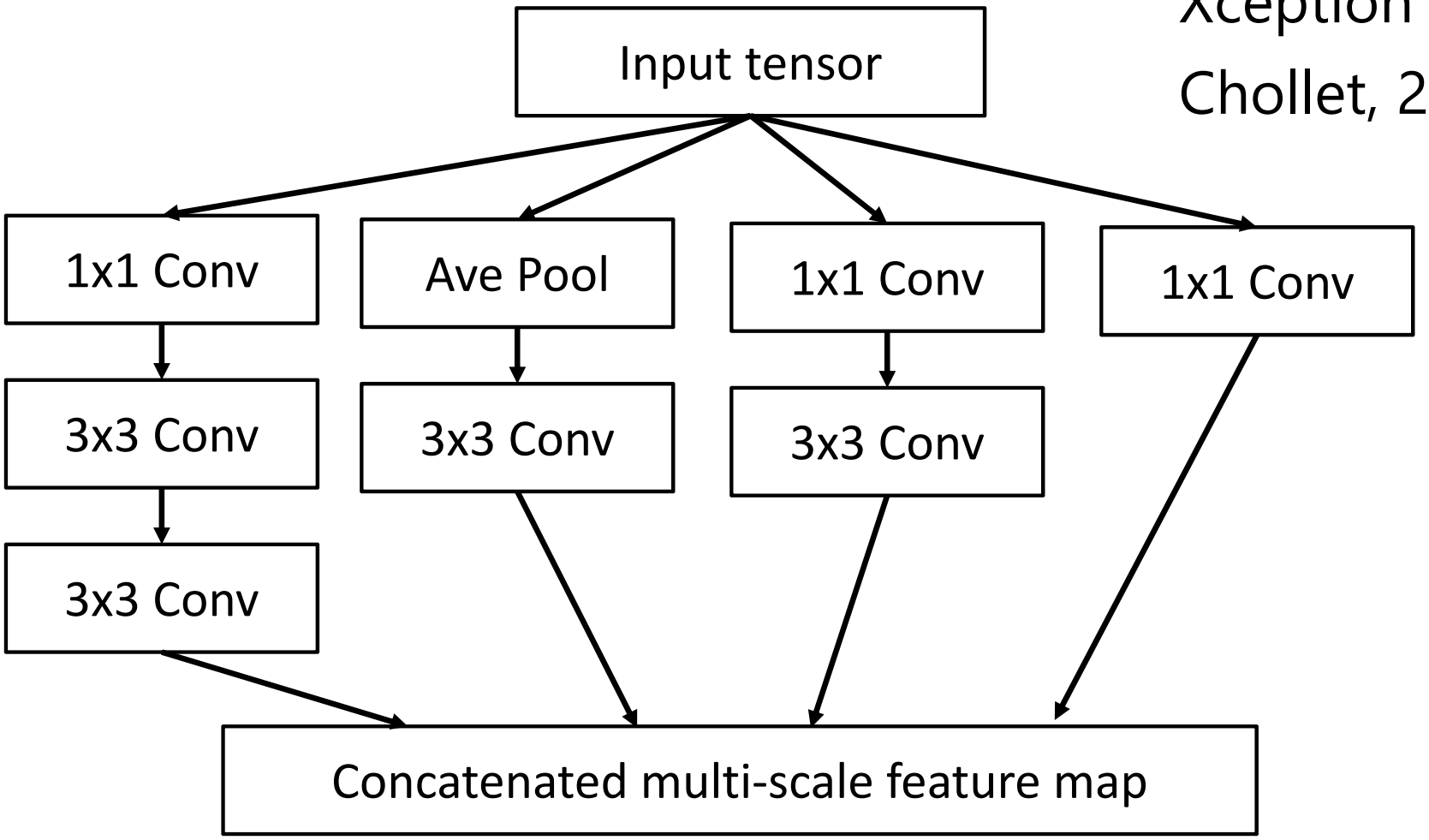
Deep and Multi-Scale Architectures

- Single convolutional layers work at single scale
- But, real-world images contain objects with different scales
- Need architecture that supports multiple scales
  - CNN layers with different scales in parallel
  - Concatenate the results
- Numerous pretrained models, using complex architectures, are now available
  - Trained on very large benchmark datasets
  - Built into deep learning frameworks; Keras, PyTorch, etc.

