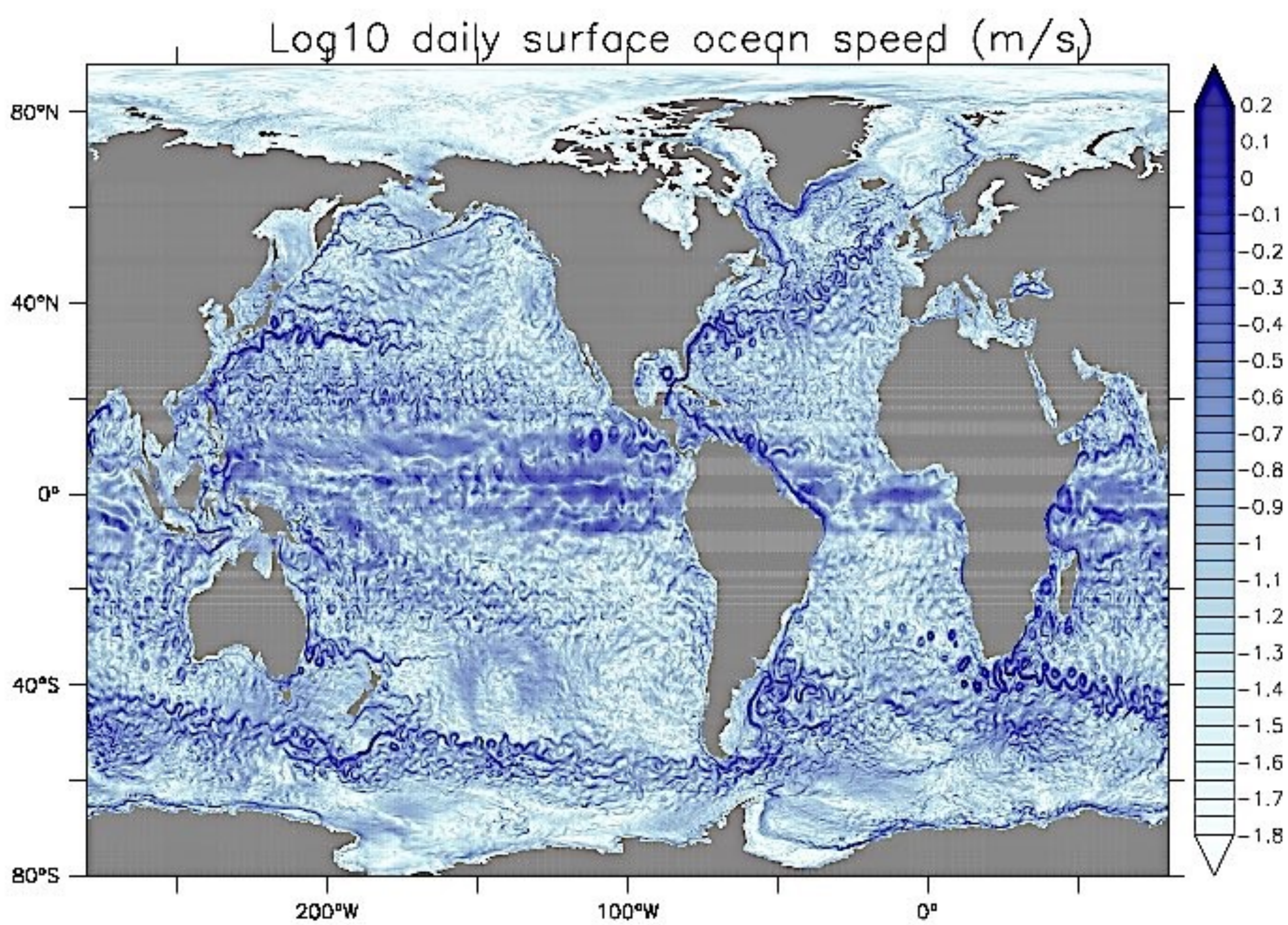
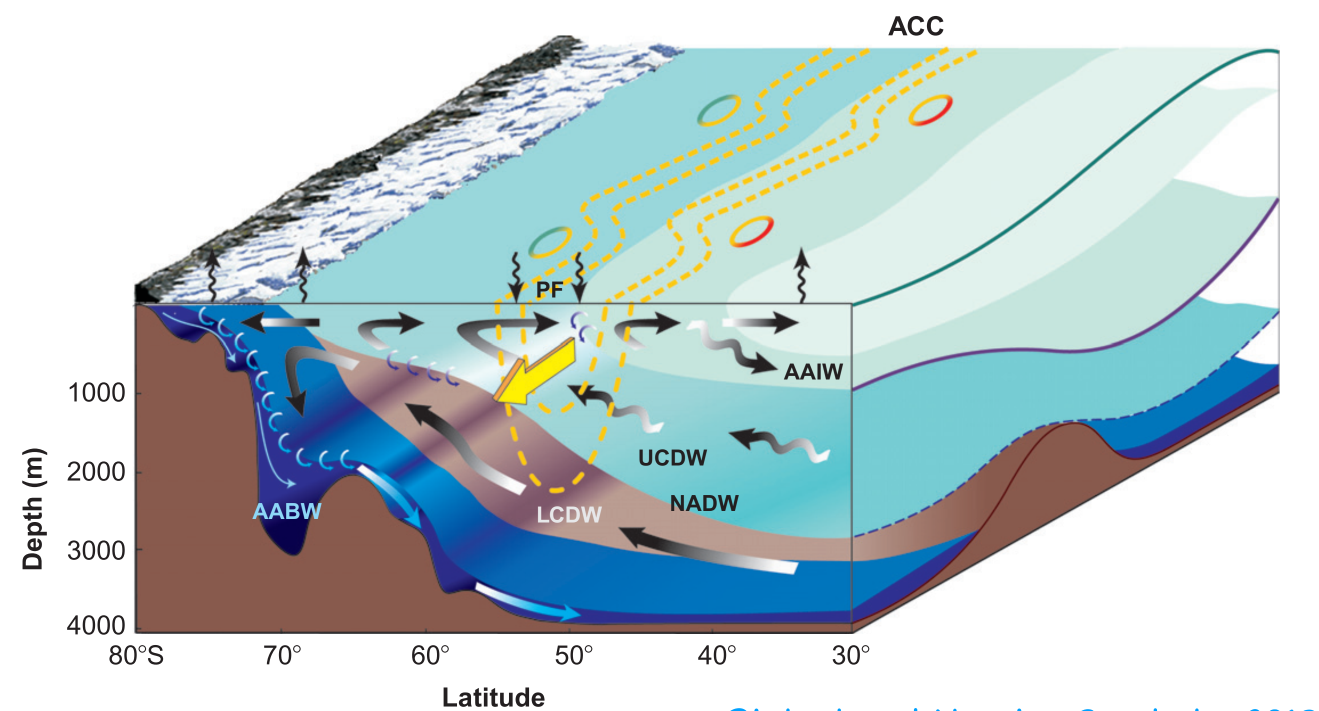
A tutorial on Southern Ocean dynamics with a focus on conceptual models

Version from 14 March 2020

Lectures given as part of Princeton University's
GEO/AOS 521 Spring 2020
Stephen Griffies



GFDL CM2.6 climate model (0.1 degree ocean grid)



Rintoul and Naveira Garabato, 2013
from Olbers and Visbeck (2005) as adopted from Speer (2000)

The purpose of conceptual models is not to fit the data but to sharpen the questions.
Samuel Karlin, mathematician

PRINCETON UNIVERSITY

Goals and Assumptions

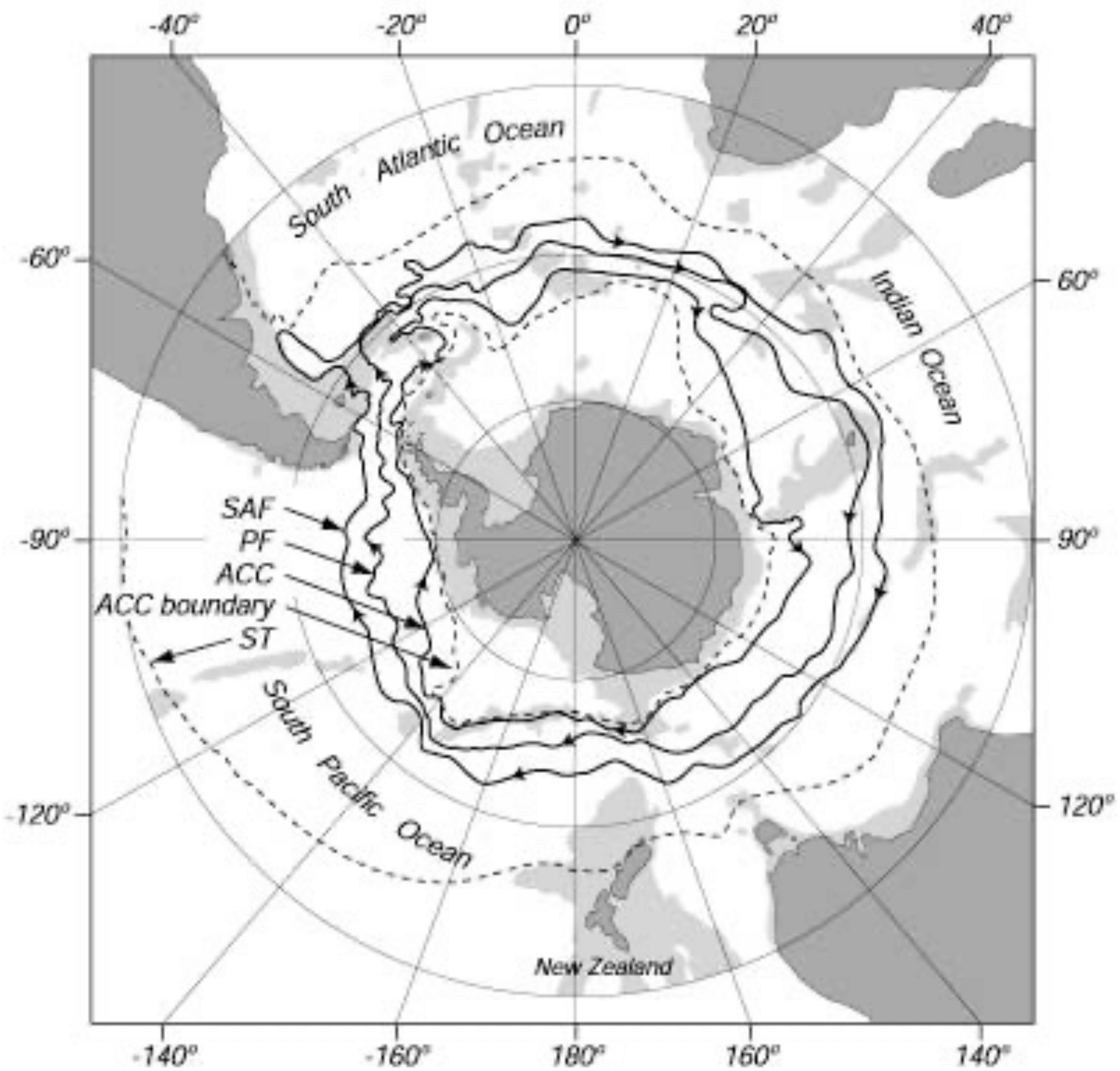
1. Provide exposure to dynamical processes relevant for understanding Southern Ocean circulation.
2. Most concepts will be presented in a manner to allow us all to be on the same playing field. Please ask questions if you get lost!
3. I will use mathematics since that is the most precise and concise means to communicate the physics. But all equations will have an English translation!
4. There is no pretense that you will be a practicing ocean fluid mechanic by the end of these lectures. But hopefully you will know a bit more of the basic concepts and language, sufficient to know the gist of "form stress", "eddy compensation", etc.
5. One specific goal of these lectures is to provide sufficient conceptual and technical insights to help understand the following review papers.
   A. Marshall and Speer, 2012: Closure of the meridional overturning circulation
   B. Rintoul and Naveira Garabato, 2013: Dynamics of the Southern Ocean Circulation
   C. Rintoul, 2018: The global influence of localized dynamics in the Southern Ocean

Outline of the Lectures

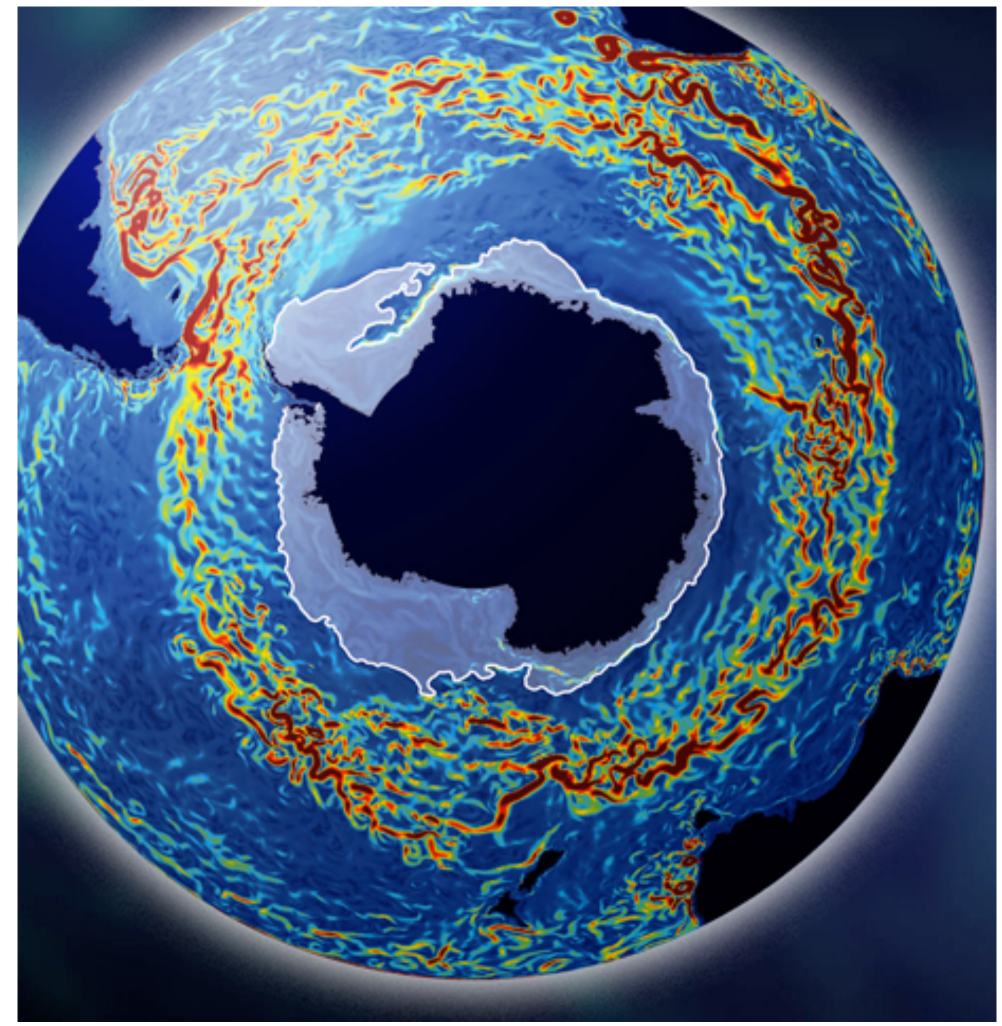
ACC Momentum Balance and the role of form stress

Southern Ocean Circulation Patterns

ACC = Antarctic Circumpolar Current



Orsi et al 1995



SOSE - surface speed (Mazloff et al 2010)

Channel geometry of the ACC emphasizes the zonal momentum balance, which contrasts to gyres that are typically studied using vorticity.

Volume transport through Drake Passage, relative to sea floor, around 137± 7 Sv (Meredith et al 2011). Including the bottom flow it is ~160Sv (Donohue et al 2016). This transport is roughly 1000 times Amazon flow. It is enough to empty the Baltic in 3 days!

Dynamics start with Newton: $\boldsymbol{F} = m\,\boldsymbol{A}$

$$\frac{\mathrm{d}(m\,\boldsymbol{v})}{\mathrm{d}t} = \boldsymbol{F} \Longrightarrow \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathcal{R}} \boldsymbol{v}\,\rho\,\mathrm{d}V\right) = \boldsymbol{F}$$

- What are the forces active in/on the Southern Ocean?
- How do these forces generate motion/circulation?

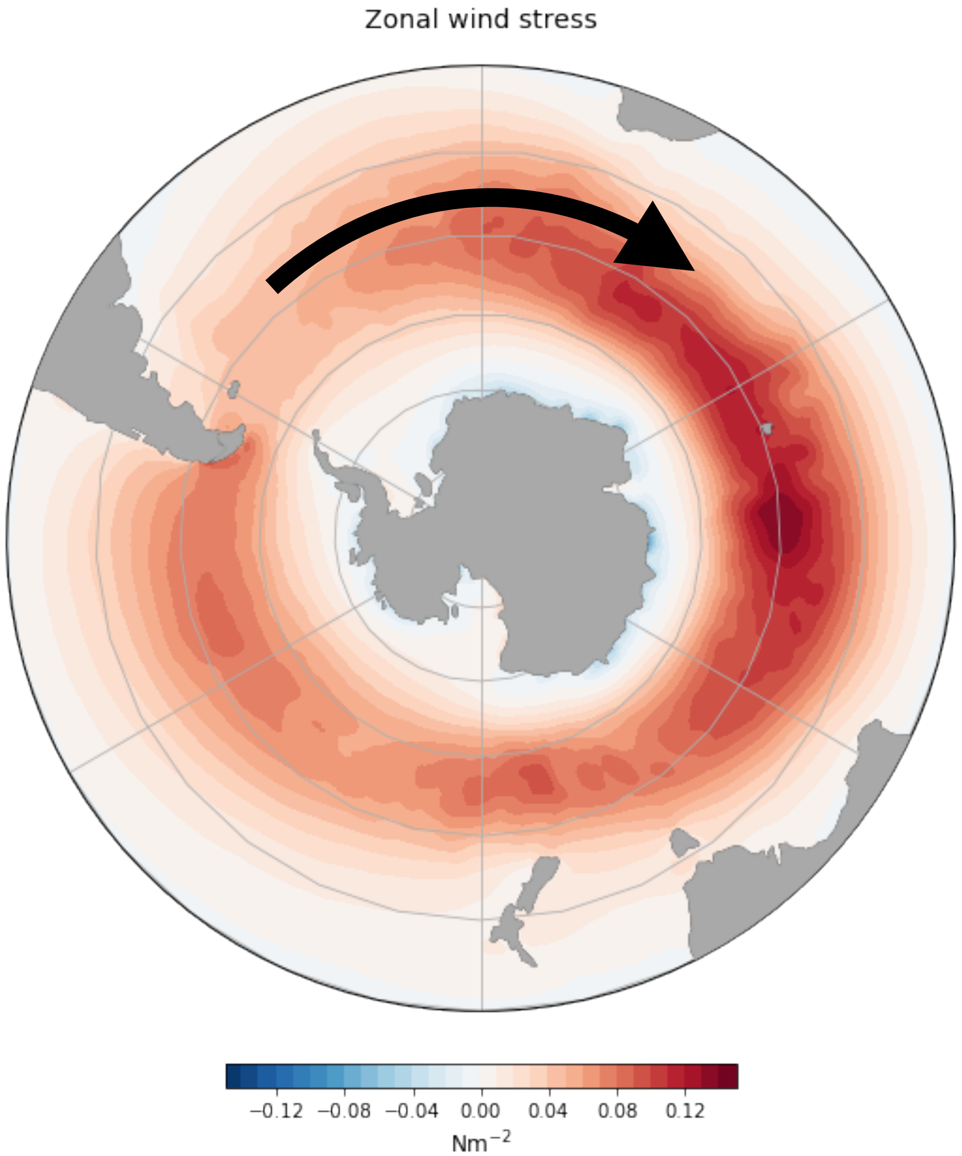
Caution: here are some "loaded" questions:

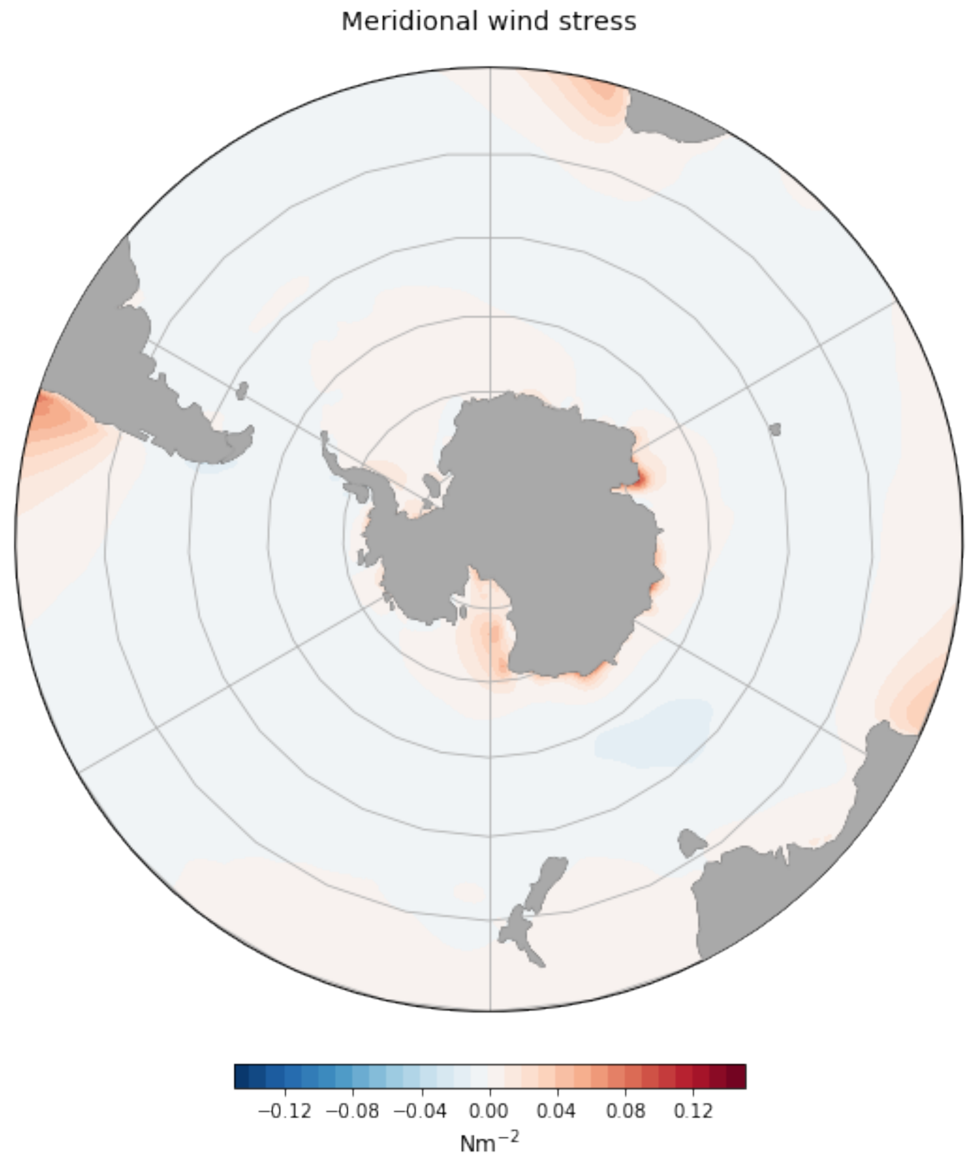
1. "What drives the circulation (car)?" (dog)
2. "What controls the circulation (car)?" (steering wheel)
3. "What is the source for the circulation (car)?" (engine)

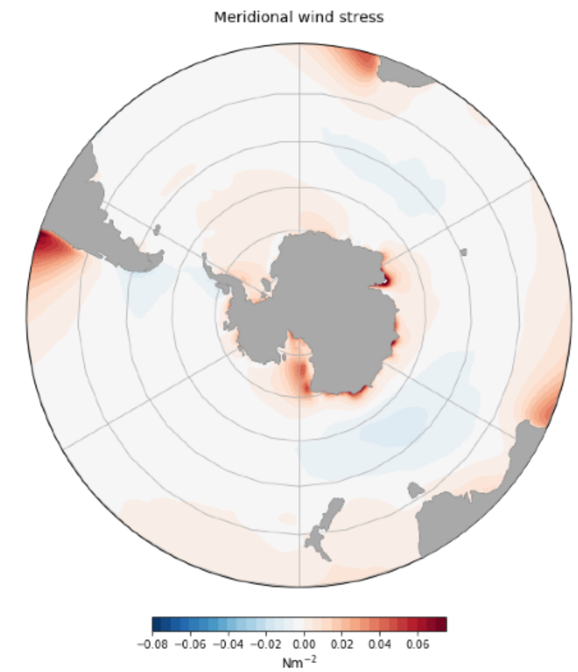
For ocean circulation, forces are not linearly superposed and there are feedbacks from the circulation on the forces.

So any attempt at linear superposition (e.g., through idealized numerical simulation) is incomplete. Nevertheless, we learn heaps by trying!

Wind Stress is Key!





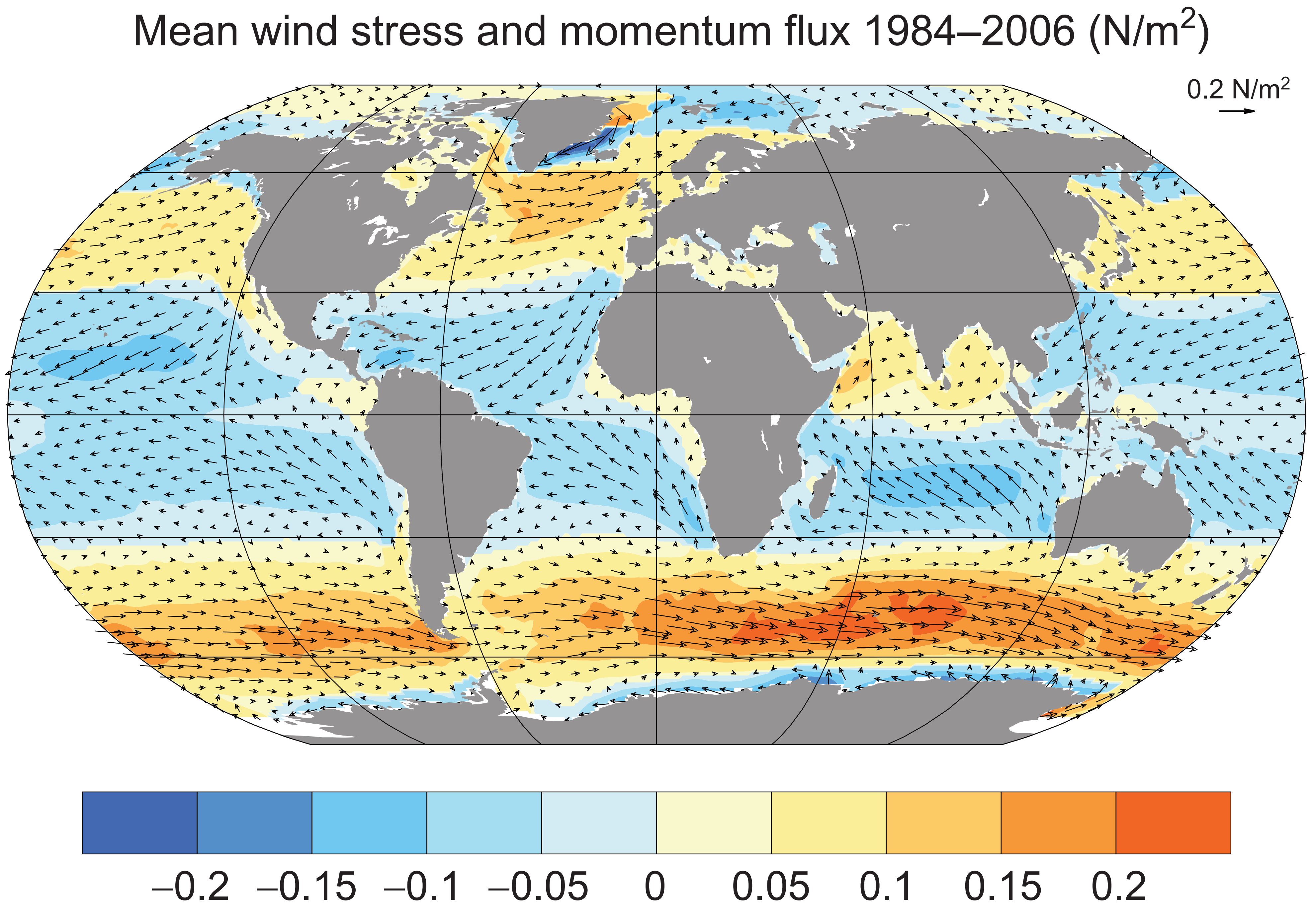


- 10-year means from ERA-interim reanalysis (atmos model + assimilation).
- Note dominance of zonal wind stress (westerlies).
- The westerlies are generally strong year round!
- Katabatic winds emphasized next to Antarctica in the meridional stress.

From the RRS J.C. Ross in the Scotia Sea
@S.M. Griffies, May 2017

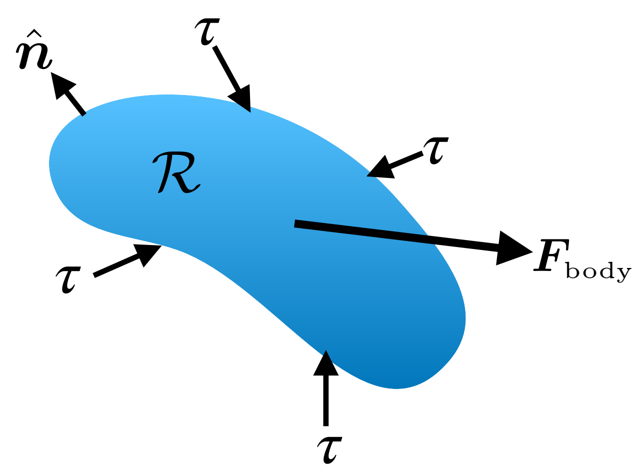
Global wind stress

Southern Ocean winds are huge!



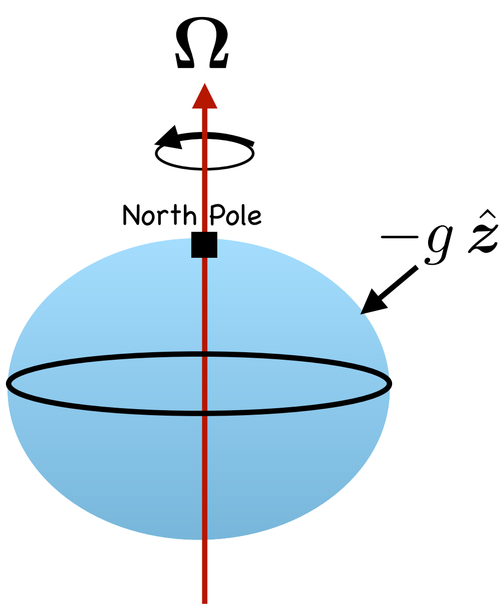
Stocker (2013) redrawn from data by Large and Yeager (2009)

Newton's Law for an arbitrary fluid region



$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathcal{R}} \boldsymbol{v}\,\rho\,\mathrm{d}V\right) = \boldsymbol{F}_{\rm body} + \boldsymbol{F}_{\rm contact}$$

Body forces from Coriolis + gravity + planetary centrifugal.



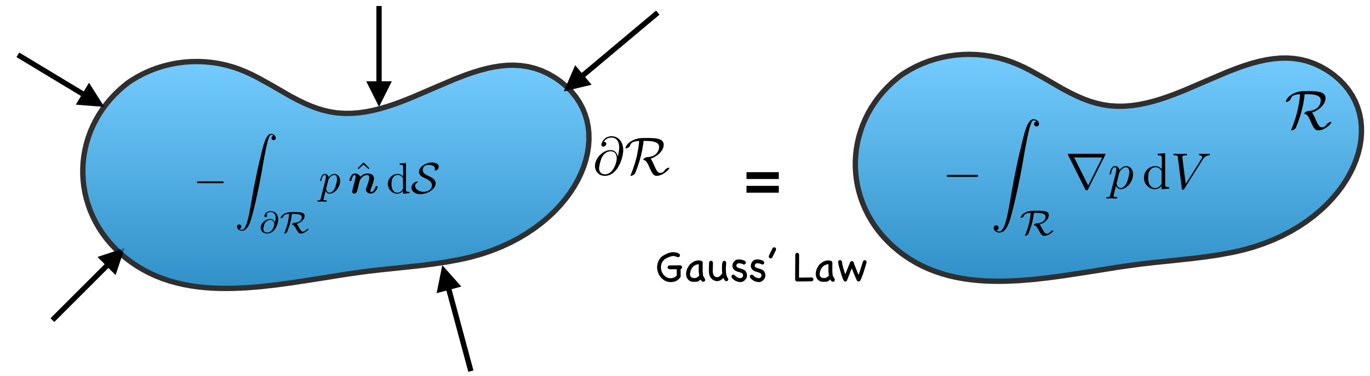
$$\boldsymbol{F}_{\rm body} = -\int_{\mathcal{R}} (2\,\boldsymbol{\Omega}\wedge\boldsymbol{v} + g\,\hat{\boldsymbol{z}})\,\rho\,\mathrm{d}V$$

Contact stresses (force per area) arise from pressure + kinetic + subgrid.

$$\boldsymbol{F}_{\rm contact} = \oint_{\partial\mathcal{R}} \boldsymbol{\tau}\,\mathrm{d}\mathcal{S} \qquad \boldsymbol{\tau} = \underbrace{-\hat{\boldsymbol{n}}\,p}_{\rm pressure}\;\underbrace{-(\rho\,\boldsymbol{v}\otimes\boldsymbol{v})\cdot\hat{\boldsymbol{n}}}_{\text{kinetic stress}}\;\underbrace{-\boldsymbol{\tau}_{\text{sub-grid}}}_{\text{Reynolds + bottom + winds}}$$

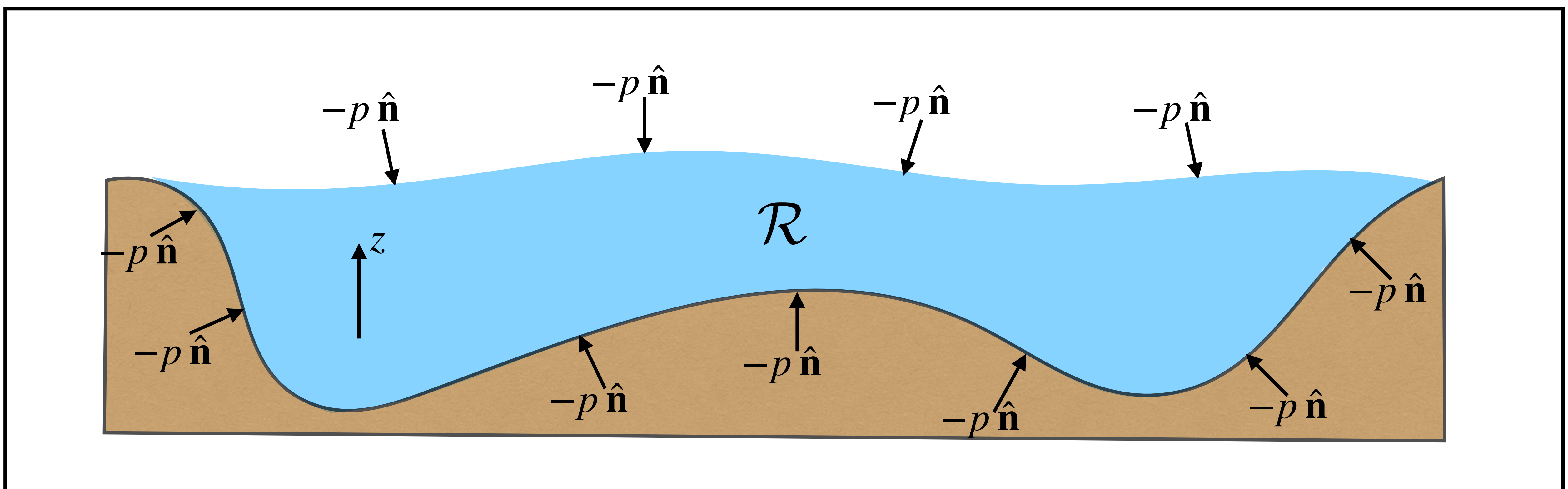
Basics of the pressure force

$$\boldsymbol{F}^{\text{pressure}} = -\int_{\partial\mathcal{R}} p\,\hat{\boldsymbol{n}}\,\mathrm{d}\mathcal{S} = -\int_{\mathcal{R}} \nabla p\,\mathrm{d}V$$

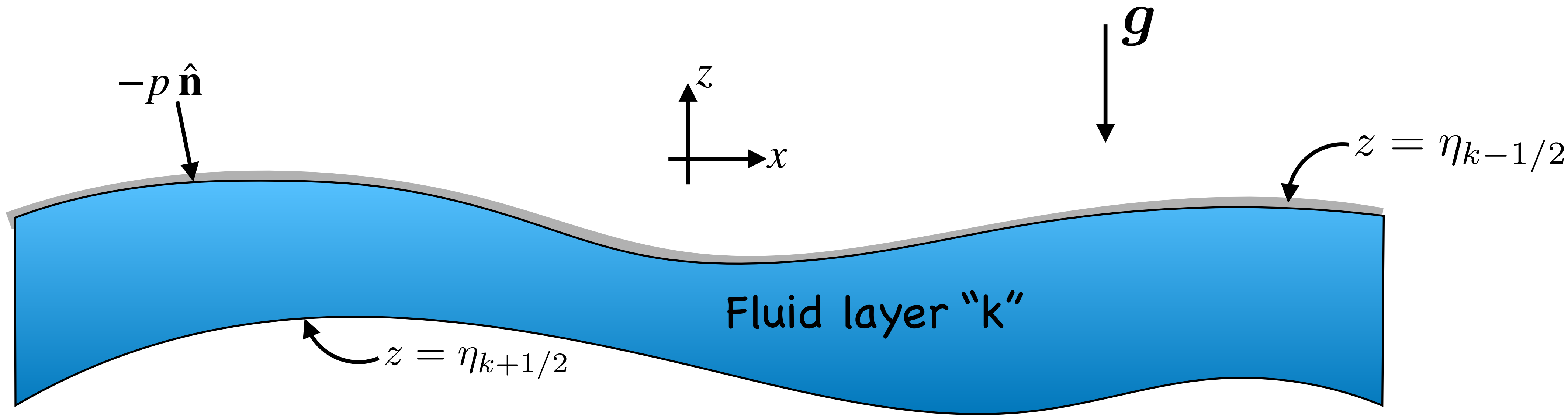


Net pressure force acting on a region can equivalently be computed as:

1. The surface integral of pressure contact force over the boundary of a region.
2. The volume integral of the pressure gradient body force acting within the region.
3. Either way, the integrated pressure force acting on the full ocean equals to that acting on its boundaries.



