ENGR2240 Lab 4 Laplace and Reactive Circuits Name:____________________________

ABSTRACT

The Laplace Transform is a convenient method to solve some ODEs by transforming them from a differential equation in the time domain to an algebraic equation in the frequency domain. We can then work with and manipulate the algebraic equation to get a solution and transform back to the time domain to obtain our solution as a function of time.

INTRODUCTION

The response of an electrical circuit with reactive elements (capacitors and inductors) can be described using Laplace transforms to solve the differential equation. The circuit can be analyzed using traditional circuit techniques but with the impedances for the components shown below in Fig. 1 (ie: jω terms become s).

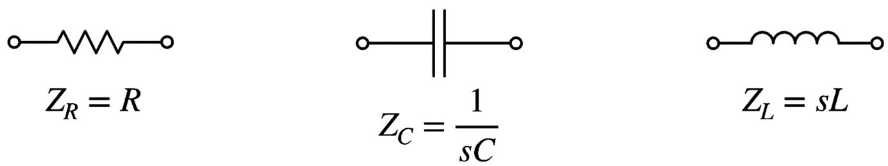

Figure 1: Impedances of circuit components in the Laplace domain (s can also be written as jω in the time domain)

The transfer function is then determined as $H(s)=V_o/V_{in}$. One can then use the Laplace of the input and the transfer function to obtain an equation for the Laplace of the solution: $V_o(s) = H(s)V_{in}(s)$. An inverse Laplace will then yield the time domain solution for the output voltage.

For example, in the RC circuit below in Fig 2, we can use Ohm's law or voltage division to write the transfer function in terms of R, C and s.

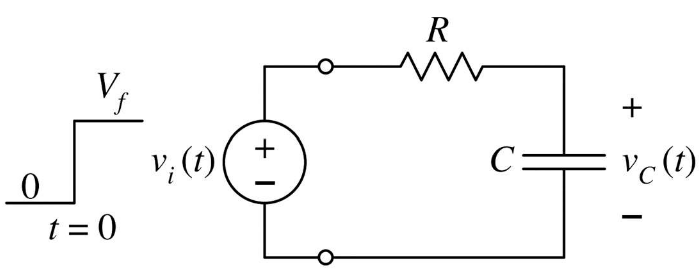

Figure 2: A series RC circuit

$$H(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{RCs + 1} = \frac{1}{RC(s + \frac{1}{RC})} \qquad \text{Eq. 1}$$

For a square wave or pulse input: $V_{in}(t) = V_f u(t-0) \quad \therefore \quad V_{in}(s) = V_f \frac{e^{0*t}}{s} = \frac{V_f}{s}$ Eq. 2

$$\text{Then, } v_o(t) = \mathcal{L}^{-1}\left\{\left(\frac{V_f}{s}\right)\left(\frac{1}{RC(s+\frac{1}{RC})}\right)\right\} \qquad \text{Eq. 3}$$

Here, partial fractions or even the convolution integral could be employed to take this inverse Laplace.

For a sinusoidal input, the input equation changes slightly to:

$$V_{in}(t) = Asin(\omega t) \text{ and thus } V\_in\ (s) = A\omega/(s^2 + \omega^2), \qquad \text{Eq. 4}$$

but the transfer function remains unchanged. This circuit with a sinusoidal input is a common passive filter. The output over the capacitor passes signals with low frequency and stops (or attenuates) high frequencies. This means that at low frequencies the output is high while at high frequencies is near zero. The "**cutoff frequency**" we define as the point at which the output is $\frac{1}{\sqrt{2}}(Max\ Value)$ which is also when it is reduced by 3 decibels from the maximum output.

A decibel is a unit of measure that utilizes the logarithmic scale. The plot of magnitude of the transfer function in decibels as well as the phase shift is called the Bode Plot. To generate a Bode Plot, the equations below are used:

$$Magnitude = -20\log_{10}\left|\frac{V_o}{V_{in}}\right| = |H(j\omega)| \qquad \text{Eq. 5}$$

$$Phase = \arg\ H(j\omega)$$

Using Python to obtain Bode Plots:

Numerically we can solve this with the help of Python packages such as SciPy and the module Signal.

```
from scipy import signal
import matplotlib.pyplot as plt
```