Deep and Multi-Scale Architectures

Xception

Chollet, 2017



LeNet5: Model

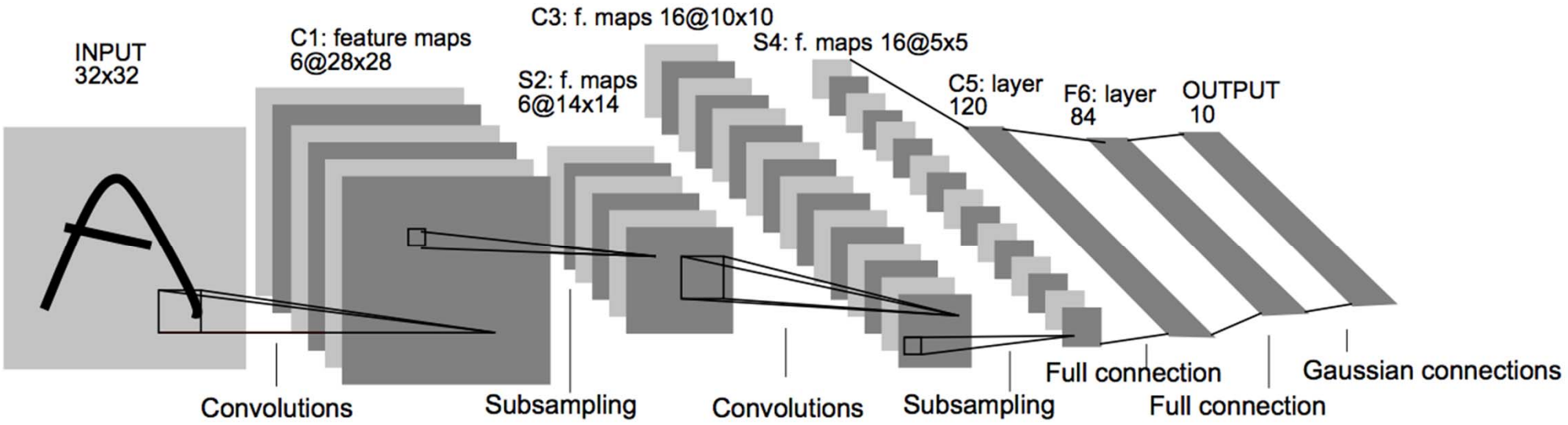


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
# import the necessary packages
from keras.models import Sequential
model = Sequential()
```

LeNet5: Input

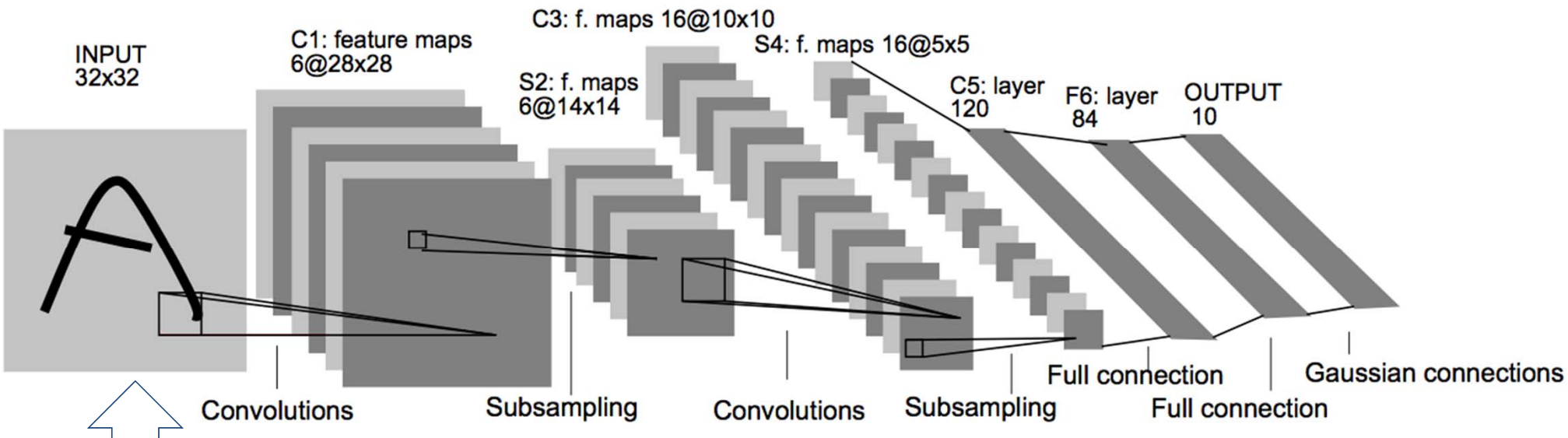


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
# import the necessary packages
from keras.datasets import mnist
(x_train, y_train), (x_test, y_test) = mnist.load_data()
```

LeNet5: Convolution

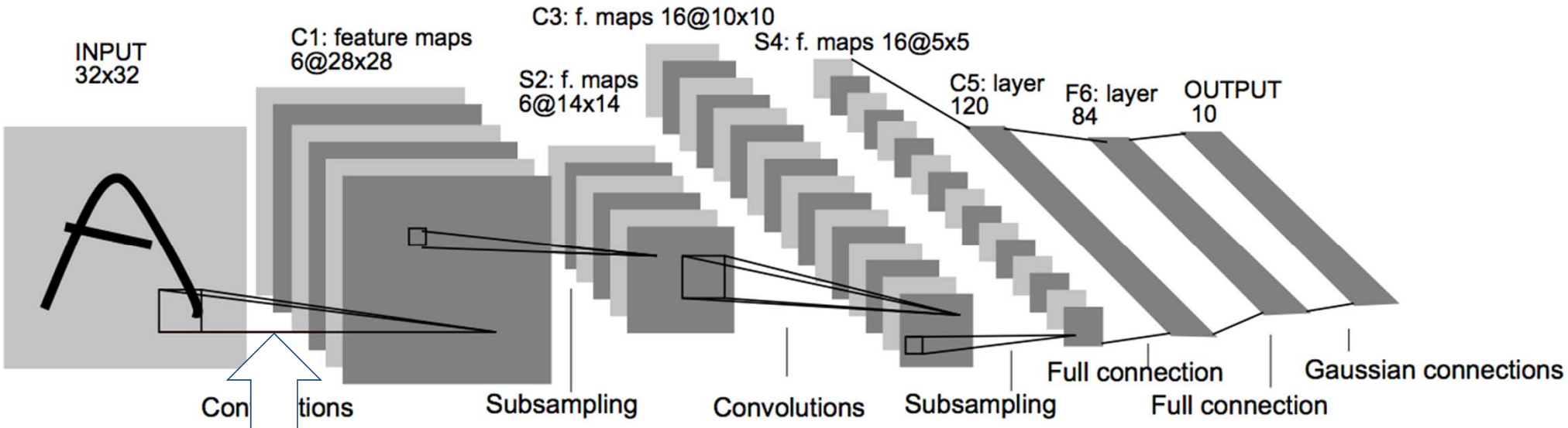


Fig. 2. Architecture o Net-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are trained to be identical.

```
model.add(Convolution2D(6, 5, 5,
                        border_mode="valid",
                        input_shape=(depth, height, width)))
```