Hydrostatic Pressure + Form Stress



What is the pressure force acting on the top surface from fluid above it?

$$\boldsymbol{F}^{\text{pressure}}_{z=\eta_{k-1/2}} = -\int_{z=\eta_{k-1/2}} p\,\hat{\boldsymbol{n}}\,\mathrm{d}\mathcal{S}$$

What are the Cartesian components of this pressure force?

$$-p\,\hat{\boldsymbol{n}}\,\mathrm{d}\mathcal{S} = p\,(-\hat{\boldsymbol{z}} + \nabla\eta_{k-1/2})\,\mathrm{d}x\,\mathrm{d}y$$

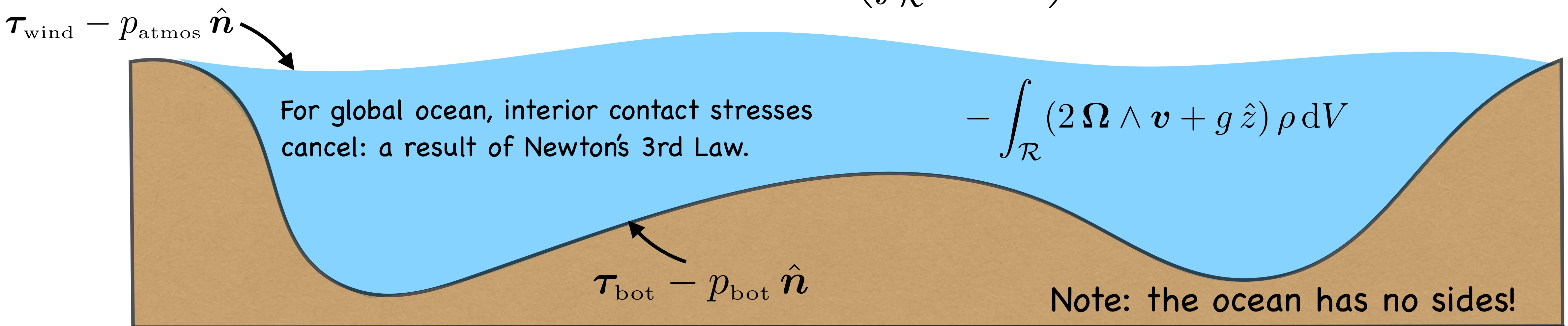
Horizontal pressure forces acting on the surface are called "form stresses". These are horizontal stresses due to the "geometric form" of the surface.

$$\boldsymbol{F}^{\text{form}}_{k-1/2} \equiv \underbrace{p\,\nabla\eta_{k-1/2}}_{\text{form stress}}\;\mathrm{d}x\,\mathrm{d}y$$

Net vertical pressure force is typically in hydrostatic balance.

$$\hat{\boldsymbol{z}}\cdot\boldsymbol{F}^{\text{pressure}}_{k} = -\int_{z=\eta_{k-1/2}} p\,\mathrm{d}x\,\mathrm{d}y + \int_{z=\eta_{k+1/2}} p\,\mathrm{d}x\,\mathrm{d}y \overset{\text{hydrostatic}}{=} g\,M_k \text{ = weight}$$

Steady state balance => $\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\mathcal{R}} \boldsymbol{v}\,\rho\,\mathrm{d}V\right) = \boldsymbol{F}_{\text{body}} + \boldsymbol{F}_{\text{contact}} = 0$



Global integrated steady momentum balance

$$\int_{\mathcal{R}} (2\,\boldsymbol{\Omega}\wedge\boldsymbol{v} + g\,\hat{\boldsymbol{z}})\,\rho\,\mathrm{d}V = \int_{z=\eta_{\text{top}}} (\boldsymbol{\tau}_{\text{wind}} - p_{\text{atm}}\,\hat{\boldsymbol{n}})\,\mathrm{d}\mathcal{S} + \int_{z=\eta_{\text{bot}}} (\boldsymbol{\tau}_{\text{bot}} - p_{\text{bot}}\,\hat{\boldsymbol{n}})\,\mathrm{d}\mathcal{S}$$

Hydrostatic Balance and Traditional Approximation form of Coriolis acceleration

$$\int_{\mathcal{R}} f\,\hat{\boldsymbol{z}}\wedge\boldsymbol{v}\,\rho\,\mathrm{d}V = \int_{z=\eta_{\text{top}}} (\boldsymbol{\tau}_{\text{wind}} + p_{\text{atm}}\,\nabla\eta_{\text{top}})\,\mathrm{d}x\,\mathrm{d}y + \int_{z=\eta_{\text{bot}}} (\boldsymbol{\tau}_{\text{bot}} - p_{\text{bot}}\,\nabla\eta_{\text{bot}})\,\mathrm{d}x\,\mathrm{d}y$$

Zonal integrated steady zonal momentum balance

Steady mass conservation means the zonal component of Coriolis force vanishes.

$$\oint_{\phi=\text{cst}} f\,v\,\rho\,\mathrm{d}z\,\mathrm{d}x = f \oint_{\phi=\text{cst}} v\,\rho\,\mathrm{d}z\,\mathrm{d}x = 0$$

At steady state, no mass accumulates across a latitude circle (ignore P-E+R). Hence, Coriolis acceleration plays no role in the zonal momentum balance for depth+zonal integrated flow.

Observational estimates (Morrow et al 1992) and simulations suggest that meridional divergence of interior kinetic/Reynolds stresses are either negligible or they speed up the jets. They are an insufficient sink of the zonal momentum.

We are thus left with the steady depth + zonal integrated zonal momentum balance:

$$\oint_{z=\eta_{\text{top}}} (\tau^{x}_{\text{wind}} + \underbrace{\cancel{p_{\text{atm}}\,\partial_x\eta_{\text{top}}}}_{\text{small}})\,\mathrm{d}x = \oint_{z=\eta_{\text{bot}}} (\underbrace{\cancel{-\tau^{x}_{\text{bot}}}}_{\text{small}} + \underbrace{p_{\text{bot}}\,\partial_x\eta_{\text{bot}}}_{\text{"mountain drag"}})\,\mathrm{d}x$$

Munk and Palmen (1951) Johnson and Bryden (1989) Olbers (1998)

What is needed for bottom drag to balance wind stress?

Force per length input by winds integrated around the Southern Ocean at 60S:

$$\oint \tau^x_{\text{winds}} \, \mathrm{d}x = L \, \langle \tau^x_{\text{winds}} \rangle \approx (24 \times 10^6 \text{ m}) \, (0.1 \text{ N m}^{-2}) = 2.4 \times 10^6 \text{ N m}^{-1}$$

This is a huge amount of momentum transferred to the Southern Ocean by winds.

Force per length from quadratic bottom drag integrated around the Southern Ocean:

$$\oint \tau^x_{\text{bott}} \, \mathrm{d}x = L \, C_{\text{d}} \, \rho_0 \, \langle u^2_{\text{bot}} \rangle \qquad C_{\text{d}} \approx 2 \times 10^{-3} \qquad \text{and} \qquad \rho_0 = 1035 \text{ kg m}^{-3}$$

Bottom drag arises from small scale turbulent processes in the bottom boundary layer and via flow interactions with small-scale topography (roughness). Quadratic bottom stress is a typical parameterization of this drag.

Wind stress balanced by bottom drag requires the following zonal ave bottom velocity

$$\langle u^2_{\text{bot}} \rangle = \frac{\langle \tau^x_{\text{winds}} \rangle}{C_{\text{d}} \, \rho_0} \Longrightarrow \sqrt{\langle u^2_{\text{bot}} \rangle} \approx 0.2 \text{ m s}^{-1}$$

This is a huge bottom velocity.

How huge? Well, if this bottom velocity is representative of the depth averaged velocity (i.e., barotropic velocity), then the Drake Passage transport is

$$T^x = \int_{\text{Drake}} \mathrm{d}y \int_{-H}^{0} \sqrt{\langle u^2_{\text{bot}} \rangle} \, \mathrm{d}z \approx 8000 \text{ Sv} \approx 50 \times T^x_{\text{measured}}$$

Conclude: For bottom drag to balance the (huge) momentum input to the Southern Ocean by the winds would require an unphysically large bottom zonal velocity. However, measurements rule out this large velocity.

NB: numerical flat bottom channel models generally have huge zonal transports!

Can bottom form stress indeed balance zonal wind stress?

AKA "mountain drag"

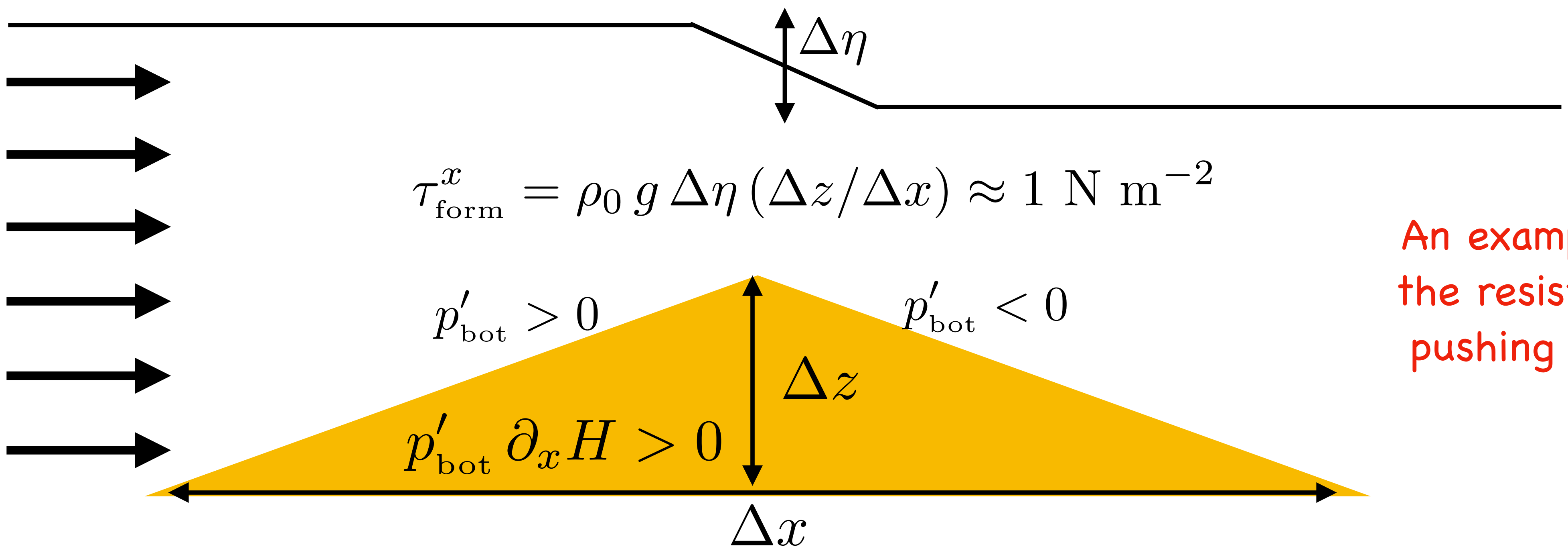
Walter Munk (1917-2019)

$$\oint_{z=\eta_{\rm top}} \tau^x_{\rm wind}\,{\rm d}x = -\oint_{z=\eta_{\rm bot}} (p_{\rm bot}\,\partial_x H)\,{\rm d}x \Longrightarrow \langle \tau^x_{\rm wind}\rangle = -\langle p'_{\rm bot}\,\partial_x H\rangle > 0$$

$$\tau^x_{\rm wind} > 0$$

$$p_{\rm bot} = \rho_0\, g\, H + p'_{\rm bot}$$

Plausibility argument: Consider a homogeneous (constant density) ocean with a 1000m high ridge extending horizontally over 1000km. Assume the free surface has a 10cm high SSH anomaly on upstream side of ridge. What is the form stress?



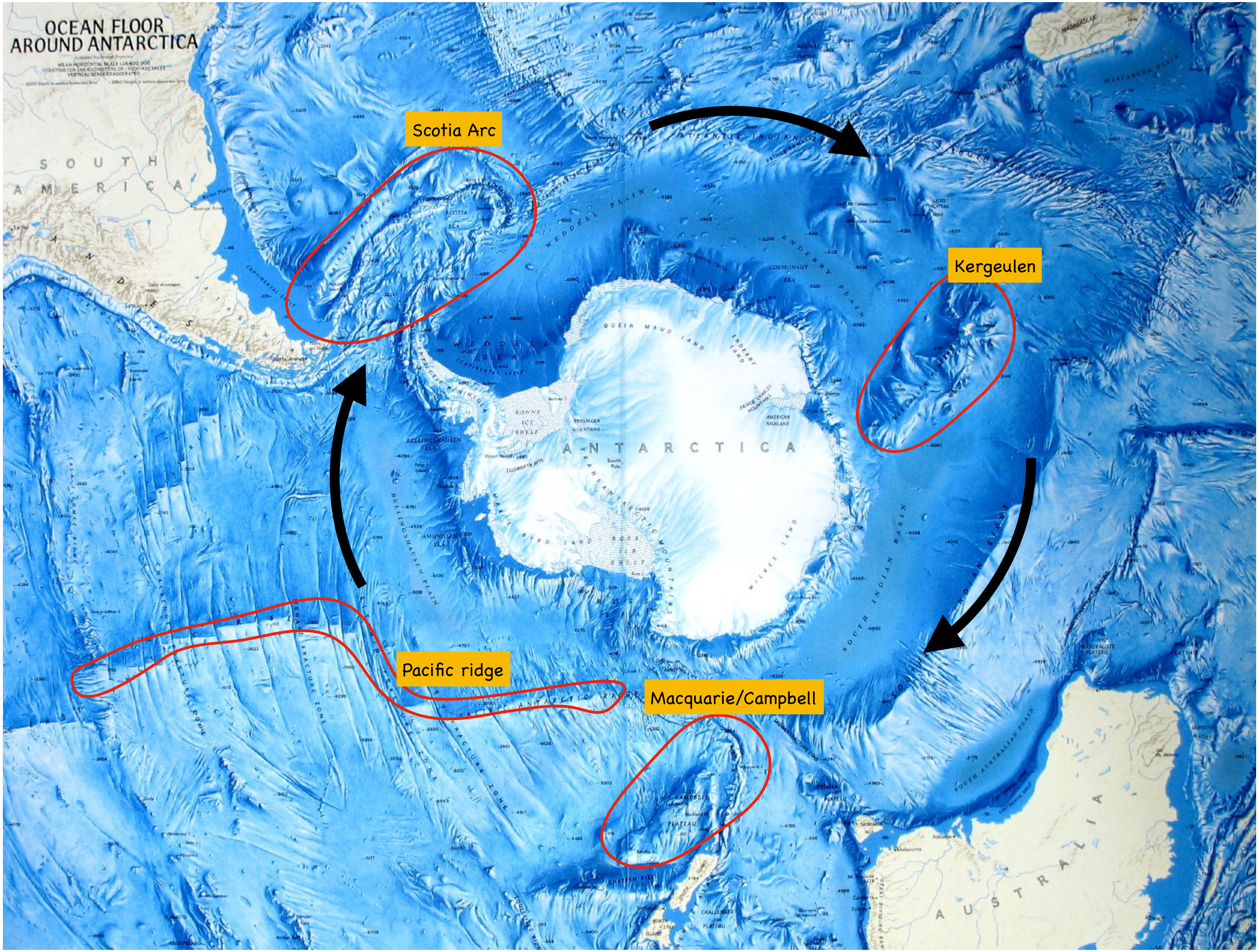
$$\tau^x_{\rm form} = \rho_0\, g\, \Delta\eta\, (\Delta z/\Delta x) \approx 1\ {\rm N\ m^{-2}}$$

An example of form stress is the resistance one gets when pushing against a solid wall.

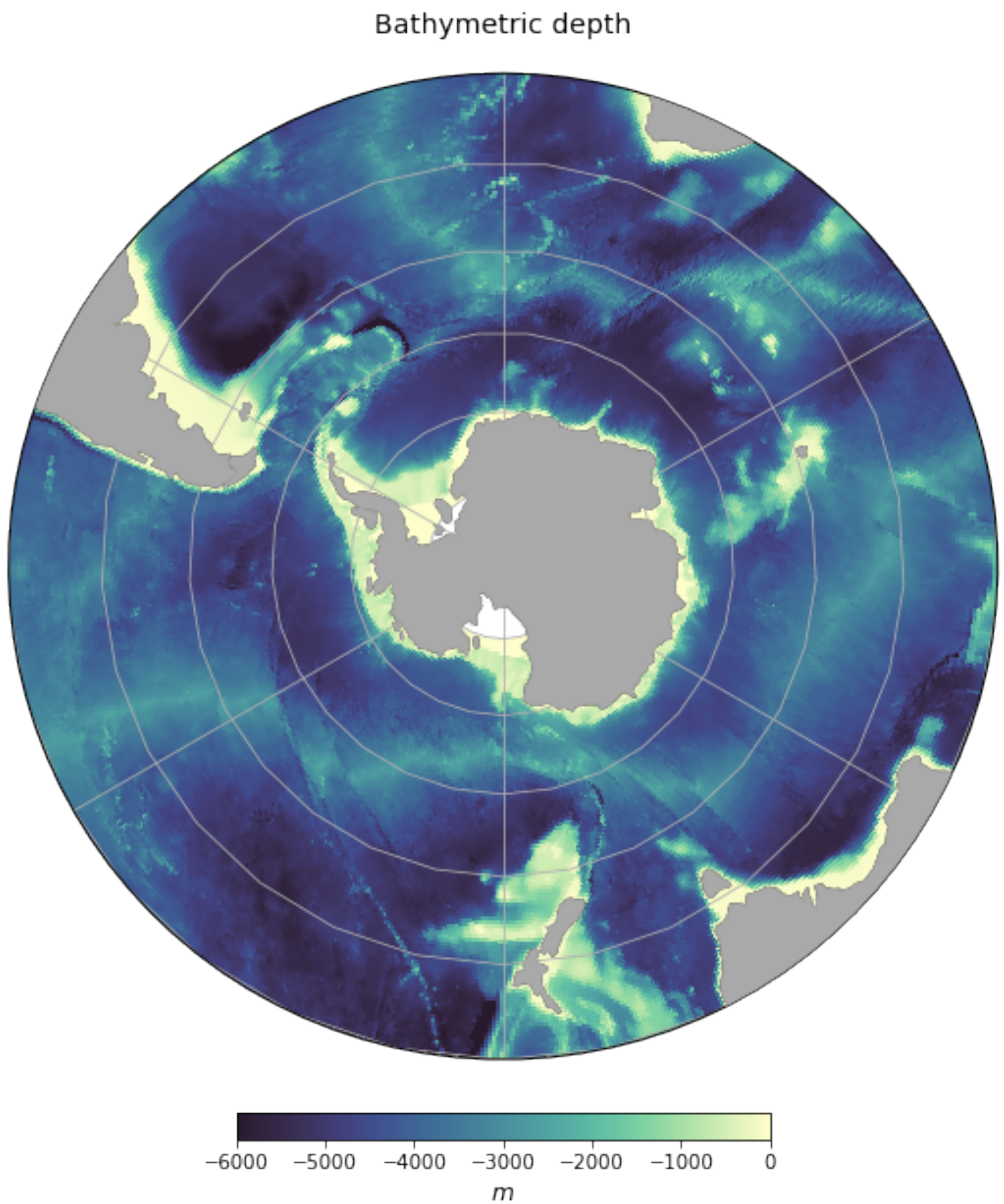
If we have topographic features with $p'_{\rm bot}\,\partial_x H > 0$ spread around the Southern Ocean, and if they are large enough, then they can provide a sink of zonal momentum. Turns out that the average zonal form stress arising from four large topographic features (Kerguelen, Scotia, Campbell, Pacific rise) and a modest bottom pressure anomaly are sufficient to have

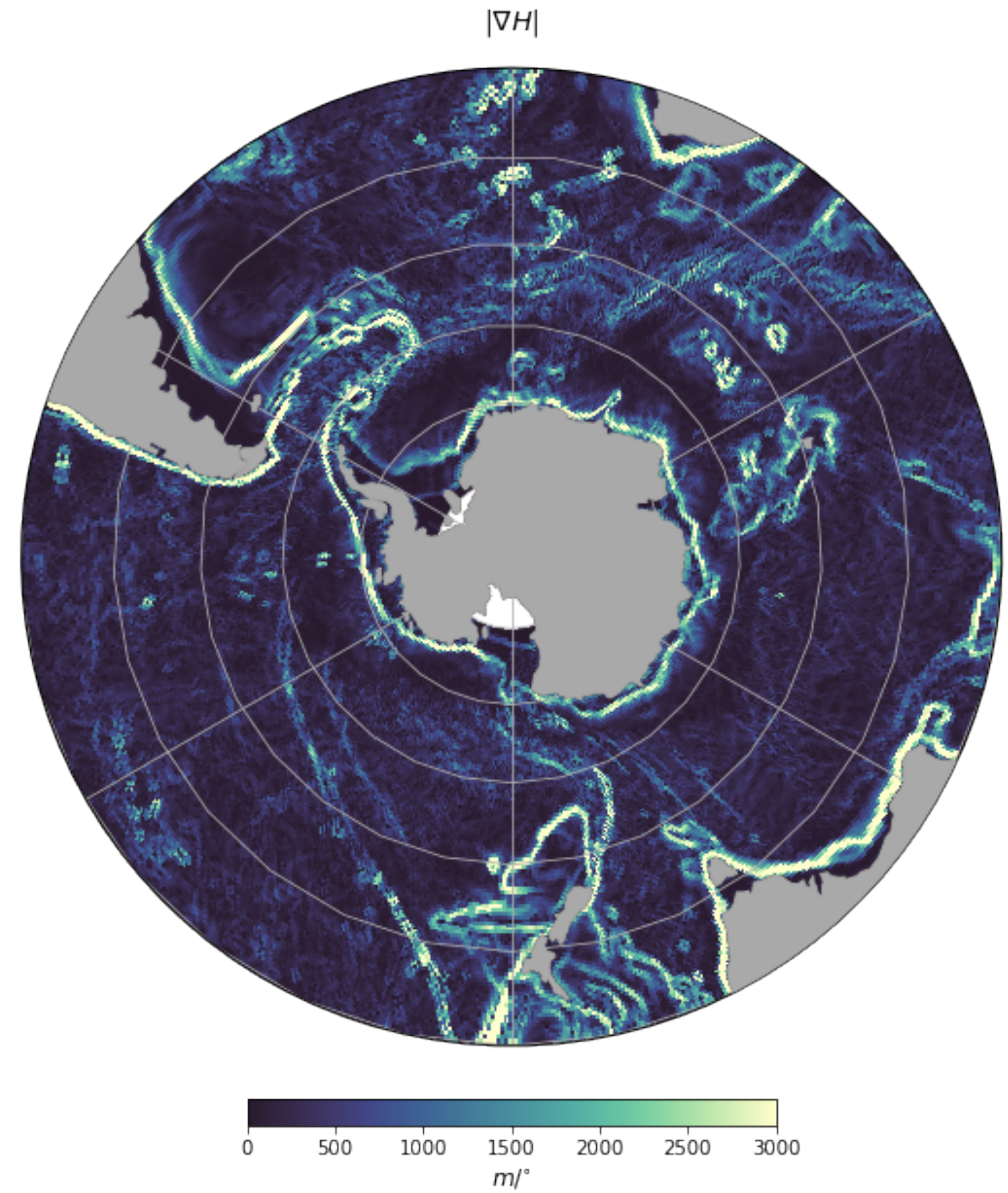
$$\langle \tau^x_{\rm form}\rangle \approx \langle \tau^x_{\rm wind}\rangle \approx 0.1\ {\rm N\ m^{-2}}$$

Bottom topography (National Geographic Society: Wikiversity)



Bottom topography + |gradient H|



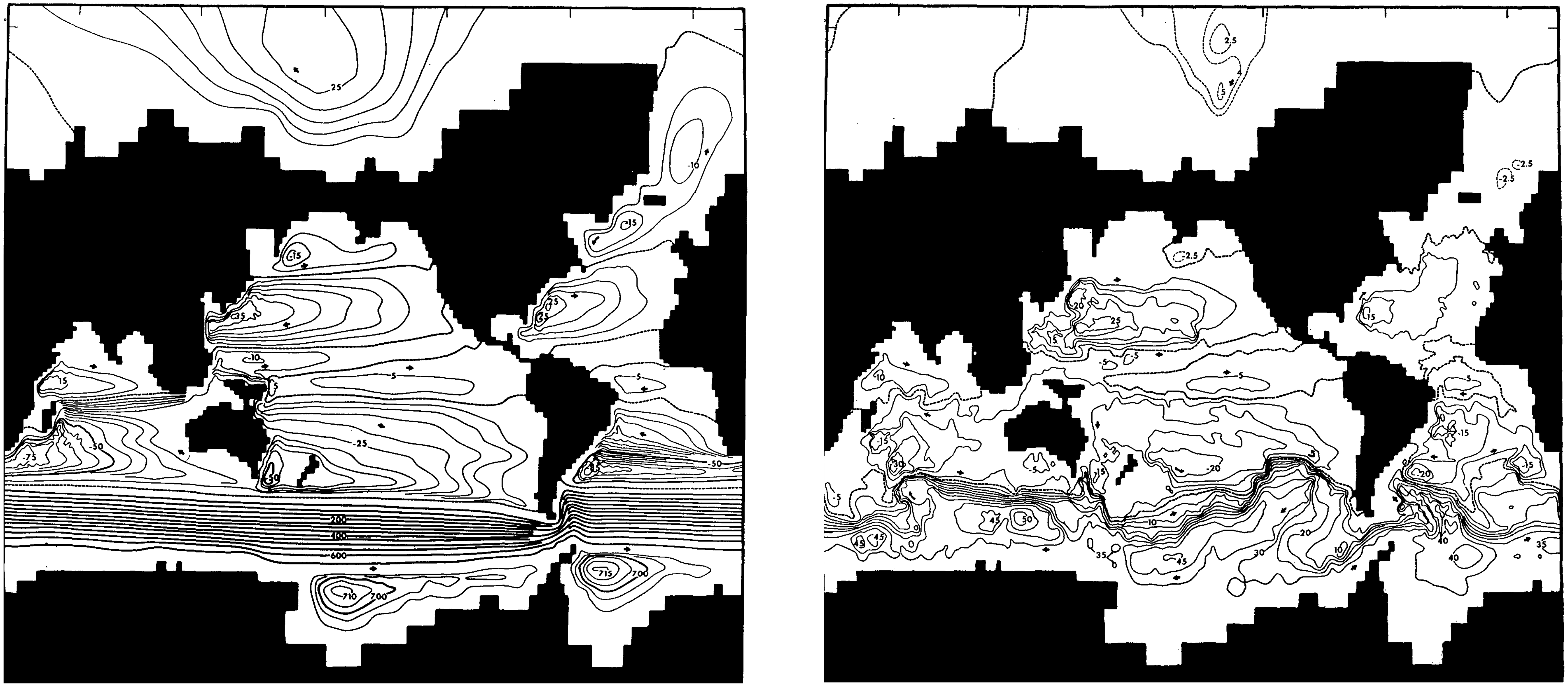


Thanks to Graeme MacGilchrist for these figures

Simulations with and without topography

no bathymetry

realistic bathymetry



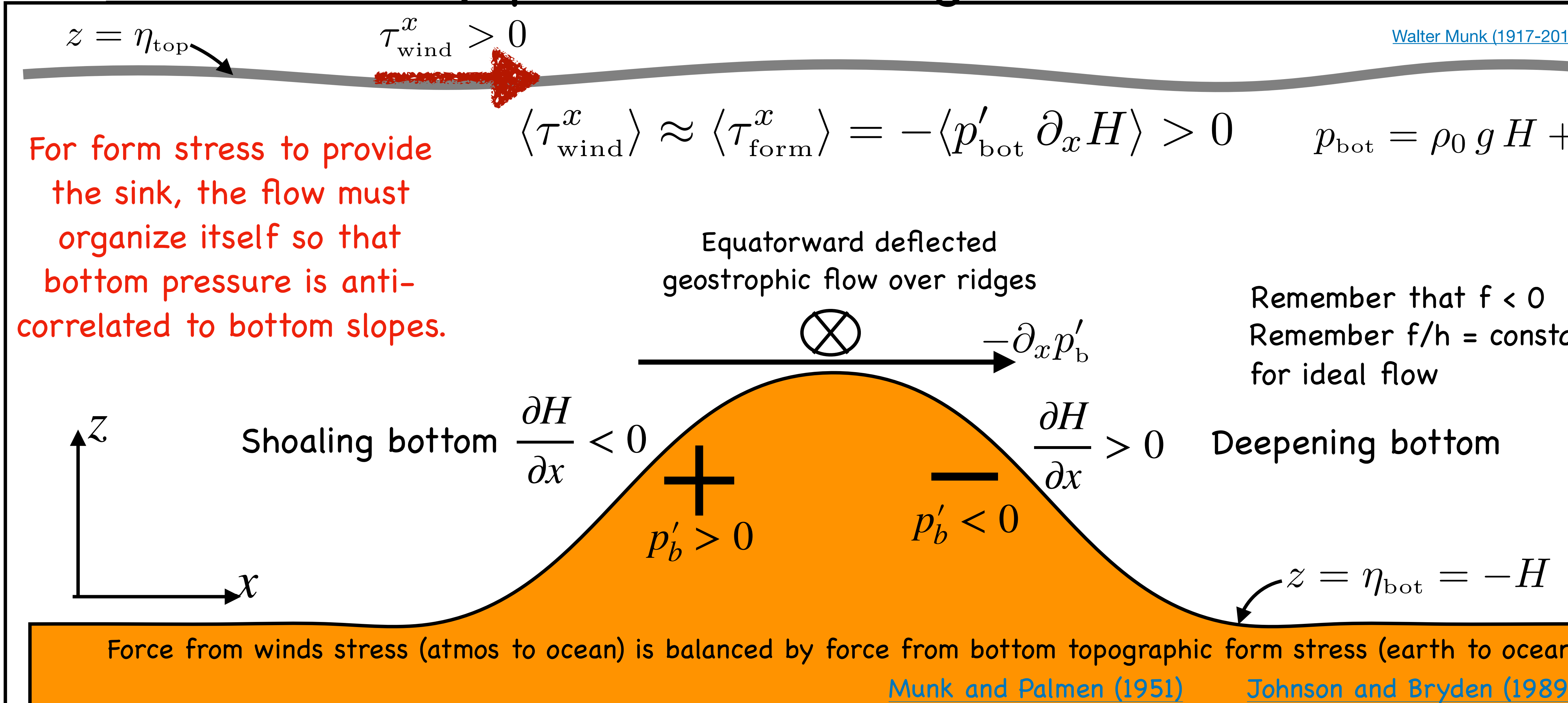
ACC > 600 Sv

ACC ~ 32 Sv

Numerical solutions with homogeneous model [(Bryan and Cox, 1972)]

Illustrates the role of topographic form stress particularly for the ACC.
Also illustrates that a homogeneous ocean is not sufficient for describing ACC flow.

Summary points concerning bottom form stress



Bottom form stress next to large-scale bottom features is the dominant sink of zonal wind stress in the ACC.

Above topography there is a zero zonal mean zonal pressure force; only have a zonal mean pressure force at and below depth of shallowest topography.

Newton's 3rd Law: Pressure force from ocean on the earth is equal and opposite to pressure force from earth on the ocean.

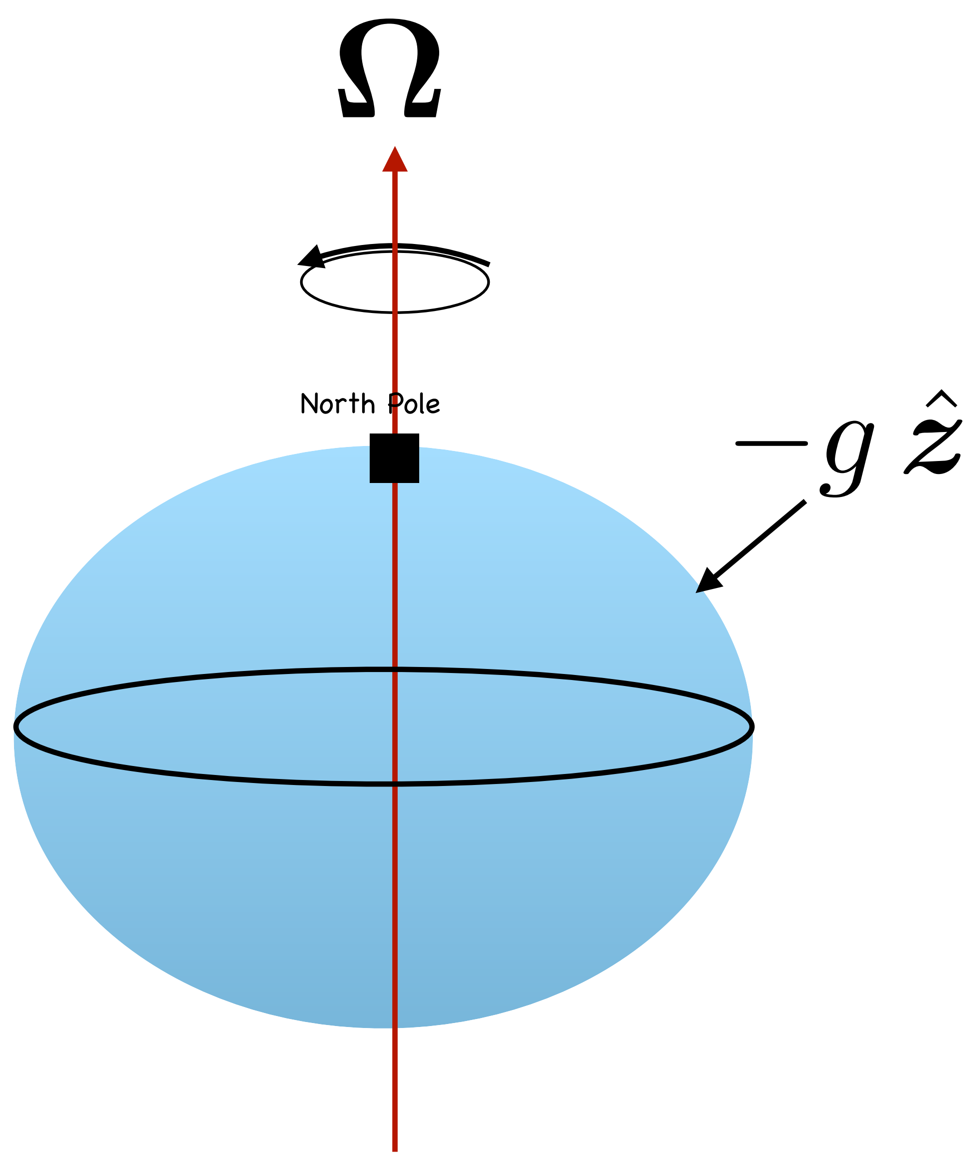
Internal friction (e.g., molecular viscosity + Reynolds stresses) and bottom drag (bottom boundary layers) are far less important than form stress. In contrast, friction is key to the dynamical balance of subtropical gyres particularly in their western boundaries; e.g., Stommel/Munk gyre theories involve bottom/side friction.

But how does zonal momentum get transferred from the ocean surface to the bottom? Geostrophic eddies...

A review of geostrophy and thermal wind

Geostrophy and thermal wind

Dominant balance for rapidly rotating fluids (geostrophy) and for rapidly rotating stably stratified fluids (thermal wind).

