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**MAT220 - Ass. 3**

**Part A:**

1. **5.1 - Linear independence of stacked vectors. Consider the stacked vectors**

**c1 = , . . . , ck = ,**

**where a1, . . . , ak are n-vectors and b1, . . . , bk are m-vectors.**

1. **Suppose a1, . . . , ak are linearly independent. (We make no assumptions about the vectors b1, . . . , bk.) Can we conclude that the stacked vectors c1, . . . , ck are linearly independent?**

β1 = … = βk = 0

β1 + … + β1 =

+ … + β1 =

=

β1c1 + … + βkck = 0 and β1 = … = βk = 0 so the vectors c1, …, ck are linearly independent

1. **Now suppose that a1, . . . , ak are linearly dependent. (Again, with no assumptions about b1, . . . , bk. Can we conclude that the stacked vectors c1, . . . , ck are linearly dependent?**

a1 = , a2 = are linearly dependent 2a1 – a2 = 0

b1 = , b2 =

c1 = = , c2 =

With this example it can be seen that even if a1, . . . , ak are linearly dependent, it does not guarantee that the stacked vectors c1, . . . , ck are linearly dependent.

1. **6.2 – Matrix notation. Suppose the block matrix**

**makes sense, where A is a p x q matrix. What are the dimensions of C?**

= p x q = p x p = q x q

p

= q x p

p

q

q

q

q

p

p

1. **6.3 – Block Matrix. Assuming the Matrix:**

**K =**

**Makes sense, which of the following statements must be true? (‘Must be true’ means that it follows with no additional assumptions.)**

1. **K is a square - True**

A = AT = B = BT =

= 4 x 4 B = = 5 x 5

1. **A is a square or wide – False, A can be tall as well**

= 5 x 5

1. **K is symmetric, i.e. KT = K – False**

K = KT =

1. **The identity and zero submatrices in K have the same dimensions – False**

The two 5 x 5 matrices show that they are not always the same. If A is not a square, then the identity and zero submatrices will not have the same dimensions.

1. **The zero submatrix is a square – True**

It can be seen from the examples above that the zero submatrix is always a square. Because AT bottom side dimension is the same as the right side dimension of A.

1. **6.6 – Matrix-vector multiplication. For each of the following matrices, describe in words how x and y = Ax are related. In each case x and y are n-vectors, with n = 3k.**
2. **A =**  = 3k x 3k

x = = 3k x 1 where

y = = = =

y is the reverse order of the elements of x, where y1 = x3k, … y3k = x1.

1. **A =** , where E is the k x k matrix with all the entries 1/k.

x = = 3k x 1 where E = = k x k

y = = =

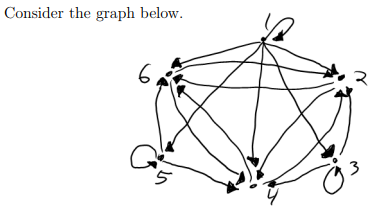
= = = 1/k

Each element of y is the k x k matrix of x’s multiplied by 1/k.

1. **6.7 – Currency exchange matrix. We consider a set of n currencies, labeled 1, . . . , n. (These might correspond to USD, RMB, EUR, and so on.) At a particular time the exchange or conversion rates among the n currencies are given by an n × n (exchange rate) matrix R, where Rij is the amount of currency i that you can buy for one unit of currency j. (All entries of R are positive.) The exchange rates include commission charges, so we have RjiRij < 1 for all i ≠ j. You can assume that Rii = 1.**

**Suppose y = Rx, where x is a vector (with nonnegative entries) that represents the amounts of the currencies that we hold. What is yi? Your answer should be in English.**

yi is the total amount of currency i obtained when all money in other currencies is exchanged to currency i and accounts for the commission charges.

**Part B:**

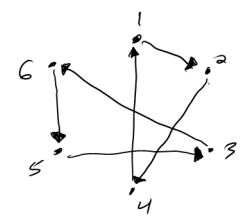
* 1. **Find the matrix representation of the graph.**

1 2 3 4 5 6

= 6 x 6

* 1. **Find the density of the graph.**

nnz(A)/mn = 18/(6)(6) = 18/36 = 1/2

1. **Consider the graph below.**
   1. **Find the matrix representation of the graph.**

1 2 3 4 5 6

= 6 x 6

* 1. **Find the density of the graph.**

nnz(A)/mn = 6/(6)(6) = 1/6

1. **Determine whether the vectors are linearly dependent or linearly independent. Explain why your answer is correct.**
   1. **u = (3, 4), v = (−6, −8)**

+ α2 = 0 🡪 -2 + 1

Linearly dependent because v = -2u

* 1. **u = (1, 5), v = (2, 3)**

α1 + α2 =

α1 + 2α2 = 0 🡪 α1 = -2α2

5α1 + 3α2 = 0 🡪 5(-2α2) + 3α2 = -10α2+ 3α2 = 0 🡪 -7α2 = 0 🡪 α2 = 0

α2 = 0 🡪 α1 + 3(0) = 0 🡪 α1 = 0

The only solution for this system is all zeros, which means the two vectors are linearly independent. There is nothing to multiply one vector by to get the other.