LeNet5: Non-Linearity

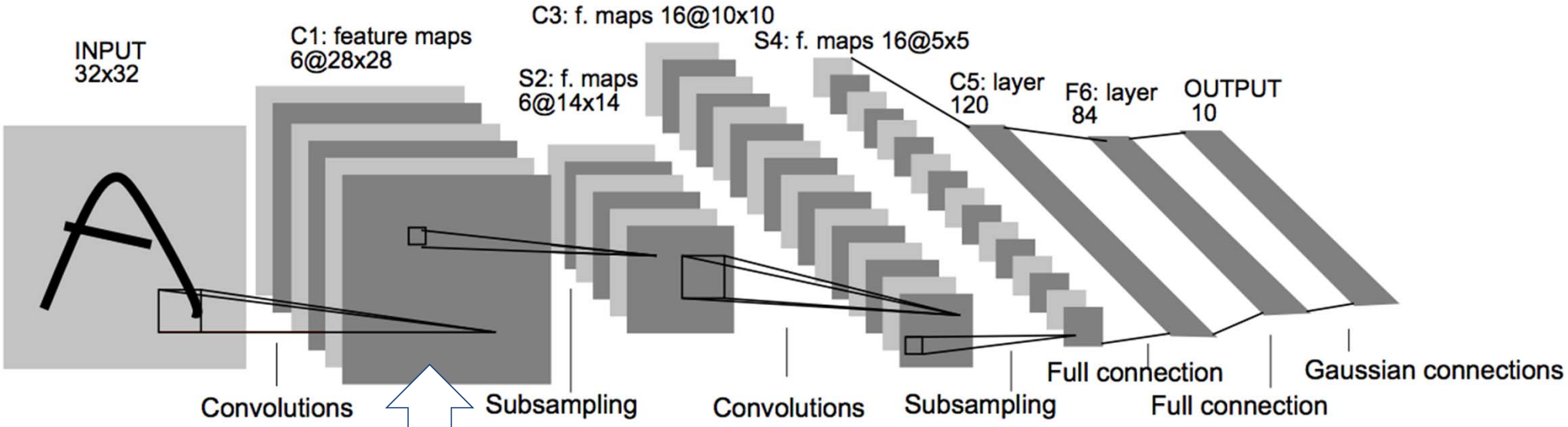


Fig. 2. Architecture of LeNet-5, a C olutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained tc identical.

```
from keras.layers.core import Activation
model.add(Activation(" sigmoid "))
```

LeNet5: Pooling

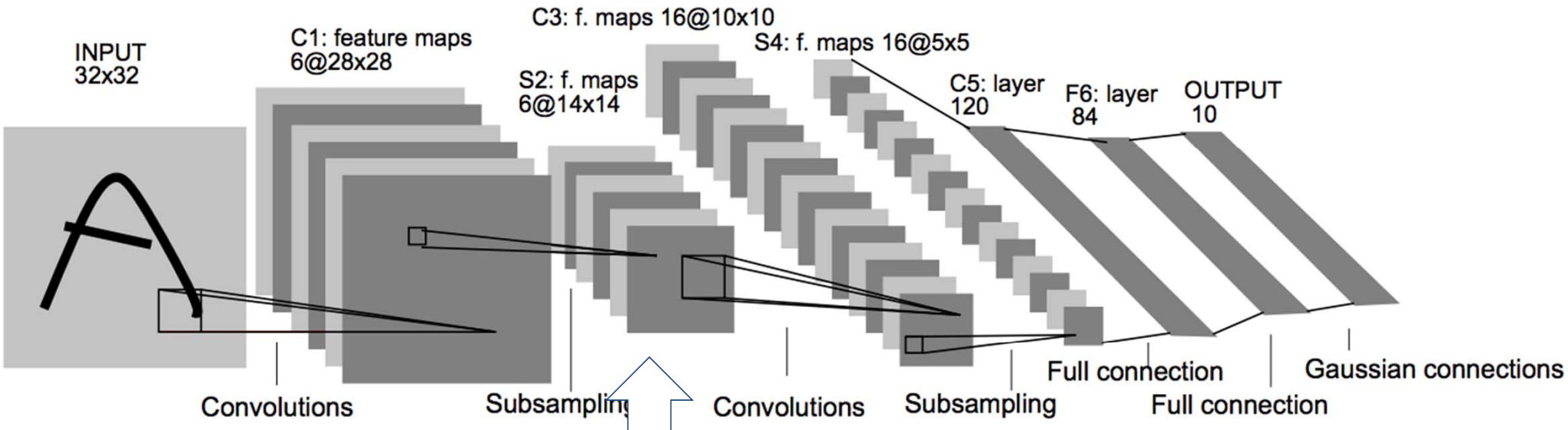


Fig. 2. Architecture of LeNet-5, a Convolutional Neural twork, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
from keras.layers.convolutional import MaxPooling2D
model.add(MaxPooling2D(pool_size=(2, 2), strides=(2, 2)))
```

LeNet5: Second convolution

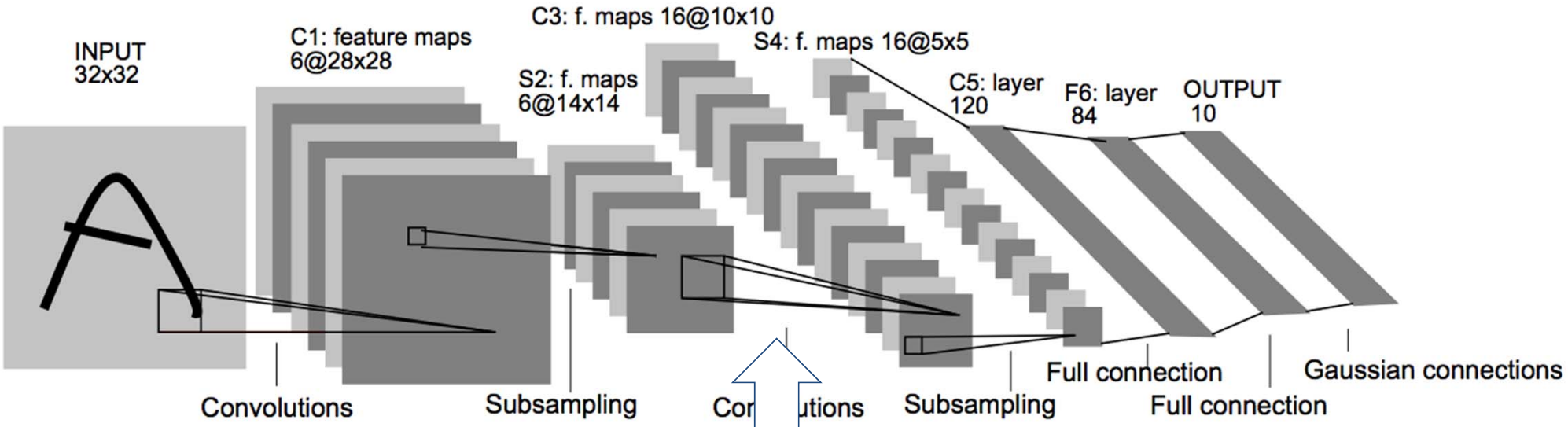


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
model.add(Convolution2D(16, 5, 5,
                        border_mode="valid"))
```