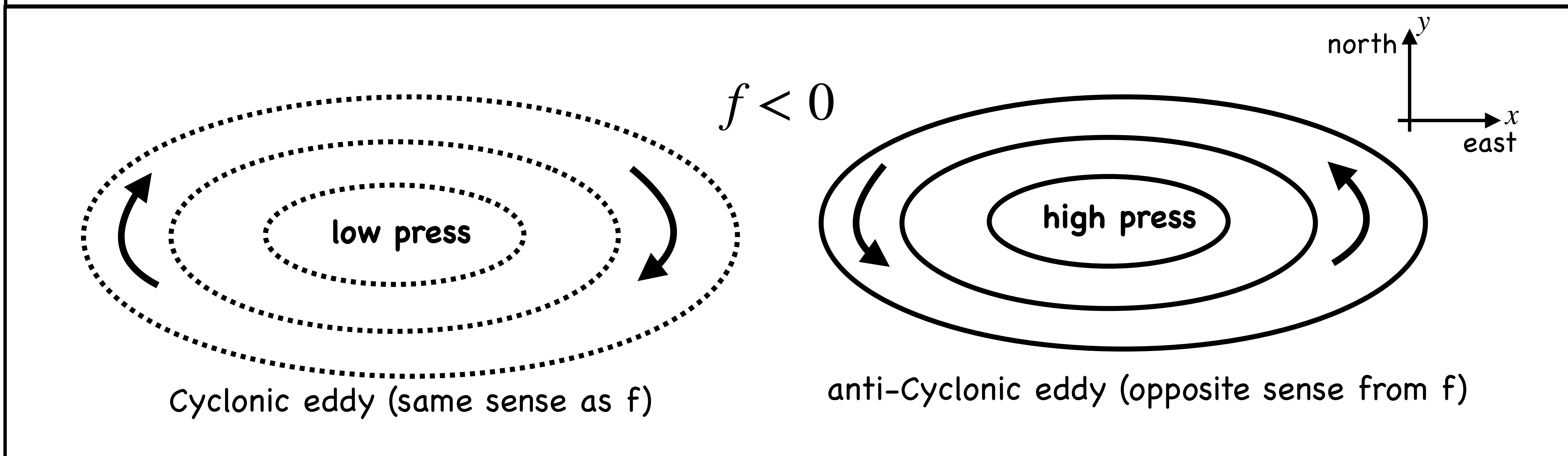


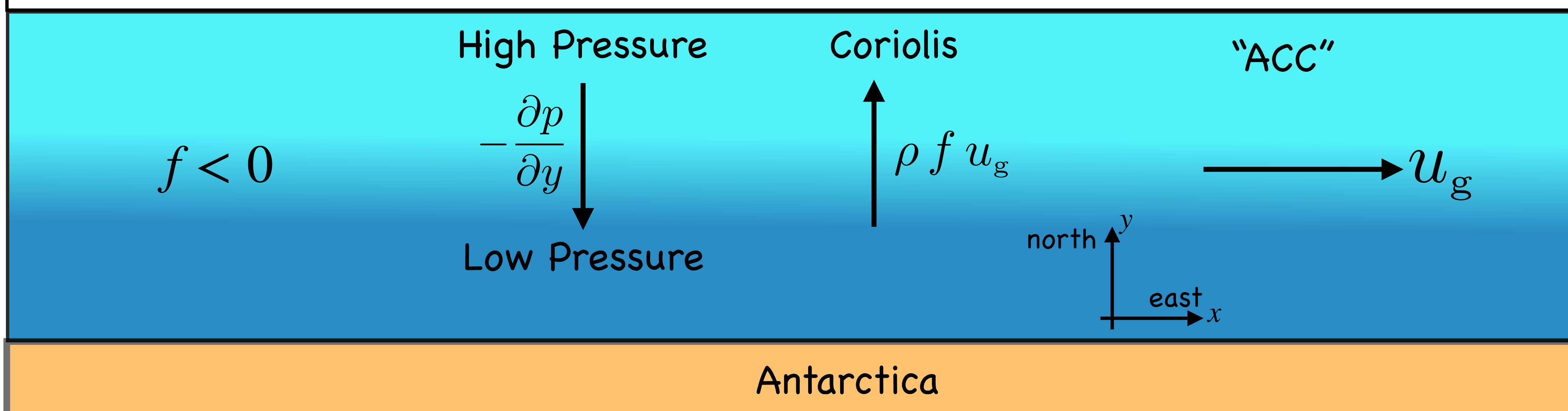
Geostrophic Balance

$$\rho\, f\, \hat{\boldsymbol{z}} \wedge \boldsymbol{u}_{\mathrm{g}} = -\nabla_z p$$ <=> Coriolis acceleration = pressure gradient

Note: buoyancy forcing modifies meridional pressure gradient to thus impact zonal geostrophic transport.

$$u_{\mathrm{g}} = -\frac{1}{f\,\rho}\frac{\partial p}{\partial y} \quad \text{and} \quad v_{\mathrm{g}} = \frac{1}{f\,\rho}\frac{\partial p}{\partial x}$$



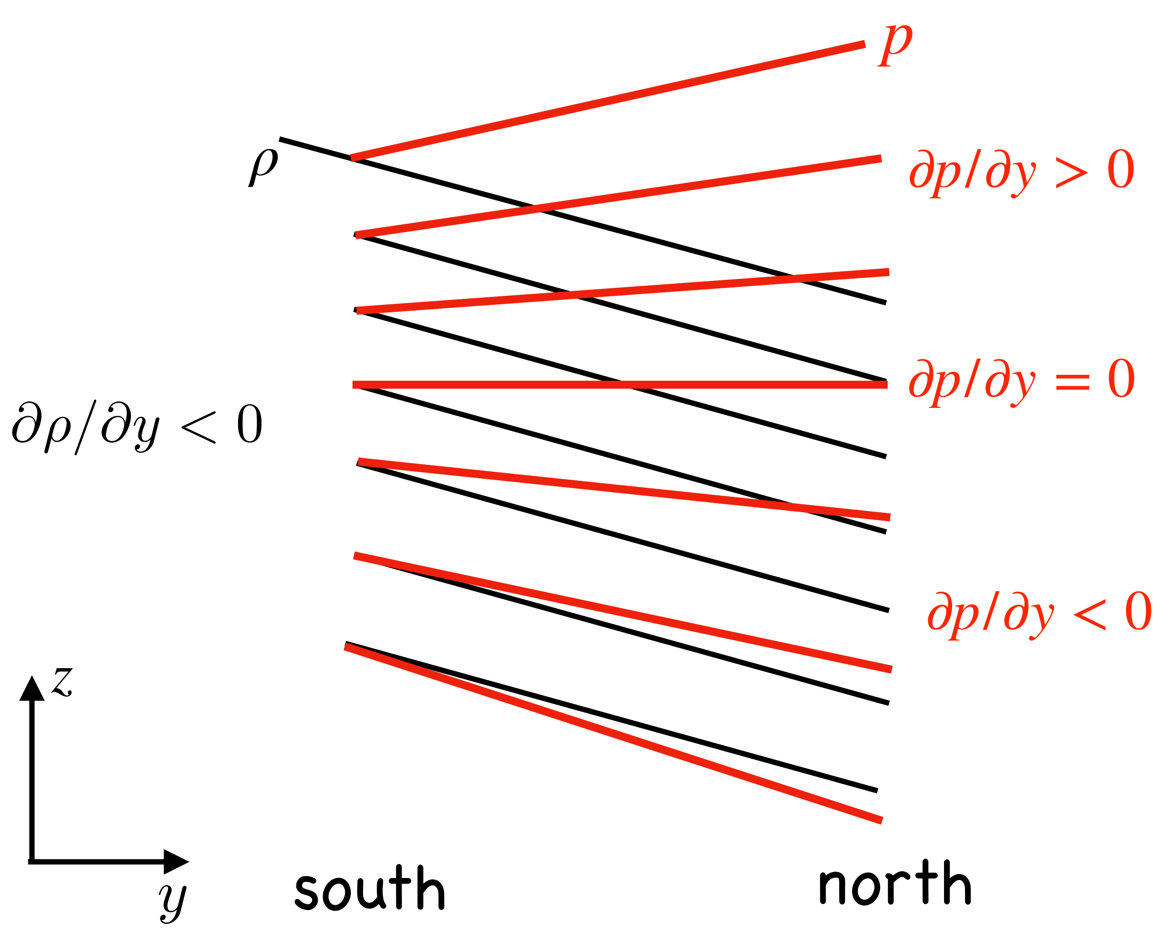


Thermal Wind = Hydrostatic + Geostrophic

Hydrostatic with horizontal density gradients

$$\frac{\partial p}{\partial z} = -\rho\, g \Longrightarrow \frac{\partial(\nabla_z p)}{\partial z} = -g\,\nabla_z \rho$$

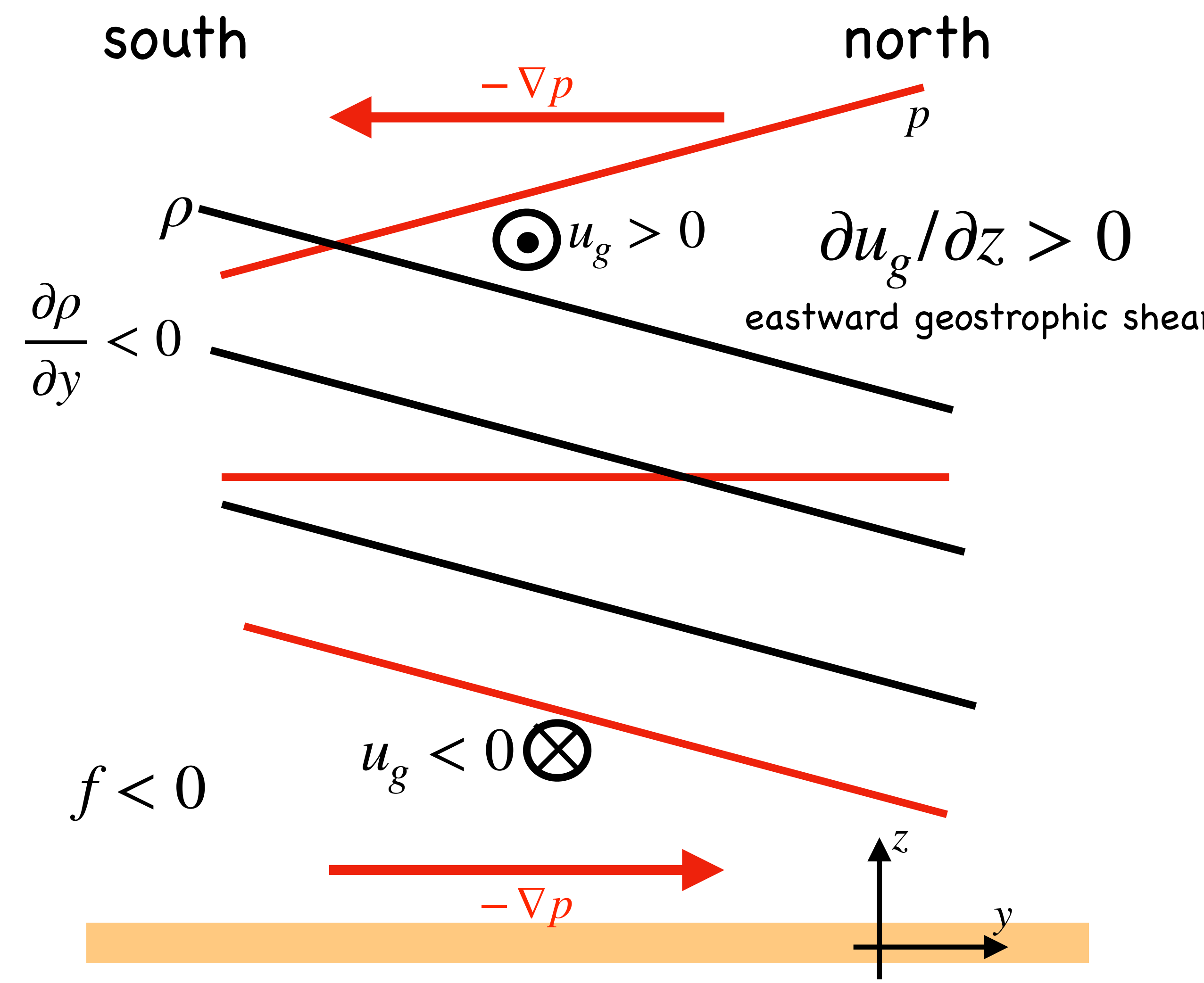
Horizontal density gradients induce a vertical shear to the horizontal pressure gradient.



Thermal wind = vertical shear in geostrophic velocity

$$f\,\frac{\partial(\rho\,\boldsymbol{u}_{g})}{\partial z} = \hat{\boldsymbol{z}} \wedge \nabla(\partial p/\partial z) = -g\,\hat{\boldsymbol{z}} \wedge \nabla\rho$$

Horizontal density gradients => vertical shear in horizontal geostrophic velocity.

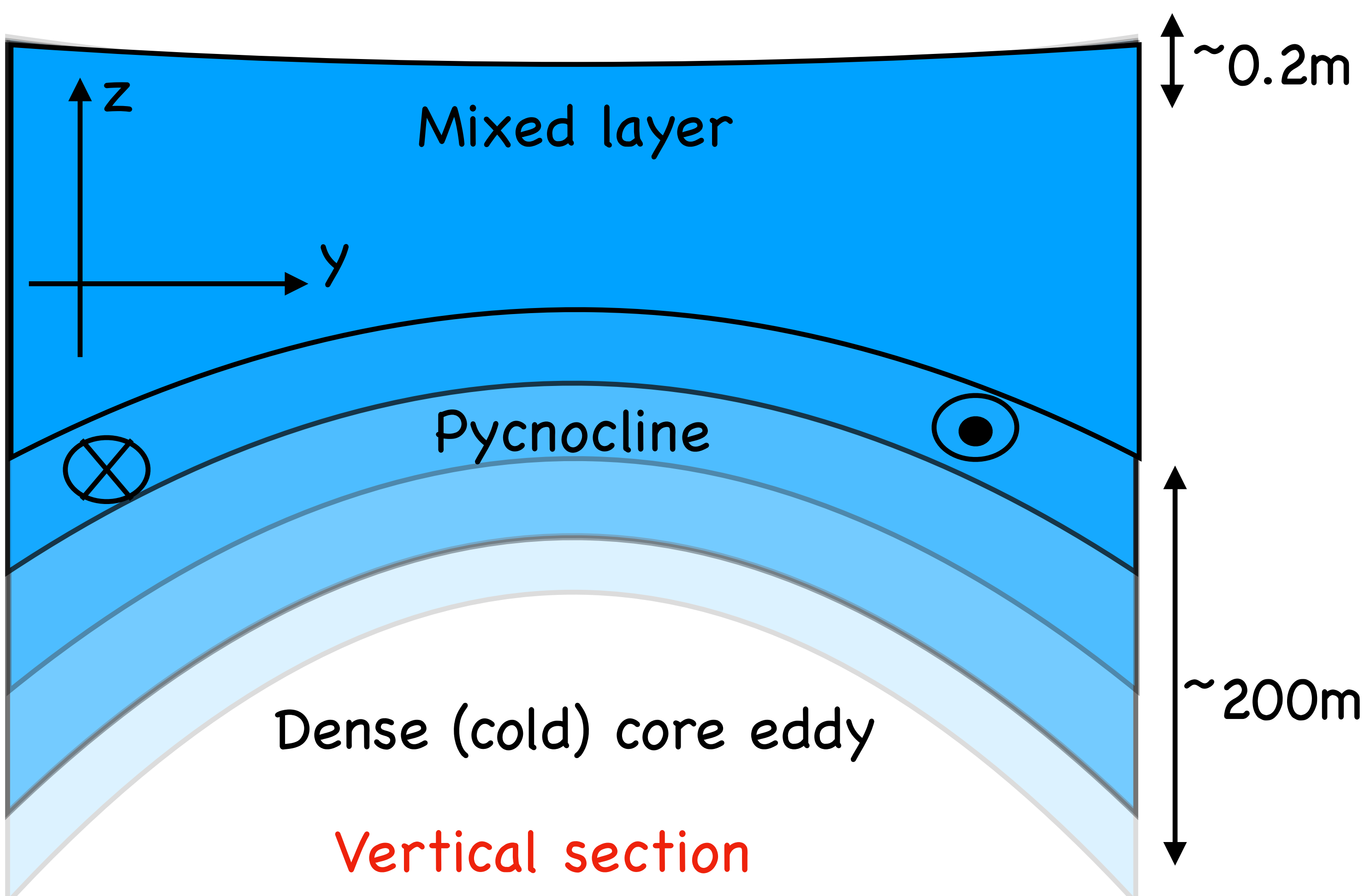


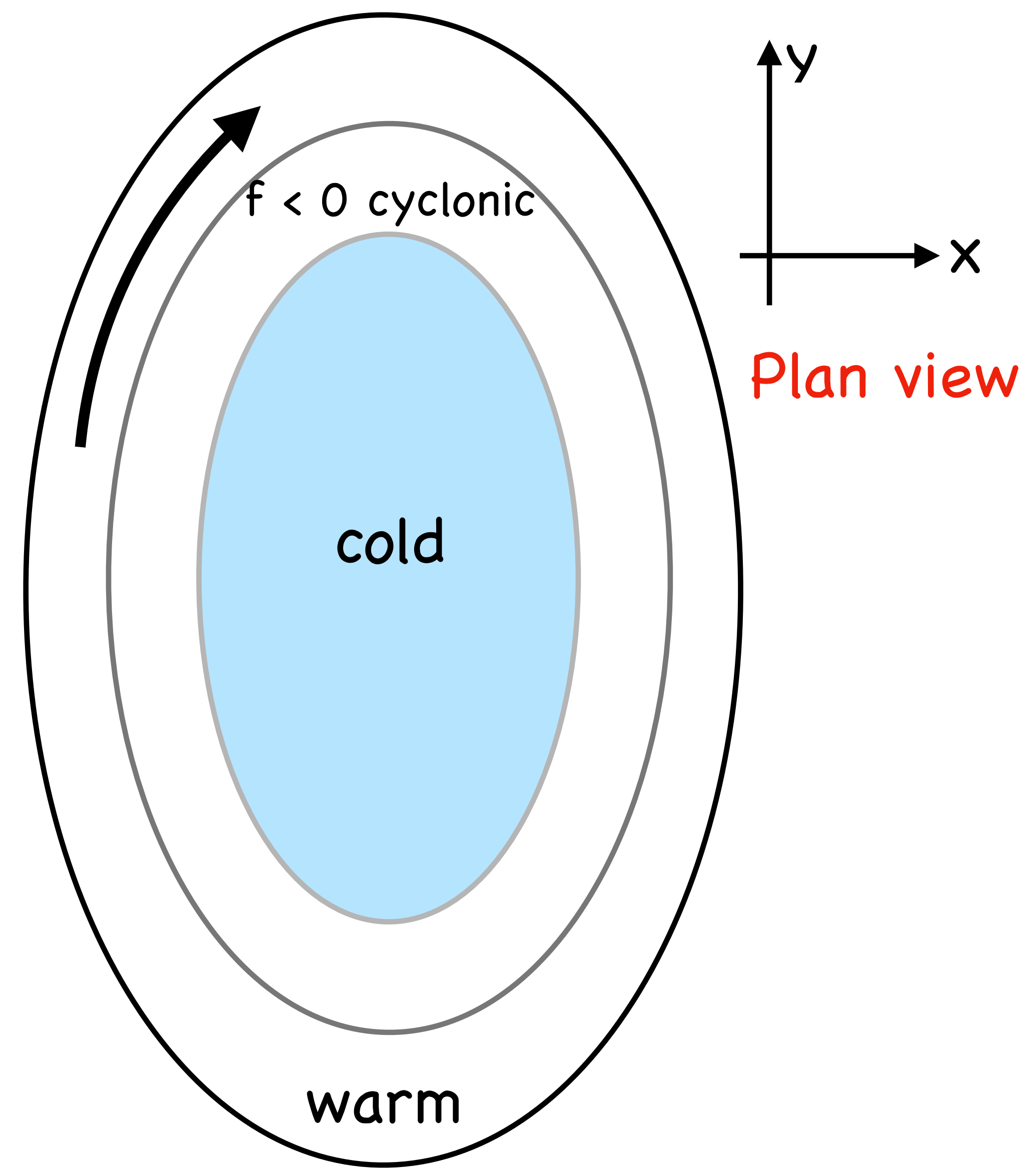
Baroclinic pressure gradient is specified by the density structure. However, density does not provide information about the depth-independent pressure gradient (barotropic) nor the corresponding depth-independent velocity field. Measuring the ACC barotropic velocity field is tough and was only recently determined by [Donohue et al (2016)](#)

Orienting motion in S.O. geostrophic eddies

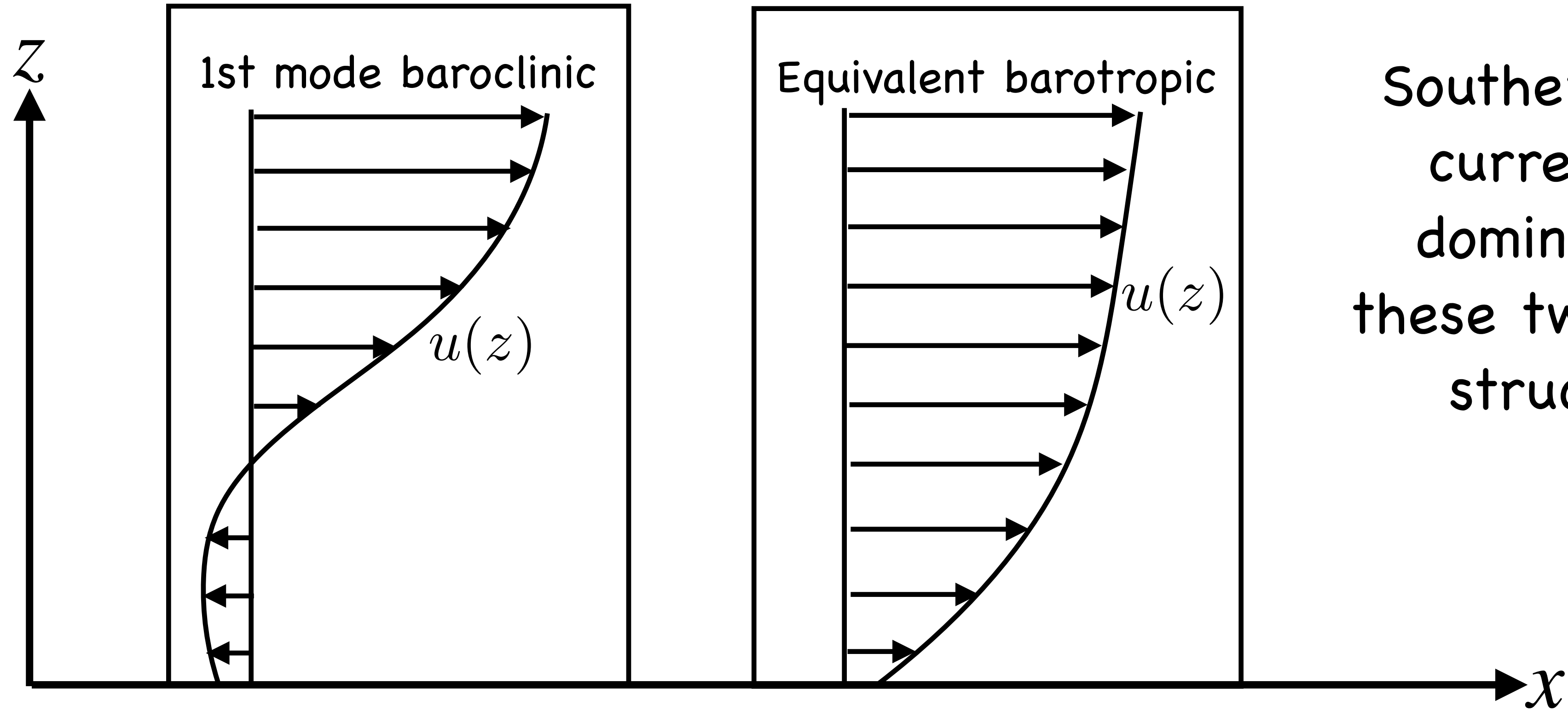
Dense (cold) core eddy with upward bowing isopycnals and associated depression in sea level (ratio roughly 100 to 1). Geostrophic balance in Southern Hemisphere means the thermal wind flow is clockwise, assuming a level of no-motion at base of the eddy.

Plan view through a dense cyclonic eddy with clockwise motion as per geostrophic balance in the southern hemisphere.

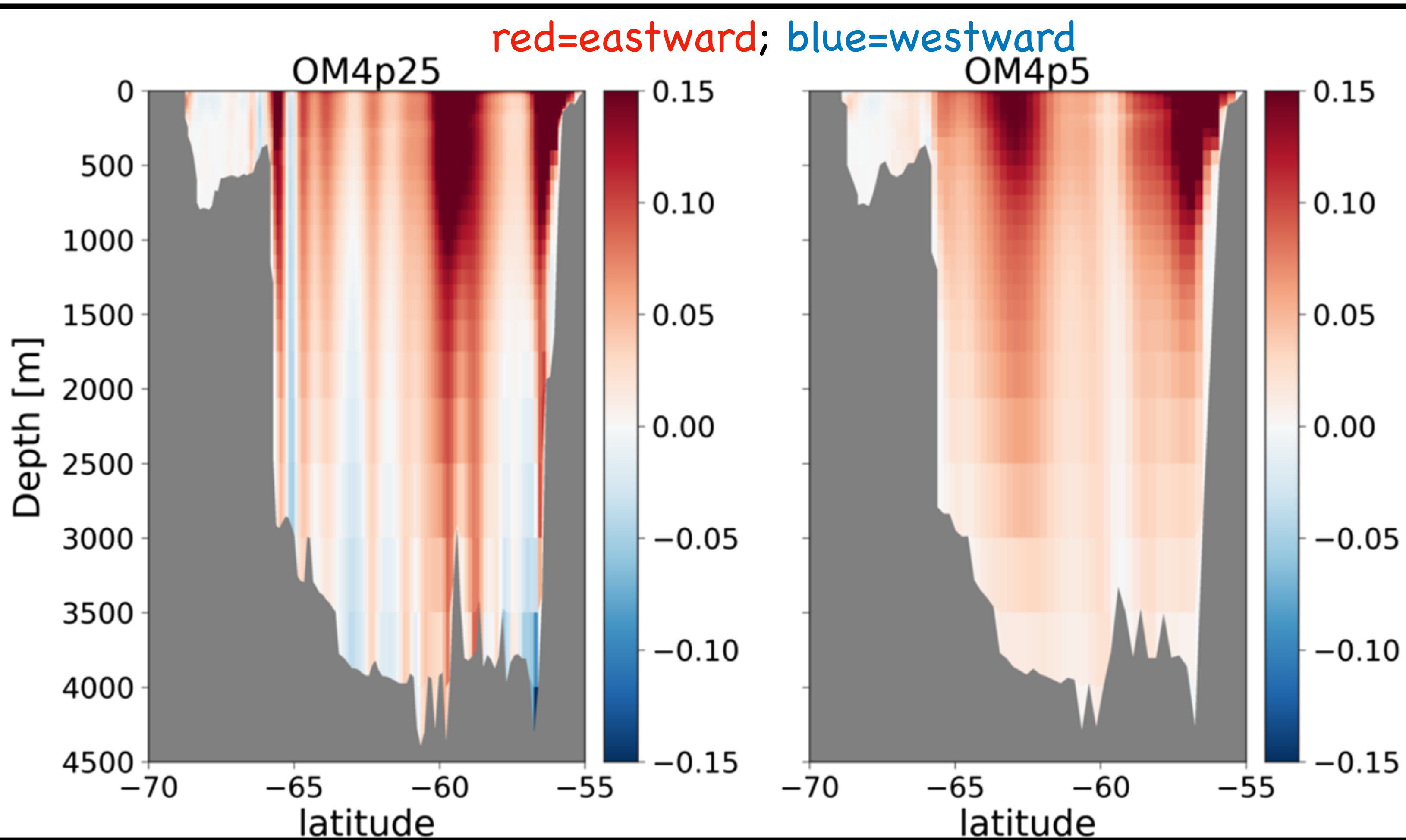




Vertical structure of (mostly) geostrophic currents



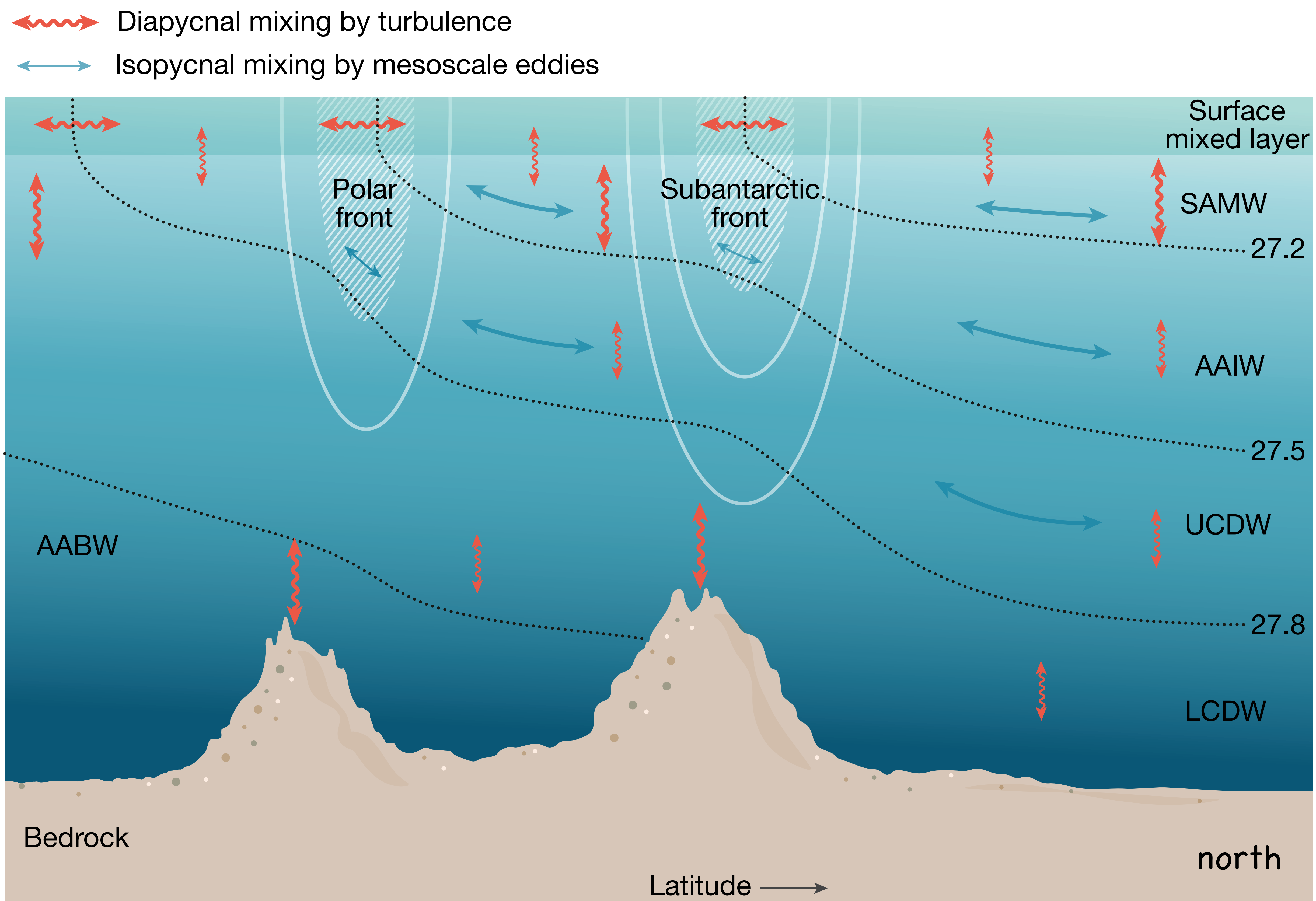
Southern Ocean currents are dominated by these two vertical structures.



Zonal velocity along the Drake Passage from GFDL models with 1/4-degree & 1/2-degree grids.

Note the enhanced jet-like features at 1/4-degree as well as the presence of a baroclinic mode-1 structure rather than the mostly equivalent barotropic in OM4p5

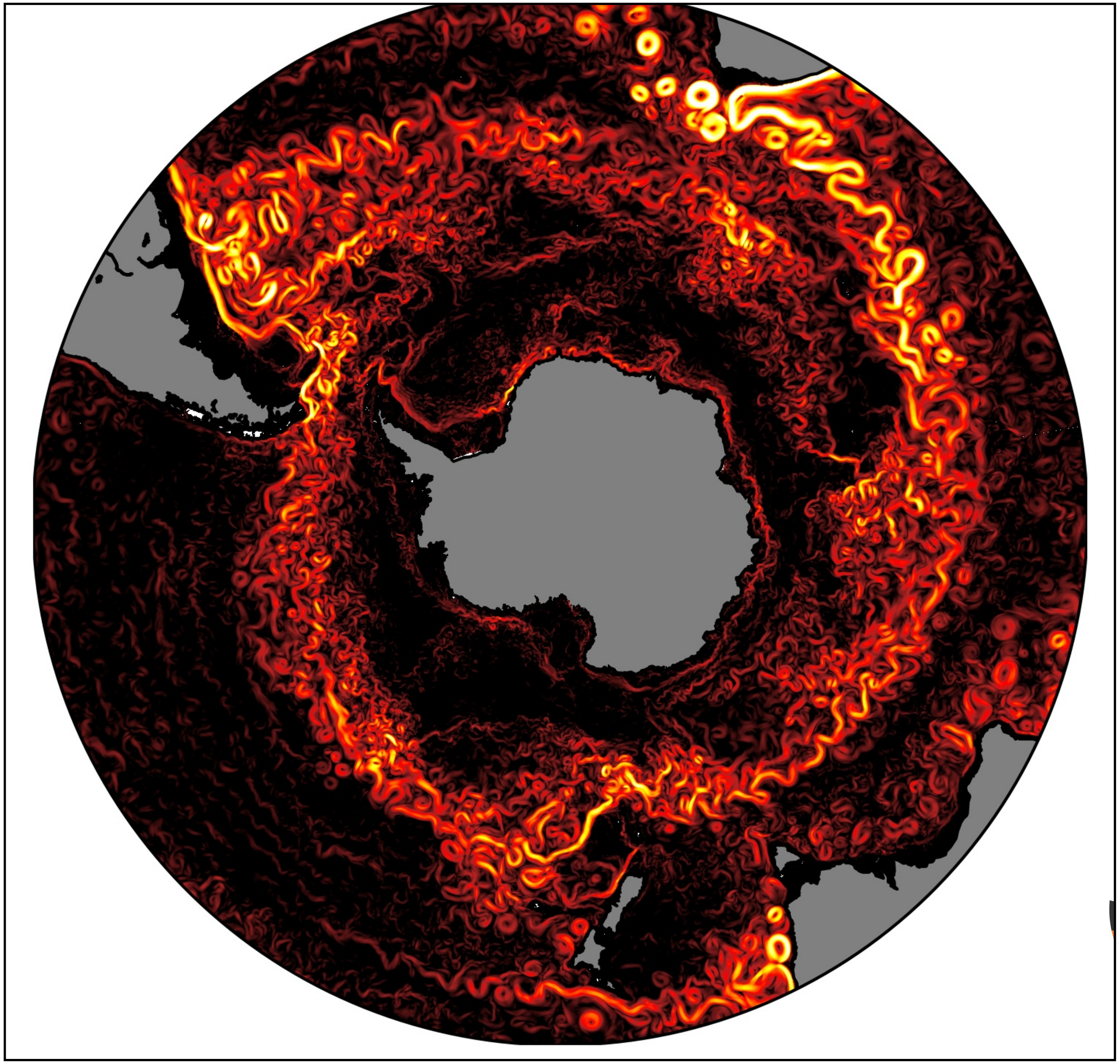
Thermal wind balanced Southern Ocean currents



Rintoul (2018)

A role for geostrophic eddies in the ACC momentum budget

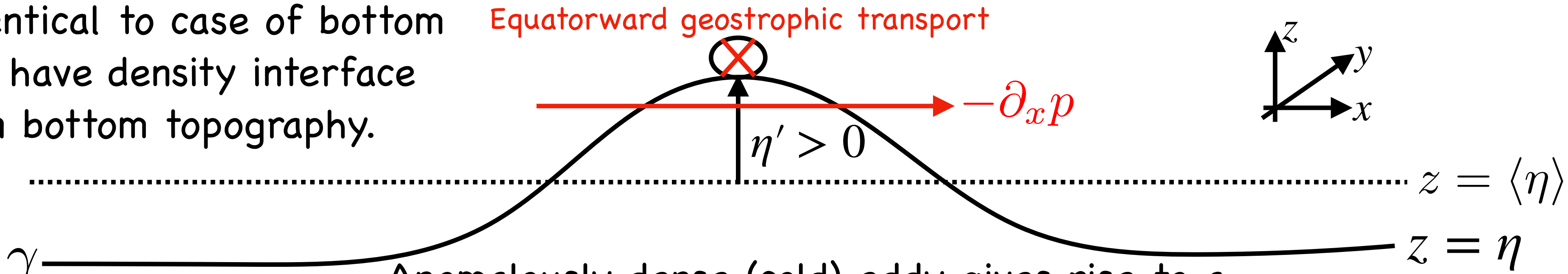
Geostrophic eddies and fronts in a simulation



GFDL CM2.6 simulation of daily mean surface speed.
Courtesy of Adele Morrison, ANU Canberra

Poleward transport of buoyancy by geostrophic eddies

Geometry and flow are identical to case of bottom form stress, only now we have density interface undulations rather than bottom topography.



Anomalously dense (cold) eddy gives rise to a positive elevation relative to the zonal mean.

