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The Fourier Transform

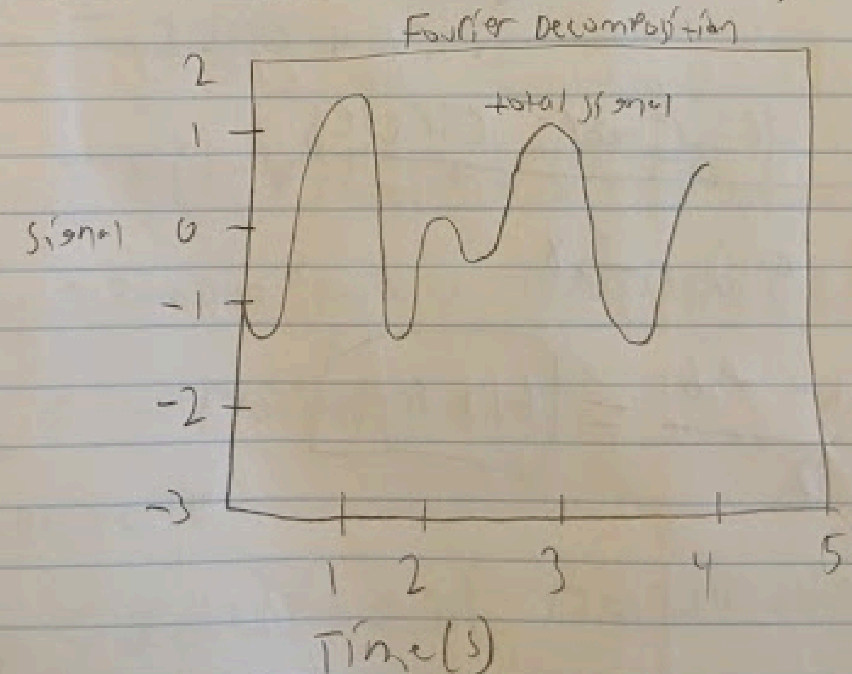
The goal is to give a general introduction to the Fourier transform and how it is typically implemented numerically so as to provide a basis for the ideas we need

$$y(t) = \sum_{j=1}^{5} y_j \sin(2\pi f_j t + \phi_j)$$

y_j is the amplitude, f_j is the frequency, ϕ_j is the phase of the jth sine wave component.

The signal above is pretty simple. Therefore it can be decomposed into a collection of sine waves.

Nearly 200 years ago, Joseph Fourier showed that virtually any signal can be written this way.



The next issue to consider is how to actually compute a fourier transform. In numerical work we are almost never given the analytic form of the signal, but instead have knowledge of its amplitude at certain discrete values of t.

$$y_m = \frac{1}{N} \sum_{n=0}^{N-1} Y_n e^{-2\pi i m n/N}$$

$$Y_n = \sum_{n=0}^{N-1} y_n e^{2\pi i m n/N}$$

The forward and inverse discrete Fourier transforms are related via

$$\sum_{n=0}^{N-1} e^{2\pi i n(m-m')/N} = N \delta_{m,m'}$$

The discrete fourier transforms are just a sum of exponential terms, so it appears to be very amenable to numerical evaluation.