LeNet5: Second NonLinearity

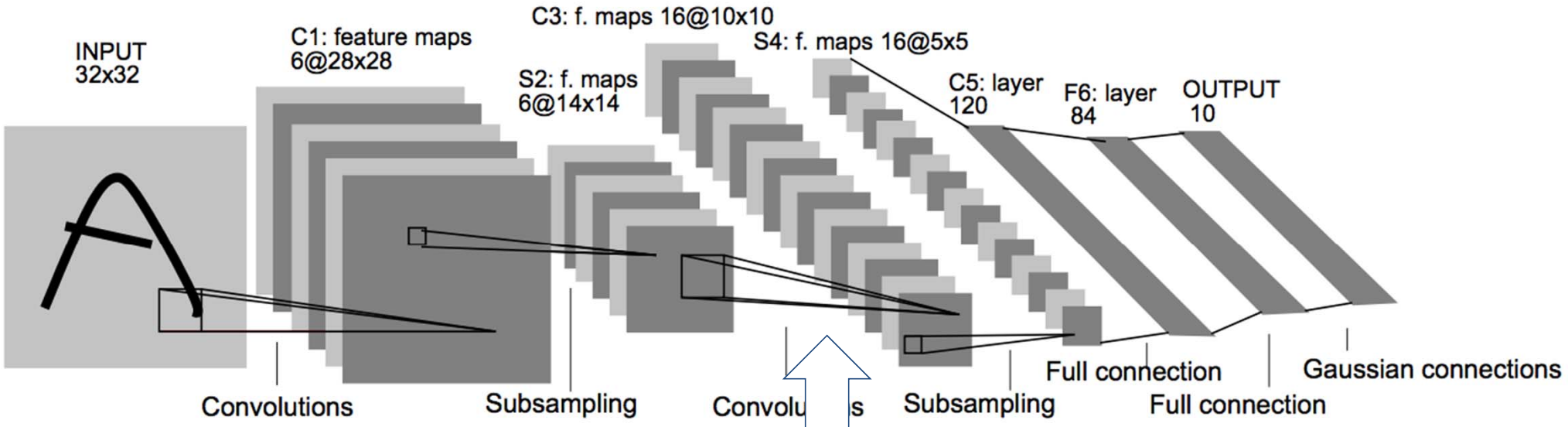


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

model.add(Activation(”sigmoid"))

LeNet5: Second Pooling

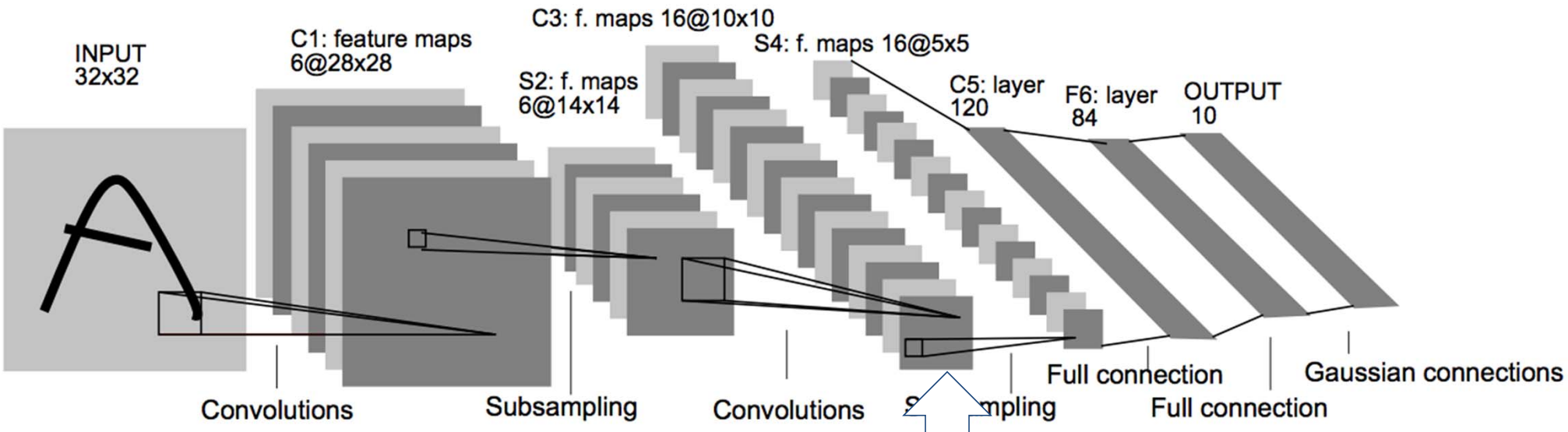


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits r nition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

model.add(MaxPooling2D(pool_size=(2, 2), strides=(2, 2)))