For geostrophic eddies, the zonal mean zonal form stress is proportional to the meridional flux of thickness:

Within each layer
$\langle v \rangle = 0$ continuity and ignore P-E+R

$$\langle \tau^x_{\rm form} \rangle = \langle p\, \partial_x \eta \rangle = -\langle \eta\, \partial_x p \rangle = -f\, \rho_0\, \langle \eta\, v \rangle = -f\, \rho_0\, \langle \eta'\, v' \rangle = (f\, \rho_0 / N^2)\, \langle b'\, v' \rangle$$

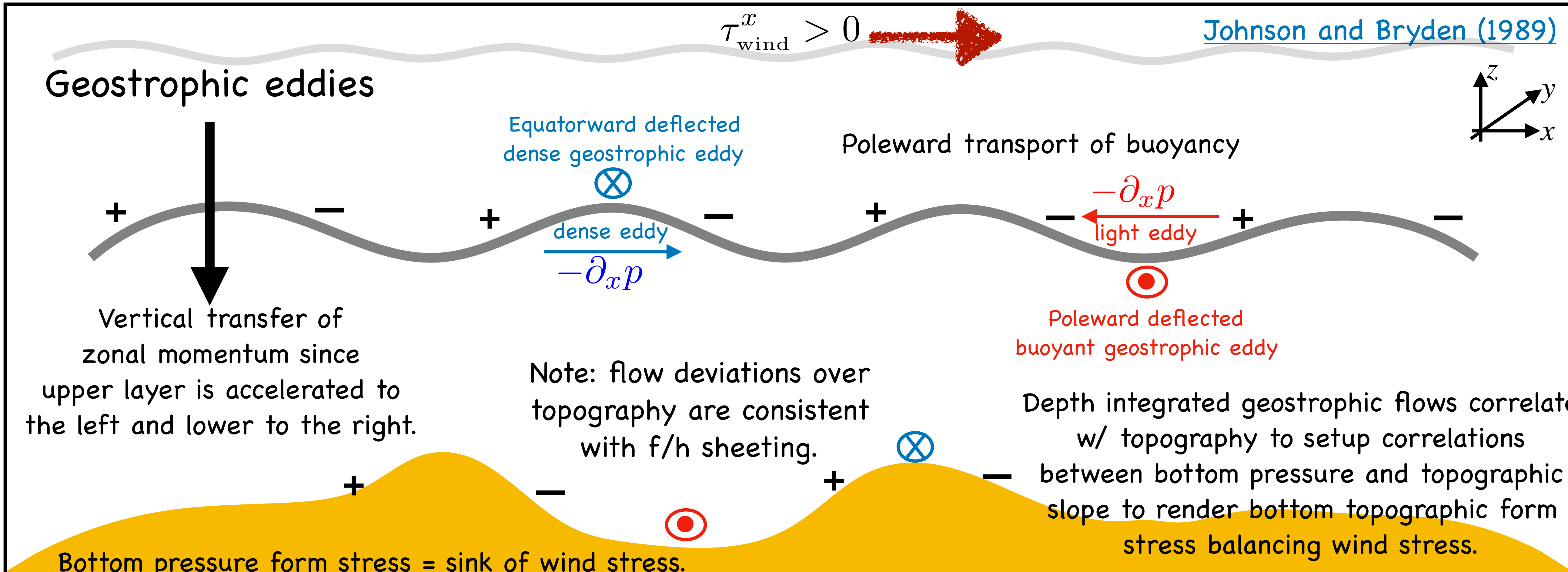
$\langle \tau^x_{\rm form} \rangle > 0$ **<=>** equatorward eddy transport of dense (cold) anomalies & conversely a poleward transport of light (warm) anomalies. Hence, the eddies transport buoyancy poleward thus reducing the equator-to-pole temperature gradient.

As in the atmosphere, ocean geostrophic (mesoscale) eddies transport buoyancy (heat) poleward. Hence, they play a key role in the earth's climate.

In contrast, time mean currents provide only a small net poleward heat transport in the Southern Ocean (deSzoeke and Levine 1981). This feature is largely due to the lack of net zonal pressure gradients above topography, thus supporting no zonal mean meridional geostrophic flow.

These characteristics greatly contrast to the mid-latitude gyres, where meridional boundaries support zonal pressure gradients that drive mean meridional flows that move heat poleward, especially in western boundaries.

Geostrophic eddies transfer momentum vertically + buoyancy poleward



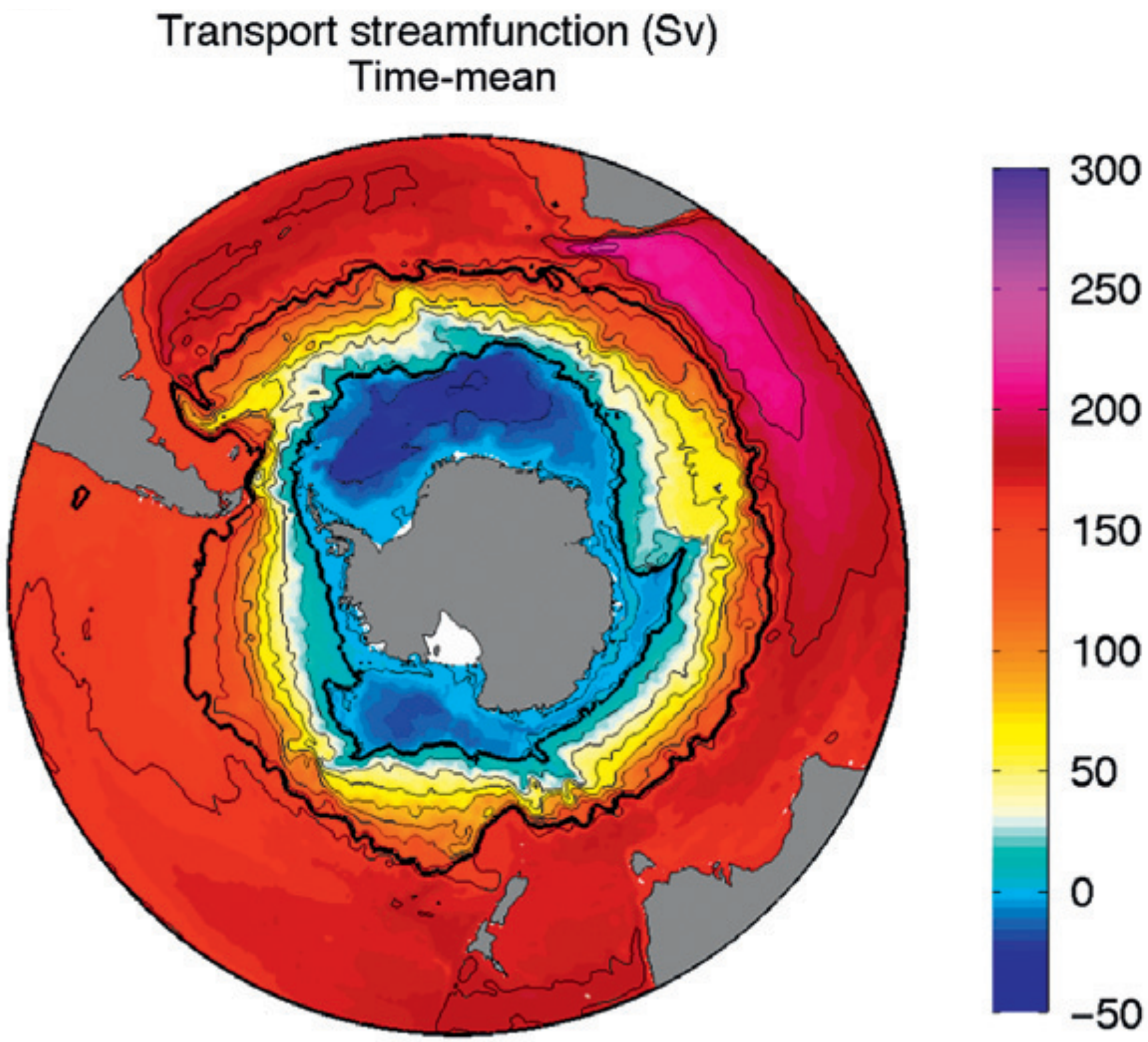
Geostrophic mesoscale eddies provide a physical mechanism for transferring zonal momentum imparted by the surface wind stress down to the bottom where it is balanced/counteracted by topographic form stress.

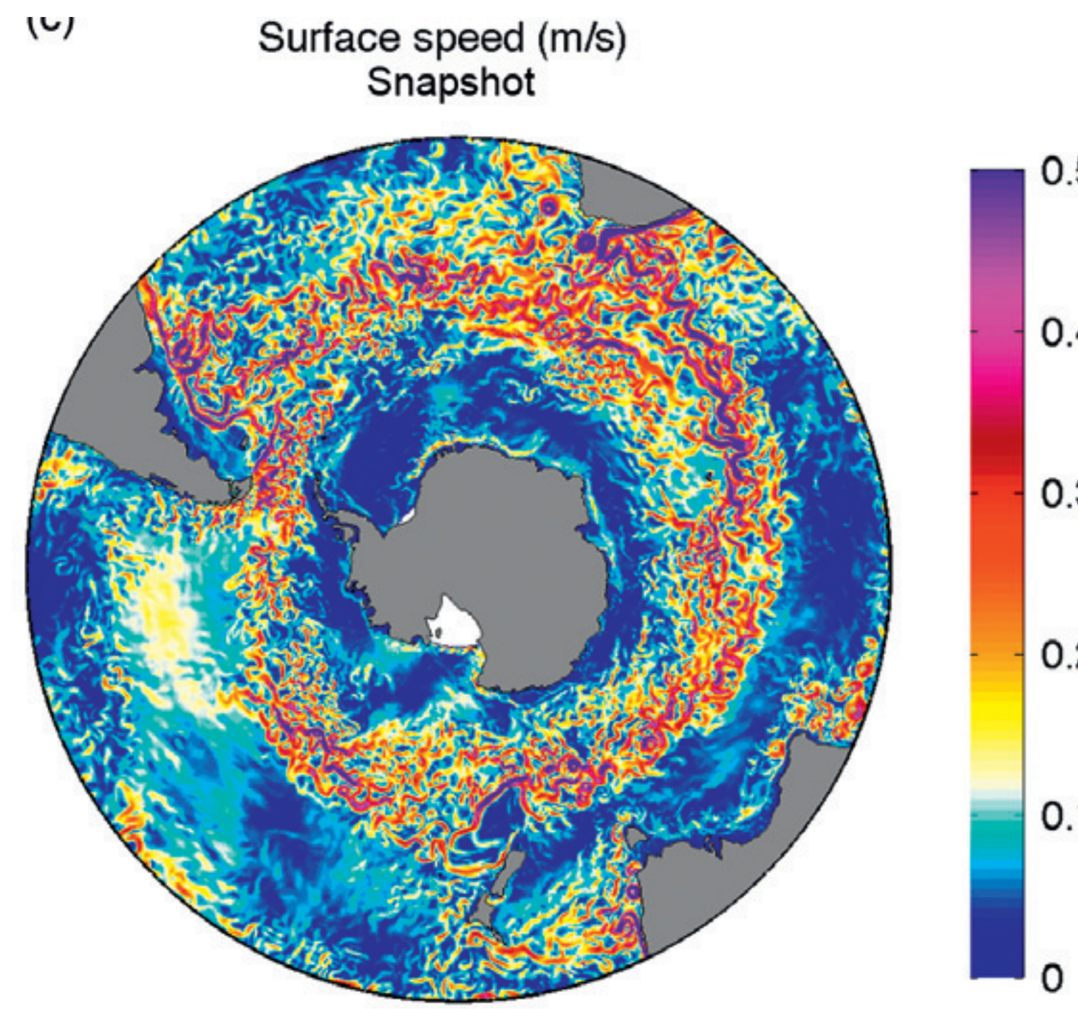
The mesoscale eddies do so through "interfacial form stress", which is the horizontal pressure stress imparted to undulating isopycnal surfaces.

The mechanism for vertical momentum transport by interfacial form stress is geometrically identical to the mechanism for topographic form stress.

Topographically induced meanders are much larger (~100s of km) than the mesoscale eddy scales (deformation radius ~20km). This situation contrasts to that in the atmospheric storm tracks where meanders and eddies are roughly same scale (Williams 2007).

Time mean depth integrated transport streamfunction





From SOSE (Mazloff et al 2010) as reported by Rintoul and Naveira Garabato (2013)

Left: Time mean streamfunction for the depth integrated flow. Right: snapshot of surface speed. Note the large scales (100-1000 km) for the meanders in the left panel, with largely follow f/H contours (H=large-scale topography), versus the smaller scales (10-100km) of the mesoscale eddy features in right panel.

Some of the controversy surrounding form stress

OCTOBER 1996 NOTES AND CORRESPONDENCE 2297

On the Obscurantist Physics of "Form Drag" in Theorizing about the Circumpolar Current*

BRUCE A. WARREN

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PAUL E. ROBBINS

Department of Physical Oceanography, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

JANUARY 1997 NOTES AND CORRESPONDENCE 209

Comments on "On the Obscurantist Physics of 'Form Drag' in Theorizing about the Circumpolar Current"

C. W. HUGHES

Proudman Oceanographic Laboratory, Birkenhead, Merseyside, United Kingdom

AUGUST 1998 NOTES AND CORRESPONDENCE 1647

Comments on "On the Obscurantist Physics of 'Form Drag' in Theorizing about the Circumpolar Current*

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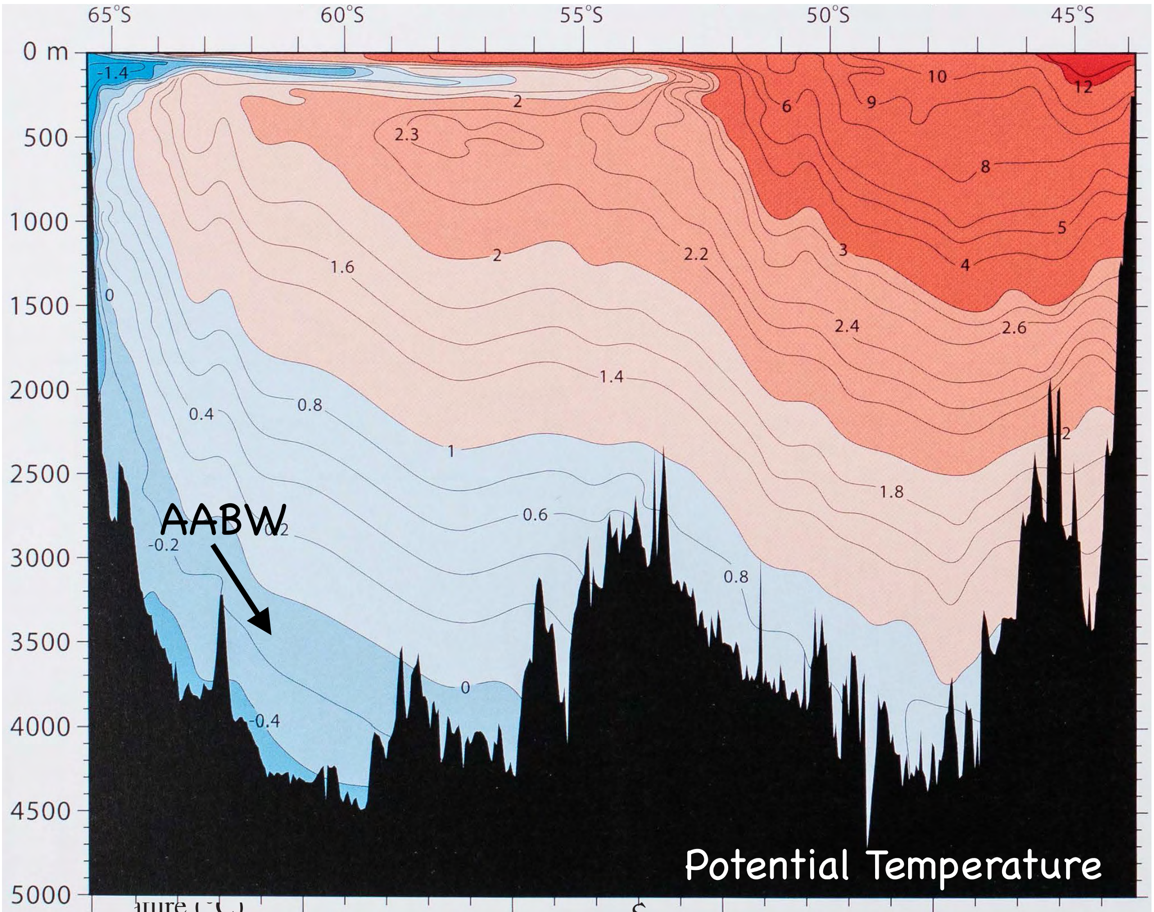
Key points from ACC zonal momentum balance

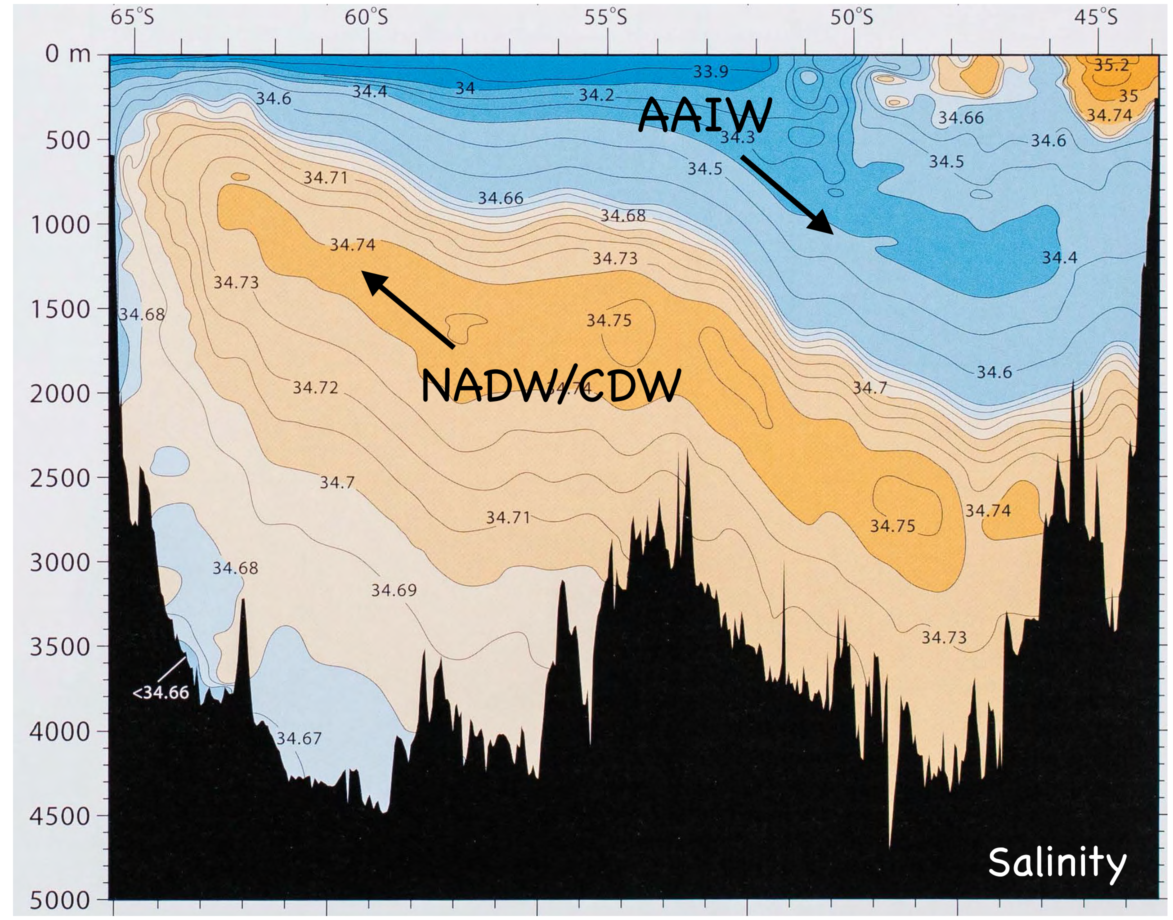
1. Zonally re-entrant geometry makes zonal momentum balance a key foundation for understanding the ACC.
   - This character contrasts with gyre circulations that rely on meridional boundaries w/ friction on the western boundary. Correspondingly, the vorticity balance (i.e., Sverdrup balance) is effective for understanding gyre dynamics.
2. Zonal momentum input by winds is, in a steady state, largely balanced by bottom pressure form stress.
   - There is little direct role for friction, either molecular/Reynolds stresses or bottom drag.
   - Large-scale geostrophic flow organizes itself so that bottom pressure perturbations are anti-correlated with bottom slope.
   - Bottom pressure form stress associate with geostrophic flow is dominated by large-scale topographic features (Scotia Arc, Kerguelen Plateau, Campbell Plateau, Pacific Rise).
3. Interfacial form stress acts on isopycnal layers through undulations created by geostrophic eddies.
   - Interfacial form stress provides the physical mechanism for transferring surface wind stress through the water column to the bottom topography.
   - Geostrophic eddies have thickness perturbations correlated with velocity perturbations to transfer eastward zonal momentum from surface downward.
4. As they transfer zonal momentum vertically, geostrophic eddies also transport buoyant waters poleward (just like atmosphere eddies).
   - For geostrophic eddies, poleward heat transport and vertical momentum transport are two sides to the same coin.

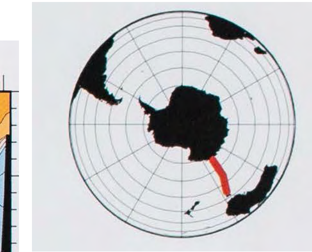
Southern Ocean meridional overturning circulation (SOMOC)

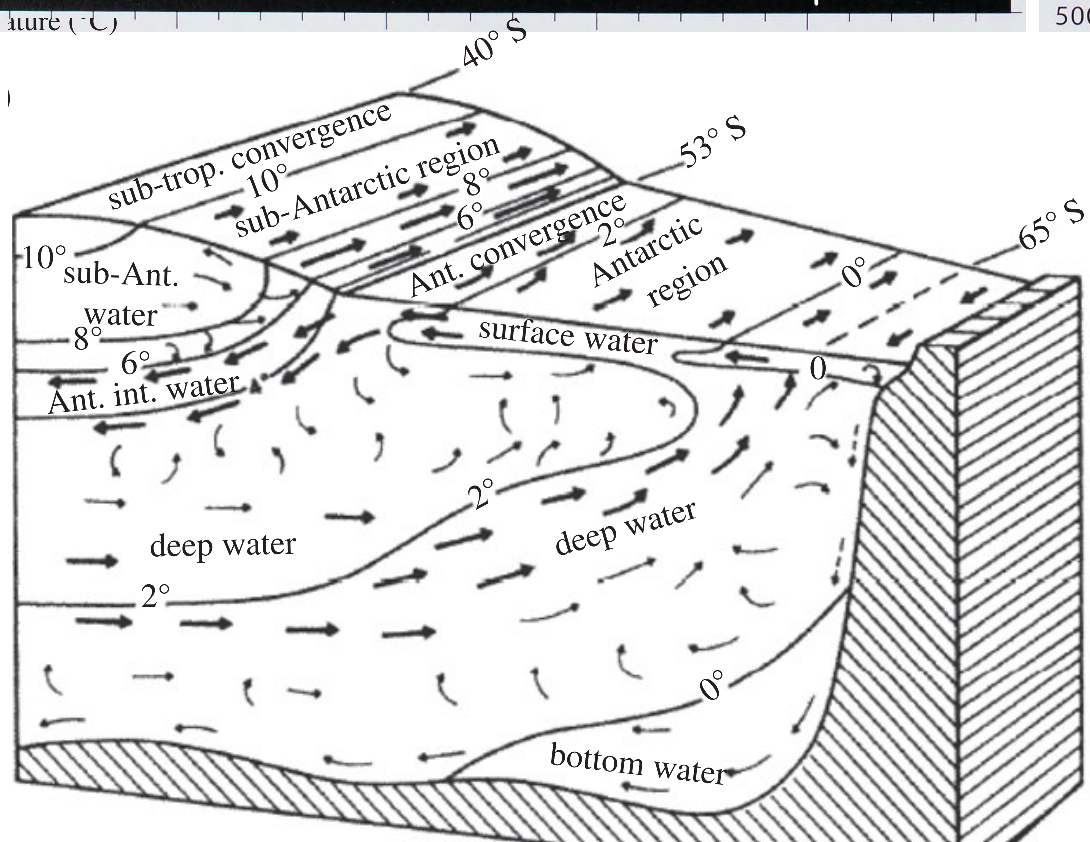
Overturning arises from watermass transformation by boundary buoyancy forcing + interior diapycnal mixing along with Ekman upwelling/downwelling.

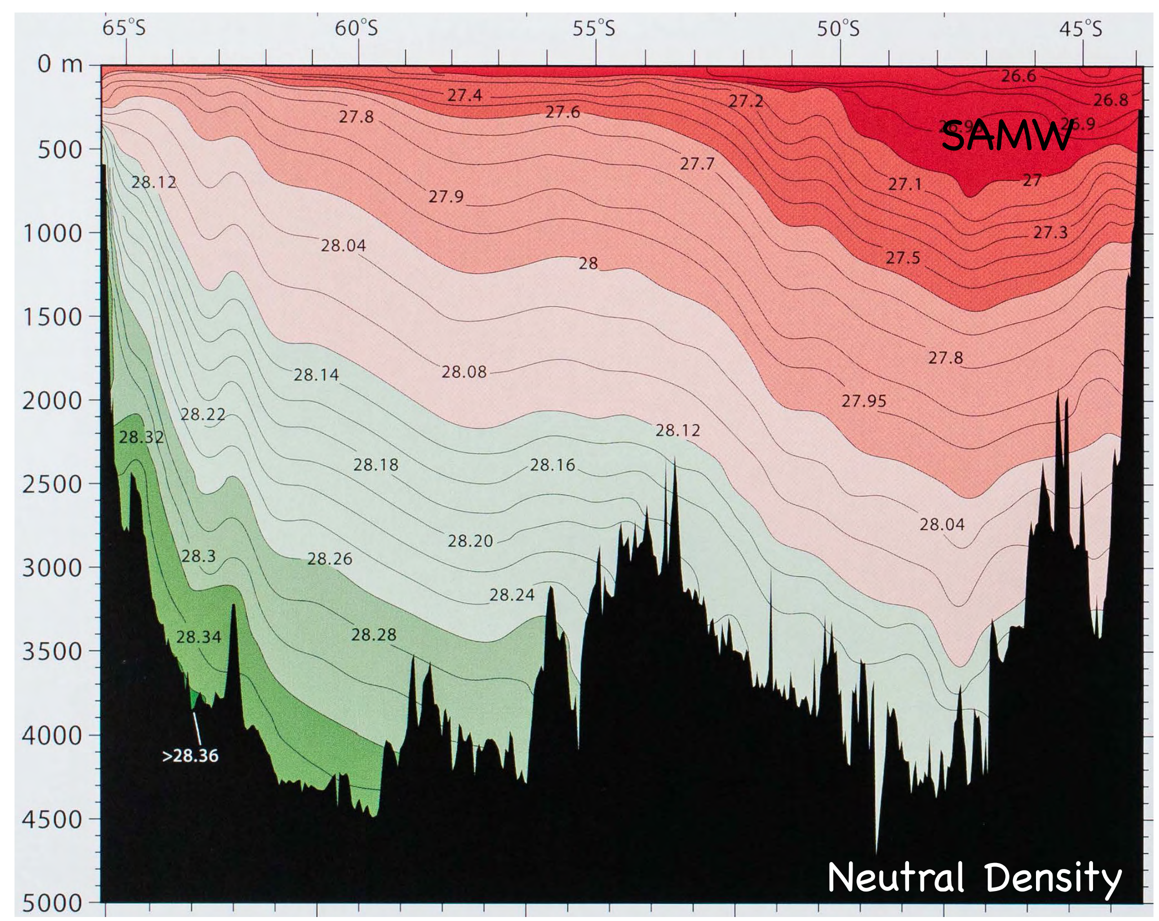
Inferring the overturning from WOCE Section S3





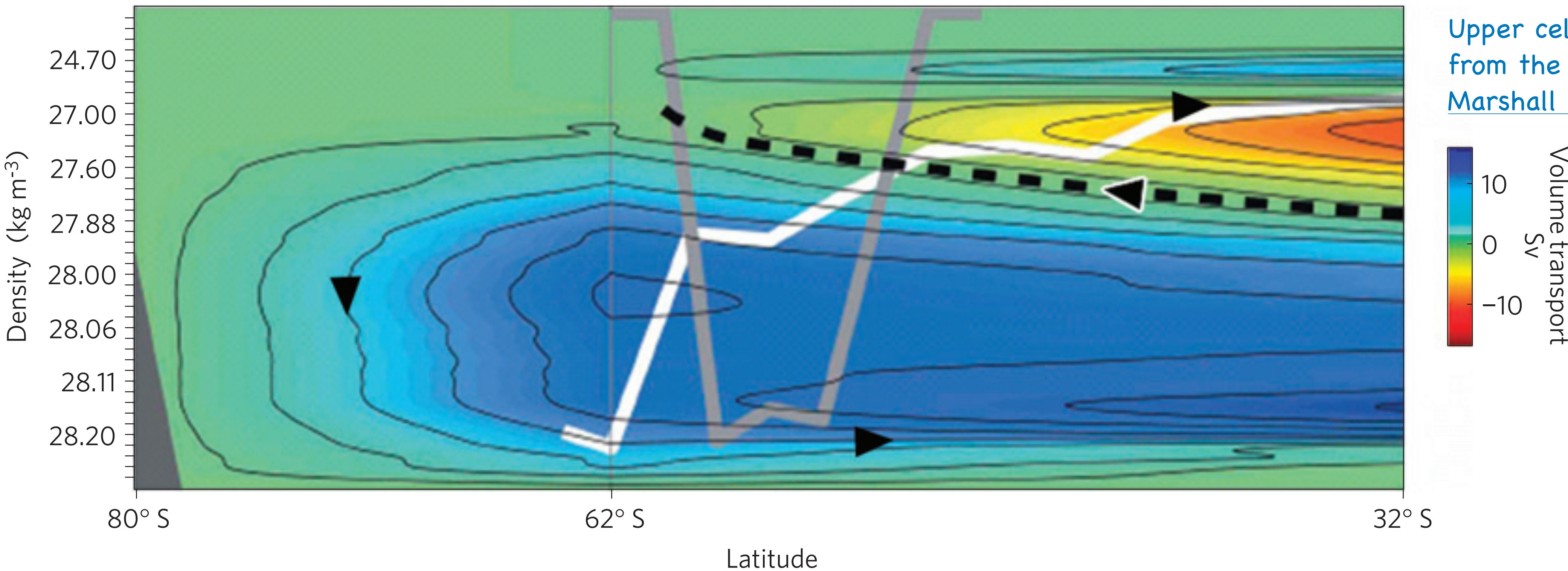




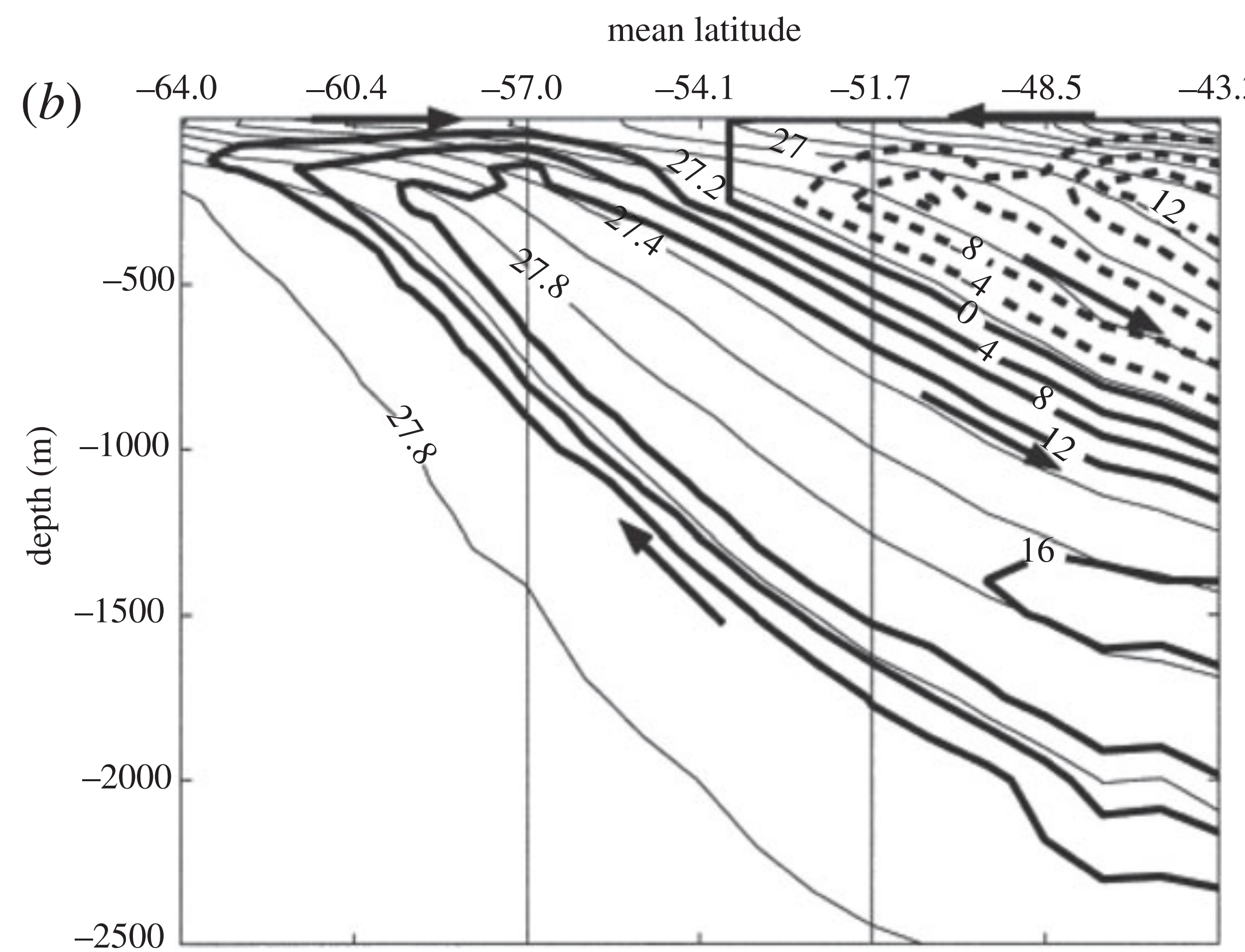


Sverdrup, Johnson, and Fleming (1942)
Inference based on watermass properties.

21st century inferences of the SOMOC



Upper cell of the SOMOC from the inversion of Marshall and Speer (2012)

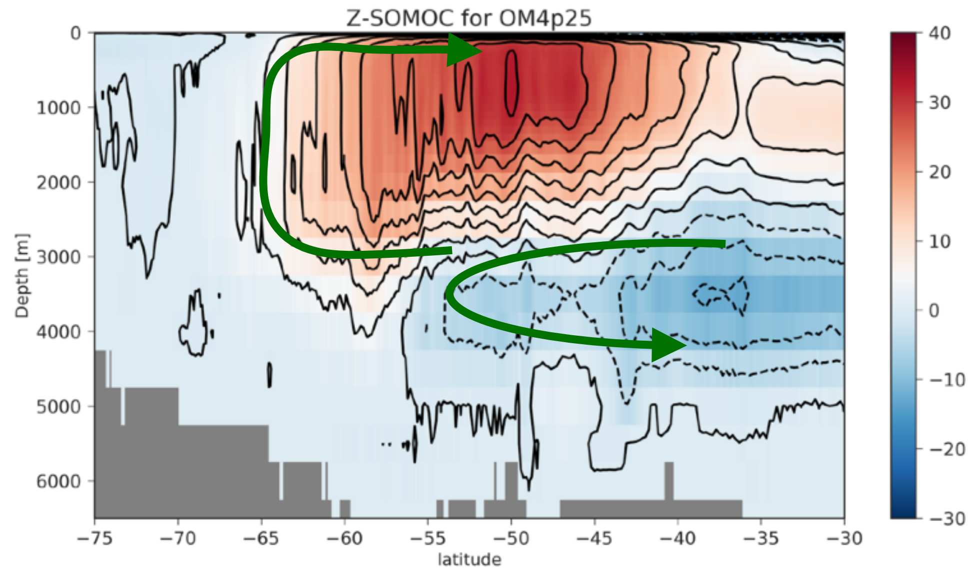


Karsten and Marshall (2002): Inference of "upper cells" based on residual mean theory + winds + surface buoyancy.

Note the two-cell circulation.

