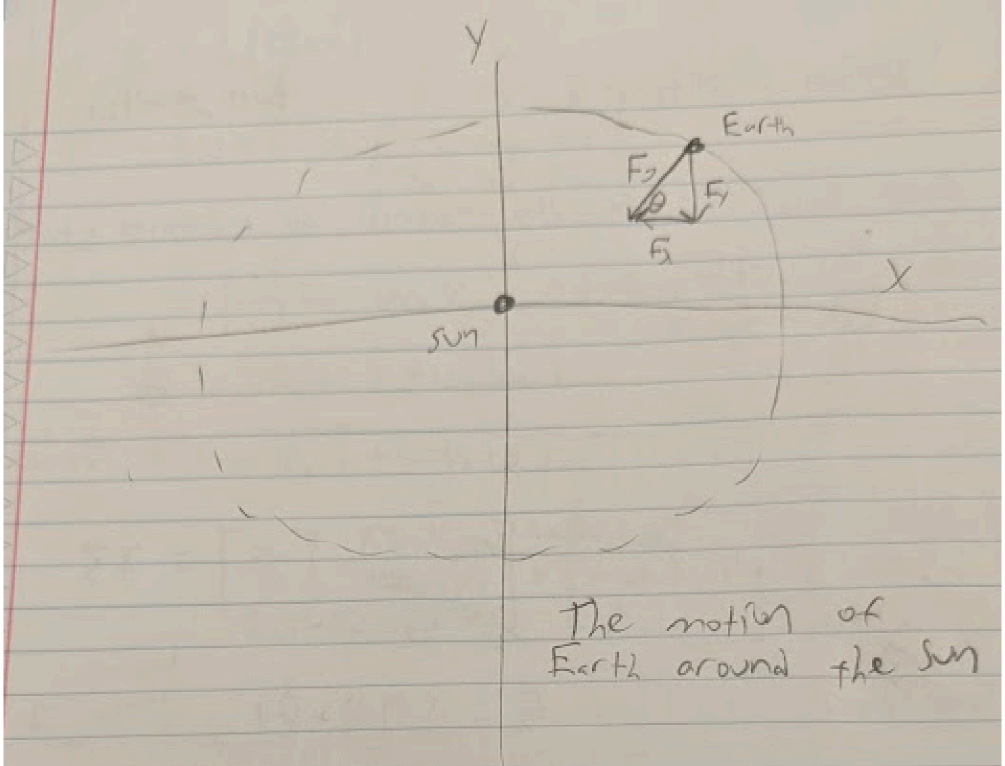
Stephen Martino
3/2/24

PHYS250
Dr. Finn

Chapter 4 Notes

Kepler's Laws

Equation 4.1: FG = (G*MS*ME)/r^2 where MS and ME are the masses of the Sun and Earth, and G is the gravitational constant.



The motion of Earth around the Sun

Newtons second law of motion gives us:

$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E} \qquad \frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E}$$

from the figure above we have:

$$F_{G,x} = -\frac{G M_S M_E}{r^2}\cos\theta = \frac{G M_S M_E x}{r^3}$$

Second order diff eq's

$$\frac{\partial v_x}{\partial t} = \frac{-G m_s x}{r^3} \qquad \frac{\partial v_y}{\partial t} = \frac{-G m_s y}{r^3}$$

$$\frac{\partial x}{\partial t} = v_x \qquad \frac{\partial y}{\partial t} = v_y$$

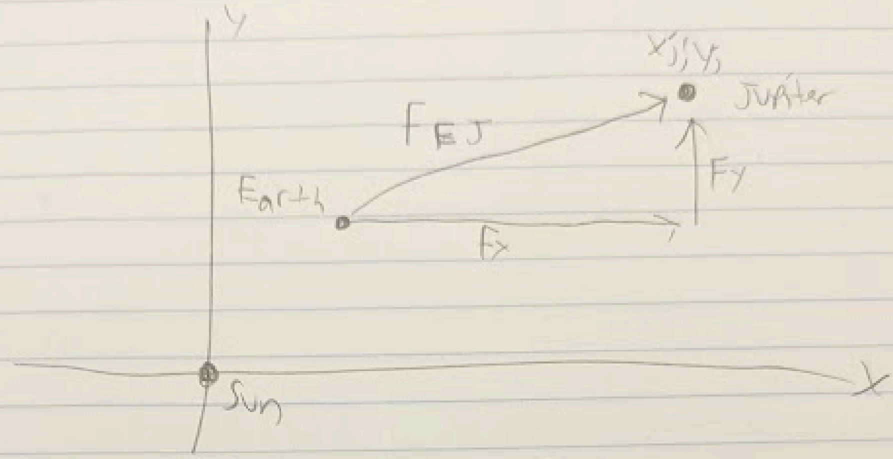
In analyzing planetary motion, it is very useful to visualize the movement of the planet graphically. This can be done by plotting the position of the planet as it becomes available during the calculation.

For a two-body system, all three of Kepler's laws are consequences of the fact that the gravitational force follows an inverse-square law. We consider a two-body system in which the interaction force depends only on the separation r. The relative motion in this system can be studied as if it were a one body problem.

Most of the planets have orbits that are very nearly circular. The planets whose orbits deviate the most from circular are Mercury and Pluto.

To this point all of our planetary simulations have involved two-body solar systems. Now we can consider some of the things that happen when there are three or more objects in the solar system. The problem of two objects interacting through the inverse-square law can be solved exactly, leading to Kepler's laws. However, if we add just one more planet to give what is known as the three body problem, an analytic theory becomes much more difficult.

FE,J = G MJ ME/(r^2*EJ)



components of the gravitational force due to Jupiter, located at x_j, y_j with Earth at x_e, y_e

writing F_{EJ} in terms of components yields

$$F_{EJ_x} = -\frac{G m_J m_E}{r^2_{EJ}} \cos\theta_{EJ} = -\frac{G m_J m_E (x_e - x_j)}{r^3_{EJ}}$$

It is interesting to see what would happen if the mass of Jupiter were increased to 100 MJ or even to 1000 MJ. However, since 1000 MJ is about equal to the mass of the Sun, the perturbation by Jupiter on the Sun would then be significant and would have to be taken into account.