Ocean circulation as a problem in mathematical & computational physics: a historical and contemporary perspective

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Traditional lands of the Bedegal people of the Eora Nation

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Goals and Assumptions

- This talk offers a taste for the mathematics and physics encountered when studying ocean circulation:
  - ⋆ A rapid-fire historical introduction to geophysical fluid mechanics.
- With Australia's finest math/science students in the audience, I assume you wish to see a few of the details.
  - ⋆ Physical oceanography is a rich sub-discipline of mathematical & computational physics with sophisticated methods used to unravel understanding and improve predictions.
- We encounter animations from state-of-the-science computational fluid mechanics simulations of ocean flows.
  - ⋆ Eye-candy to illustrate the richness, complexity, and beauty of ocean flows.

Some maths/physics relevant to ocean circulation

- Linear & nonlinear partial differential equations of continuum mechanics;
- Geometrical/tensor methods to pose the governing equations on a rotating and stratified sphere;
- Numerical methods for computer simulations used as a tool for experimental investigations;
- Geometrical/tensor methods to analyze the space-time properties of complex and multi-scale flows emerging from the governing equations;
- Dynamical systems methods to conceptually frame multi-scale linear, nonlinear, and chaotic behaviour of currents, waves, instabilities, coherent structures, and turbulence.
- Statistical methods for robust analysis and probabilistic methods for prediction;
- Big data science tools to reveal underlying patterns to help diagnose physical processes from within the huge data streams

Animation: sea surface temperature and currents

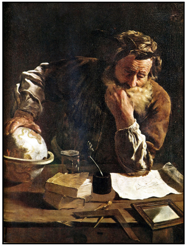
Computer model simulation of sea surface temperature from the NOAA/GFDL/Princeton climate model CM2.6 run on the Gaea supercomputer of the US Department of Energy.

Outline

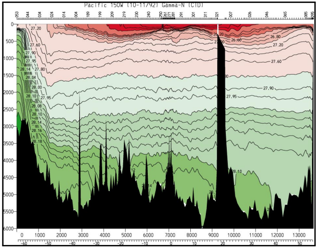
Outline

1. Building the math/physics framework

Archimedes of Syracuse: buoyancy



Archimedes (287 - c.212 BC)



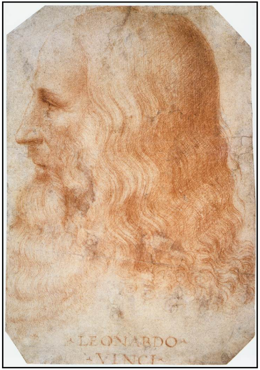
Pacific buoyancy (Wunsch & Ferrari 2004)

- “Eureka Moment”: an understanding of how the buoyancy force on an object in a liquid equals to the weight of displaced liquid.

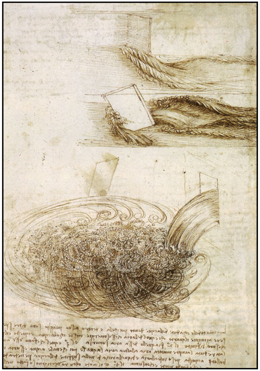
$$\boldsymbol{F}^{\text{buoyancy}} = \hat{\boldsymbol{z}}\, g\, \rho_{\text{liquid}}\, V_{\text{displace}}. \tag{1}$$

- Gravity stratifies according to buoyancy: light water above heavy.
- Buoyancy is built into the hydrostatic balance: $\mathrm{d}p/\mathrm{d}z = -\rho\, g$: pressure at a point equals to the weight per area above the point.

Leonardo di ser Piero da Vinci: visualizing fluid flow



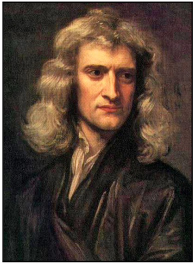
Leonardo da Vinci (1452-1519)



Flow around obstacles & entering a pool

- Leonardo's observations and visualizations of laminar and turbulent fluid flows were both insightful and prescient.
- Detail in his sketches suggest an understooding of vorticity & vortex tubes: $\boldsymbol{\omega} = \nabla \wedge \boldsymbol{v}$.