LeNet5: Third Convolution

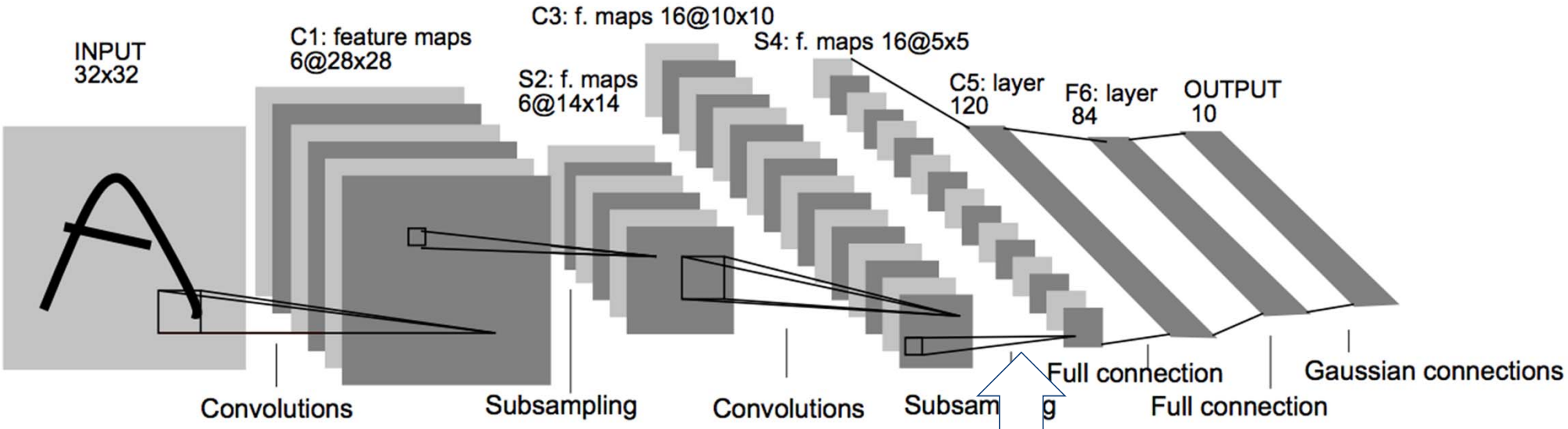


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recogni Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

model.add(Convolution2D(120, 1, 1, border_mode='valid'))

LeNet5: Flattening

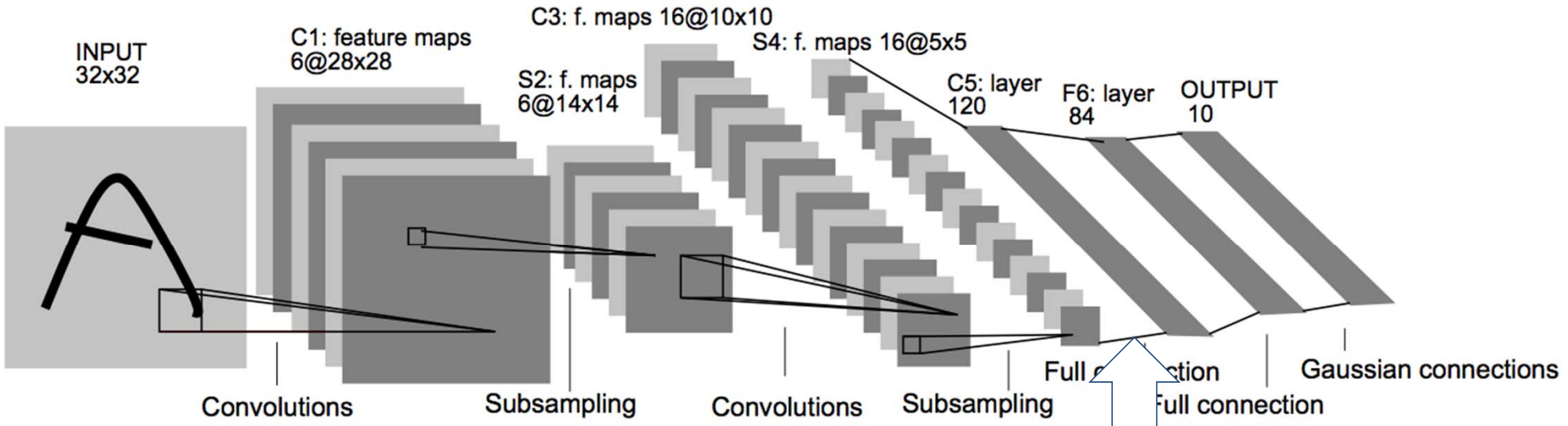


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
from keras.layers.core import Flatten
model.add(Flatten())
```

LeNet5: Dense Connections

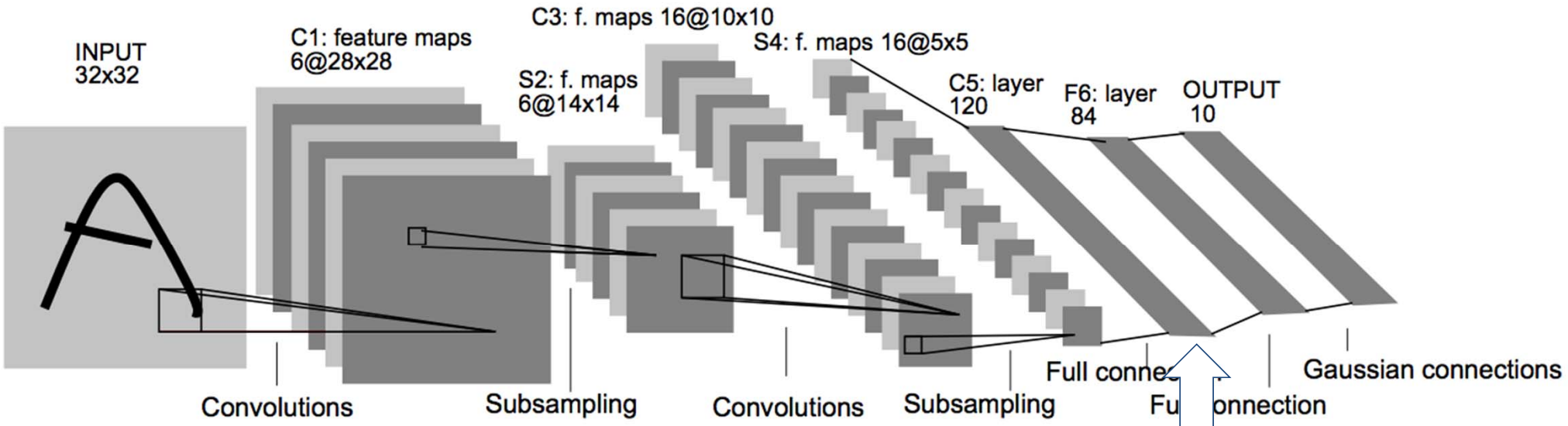


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature map, i.e. a set of units whose weights are constrained to be identical.

