

### Chapter 3 Notes

Examples of oscillatory phenomena can be found in many areas of physics, including the motion of electrons in atoms, the behavior of currents and voltages in electronic circuits, and planetary orbits. In the next several sections, we will explore some of the interesting effects that occur in real oscillatory systems. We begin with a simple pendulum and consider how to treat simple harmonic motion numerically. We then generalize our pendulum model to include the effects of friction and nonlinearities and find that they give rise to the possibility of chaotic behavior. One example of a simple pendulum is a particle of mass  $m$  connected by a massless string to a rigid support. We let  $\theta$  be the angle that the string makes with the vertical and assume that the string is always taut. We also assume that there are only two forces acting on the particle, gravity and the tension of the string. It is convenient to consider the components of these forces parallel and perpendicular to the string. The parallel forces add to zero, since we assume that the string doesn't stretch or break. The equation of motion describes a frictionless pendulum, so we begin by adding some damping. The manner in which friction enters the equations of motion depends on the origin of the friction. Possible sources of friction include the effective bearing where the string of the pendulum connects to the support, air resistance, etc. In many cases this damping force is proportional to the velocity, and that is the assumption we will make here. The frictional force we will employ thus has the form  $-q(d\theta/dt)$ , since the velocity of the pendulum is  $L(d\theta/dt)$ . Here  $q$  is a parameter that measures the strength of the damping, and the minus sign guarantees that this force will always oppose the motion of the pendulum. Now that we have a numerical method that is suitable for various versions of the simple pendulum problem, and armed with some understanding of what might or might not happen when dissipation, an external driving force, and or nonlinearity is present, we are ready to take on a slightly more complicated and also more interesting situation.