Newton & LaPlace: mechanics & math physics



Newton (1642-1727)

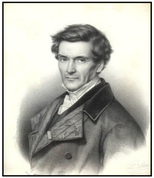
LaPlace (1749-1827)

- Calculus was formulated by Newton and LaPlace.
- Newton's 2nd Law of Motion: fundamental to all classical mechanical systems, including fluids

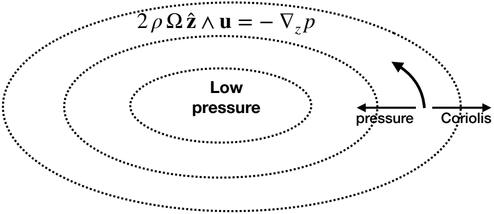
$$\boldsymbol{F} = \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} \overset{\text{M constant}}{=} M\,\frac{\mathrm{d}^2\boldsymbol{X}}{\mathrm{d}t^2} = M\,\boldsymbol{A} \qquad \left(\boldsymbol{P} = \frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t}\right). \tag{2}$$

- Both made fundamental discoveries in the theory of tides.

Coriolis: motion in a rotating reference frame



Coriolis (1792-1843)



Geostrophic balance around a northern hemisphere low

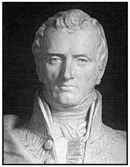
- Motion in a rotating reference frame introduces a non-inertial force that deflects motion to the right in northern hemisphere and left in the southern hemisphere

$$\boldsymbol{F}_{\text{Coriolis}} = -2\,\rho\,\boldsymbol{\Omega}\wedge\boldsymbol{v}. \tag{3}$$

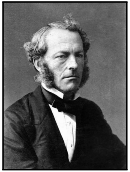
- Allows for steady *geostrophic balance* between pressure and Coriolis forces

$$\boldsymbol{F}_{\text{Coriolis}} + \boldsymbol{F}_{\text{pressure}} = 0 \Longrightarrow 2\,\rho\,\boldsymbol{\Omega}\wedge\boldsymbol{v} = -\nabla p. \tag{4}$$

Fluid dynamical equations for ocean motion



Navier (1785-1836)



Stokes (1819-1903)

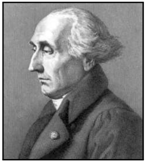
- Newton and LaPlace's calculus and mechanics is the foundation to the dynamical equation for a continuous fluid media.
- Adding Coriolis force yields the ocean dynamical equations

$$\underbrace{\frac{\partial \boldsymbol{v}}{\partial t}}_{\text{local acceleration}} + \underbrace{(\boldsymbol{v} \cdot \nabla)\,\boldsymbol{v}}_{\text{advection by flow}} + \underbrace{2\,\boldsymbol{\Omega} \wedge \boldsymbol{v}}_{\text{Coriolis}} = \underbrace{-\frac{1}{\rho}\nabla p}_{\text{pressure}} + \underbrace{\rho\, g\, \hat{z}}_{\text{gravity}} + \underbrace{\nabla \cdot \boldsymbol{\tau}_{\text{stress}}}_{\text{friction}} \tag{5}$$

- This equation is relevant for nearly all observed fluid motions from galaxies to blood.