```
from keras.layers.core import Dense
model.add(Dense(84))
model.add(Activation("sigmoid"))
```

LeNet5: Dense Connections

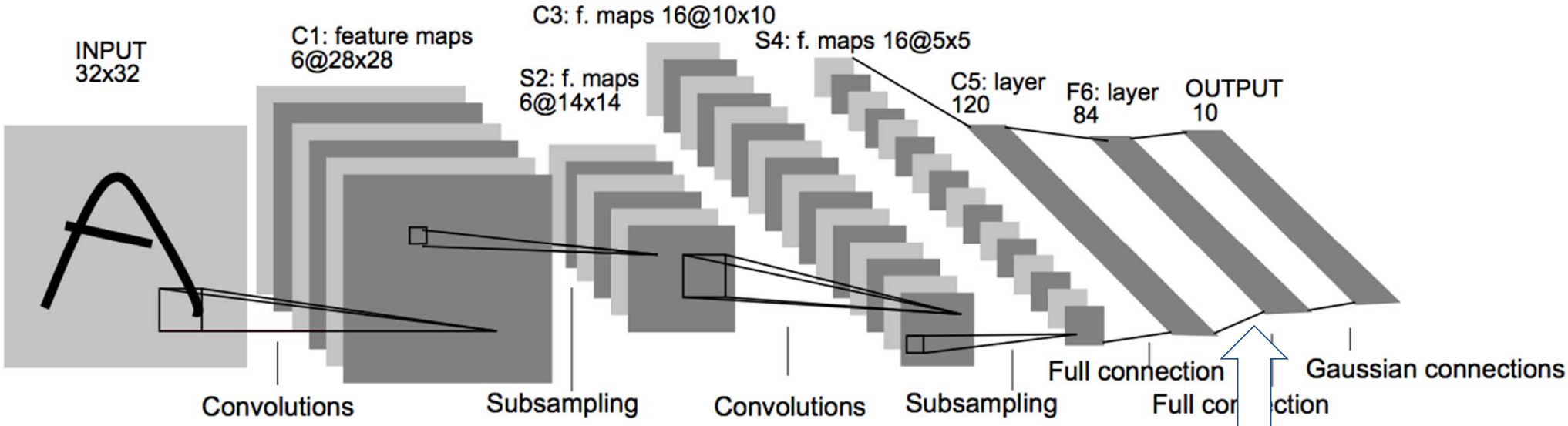


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a ture map, i.e. a set of units whose weights are constrained to be identical.

model.add(Dense(10))

LeNet5: Dense Connections

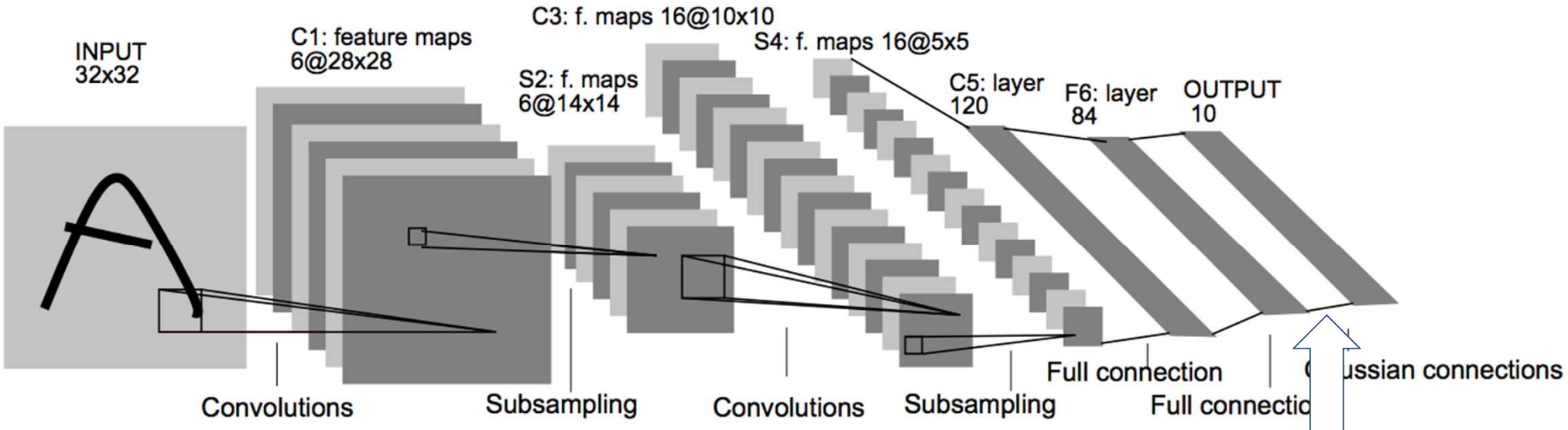


Fig. 2. Architecture of LeNet-5, a Convolutional Neural Network, here for digits recognition. Each plane is a feature ap, i.e. a set of units whose weights are constrained to be identical.

model.add(Activation('softmax'))