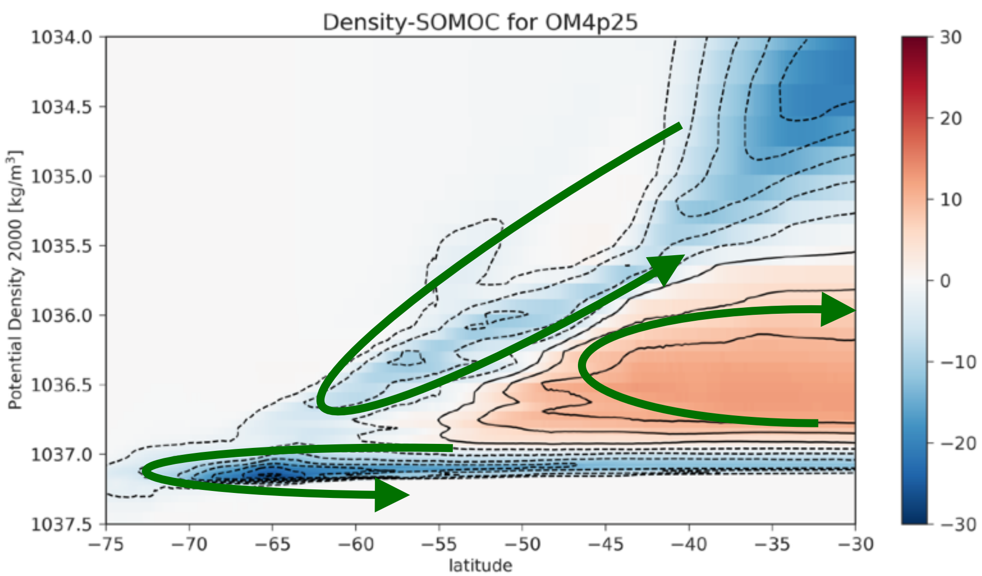
sigma-0

Three views of the SOMOC from GFDL OM4



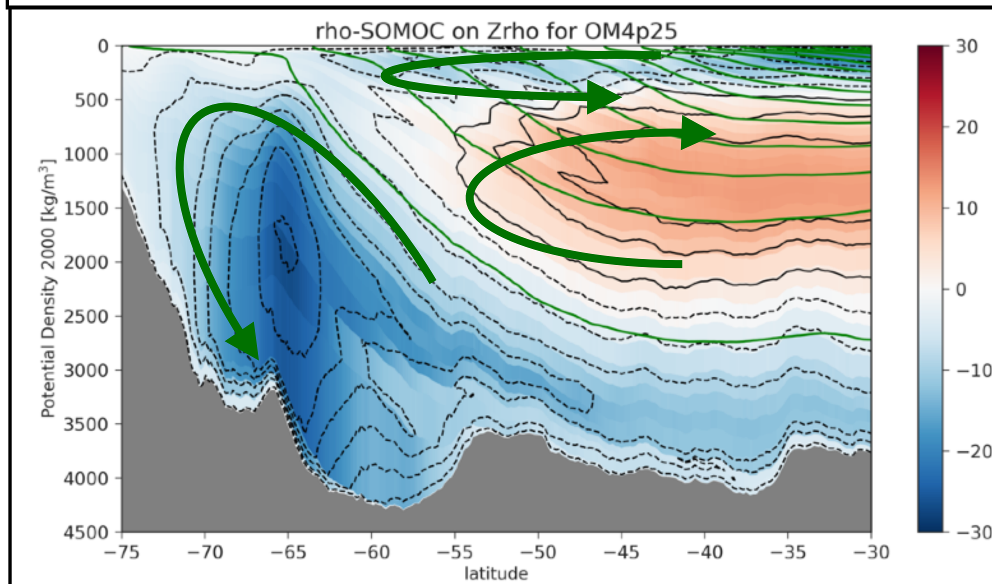
$$\Psi(y,z) = -\oint \mathrm{d}x \int_{-H}^{z} v(x,y,z')\,\mathrm{d}z'$$

Meridional-depth SOMOC



$$\Psi(y,\gamma) = -\oint \mathrm{d}x \int_{-H}^{z_\gamma} v(x,y,z')\,\mathrm{d}z'$$

Meridional-density SOMOC

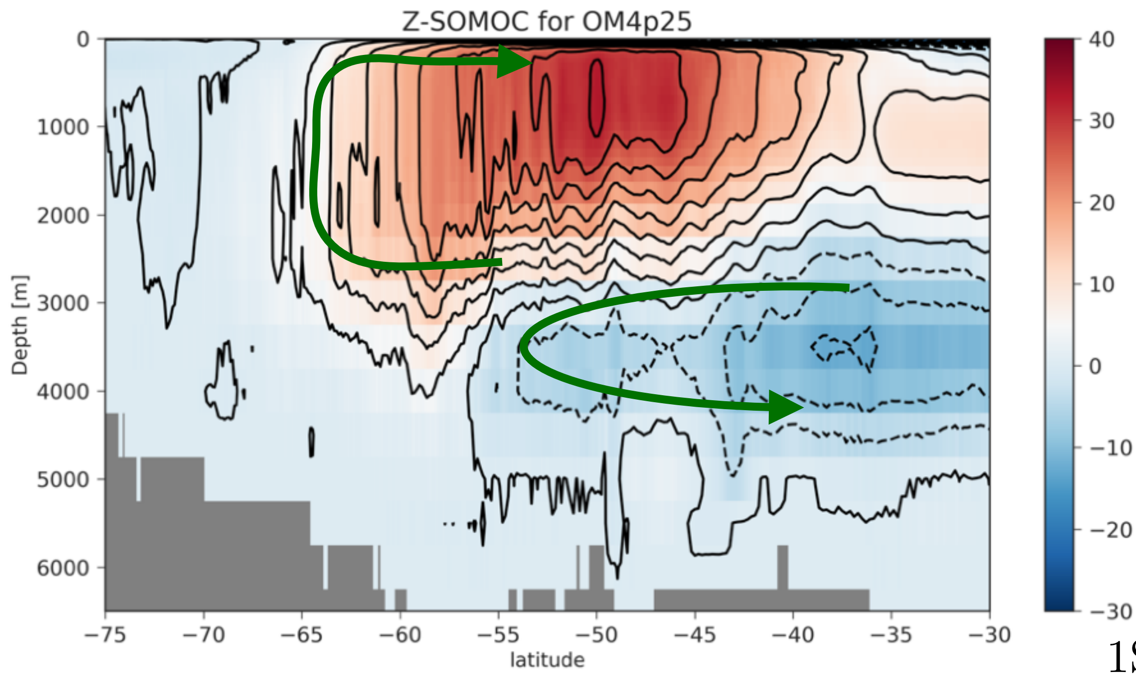


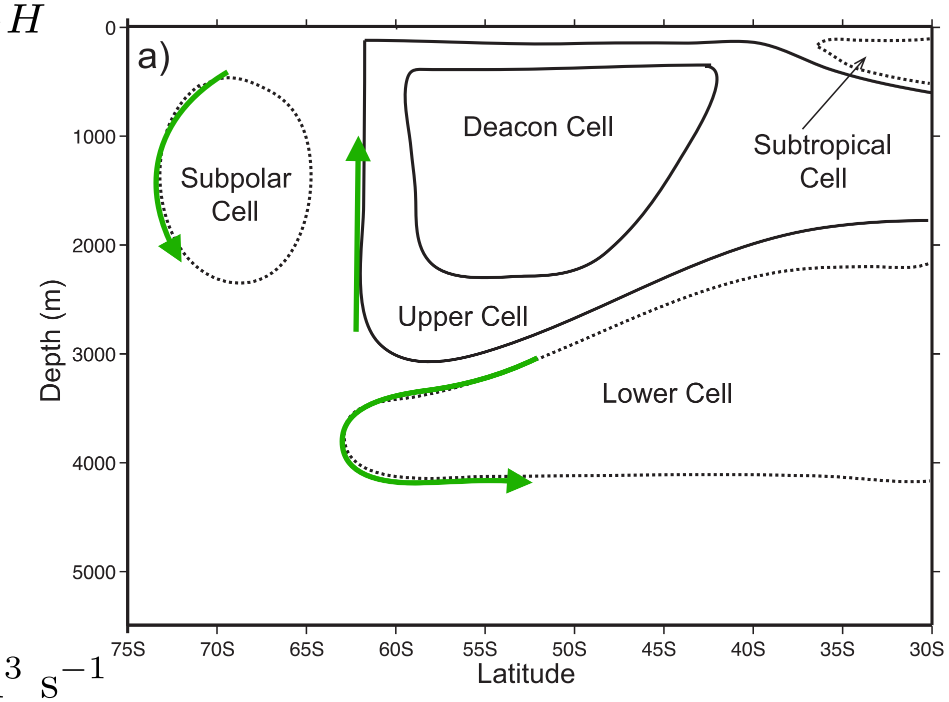
$$\Psi(y,\gamma) \Longrightarrow \Psi(y,\langle z_\gamma\rangle)$$

Meridional-density SOMOC mapped to zonal mean depth of the density surface.

SOMOC Taxonomy

$$\Psi(y,z) = -\oint \mathrm{d}x \int_{-H}^{z} v(x,y,z')\,\mathrm{d}z'$$



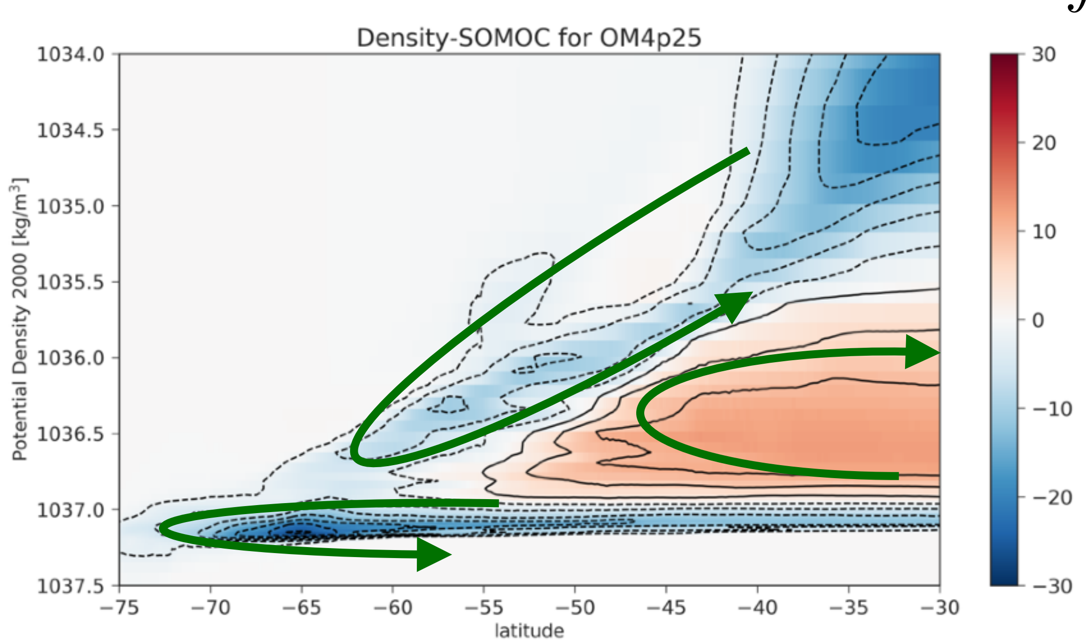


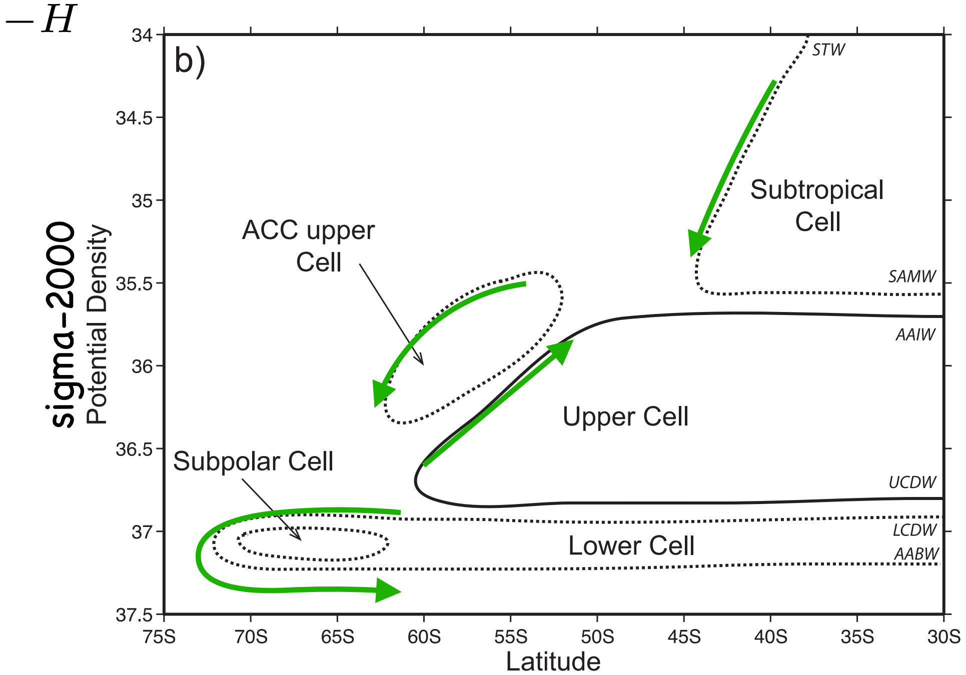
$1\mathrm{Sv} = 10^6\ \mathrm{m}^3\ \mathrm{s}^{-1}$

Model circulation from Adcroft et al, (2019)

Schematics from Farneti et al, (2015)

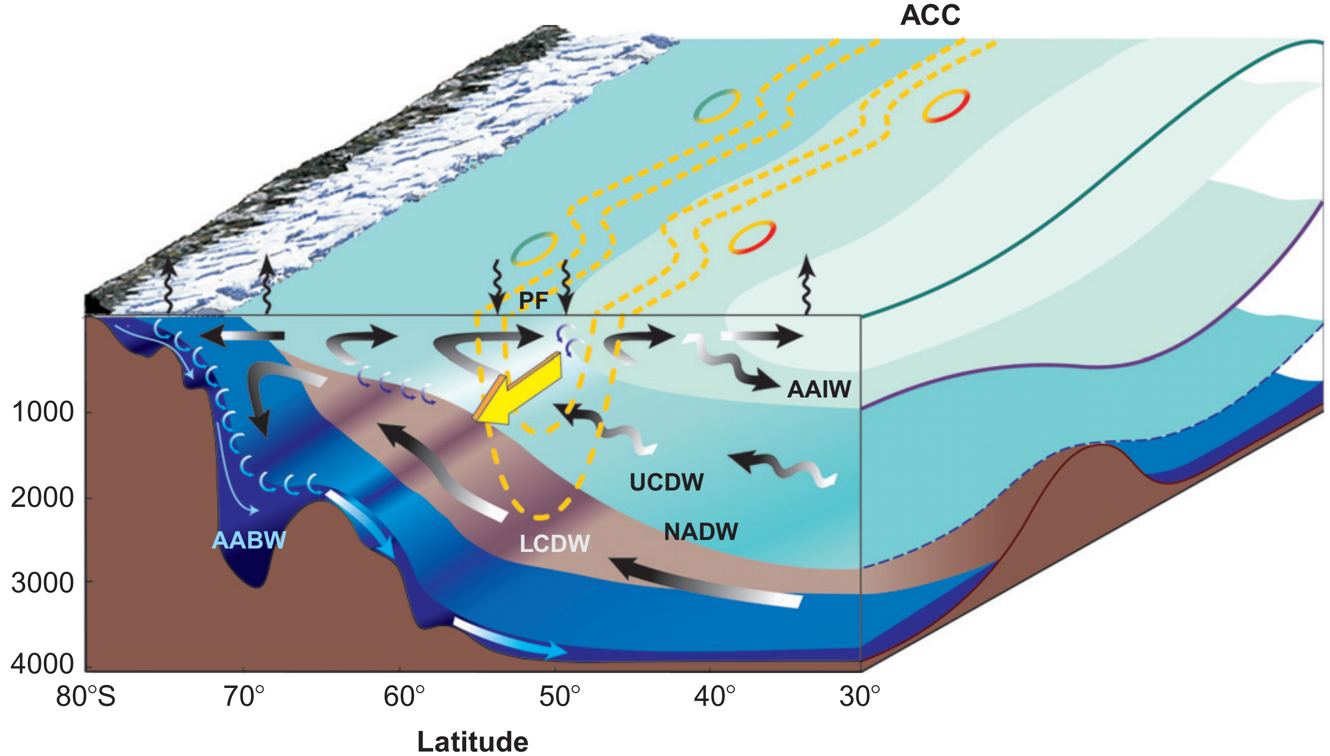
$$\Psi(y,\gamma) = -\oint \mathrm{d}x \int_{-H}^{z_\gamma} v(x,y,z')\,\mathrm{d}z'$$

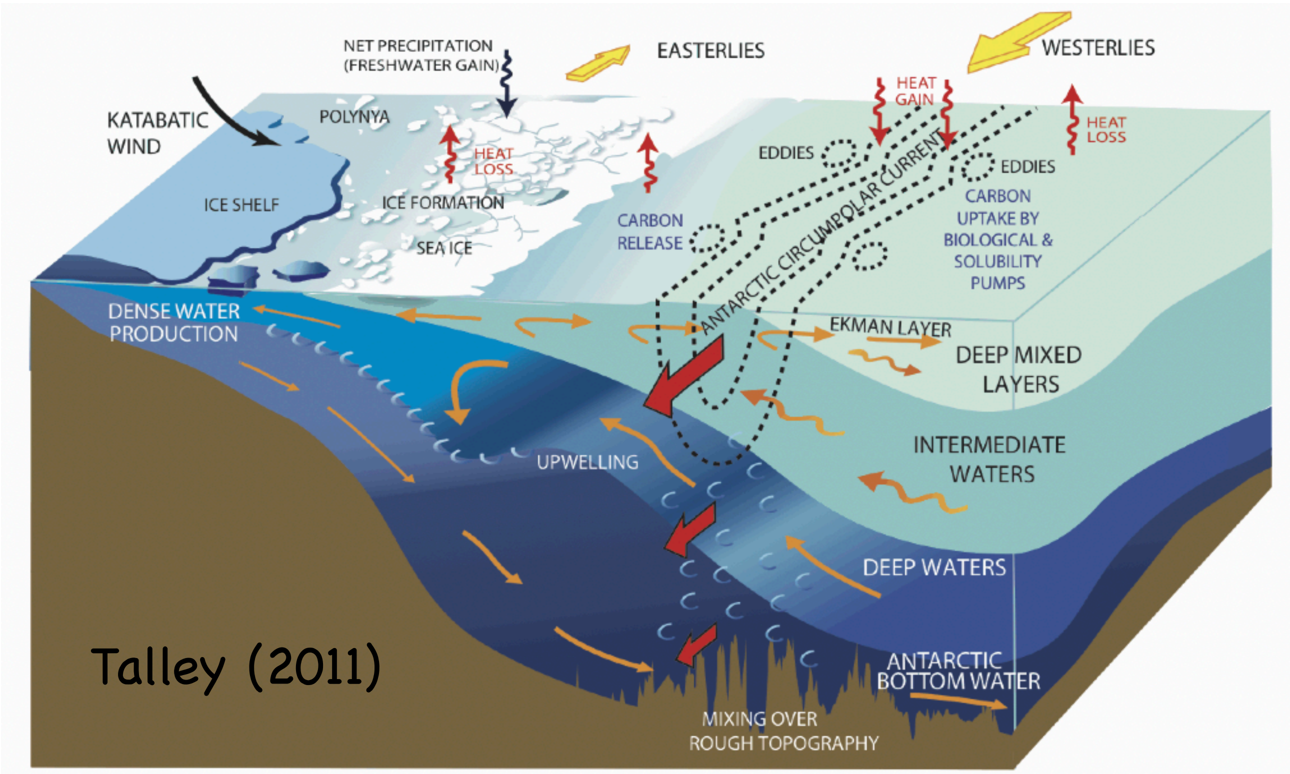


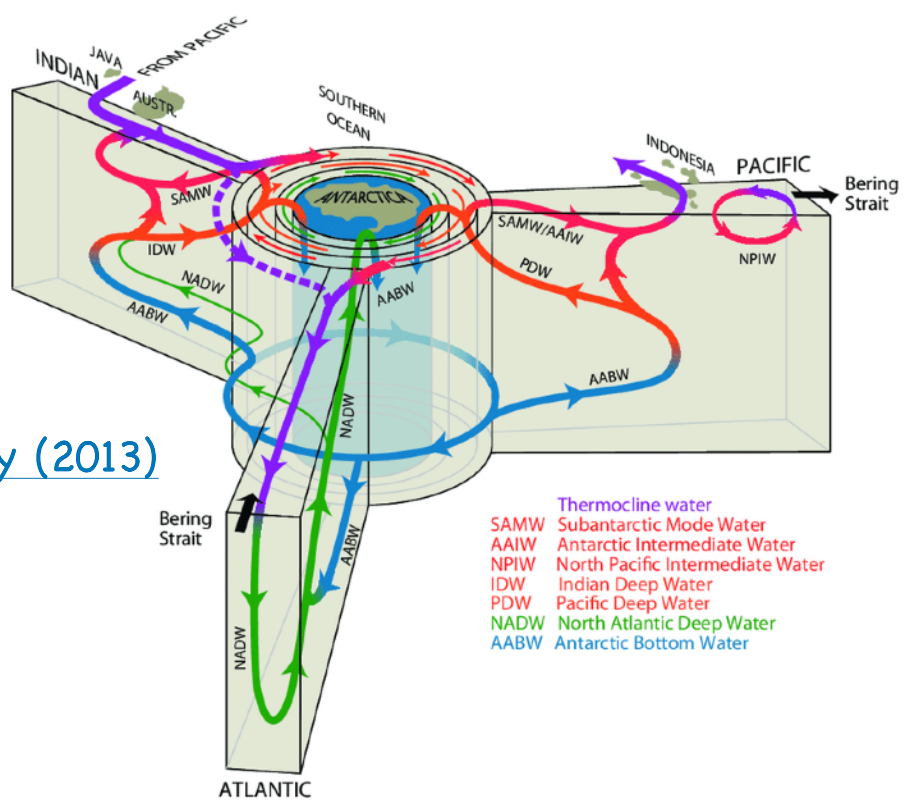


Schematics of the SOMOC

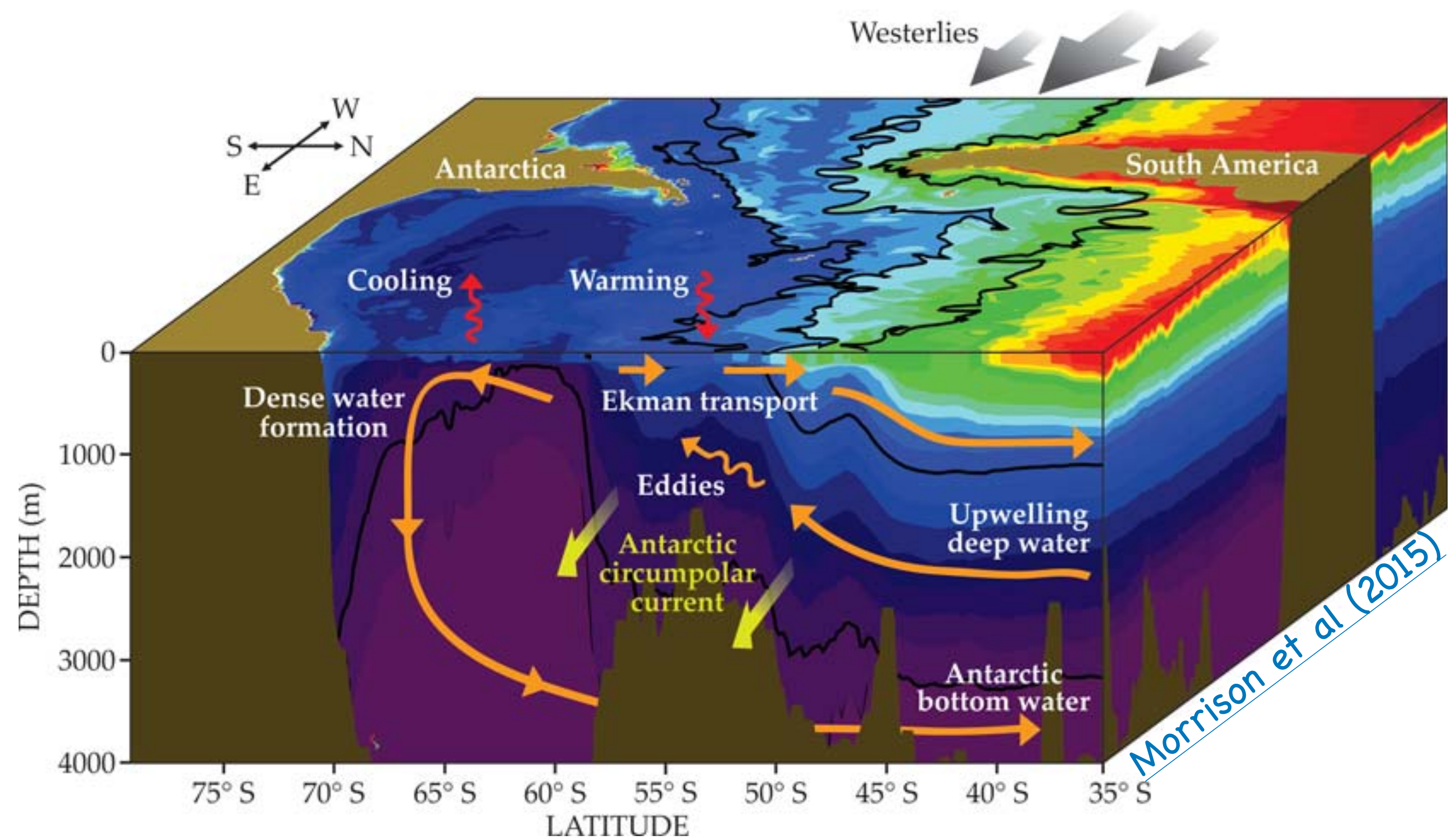
Rintoul and Naveira Garabato, 2013 taken from Olbers and Visbeck (2005) and as adapted from Speer et al (2000)







Talley (2013)

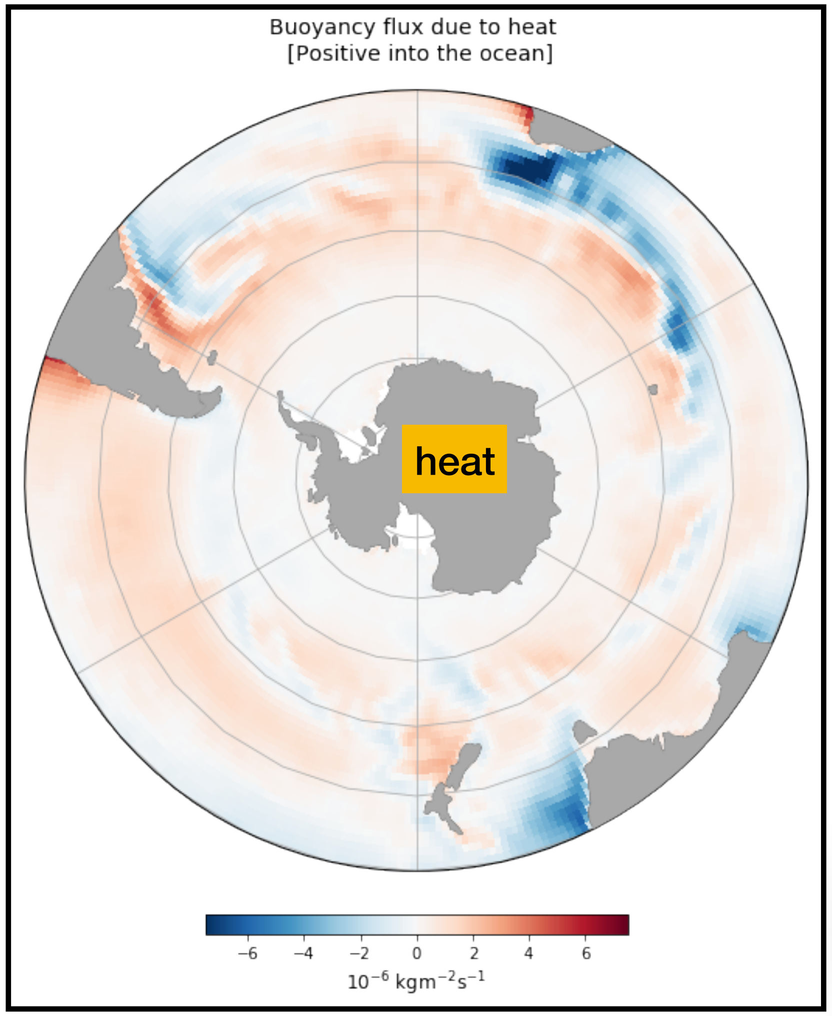


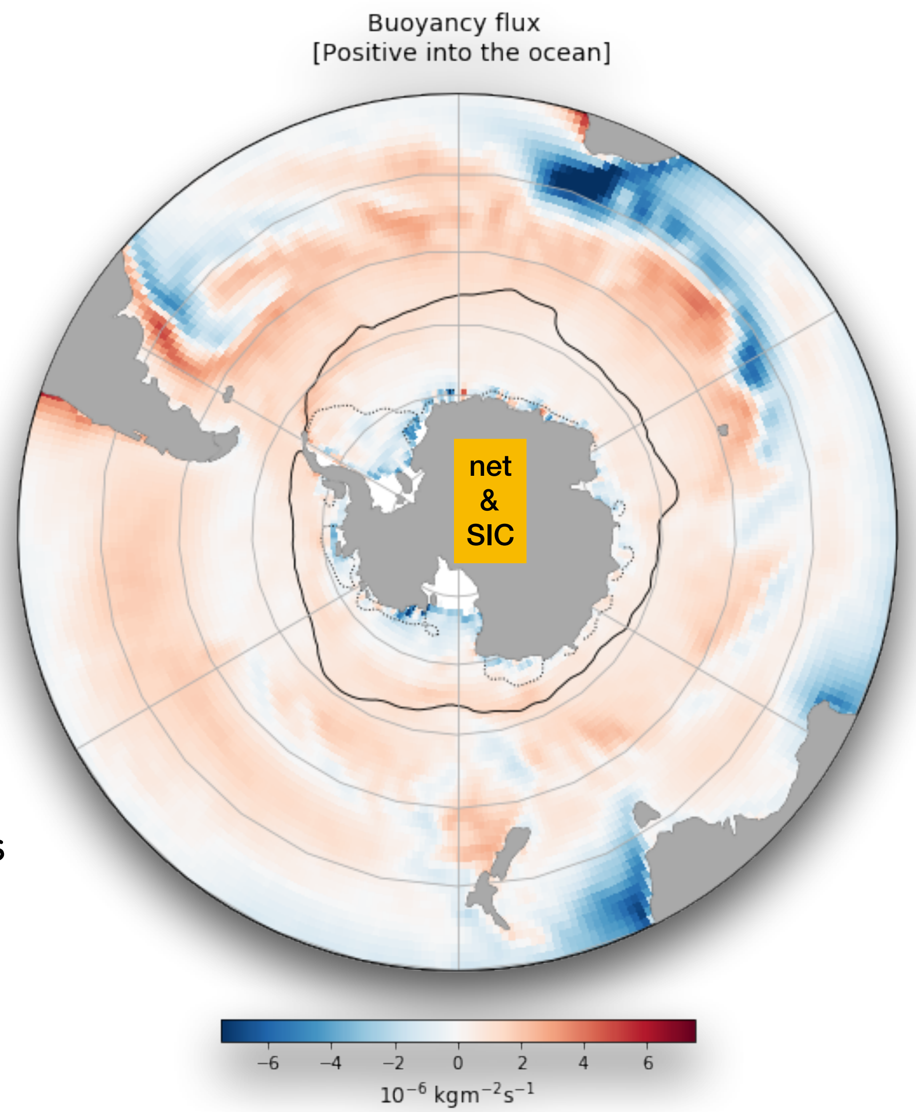
Morrison et al (2015)

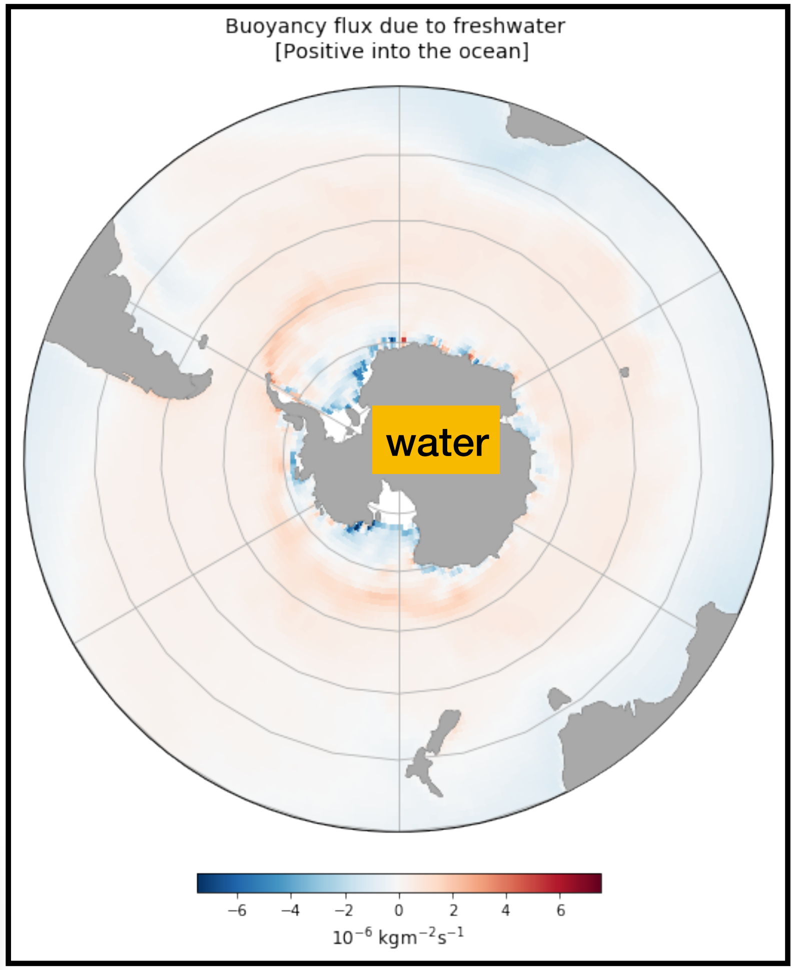
Surface buoyancy fluxes:

Provides surface watermass transformation

Positive buoyancy flux adds buoyancy to the ocean => seawater gets lighter.