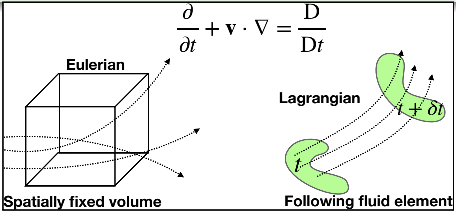
Euler and Lagrange: dual views of fluid motion

Euler (1707 - 1783)

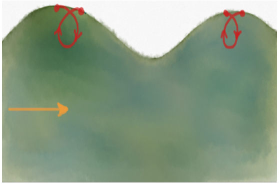


Lagrange (1736-1813)

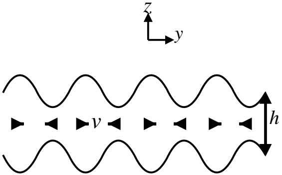
- Eulerian frame: fluid viewed/measured in a frame fixed in space (*laboratory frame*).
- Lagrangian frame: fluid viewed/measured in a frame fixed on a fluid element (*material frame*).



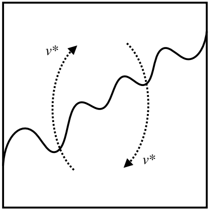
Transport by waves and eddies: Stokes Drift



Surface Stokes drift



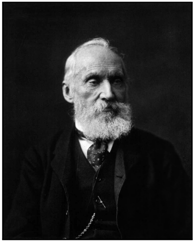
Layer thickness transport



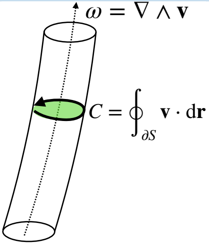
APE release

- Stokes identified how waves can move matter: *Stokes drift*.
  - Think of motion just outside the breaking surf-zone.
- Stokes drift leads to the “bolus” transport within buoyancy layers.
  - Think of how a snake swallows its meal.
- A generalized version of Stokes drift arises as sloping buoyancy surfaces flatten and release potential energy during baroclinic instability.

Kelvin and Helmholz: circulation and vorticity



Kelvin (1824-1907)



circulation & vorticity

Helmholtz (1821-1894)

- Circulation is related to vorticity via Stokes' Theorem

$$C = \oint_{\partial S} \boldsymbol{v} \cdot \mathrm{d}\boldsymbol{r} \overset{\text{Stokes Thm}}{=} \int_S (\nabla \wedge \boldsymbol{v}) \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S = \int_S \boldsymbol{\omega} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S. \qquad (6)$$

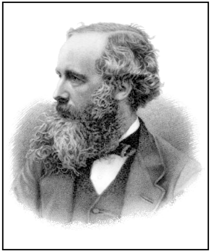
- Kelvin's Circulation Theorem for inviscid baroclinic flow:

$$\frac{\mathrm{D}C}{\mathrm{D}t} = \int_S \underbrace{\rho^{-2} \, (\nabla \rho \wedge \nabla p)}_{\text{baroclinicity}} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S. \qquad (7)$$

Animation: Kelvin-Helmholz instability leading to turbulent mixing

Computer simulation of Kelvin-Helmholz instability (shear instability) using code from the Dedalus Project (an open source partial differential equation solver using spectral methods).

Maxwell and Gibbs: Thermodynamics



Maxwell (1831-1879)

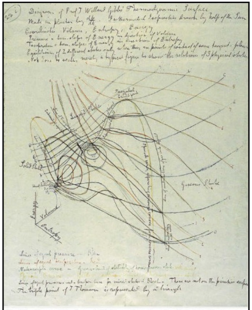
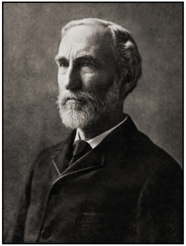


Plate IV. Diagram of the lines on Gibbs' thermodynamic surface, 8 July 1875 (Number 564).

Maxwell: H_2O P-V-T phase diagram

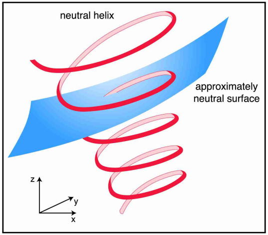


Gibbs (1839-1903)

- Thermodynamics provides relations between energy, entropy, enthalpy, heat, work, etc.
- Developed a geometric view of thermodynamic phase spaces.