ResNet

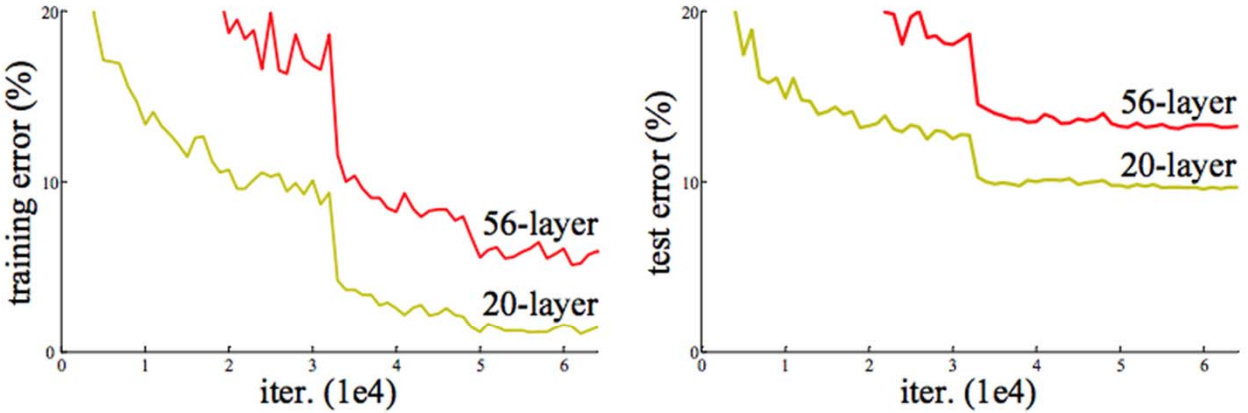


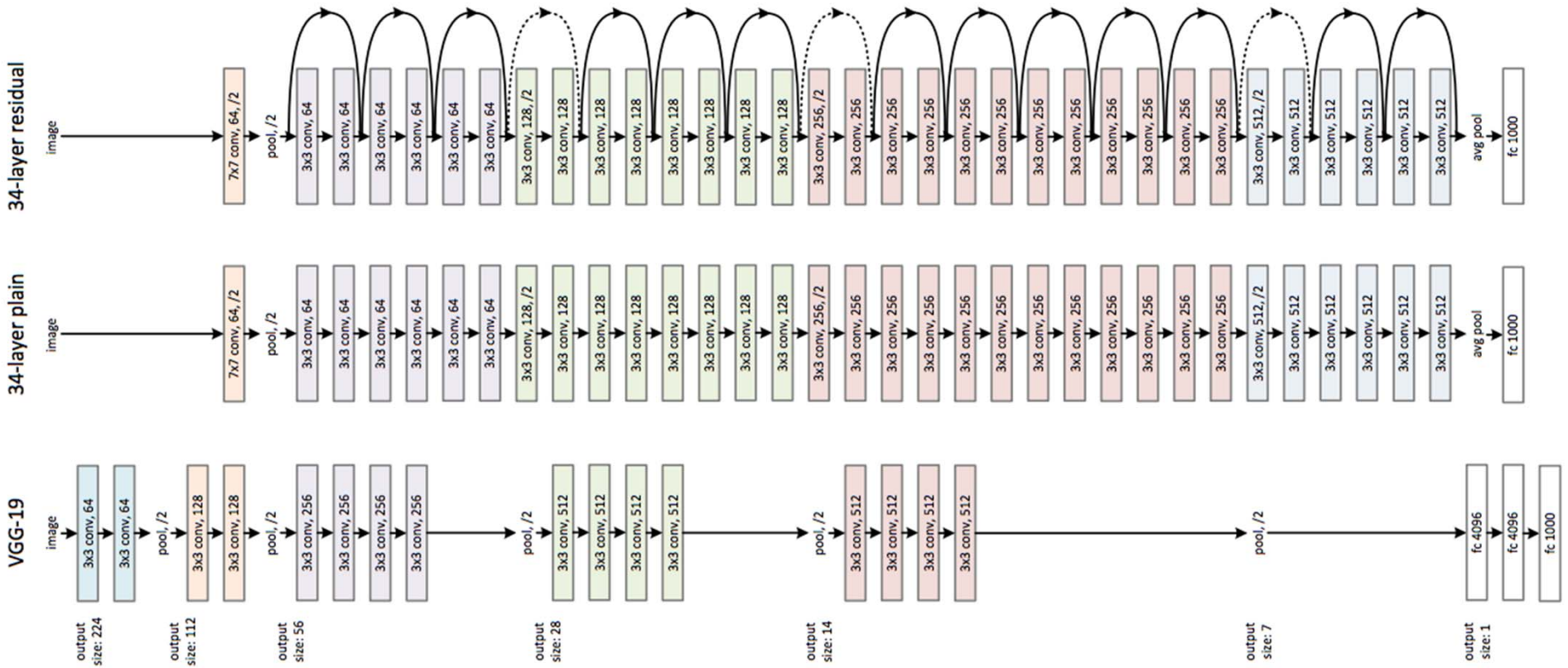
Figure 1. Training error (left) and test error (right) on CIFAR-10 with 20-layer and 56-layer "plain" networks. The deeper network has higher training error, and thus test error. Similar phenomena on ImageNet is presented in Fig. 4.

https://arxiv.org/pdf/1512.03385.pdf

- How can the behavior in the above charts be explained?
  - Vanishing gradients?
  - Overfitting?
  - Representation power?

ResNet Representation Power

https://arxiv.org/pdf/1512.03385.pdf



ResNet: Residual Learning Building Block

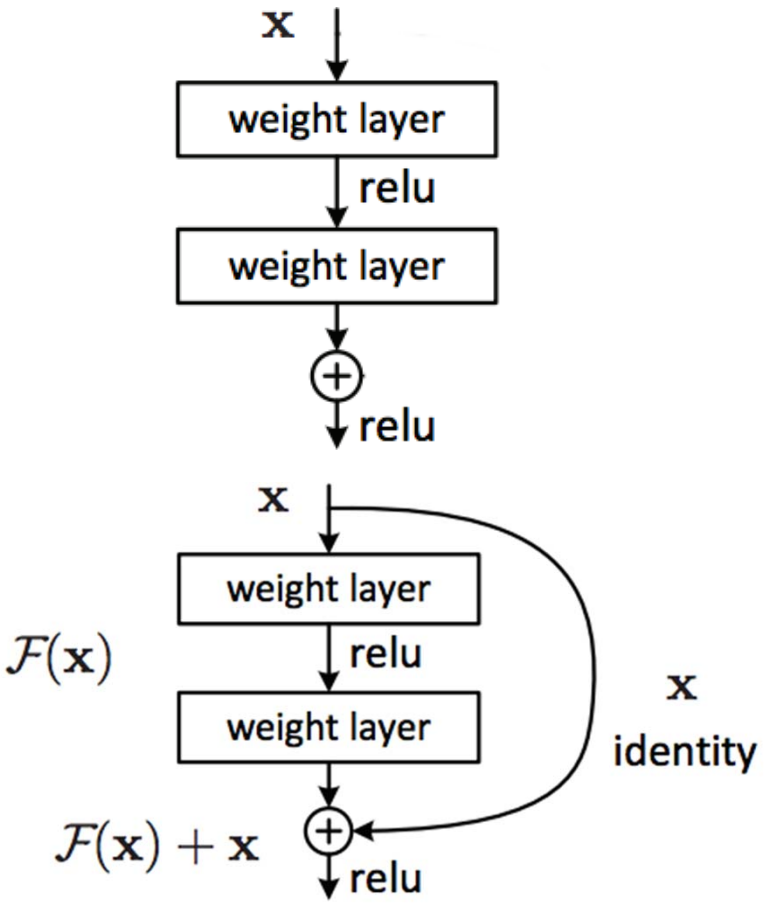