Includes impact from sea-ice melt (courtesy A. Haumann)

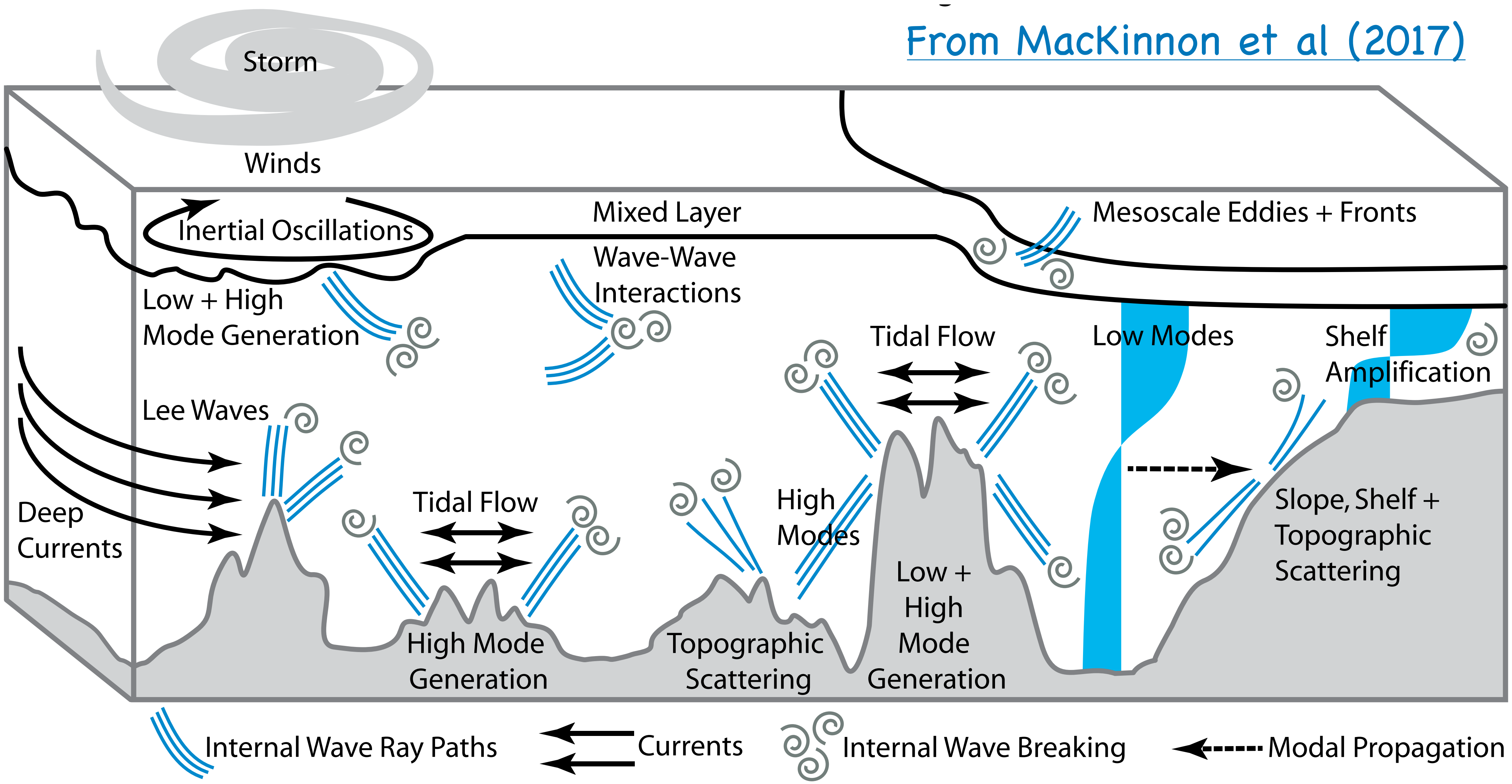
Thanks to Graeme MacGilchrist for these figures

10-year means from ERA-interim reanalysis (atmos model + assimilation)

The zoo of mixing from breaking gravity waves:

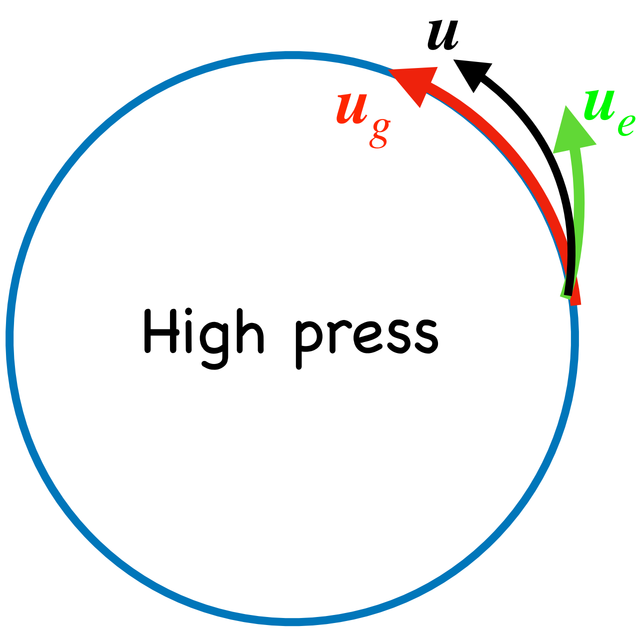
Provides interior ocean watermass transformation

From MacKinnon et al (2017)

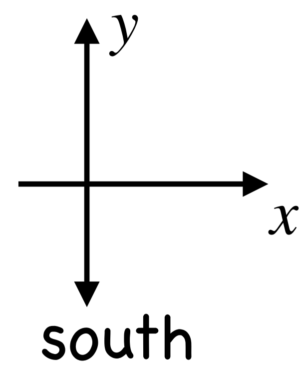


The role of winds:

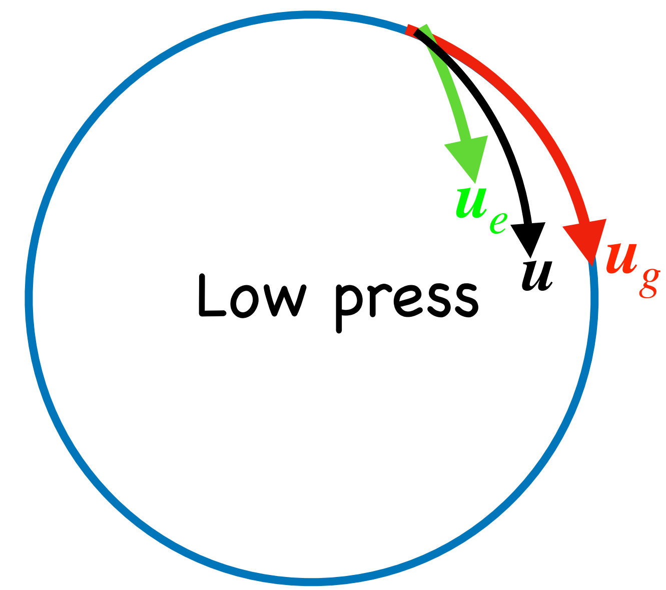
Ekman transport

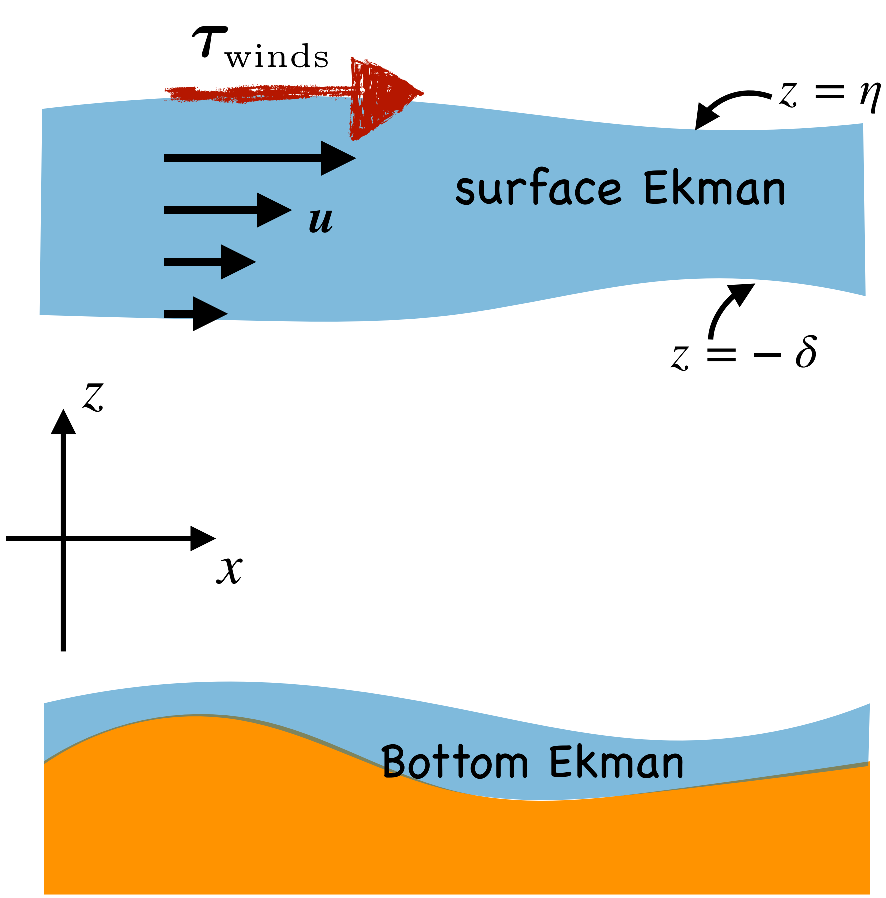


$$\rho_0\, f\, \hat{\boldsymbol{z}} \wedge (\boldsymbol{u}_{\rm g} + \boldsymbol{u}_{\rm e}) = -\nabla_z p + \frac{\partial \boldsymbol{\tau}}{\partial z}$$



$$f < 0$$





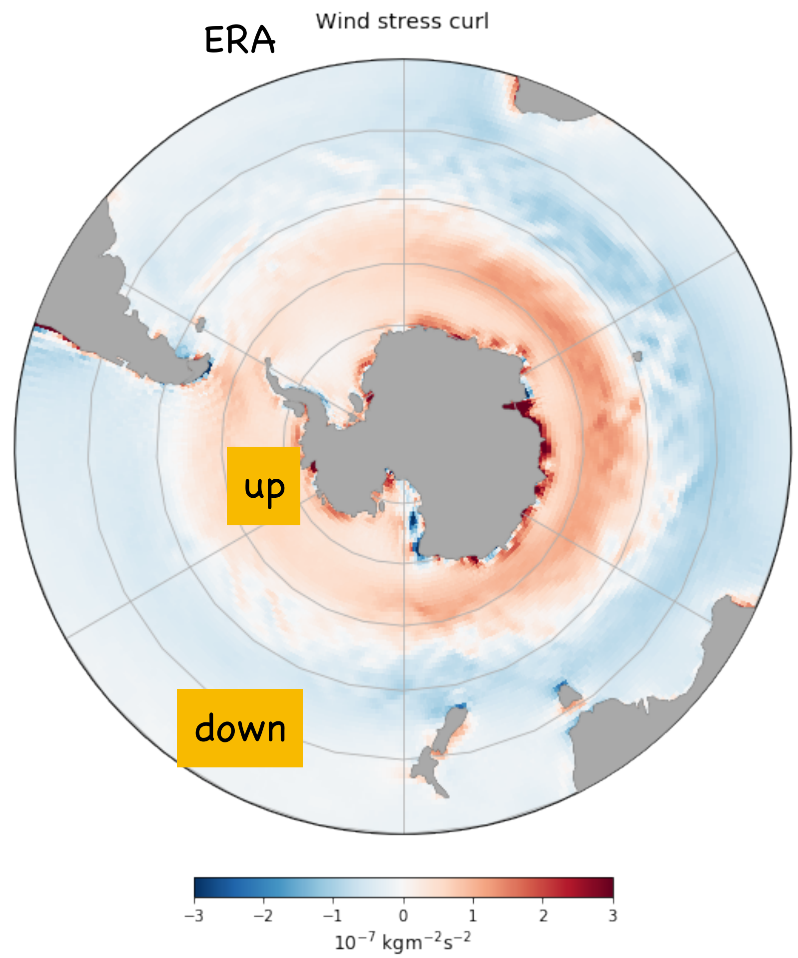
$$\boldsymbol{M}_{\rm e} = \rho_0 \int_{-\delta}^{\eta} \boldsymbol{u}_{\rm e}\, {\rm d}z \approx -\hat{\boldsymbol{z}} \wedge (\boldsymbol{\tau}_{\rm winds}/f) \Longrightarrow M^y_{\rm e} = -\tau^x_{\rm winds}/f$$

Ekman transport is set by strength of the wind stress and it is aligned to left of the stress (f < 0)

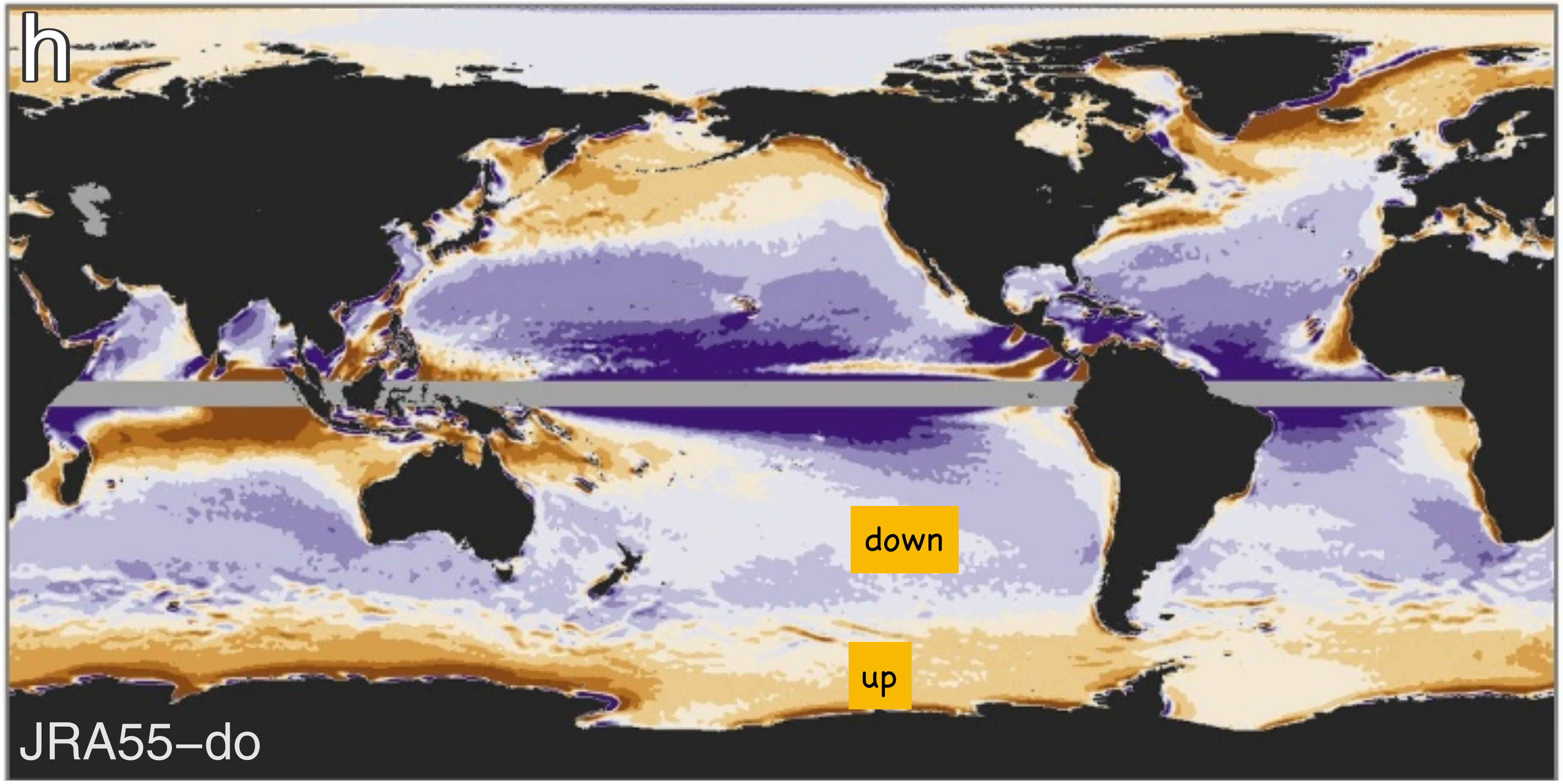
$$\nabla \cdot \boldsymbol{M}_{\rm e} = \hat{\boldsymbol{z}} \cdot [\nabla \wedge (\boldsymbol{\tau}/f)] \Longrightarrow \nabla \cdot \boldsymbol{M}_{\rm e} \approx -f^{-1}\, \partial_y\, \tau^x$$

Divergence of Ekman transport is determined by curl of (wind stress/f), which is approximated by the meridional derivative of the zonal wind stress.

Divergence of Ekman transport: $\nabla \cdot M_e = \hat{z} \cdot [\nabla \wedge (\boldsymbol{\tau}/f)]$



Thanks to Graeme MacGilchrist for this figure



Taboada et al (2019)

Positive: Ekman transport diverges so to bring water upward to satisfy continuity.

Negative: Ekman transport converges, which pushes water down into the interior.

Ekman upwelling occurs in the southern part of the Southern Ocean, pulling water up and then transporting it to the north where it experiences Ekman downwelling and is pushed into the interior. This process converts NADW into Antarctic Intermediate and Mode waters (there is also help from buoyancy forcing).

Equations for steady meridional motion

Steady linear zonal Boussinesq momentum equation sans horizontal Reynolds stresses.

$$-f\,v = \frac{1}{\rho_0}\left[-\frac{\partial p}{\partial x} + \frac{\partial \tau^x}{\partial z}\right]$$

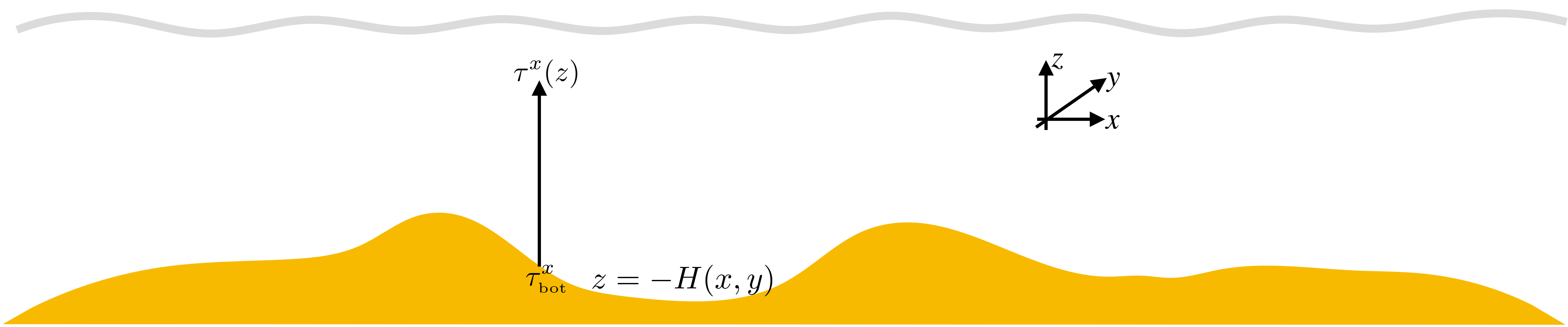
Coriolis acceleration balances zonal pressure gradient + vertical transfer of zonal friction stresses.

Depth integrate from ocean bottom to an arbitrary z.

$$-f\rho_0 \int_{-H}^{z} v\,\mathrm{d}z' = -\int_{-H}^{z} \frac{\partial p}{\partial x}\,\mathrm{d}z' + [\boxed{\tau^x(z)} - \tau^x_{\text{bot}}]$$

Dominated by winds in Ekman layer.

Vertical integral of frictional stress derivative leaves stress acting at z minus stress at the bottom.



Leibniz Rule for the horizontal pressure gradient

$$-f\rho_0 \int_{-H}^{z} v\,\mathrm{d}z' = -\frac{\partial}{\partial x}\int_{-H}^{z} p\,\mathrm{d}z' + [\tau^x(z) - \tau^x_{\text{bot}} + \boxed{p(z)\,\partial_x z + p_{\text{bot}}\,\partial_x H}]$$

Exposes interfacial and bottom form stresses.

Equations for steady meridional overturning

Zonal integrate on constant z-surface and introduce the z-space (Eulerian) MOC

small

$$f\rho_0\Psi(y,z) = \oint \boxed{[\tau^x(z)} - \cancel{\tau^x_{\text{bot}}} + p_{\text{bot}}\,\partial_x H]\,\mathrm{d}x$$

Includes Ekman transport when in upper ocean. Drives the "Deacon Cell".

Recall $\Psi(y,z) = -\oint \mathrm{d}x \int_{-H}^{z} v(x,y,z')\,\mathrm{d}z'$

Eulerian overturning is dominated by wind stress (Ekman) and bottom form stress.

Full depth integrated meridional transport with weak bottom friction

$$f\rho_0\Psi(y,0) = 0 \Rightarrow \oint [\tau^x_{\text{winds}} + p_{\text{bot}}\,\partial_x H]\,\mathrm{d}x = 0$$

As per zonal momentum balance.

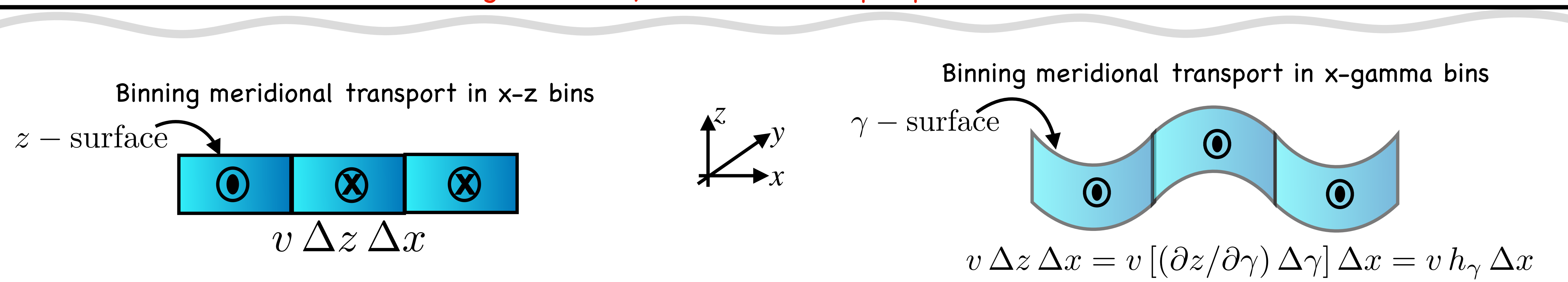
Zonal integrate on an isopycnal + introduce density-space MOC.

small

$$f\rho_0\Psi^\gamma(y,z_\gamma) = \oint [\tau^x(z_\gamma) - \cancel{\tau^x_{\text{bot}}} + \boxed{p(z_\gamma)\,\partial_x z_\gamma} + p_{\text{bot}}\,\partial_x H]\,\mathrm{d}x$$

Exposes isopycnal form stress. Recall discussion of correlations within geostrophic eddies.

New term relative to Eulerian overturning: isopycnal form stress. It partly counter-acts the wind-driven Deacon Cell, though note they have distinct depth profiles.

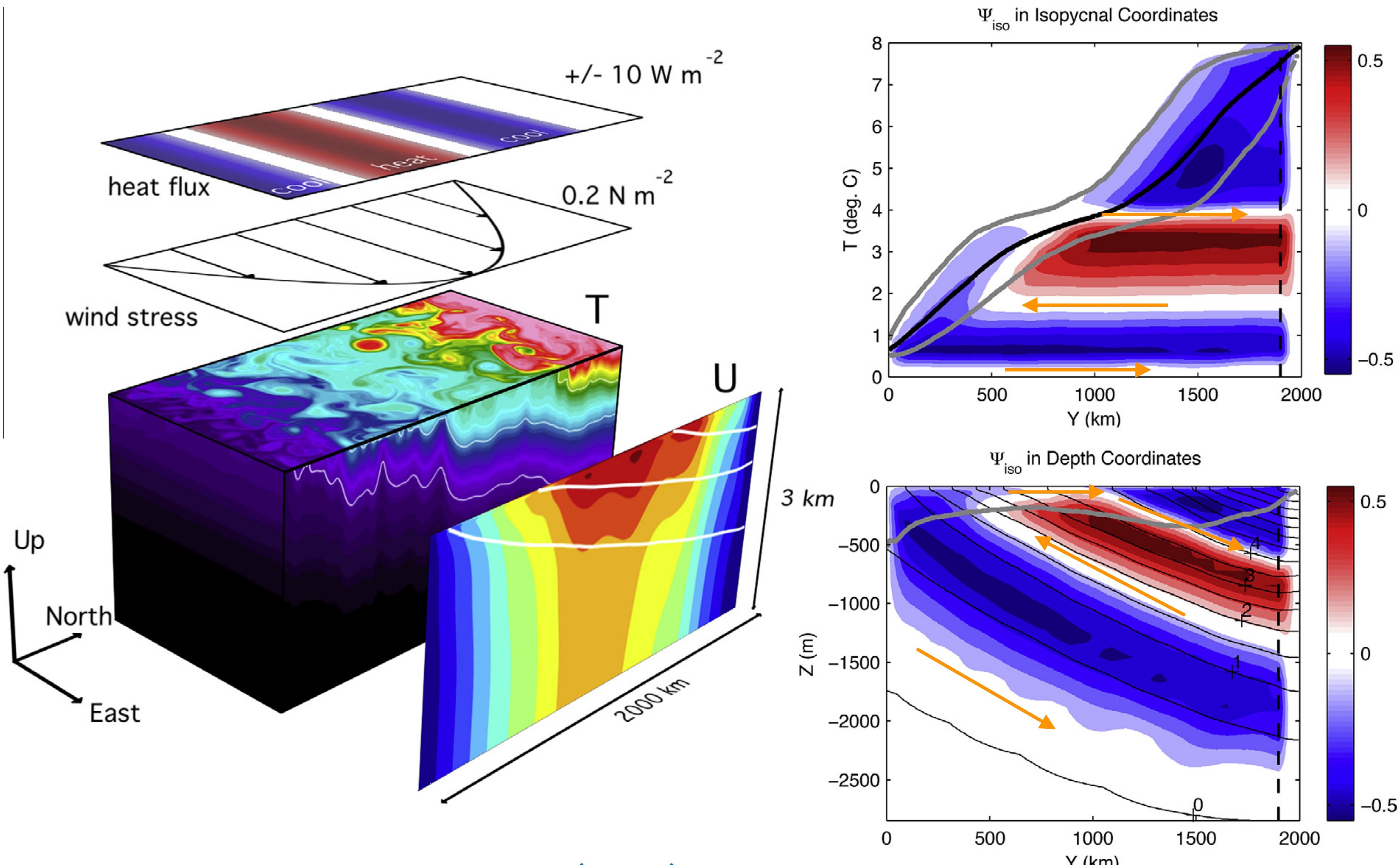


Illustrating how the integrals are computed

A process model of the Southern Ocean overturning

Illustrating how heat + winds create the SOMOC in a flat bottom channel

Link to Abernathey's channel model

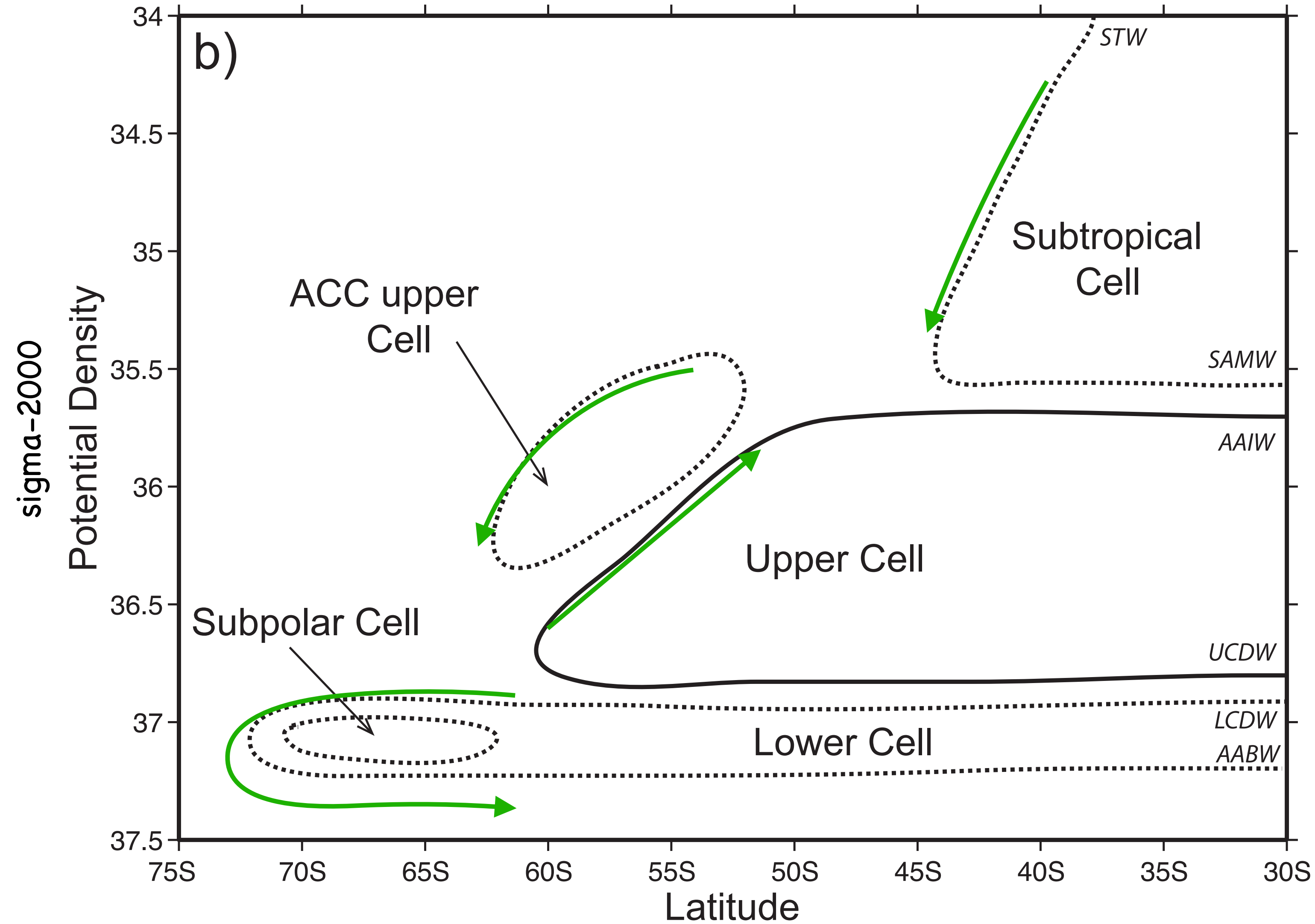


Abernathey et al (2013)

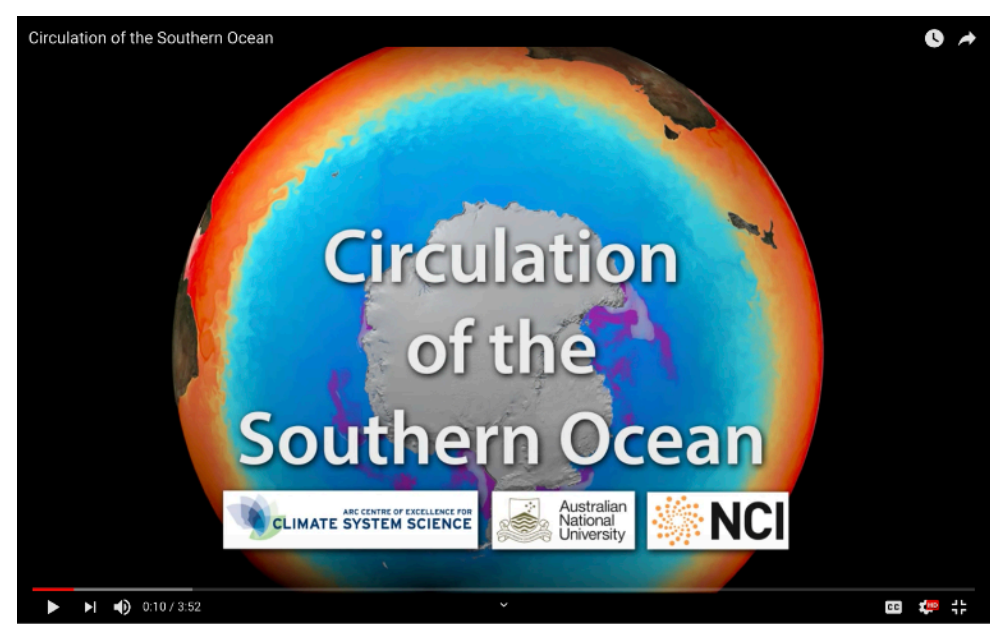
Bottom Cell: forcing on the continental shelf

Thus far we have mostly focused on the upper cells. What about the lower cell and the AABW?

This cell is largely forced by processes active on the continental shelf associated with strong surface buoyancy forcing (katabatic winds, brine rejection, etc.)



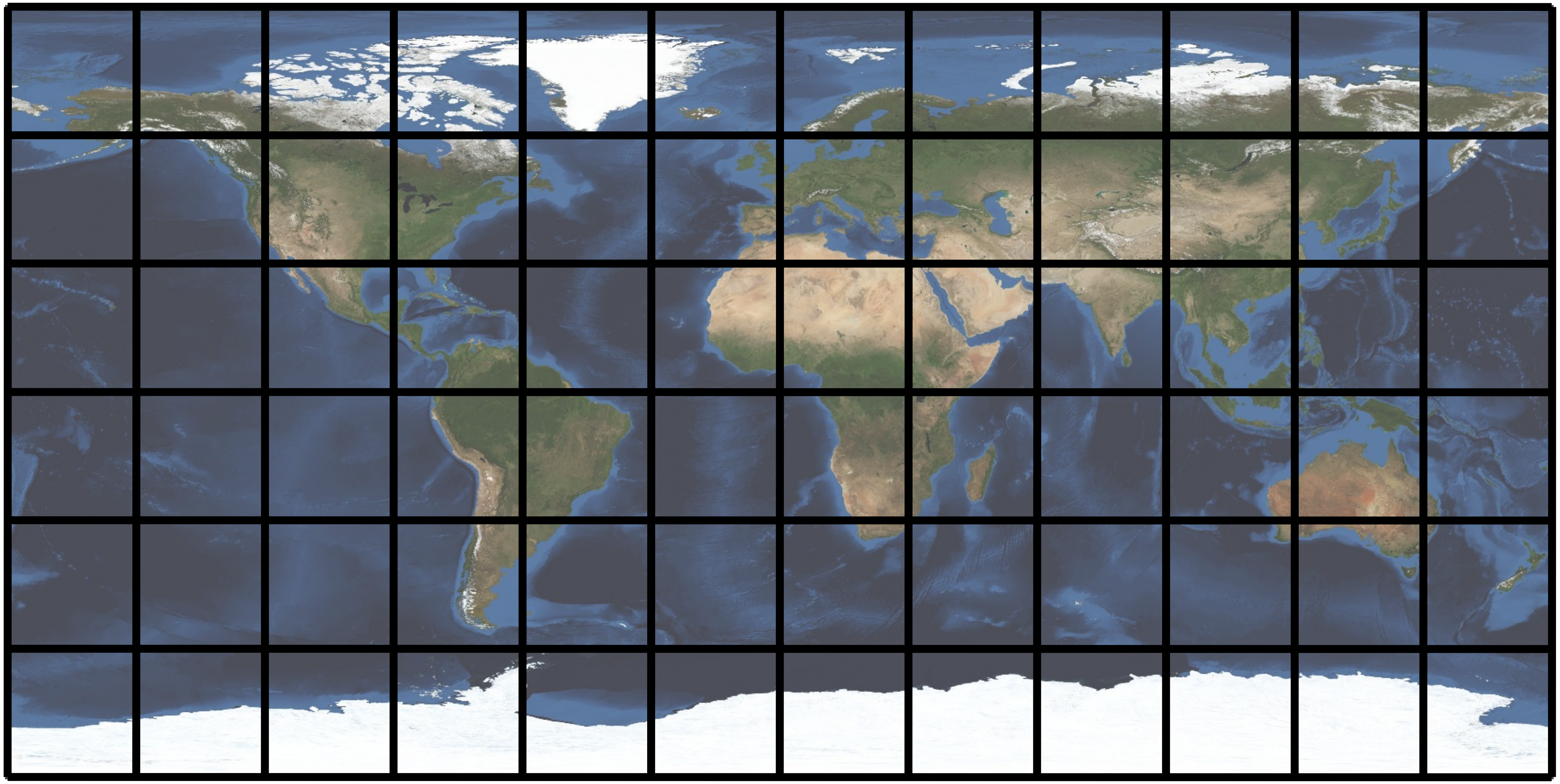
[Animation from a simulation]() of the global ocean from Australian collaborators using the GFDL ocean/ice model (based on CM2.6) run at ~5km resolution in the Southern Ocean. The simulation emphasizes the importance of shelf processes.



Mesoscale eddy parameterizations and the residual mean circulation

The need for ocean mesoscale eddy parameterizations

In terms of geostrophic eddy resolution, a 1° ocean model is roughly a 30° atmosphere model.



(after Peter Killworth)

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."

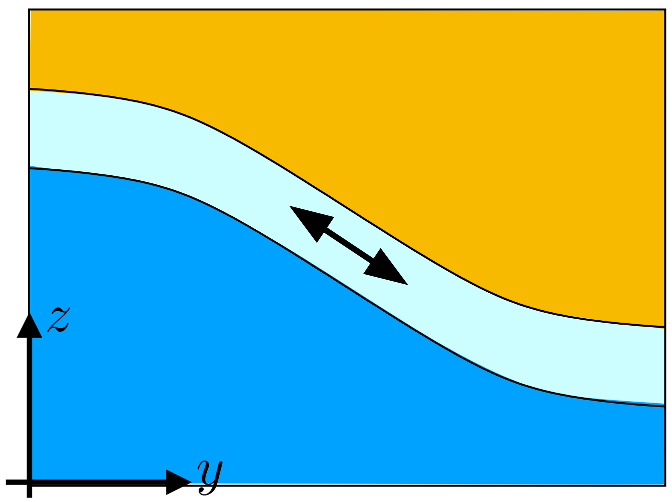
(Horace Lamb, 1932)

Mesoscale Eddy Parameterizations acting on tracers

$$\frac{\partial C}{\partial t} + (\boldsymbol{v} + \boldsymbol{v}^{\text{eddy}}) \cdot \nabla C = -\nabla \cdot \boldsymbol{F}^{\text{diffusion}}$$

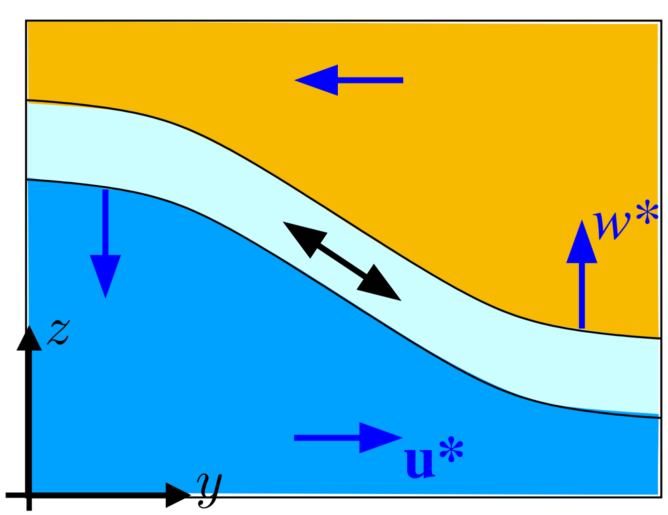
There are two key processes associated with ocean mesoscale eddies.

1. Eddy stirring of tracers along neutral directions leading to a downscale cascade of tracer variance. This process is parameterized as downgradient "neutral diffusion".



Montgomery (1938), Solomon (1971), Redi (1982), Olbers et al (1985), Griffies et al (1998)

2. Adiabatic sink of available potential energy (APE) that flattens isopycnals. It is generally parameterized by eddy (Stokes) advection or equivalently skew diffusion.



Gent & McWilliams (1990), Gent et al (1995), Griffies (1998)

The "GM90" parameterization leads to poleward flux of buoyancy (heat) and equivalently to a downward flux of horizontal momentum. Recall our earlier discussion of mesoscale eddies and isopycnal form stress.