McDougall: seawater thermodynamics

McDougall (UNSW professor)



Neutral helicity

- Applications of thermodynamics to seawater were surprisingly simplistic until the seminal work of McDougall and colleagues during 20th and 21st centuries.
- Pioneered studies into the geometry and topology of seawater thermodynamics and ocean mixing.

Bjerknes, Rossby, Charney: geophysical fluid mechanics

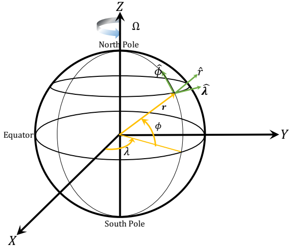
Bjerknes (1862-1951)

Rossby (1898-1957)

Charney (1917-1981)

- Bjerknes: atmospheric motion can be described using laws of thermodynamics + fluid mechanics.
- Rossby: wave patterns seen on weather maps arise from latitudinal dependence of the Coriolis force found on a spherical planet: *Rossby waves*.
- Charney (also Eady): *baroclinic instability* = hydrodynamic instability feeding off available potential energy of a rotating stratified fluid: space-time scale for weather patterns; ocean eddies; macro-turbulence.

Ocean mechanics = thermo-fluid mechanics on a rotating & gravitating sphere



$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} + 2\,\boldsymbol{\Omega} \wedge \boldsymbol{v} = -g\,\hat{\boldsymbol{z}} - \rho^{-1}\,\nabla p + \boldsymbol{F} \qquad \text{momentum (dynamics)} \qquad (8)$$

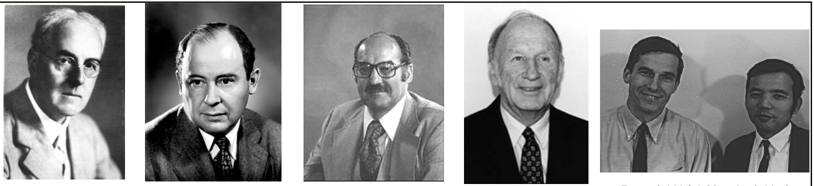
$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho\,\nabla \cdot \boldsymbol{v} \qquad \text{continuity (kinematics)} \qquad (9)$$

$$\rho\,\frac{\mathrm{D}\theta}{\mathrm{D}t} = -\nabla \cdot \boldsymbol{J}(\theta) \qquad \text{enthalpy (thermodynamics)} \qquad (10)$$

$$\rho\,\frac{\mathrm{D}S}{\mathrm{D}t} = -\nabla \cdot \boldsymbol{J}(S) \qquad \text{salinity (matter)} \qquad (11)$$

$$\rho = \rho(S,\theta,p). \qquad \text{equation of state (thermodynamics)} \qquad (12)$$

General circulation project



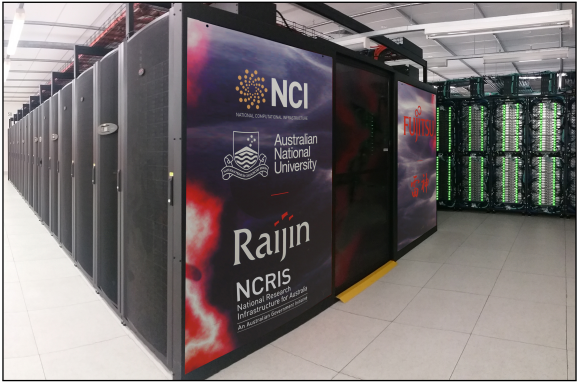
Richardson (1881-1953) | von Neuman (1903-1957) | Smagorinsky (1924-2005) | Lorenz (1917-2008) | Bryan (1929-) & Manabe (1931-)

- Developed numerical methods to turn the continuum partial differential equations into discrete algebraic equations suitable for computer simulations.
- Developed *filtered* equations that target waves and eddying patterns relevant for weather and *general circulation*.
- Exposed the chaotic nature of weather and macro-turbulence (chaos theory).
- Computer models provide an experimental tool to study the atmosphere and ocean.