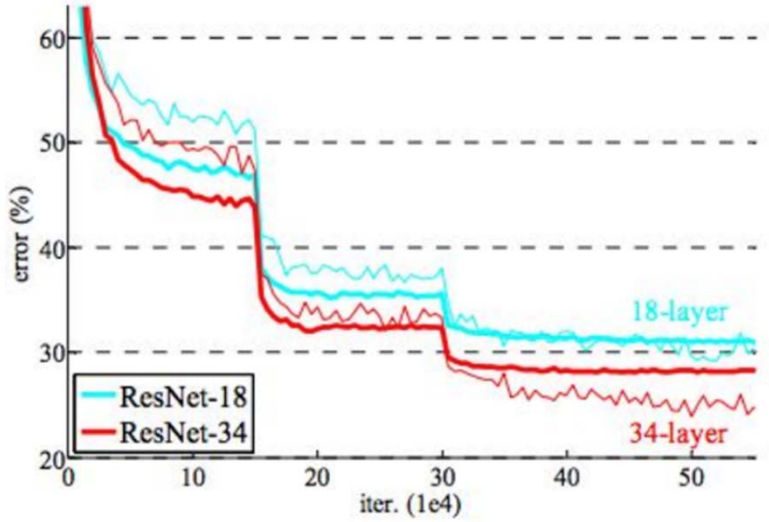


Figure 2. Residual learning: a building block.



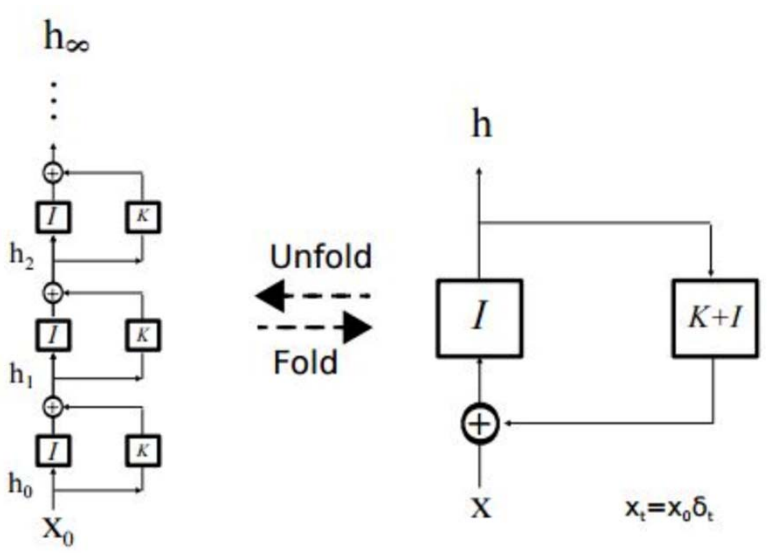
With residuals, the 34-layer network outperforms the 18 layer.

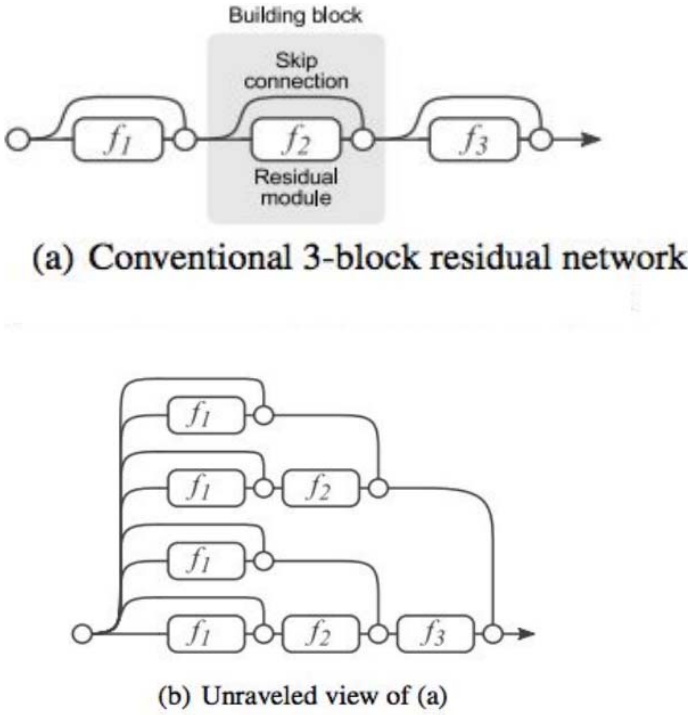
0	0	0
0	1	0
0	0	0

With 'SAME' padding, this will output the same feature map it receives as input

Why do ResNets work?

- Resembles ensembling shallower networks
- Can model recurrent computations
- Learning unrolled iterative estimations





Qianli Liao, Tomaso Poggio,Bridging the Gaps Between Residual Learning, Recurrent Neural Networks and Visual Cortex,