Residual mean overturning circulation

$$\Psi^{\dagger}(y,z) = -\oint \mathrm{d}x \int_{-H}^{z} (v + v^{\mathrm{eddy}})\,\mathrm{d}z' = \Psi^{\mathrm{Eulerian}} + \Psi^{\mathrm{eddy}}$$

$$\Psi^{\mathrm{Eulerian}} = -\oint \tau^{x}_{\mathrm{winds}}/(\rho_0\, f)\,\mathrm{d}x$$ Marshall and Radko (2003)

$$\Psi^{\mathrm{eddy}} = -\oint \kappa S^{y}\,\mathrm{d}x = \oint \frac{\kappa\,\partial_y b}{N^2}\,\mathrm{d}x$$ Gent et al (1995)

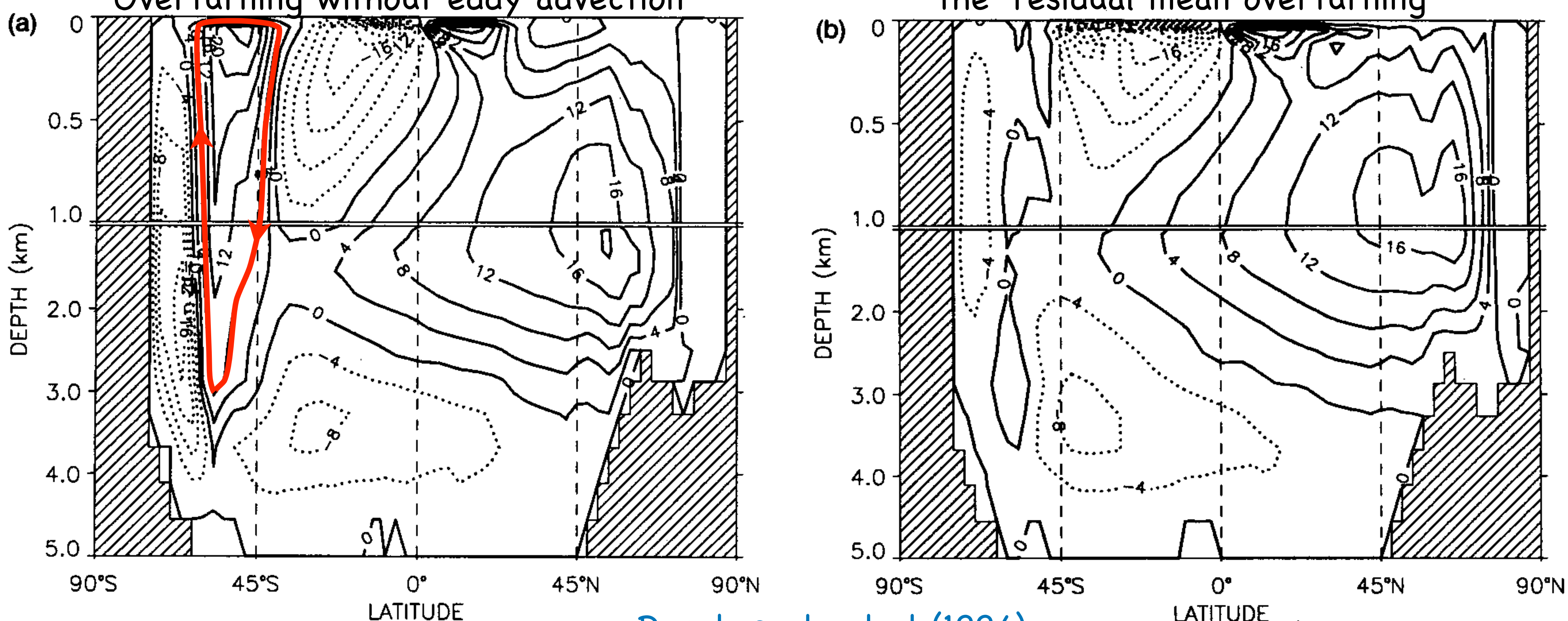
If $\Psi^{\mathrm{Eulerian}} + \Psi^{\mathrm{eddy}} \approx 0$

then $\langle S^{y}\rangle \approx -\langle \tau^{x}_{\mathrm{winds}}/(\rho_0\, f\, \kappa)\rangle$

Recall the interfacial form stress.

Overturning without eddy advection

Overturning with eddy advection: the “residual mean overturning”



$\overline{\mathbf{u}}$

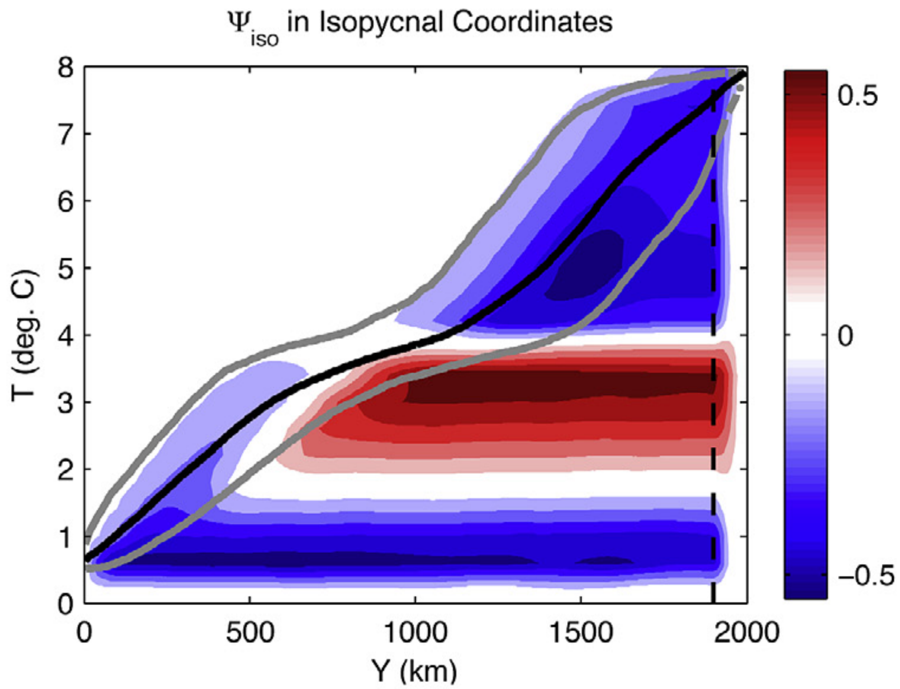
Danabasoglu et al (1994)

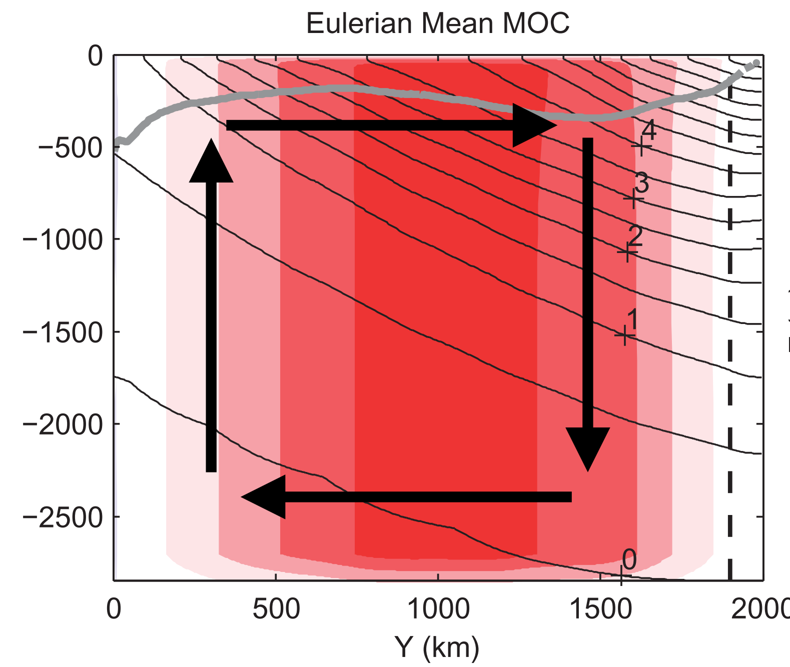
$\overline{\mathbf{u}} + \mathbf{u}^{*}$

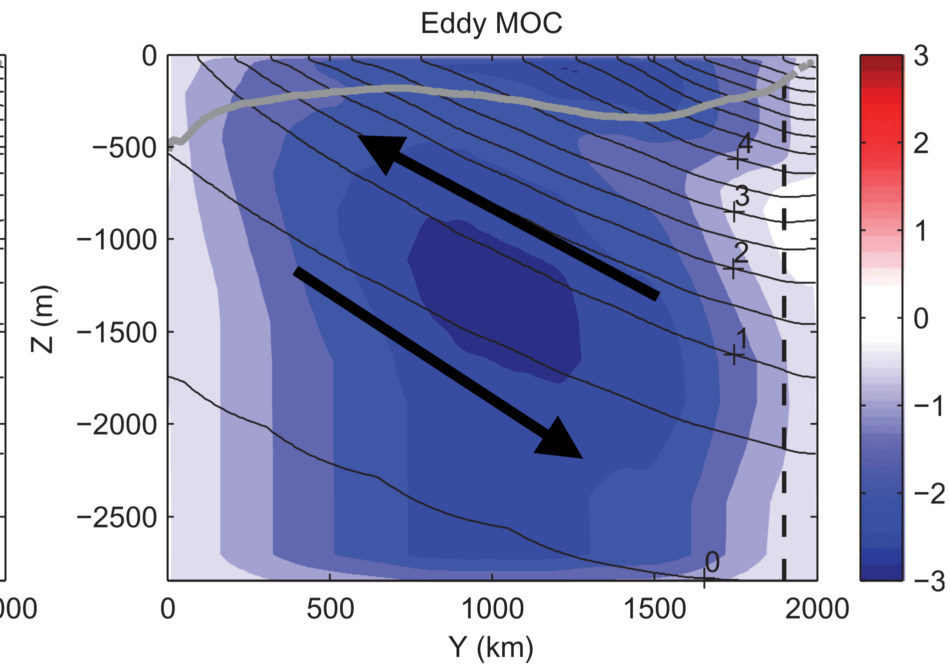
Note cancellation of the Deacon Cell in residual mean. The degree of cancellation is a function of the strength and vertical structure of the eddy-induced advection; diapycnal mixing; boundary buoyancy forcing (zero residual mean for adiabatic circulation). How much cancellation occurs in the real ocean remains a research question.

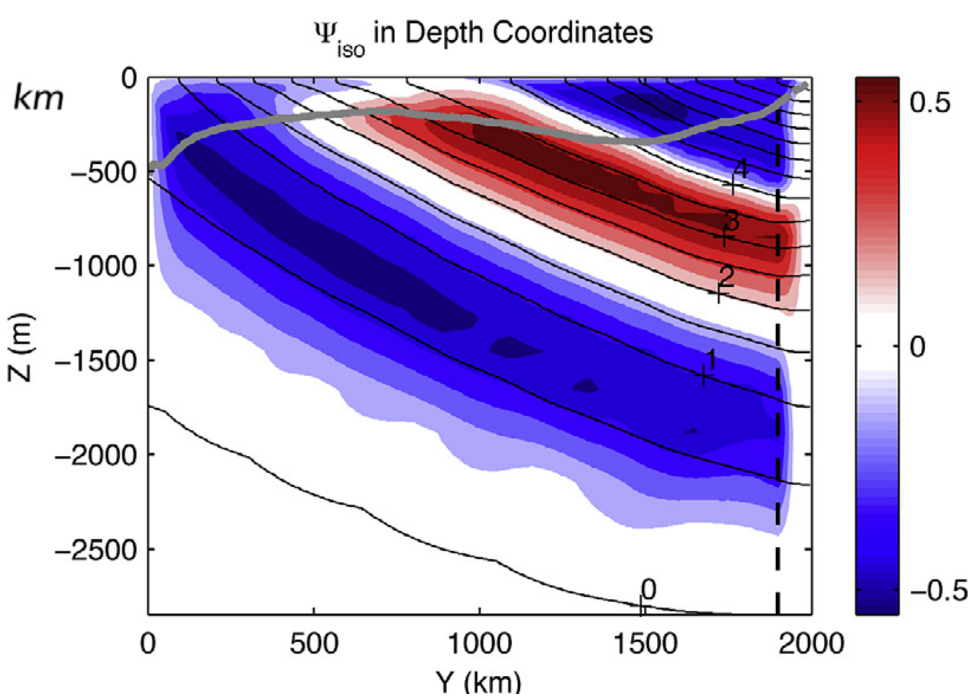
Residual mean = Eulerian + Eddy

Note the distinct structure of the Eulerian versus eddy overturning.









Without topography, the zonal momentum balance closes by having the circulation reach to the bottom.

Mean meridional flow is w/i surface + bottom Ekman layers.

Watermasses are transformed in the surface + bottom layers.

Eddy-induced overturning is oppositely directed to the wind driven circulation, but its structure is slightly distinct thus leading to a nonzero residual mean.

With topography the overturning closes near the depth of the sills where form drag sucks momentum from the ACC.

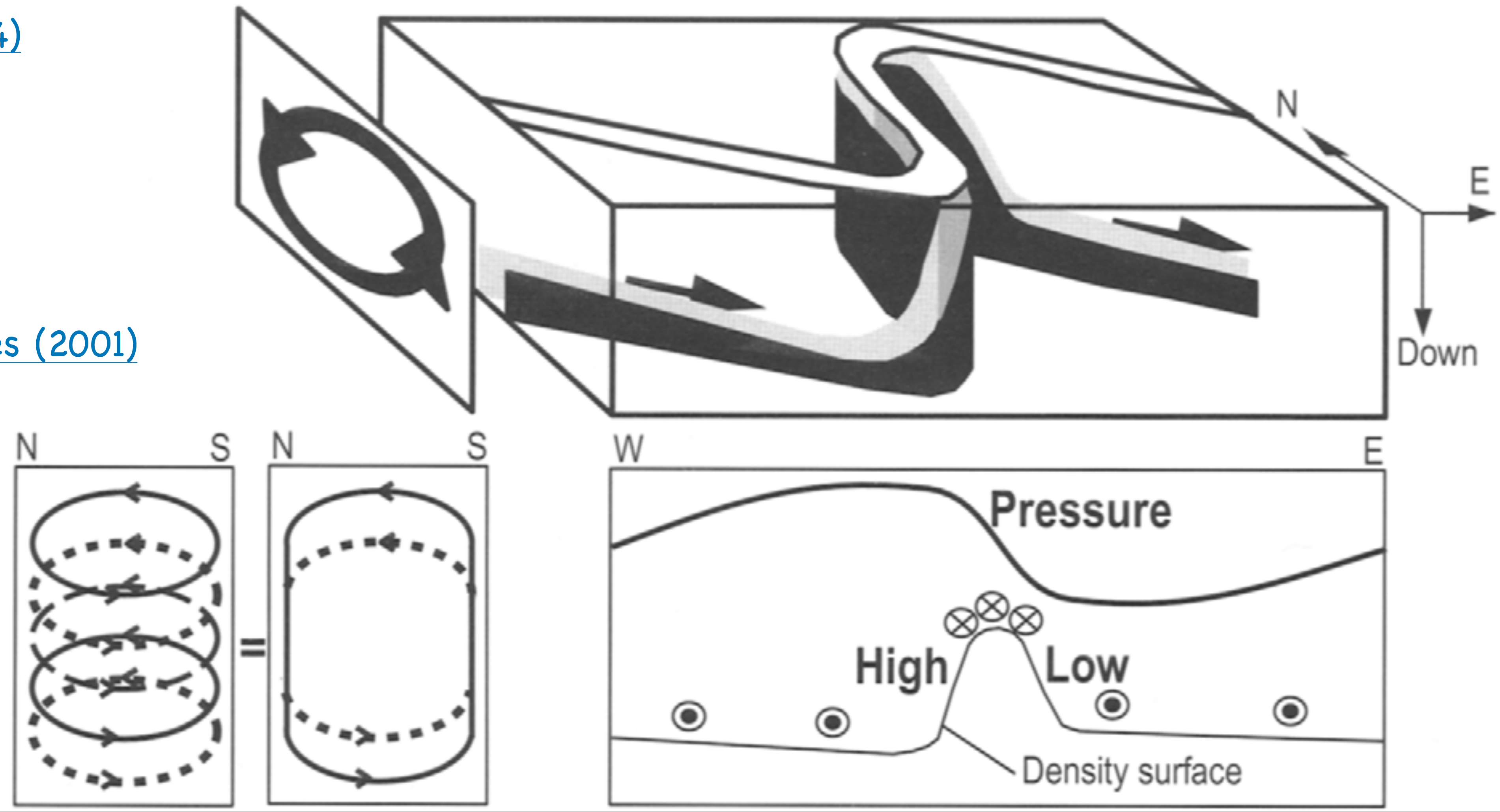
Abernathey et al (2011)

Kinematics of the Deacon Cell

Doos and Webb (1994)

Karoly et al (1997)

Rintoul, Olbers, Hughes (2001)



The ACC exhibits meridional meanders when encountering standing meanders that possess undulations in density surfaces; like a topographic obstacle.

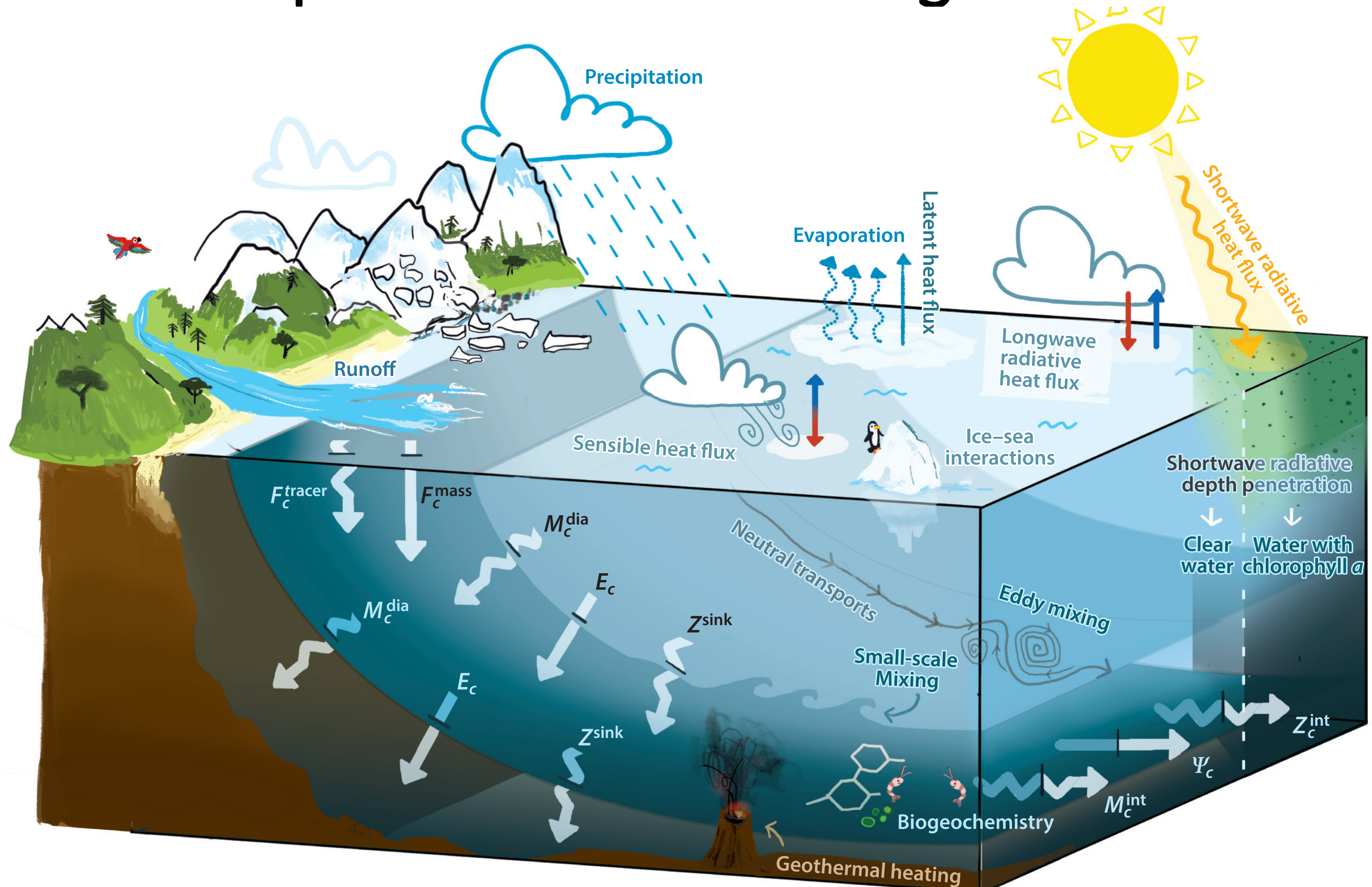
Geostrophic flow is poleward on the flanks of density ridges and equatorward on the crests.

If projected onto depth-latitude plane, meanders appear as an overturning circulation. But the interior flow is actually dominated by motion along isopycnals. Motion without buoyancy forcing and without interior mixing occurs along isopycnals.

With weak interior mixing we see that the "Deacon Cell" is largely missing from the density-meridional overturning circulation; similar to the atmospheric Ferrel Cell.

Elements of Watermass Transformation Analysis

The zoo of processes affecting transformation

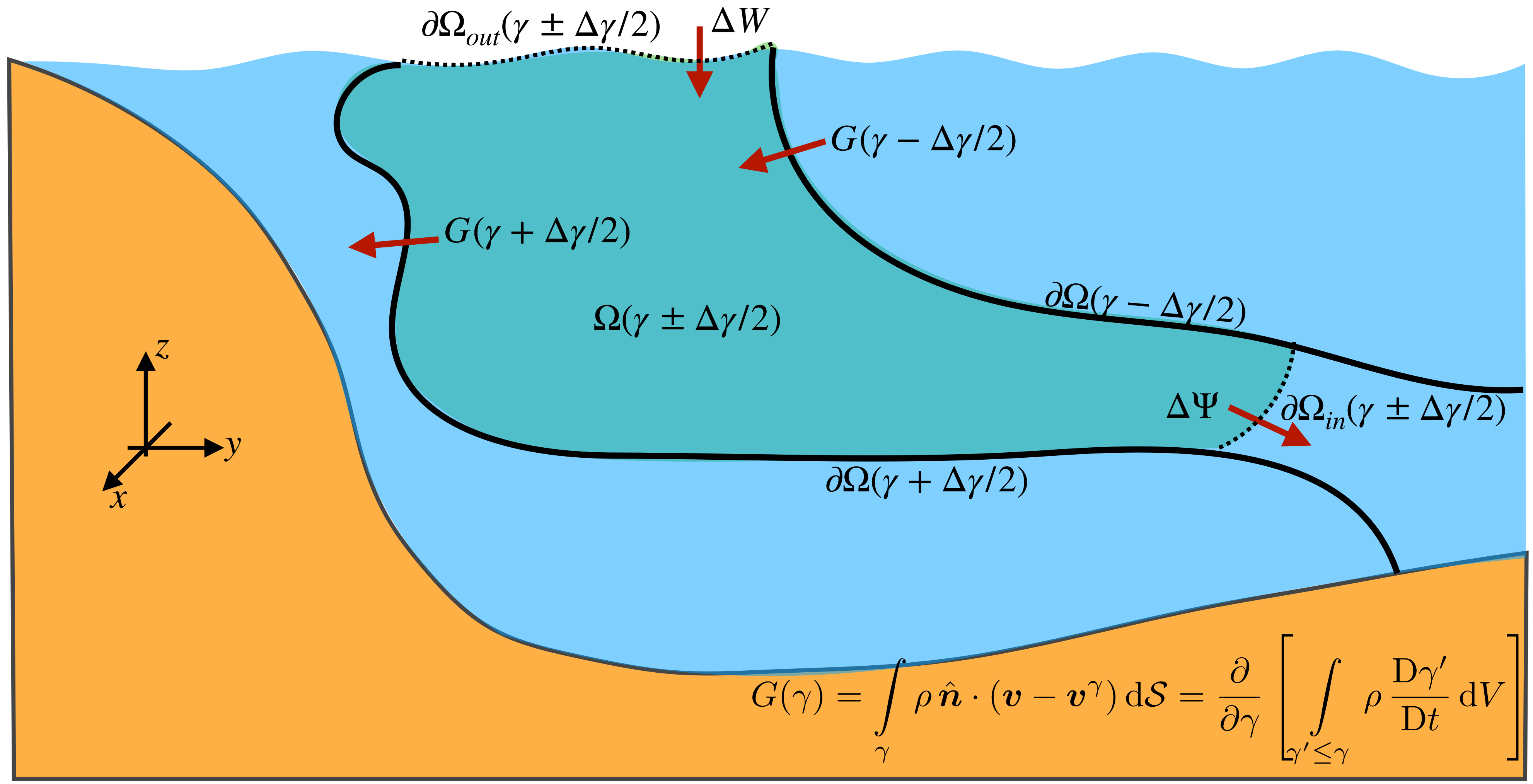


Groeskamp et al (2019)

- Watermass transformation analysis is a formalism for computing mass and tracer budgets over fluid layers rather than fixed boxes.
- It is a framework for inferring overturning circulation patterns and for attributing the circulation according to irreversible physical processes that cause the transformation.

Diapycnal transformation:

Arises from fluid motion relative to density interfaces



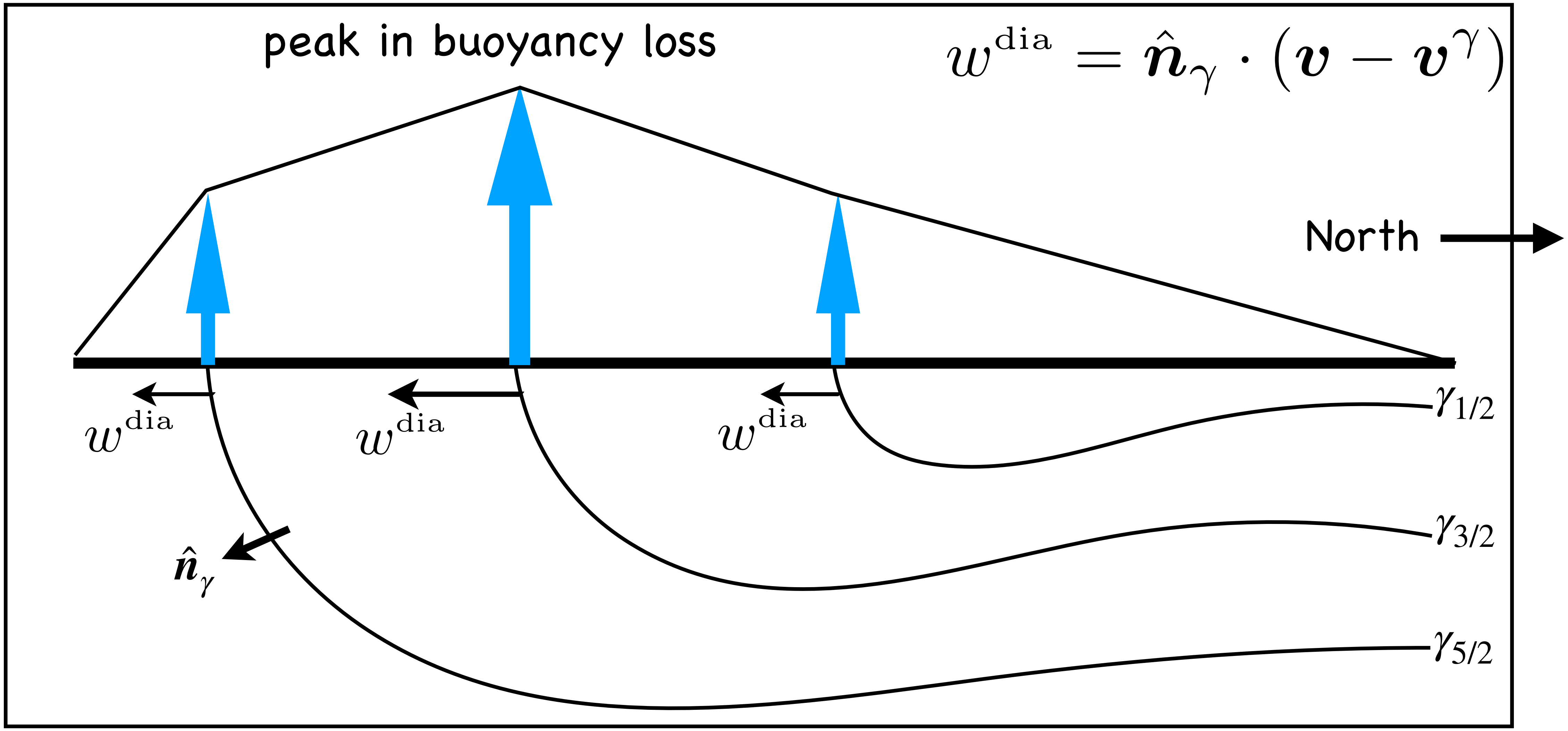
$$\underbrace{\frac{\mathrm{d}\Delta M}{\mathrm{d}t}}_{\text{mass change}} + \underbrace{\Delta\Psi}_{\text{circulation}} = \underbrace{\Delta W}_{\text{surface flux}} \underbrace{-[G(\gamma - \Delta\gamma/2) - G(\gamma + \Delta\gamma/2)]}_{\text{convergence of transformation}}$$

A steady circulation in the absence of surface water fluxes is driven solely by diapycnal transformation.

$$\Delta\Psi = -[G(\gamma - \Delta\gamma/2) - G(\gamma + \Delta\gamma/2)]$$

Conversely, there is no steady density-space circulation in the absence of diapycnal transformation.

Transformation from surface buoyancy forcing



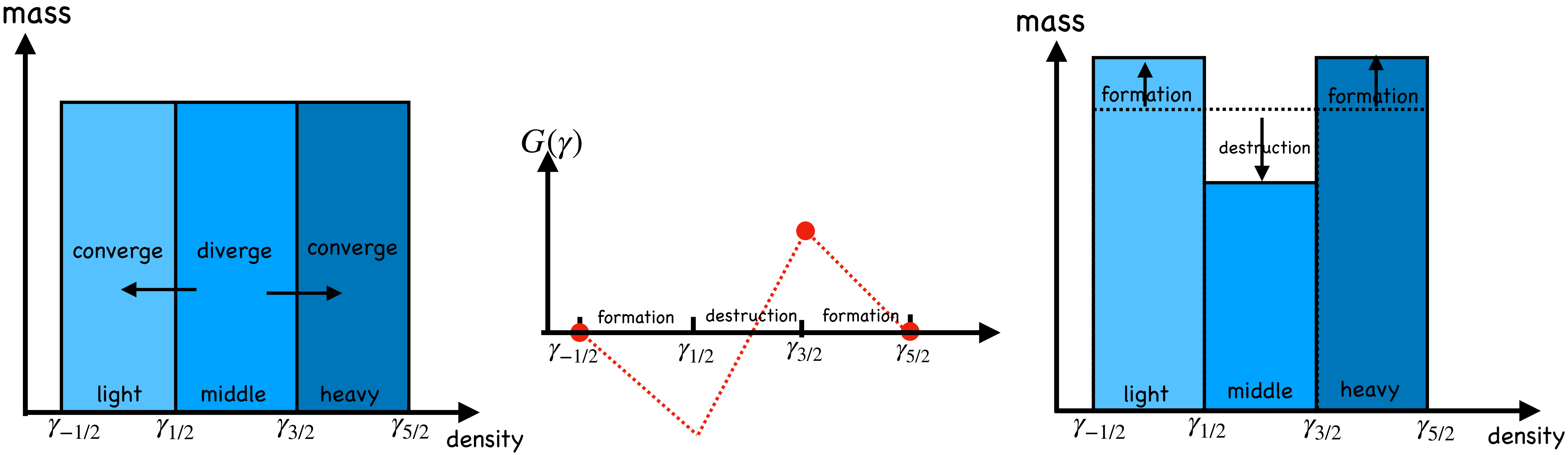
Water crosses density layers when there is flow relative to the layer interface.

This flow can occur by fluid moving, interface moving, or a bit of both.

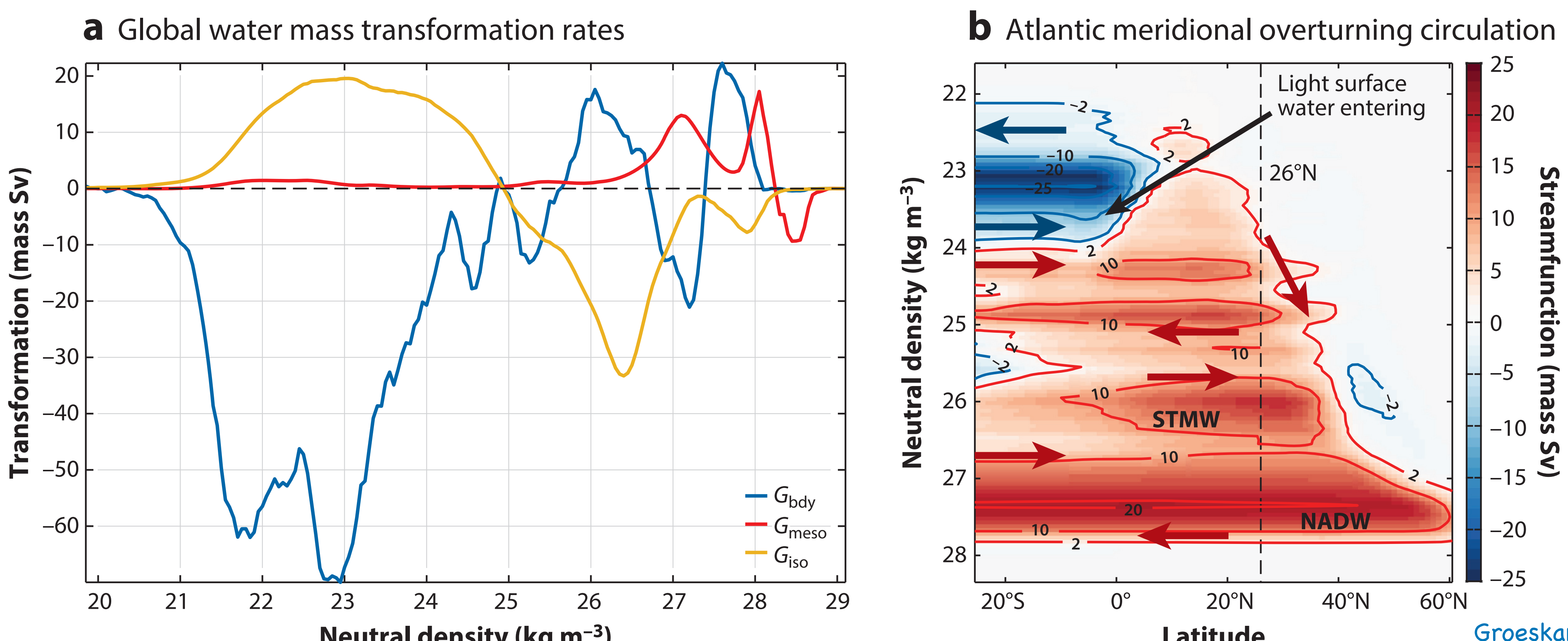
In this example, surface buoyancy loss causes density surfaces to move northward thus causing water to cross density interfaces towards the south.

Transformation <=> changes to density distribution

Water crossing density interfaces changes the density-space census.



Example density-space circulation as inferred from watermass distribution changes.



Groeskamp et al (2019)

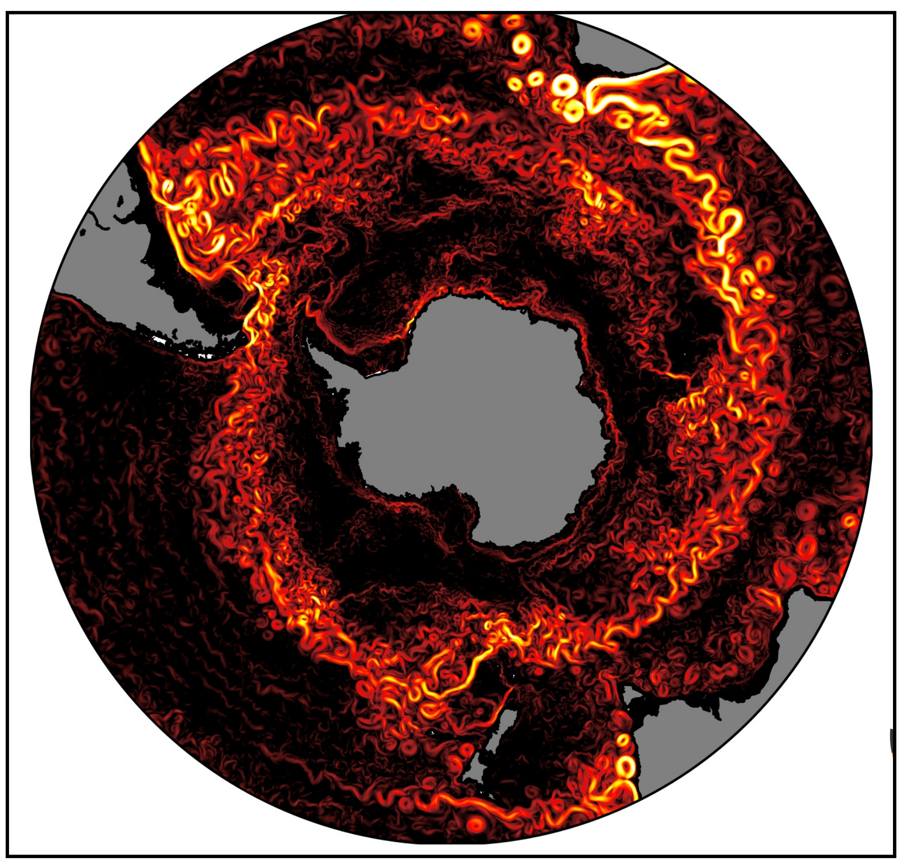
Some Further Ongoing Research

Eddy Saturation

Eddy saturation: when positive perturbations to zonal wind stress act to increase eddy energy but does not alter net zonal transport. Meredith and Hogg (2006) identify a time lag of ~few years associated with baroclinic eddies extracting newly added APE from wind perturbations.