Outline

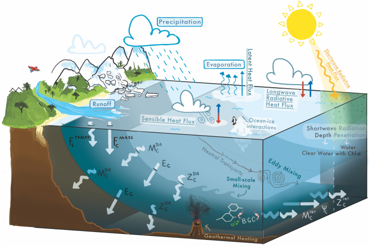
2 Emergent patterns and fluctuations

Animation: fluid dynamics in the Southern Ocean



Computer model simulation of flow regimes in the Southern Ocean from the Australian ACCESS-OM2 0.1 degree ocean/sea-ice model run on the NCI Raijin supercomputer in Canberra, AUS.

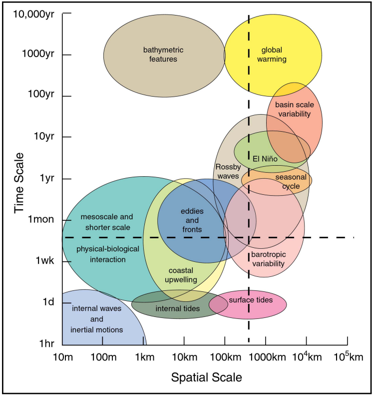
There’s a zoo of physical ocean processes



Groeskamp et al (2019)

- The ocean is a forced-dissipative system with cascades of energy across space-time scales.
- Enhanced resolution of observations (e.g., satellite) and simulations generally reveals refined layers of phenomenology.
  - ⋆ The more we look, the more we see!
- Direct implications for representation and parameterization of the ocean’s role in climate and ecosystems.

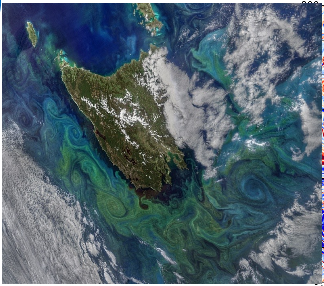
Space-time diagram of ocean dynamical processes



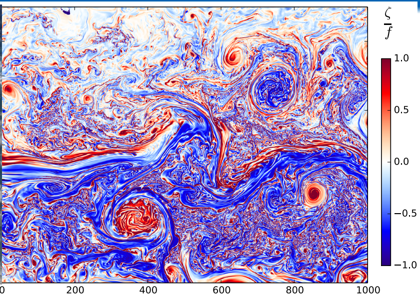
Chelton (2001)

- Broad range of space-time scales.
- Absence of a clear spectral gap a result of nonlinear cascade of turbulent flows across scales.
  - ⋆ There is no desert.
- The absence of a spectral gap introduces some difficult problems to the turbulence closure problem in geophysical flows.

Macro-scale turbulence: mesoscale + submesoscale



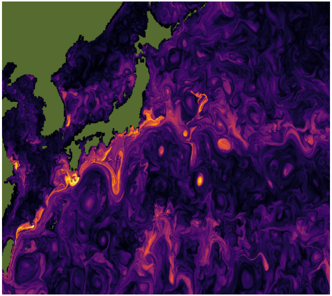
Phytoplankton bloom from NASA GSFC



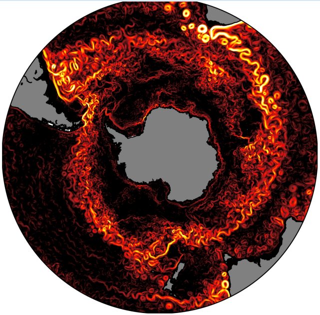
Simulation (Gula et al 2015) ($\Delta = 500$m)

- Satellite measurements reveal the multi-scale structure of ocean macro-turbulence.
- Simulations allow us to study the mechanisms.
  - ⋆ Primary eddy features ("mesoscale eddies") initiate secondary "submesoscale instabilities" that lead to filaments along the edges of the eddies.
  - ⋆ Submesoscale eddies and fronts have strong vertical motions that bring deep nutrients into the upper photic (light) zone.