* 1. **u = (1, 0, 1), v = (0, 2, 3), w = (4, 5, 0)**

α1u + α2v + α3w = 0

α1 + α2 + α3 =

(α1, 0, α1) + (0, 2α2, 3α2) + (4α3, 5α3, 0) = (0, 0, 0)

(α1 + 0 + 4α3, 0 + 2α2 + 5α3, α1 + 3α2 + 0) = (0, 0, 0)

α1 + 4α3 = 0 🡪 α1 = -4α3

2α2 + 5α3 = 0 🡪 α2 = -5/2α3

α1 + 3α2 = 0 🡪-4α3 + 3(-5/2α3) = 0 🡪 -4α3 + -15/2α3 = 0 🡪 -23/2α3 = 0

α3 = 0 🡪 α1 + 4α3 = 0 🡪 α1 + 4(0) = 0 🡪 α1 = 0

α3 = 0 🡪 2α2 + 5α3 = 0 🡪 2α2 + 5(0) = 0 🡪 2α2 = 0

α1 = α2 = α3 = 0, therefore linearly independent

* 1. **u = (1, 1, 2), v = (1, 0, −3), w = (5, 2, −5)**

α1u + α2v + α3w = 0

α1 + α2 + α3 =

(α1, α1, 2α1) + (α2, 0, -3α2) + (5α3, 2α3, -5α3) = (0, 0, 0)

(α1 + α2 + 5α3, α1 + 0 + -3α3, 5α1 + 2α2 + -5α3) = (0, 0, 0)

α1 + α2 + 5α3 = 0 🡪 3α3 + α2 + 5α3 = 0 🡪 α2 = -8α3

α1 + 0 + -3α3 = 0 🡪 α1 = 3α3

5α1 + 2α2 + -5α3 = 0 🡪5(3α3) + 2(-8α3) – 5α3 = 0 🡪 15α3 – 16α3 – 5α3 = 0 🡪 -6α3 = 0

α3 = 0 🡪 α1 + α2 + 5(0) = 0 🡪 α1 + α2 = 0 🡪 0 + α2 = 0 🡪 α2 = 0

α3 = 0 🡪 α1 + 0 + -3(0) = 0 🡪 α1 = 0

α1 = α2 = α3 = 0, therefore linearly independent

1. **Find the dimension of the subspace generated by the given vectors (the dimension of the span of the vectors or, in yet other words, the dimension of the subspace of all linear combinations of the vectors). Argue why your answer is correct. (Hint: These are the same vectors as the previous problem, so you have already shown some work in this direction.)**
   1. **u = (3, 4), v = (−6, −8)**

basis: (3, 4) or basis: (-6, -8)

1 because u and v and linearly independent and are in the same dimension.

* 1. **u = (1, 5), v = (2, 3)**

basis: (1, 5), (2, 3)

2 because u and v are both linearly independent subsets

* 1. **u = (1, 0, 1), v = (0, 2, 3), w = (4, 5, 0)**

basis: (1, 0, 1), (0, 2, 3), (4, 5, 0)

3 because u, v, and w are all linearly independent.

* 1. **u = (1, 1, 2), v = (1, 0, −3), w = (5, 2, −5)**

basis: (1, 1, 2), (1, 0, -3), (5, 2, -5)

3 because u, v, and w are all linearly independent.

1. **Let A =, B = , and v = . Calculate the following:** 
   1. **½B**

½ =

* 1. **2B − 3A**

2 – = – =

* 1. **AT**

T =

* 1. **Av**

= = =

* 1. **BTv**

T = = = =

* 1. **||A||**

||||= √(32 + 12 + 52 + 22) = √(9 + 1 + 25 + 4) = √(39)

* 1. **||ATv||**

||T|| = |||| = = =

= |||| = √(12 + (-1)2) = √2

* 1. **||B + 2I||**

|| + 2|| = ||+ || = ||||

= |||| = √((-4)2 + 02 + 02 + 12) = √(16 + 1) = √(17)