Eddies continue to transfer momentum vertically to the bottom, yet the mean flow is unaffected by changes to the eddy energy.

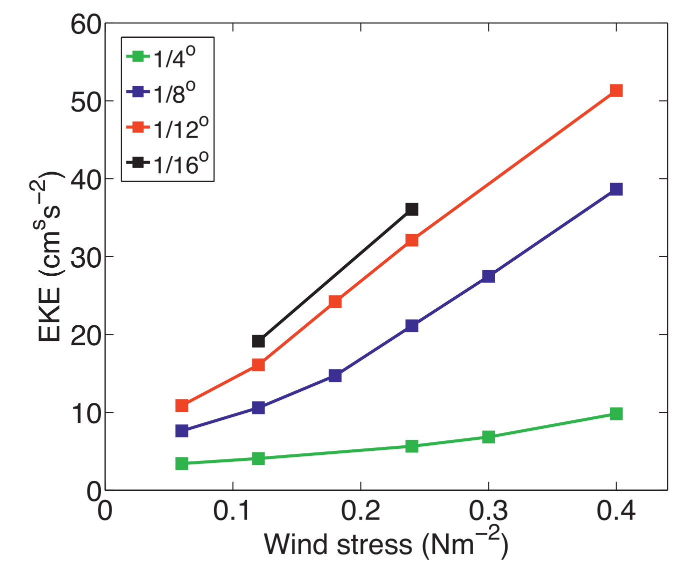
Idea suggested by Straub (1993) with numerical model results [e.g., Hallberg and Gnanadesikan (2006), Munday et al (2013), Morrison and Hogg (2013)] supporting partial eddy saturation. Meredith and Hogg (2006) suggest it holds based on observations.

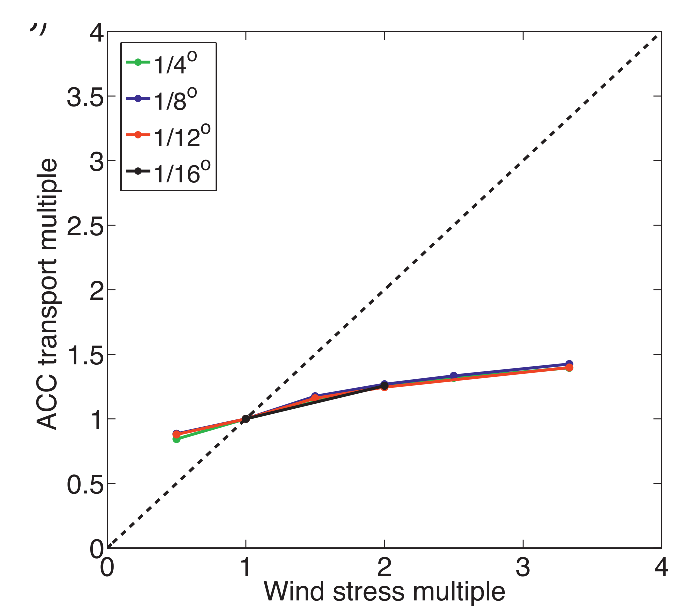


Surface ocean speed from GFDL CM2.6
Courtesy Adele Morrison

Morrison and Hogg (2013)

Results from idealized channel simulations show increased eddy energy with refined resolution but nearly constant depth integrated zonal transport.



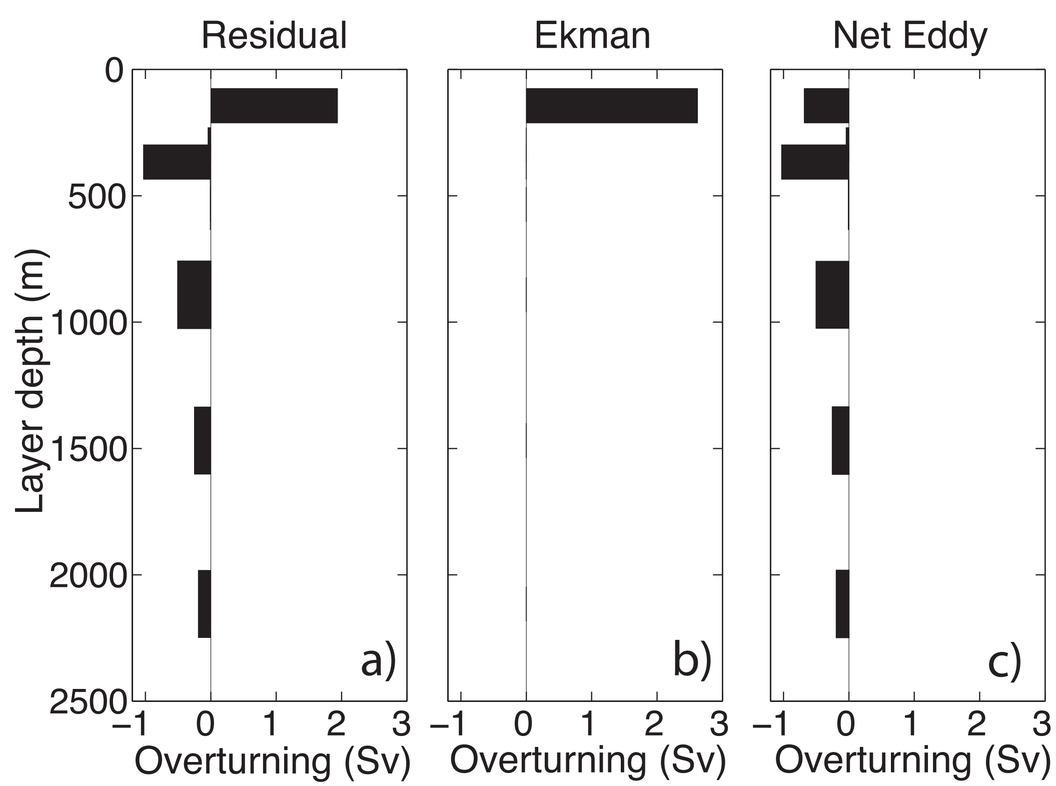


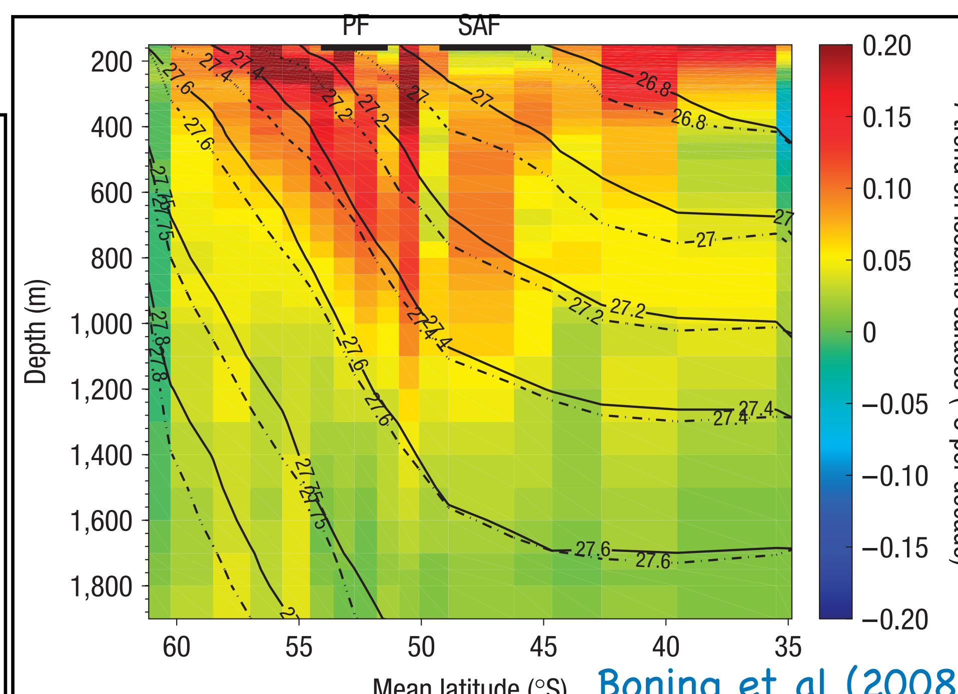
Eddy compensation + eddy saturation

- Eddy Saturation: Increased winds increase eddy energy but do not alter mean zonal transport, thus suggesting the zone transport is only a weak function of the winds.
- Eddy compensation: Increased winds increase isopycnal slopes but enhanced eddy activity counteracts the slope increase to leave slopes relatively unchanged.
- Key papers:
  Hallberg and Gnanadesikan (2006)
  Boning et al (2008)
  Meredith et al (2012)
  Morrison and Hogg (2013)
  Munday et al (2013)
  Farneti et al (2015)

We do not know how much the real ocean is eddy compensated or eddy saturated. Models show more saturation than compensation, but what is the correct amount is unknown. Obs show some of both but with large uncertainties.

Winds directly affect Ekman layer whereas eddies act deeper. So we should not expect eddy compensation to be one-to-one related to eddy saturation. In fact, we see more compensation with enhanced resolution whereas saturation is nearly constant. Morrison and Hogg (2013)

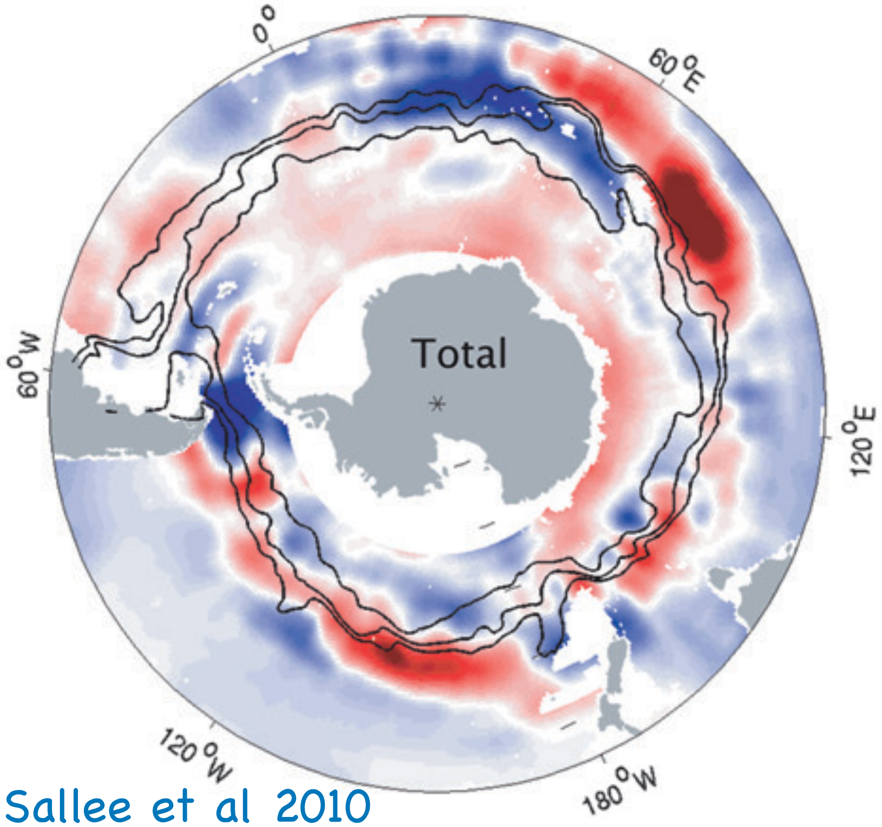




Boning et al (2008)

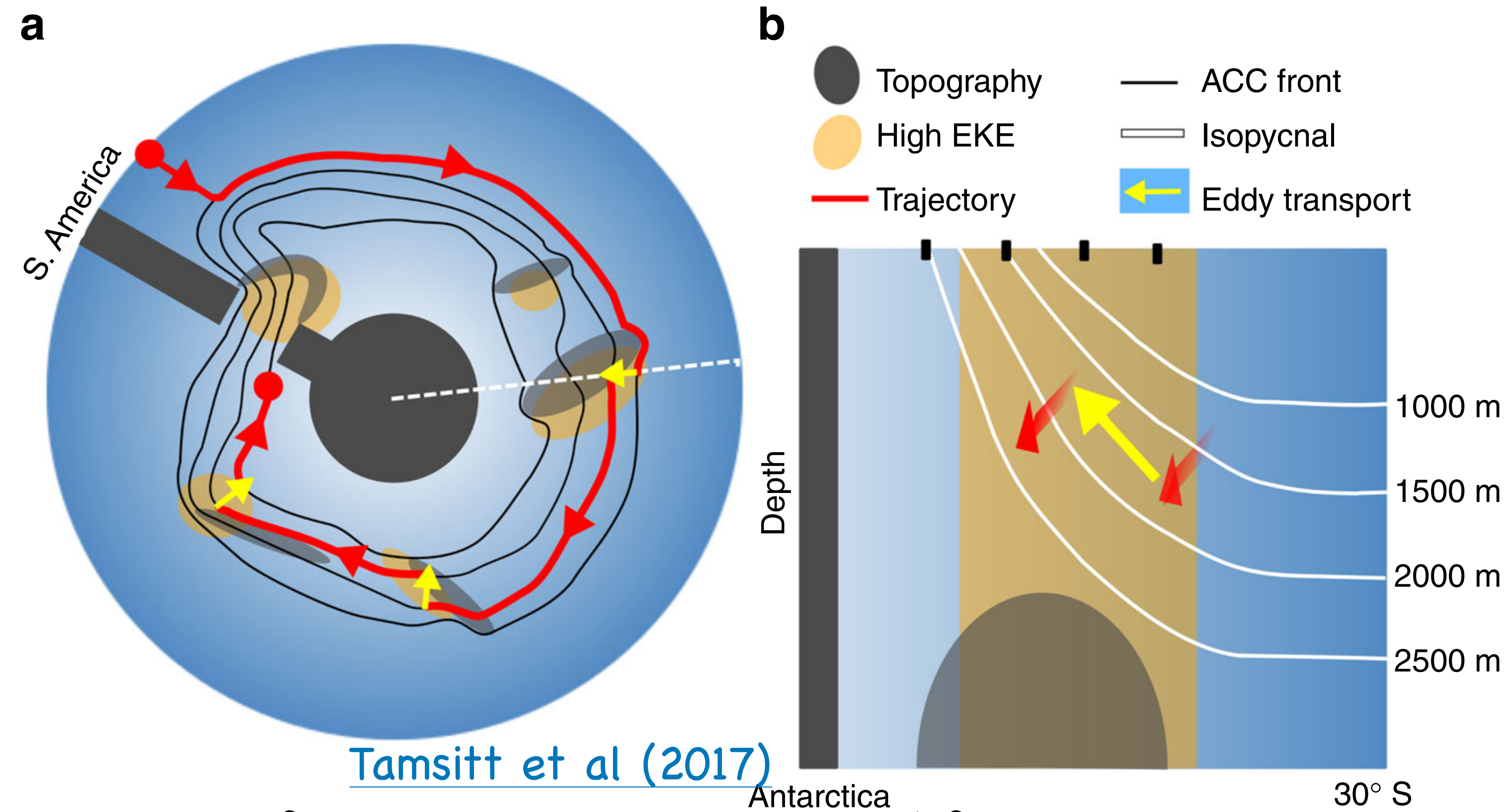
Mild changes to isopycnal slopes, suggesting eddy compensation.

Zonal asymmetry: hot spots for eddy activities and cross-front transport



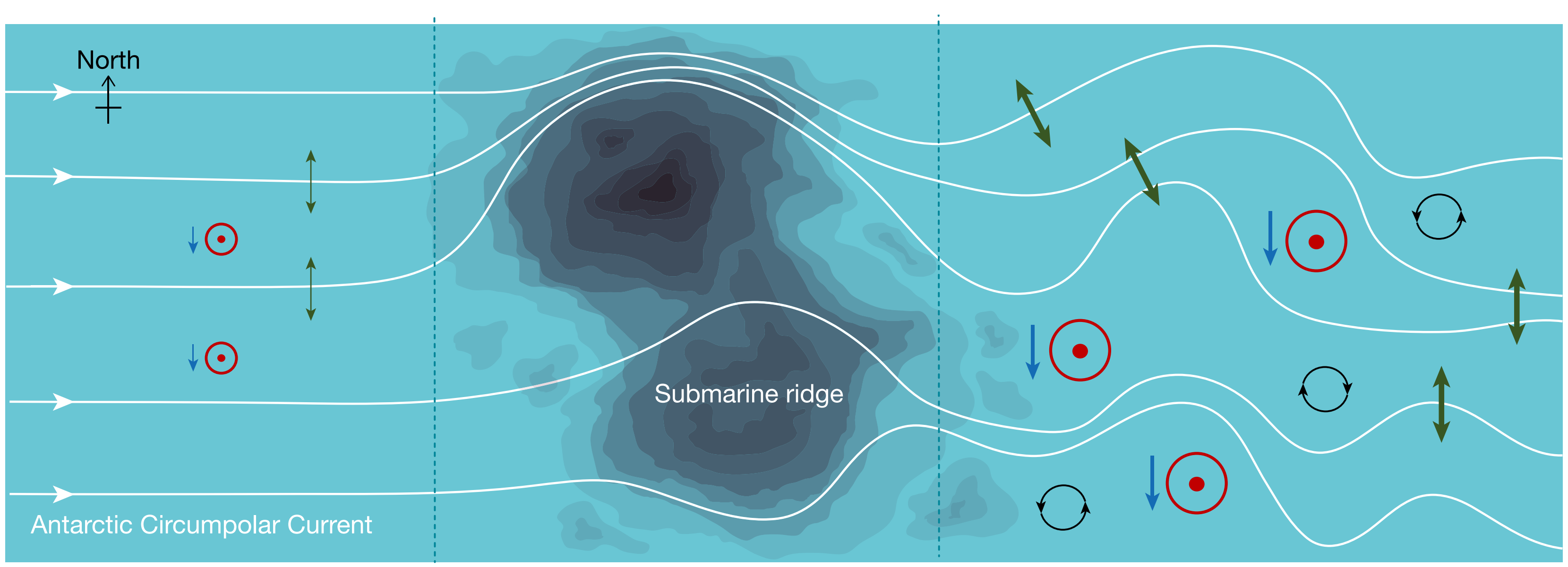
Sallee et al 2010

Subduction patterns estimated from obs.



Tamsitt et al (2017)

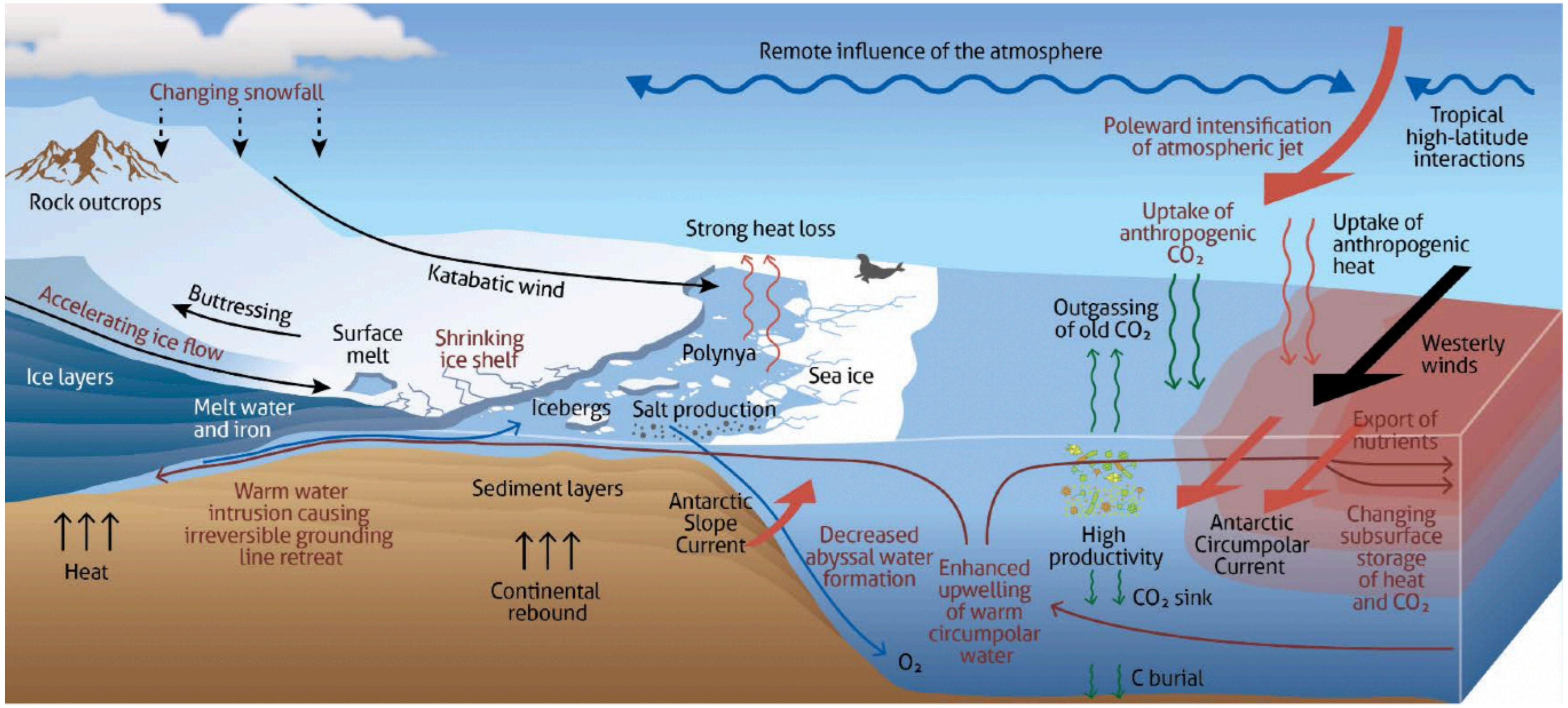
Schematic of particle pathways estimated from GFDL CM2.6



Rintoul (2018) schematic illustrating the leading order role of topography in the ACC.

There is growing evidence for the importance of longitudinal structure in the Southern Ocean, thus motivating theories and models to move beyond the zonal mean paradigm (e.g., Dufour et al 2012, Thompson and Naveira Garabato 2014).

Antarctic shelf shelves, slope current, & sea level

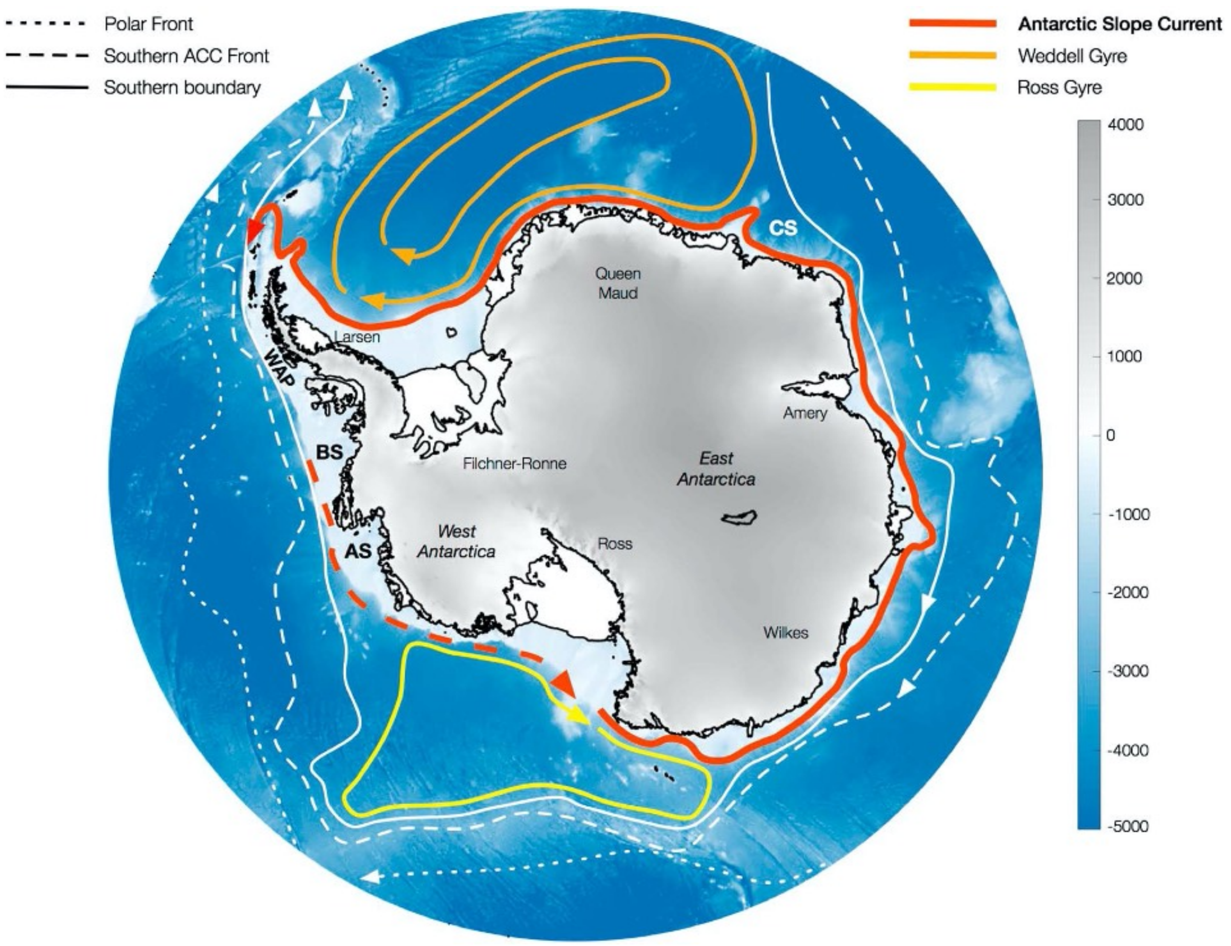


From an Australian proposal for the Antarctic Science Center of Excellence

How does relatively warm CDW reach to the base of ice-shelves?

What are the dynamical balances affecting the Antarctic Slope Current and how does it interact with the ACC?

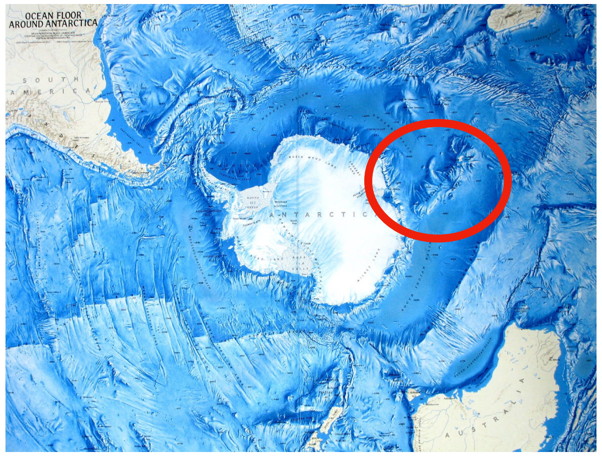
What will happen under climate change, with changing winds and buoyancy forcing affecting the ACC, the ASC, and the ice shelves?

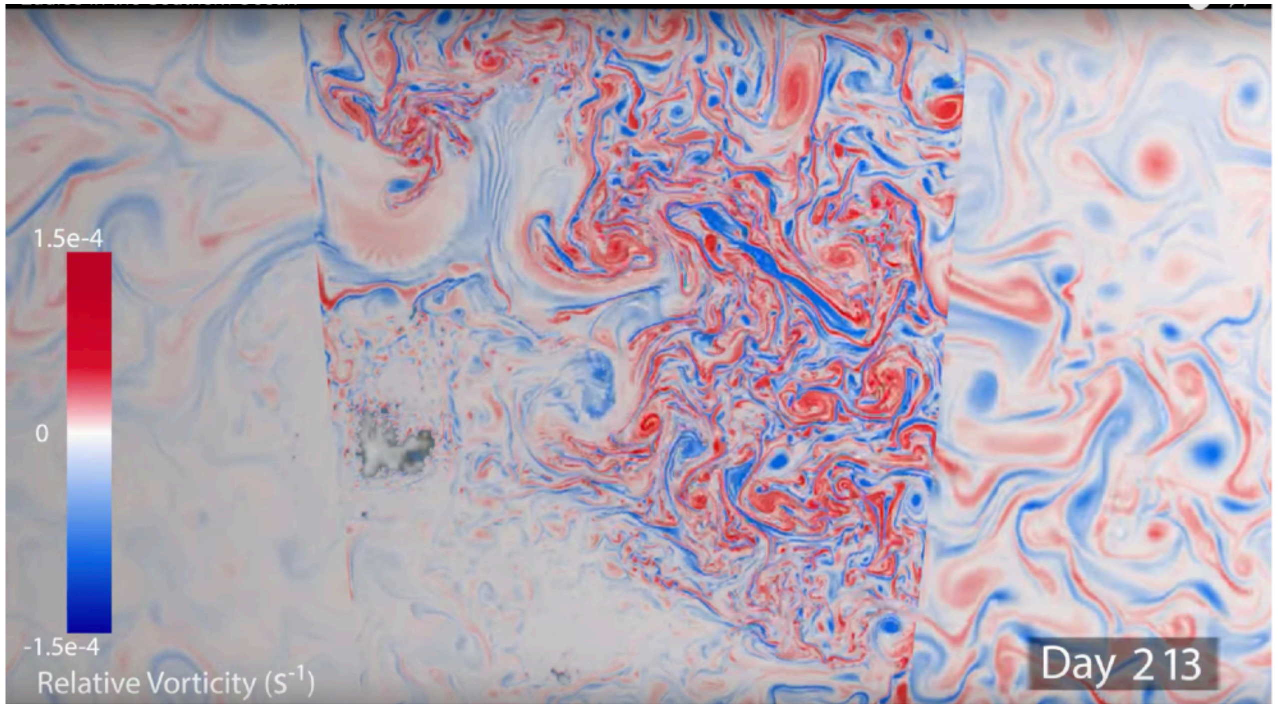


Thompson et al (2018)

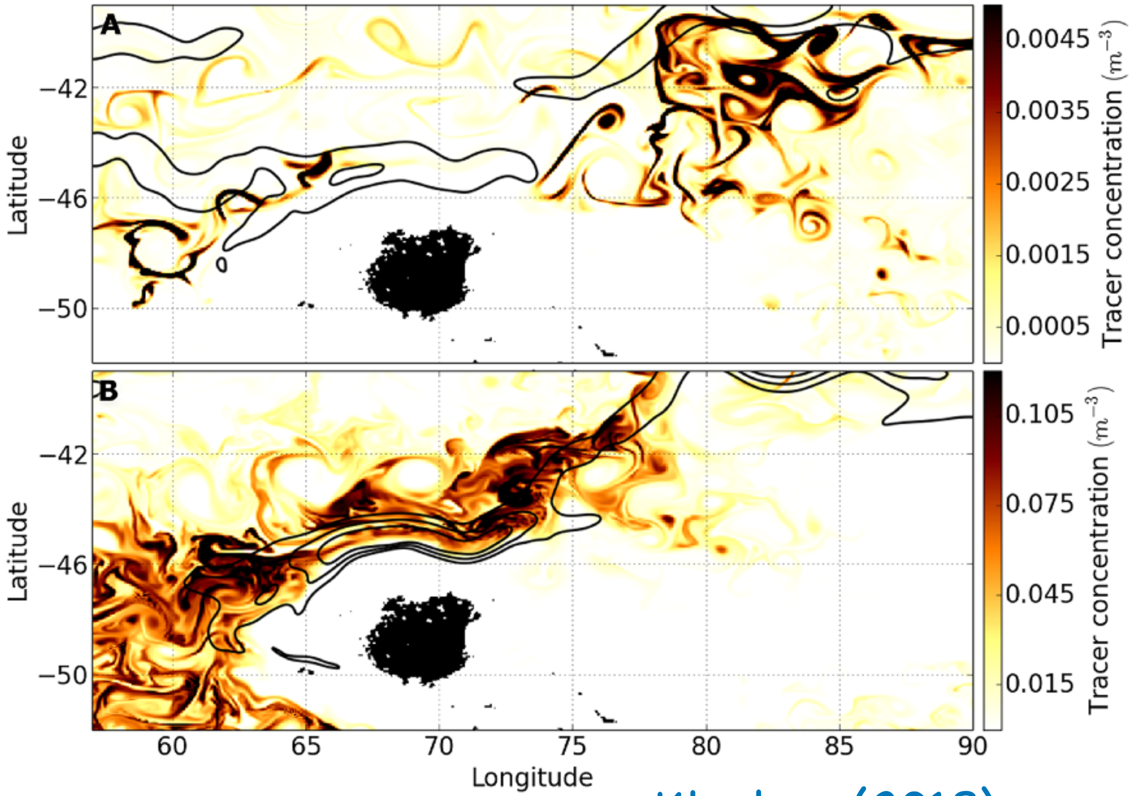
Mesoscale <—> Submesoscale

Here is a link to a movie from a numerical simulation illustrating the extremely energetic eddies found near Kerguelen Plateau, where eddies are highly energized and where bottom form stress is particularly strong.





Rosso et al (2015)



Klocker (2018)

Klocker (2018) argues that submesoscale jet features greatly enhance the vertical ventilation of tracers in the Southern Ocean down to ~500m-1000m depth, possibly affecting AAIW ventilation. He also argues that interior Reynolds stresses from these jets are NOT negligible, thus affecting the momentum balance in the Southern Ocean.

Math notation

Math notation I

$(x, y, z) =$ Cartesian triad with x=east, y= north, z=up

$\boldsymbol{F} = \boldsymbol{F}_{\text{body}} + \boldsymbol{F}_{\text{contact}} =$ force vector decomposed into body + contact forces

$\boldsymbol{v} = (u, v, w) =$ velocity vector

$\rho =$ *in situ* density

$\rho \, \mathrm{d}V =$ mass of a fluid element

$\boldsymbol{\tau} =$ stress vector (force per unit area)

$-2\,\boldsymbol{\Omega} \wedge \boldsymbol{v} \approx -2\,\hat{\boldsymbol{z}}\, f \wedge \boldsymbol{v} =$ Coriolis acceleration w/ $f = 2\,\Omega\,\sin\phi$ & $\phi =$ latitude

$g =$ effective gravitational acceleration $\approx 9.8\ \mathrm{m}^2\ \mathrm{sec}^{-2}$

$\int_{\mathcal{R}} \rho \, \mathrm{d}V =$ integrated mass over a fluid volume denoted by $\mathcal{R}$

$\oint_{\partial\mathcal{R}} \mathrm{d}\mathcal{S} =$ integrated area over the closed boundary of a fluid volume written as $\partial\mathcal{R}$

$\hat{\boldsymbol{n}} =$ vector of unit length defining the "outward" direction on a surface

$-(\rho\,\boldsymbol{v} \otimes \boldsymbol{v}) \cdot \hat{\boldsymbol{n}} = -\rho\,\boldsymbol{v}\,(\boldsymbol{v} \cdot \hat{\boldsymbol{n}}) =$ kinetic stress in direction $\hat{\boldsymbol{n}}$ (mostly ignored here)

$-p\,\hat{\boldsymbol{n}} =$ compressive pressure acting on a surface w/ outward norm $\hat{\boldsymbol{n}}$

$\nabla p =$ pressure gradient force per volume acting at point

$z = \eta(x, y, t) =$ vertical position of a surface such as an isopycnal

$z = \eta_b(x, y) = -H(x, y) =$ vertical position of bottom topography

$p_{\text{bot}} =$ bottom pressure $\quad p_{\text{atm}} =$ atmospheric pressure

Math notation II

$$\tau^x_{\text{form}} = -p_{\text{bot}}\,\partial_x H = \text{topographic form stress w/ } \partial_x H = \frac{\partial H}{\partial x} = \text{bottom x-slope}$$

$$\oint A\,\mathrm{d}x = R_{\text{earth}}\cos\phi \oint A\,\mathrm{d}\lambda = \text{zonal integral around the planet on a constant latitude circle}$$

$$\langle A \rangle = \frac{\oint A\,\mathrm{d}x}{\oint \mathrm{d}x} = \text{zonal mean of an arbitrary function } A$$

$$\langle \partial_x A \rangle = 0 \;\; \text{zonal mean of a zonal derivative vanishes if periodic in zonal direction}$$

$$p'_{\text{bot}} = p_{\text{bot}} - \rho_0\, g\, H = \text{bottom pressure relative to pressure at bottom of homogeneous fluid}$$

$$\boldsymbol{C} \wedge \boldsymbol{D} = \text{vector cross product}$$

$$\gamma = \text{neutral density}$$

$$b' = \text{anomalous buoyancy of a mesoscale eddy}$$

$$\eta' \approx -b'/N^2 = \text{eddy height fluctuation related to buoyancy fluctuation w/ } N^2 = \text{squared buoyancy frequency}$$

$$S^y = -\frac{\partial_y b}{\partial_z b} = -\partial_y b/N^2 = \text{meridional slope of a buoyancy (isopycnal) surface}$$