Coherent structures + turbulent soup = order in chaos



LAVD field from Tarshish et al (2018)



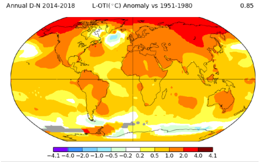
Surface current speed (courtesy A. Morrison, ANU)

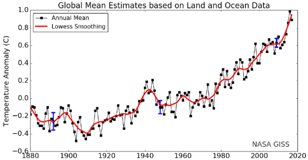
- We see coherent structures within a background turbulent soup.
  - ⋆ Quasi-geostrophic turbulence cascades energy to large scale.
  - ⋆ In some regions the cascade focuses on coherent structures: Cascade animation from R. Abernathey, Columbia Univ.
  - ⋆ Maximize information entropy while conserving energy.
- Non-autonomous dynamical systems methods to objectively identify coherent structures and measure impacts on transport.

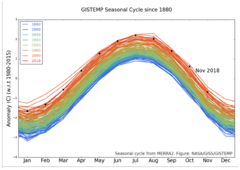
Outline

3 Climate change and sea level

Earth is warming due to fossil fuel burning (IPCC)



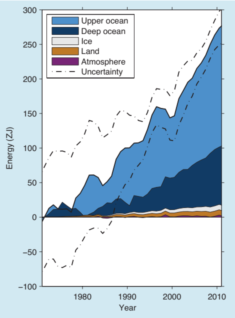




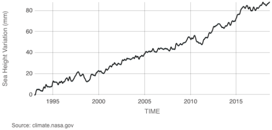
NASA analysis

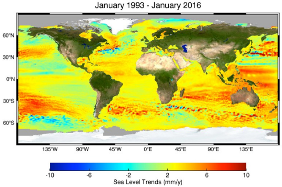
- Consistent pattern of global and regional warming of surface temperatures.
- Each season is warming.
- Northern high latitudes are warming the most, reflecting ice-albedo feedback as a result of melting Arctic sea-ice.
- North Atlantic cooling might reflect slow-down of Atlantic overturning circulation.

Global warming = ocean warming ⇒ sea level rises



IPCC AR5





NASA analysis

Anny Cazanave

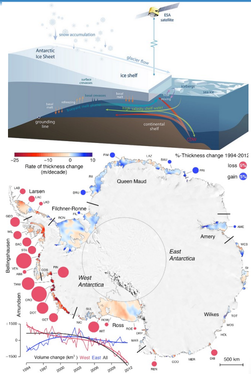
John Church

Jonathan Gregory

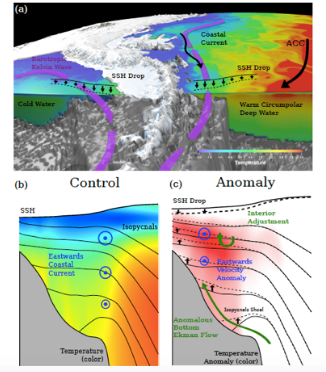
Sea level scientists

- Ocean warming raises sea level (thermosteric + land ice melt)
- Regional patterns associated with ocean dynamical processes: winds, regional heating, changes to circulation.
- Since 1970, ocean is heating at a rate equivalent to the power released by roughly 3 to 6 Hiroshima bombs per second.

Winds, waves, and warming Antarctic ice shelves



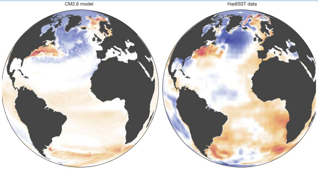
Paolo et al (2015)

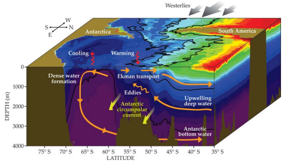


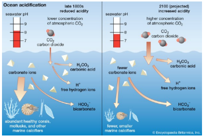
Spence et al (2017)

- Complex array of physical processes at the base of ice shelves.
- Broad trends in ice shelf melting over recent 20 years.
- Models and theory suggest that winds and waves can act to alter interior pressure gradients that in certain regions lead to movement of warm off-shore water towards ice shelves.

Further big questions of ocean climate science







Srokosz (2012) | Caesar et al (2018) | Morrison et al (2015) | Encyclopaedia Britannica

- Slowdown of Atlantic overturning circulation, with impacts on poleward/vertical heat and carbon transport, European climate, and North American east coast sea level;
- Southern Ocean ventilation of heat and carbon with possible changes due to changing winds, precipitation, and heating;
- Ocean heat waves, acidification, and impacts on marine life.

Outline

4 Closing comments

Summary

- Ocean circulation emerges from the underlying thermo/mechanical equations for a fluid on a gravitating and rotating sphere.
- Ocean circulation studies involve a synergy between observations (satellites, ships, floats), theory (maths, physics), and simulations (numerics, computer science).
- Simulations provide oceanographers with an experimental tool to reveal a mechanistic understanding of the complex multi-scale fluid flow.
- Civilization is presenting the earth with a (relatively new) geophysical force whose impacts are felt by the ocean and atmosphere:
  - ⋆ The carbon burning experiment.
  - ⋆ Humanity does not control climate but we do affect it.
  - ⋆ Unraveling the mechanisms for both natural and anthropogenic fluctuations in the ocean/atmosphere climate system is fundamental to 21st century earth science.

Some words for the aspiring mathematical scientist

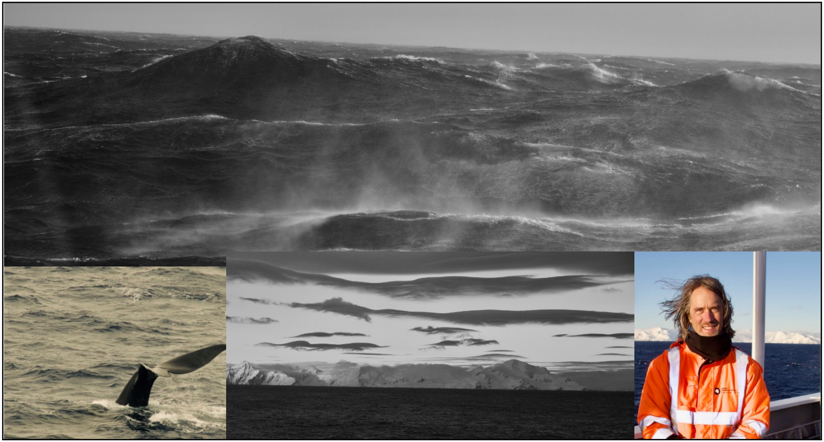
- ALBERT EINSTEIN: The important thing is not to stop questioning. Curiosity has its own reason for existence.
- R. BUCKMINSTER FULLER: You never change things by fighting the existing reality. To change something, build a new model that makes the existing model obsolete.
- GAUTAMA BUDDHA: Do not believe in anything simply because you have heard it. Do not believe in anything merely on the authority of your teachers and elders. But after observation and analysis, when you find that anything agrees with reason and is conducive to the good and benefit of one and all, then accept it and live up to it.
- F. SCOTT FITZGERALD: One should be able to see that things are hopeless and yet be determined to make them otherwise.

A suite of further cutting-edge animations

- ⋆ Process studies using Dedalus
- ⋆ Multi-scale flows from J. Gula using ROMS
- ⋆ Atlantic simulations from NEMO at < 2km grid spacing
- ⋆ NASA global surface circulation using MITgcm

Many thanks for your time and attention

It has been an honour & pleasure to prepare & to present this lecture.



Photos from the Weddell Sea and Scotia Sea, autumn 2017 on the RRS Ross