We will use the coefficients of the transfer function to enter into the command:

```
sys = signal.TransferFunction([A, B, C], [D, E, F])
```

where A-E are the constant coefficients of the numerator (first square bracket) and denominator (second square bracket) of the transfer function which has the form $H(s) = \frac{As^2+Bs+C}{Ds^2+Es+F}$

We then use the signal.bode command to generate the magnitude and phase based on this transfer function using:

```
w, mag, phase = signal.bode(sys)
```

From there it is simply a matter of plotting the magnitude (mag) and phase as we have done in previous labs. We will use a semilog plot (logarithmic in frequency on the x-axis)

```
plt.figure()
plt.semilogx(w, mag)    # Bode magnitude plot
plt.figure()
plt.semilogx(w, phase)  # Bode phase plot
plt.show()
```

PRELAB CALCULATIONS:

1. Use convolution to analytically solve Eq. 3 above for the circuit components given in Fig. 3 below:

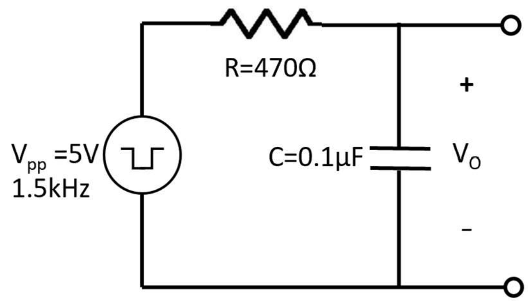

Figure 3: Circuit 1, a series RC with a square wave input

2. Repeat questions 1 using partial fractions and the Laplace table to analytically solve Eq. 3 for the circuit values given in Fig. 3.

EXPERIMENT:

3. Construct the circuit shown in Fig. 3 and display both the input and the output on the oscilloscope (Ch1 and Ch2 respectively).
    a. Zoom in so that you can easily see one full cycle. Take a picture or save data to a flash drive.
    b. Verify that your solution from the prelab is seen experimentally by sampling several data points.

4. Switch the input to a sinusoidal waveform. Set the frequency to 200Hz
    a. Measure the voltage of the output waveform using the "Measure" function on the oscilloscope. Do not worry about the phase – you will explore this in circuits classes.
    b. Repeat the measurement at frequencies 500Hz, 1000Hz, 2000Hz, 5000Hz, 10kHz, 20kHz, and 100kHz. Plot these against a logarithmic scale for frequency.
    c. Determine the cutoff frequency at which your output is $\frac{1}{\sqrt{2}}(Max\ Value)$
    d. Compare to the analytical value of the cutoff frequency given by $\frac{1}{2\pi RC}$

5. Use Python to generate a Bode Plot for the circuit in Fig. 3 and compare to your measurements.

6. Construct the RLC circuit below with a square wave input of $5V_{pk\text{-}pk}$ at a frequency of 400Hz:

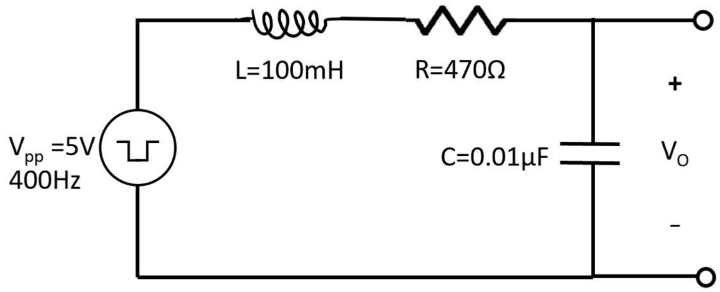

Figure 4: Circuit 2, a series RLC with a square wave input

7. Display both the input and the output on the oscilloscope (Ch1 and Ch2, respectively).
   a. Zoom in so that you can easily see one full cycle. Take a picture or save data to a flash drive.
   b. Determine if the system is underdamped or overdamped.
   c. Replace the resistor with a 1kΩ resistor and repeat steps a&b
   d. Replace the resistor with a 4.7kΩ resistor and repeat steps a&b
8. Switch the input to a sinusoidal waveform. Set the frequency to 1000Hz and replace the 470Ω resistor.
   a. Measure the voltage of the output waveform using the "Measure" function on the oscilloscope. Do not worry about the phase – you will explore this in circuits classes.
   b. Repeat the measurement at enough appropriate frequencies to obtain the shape of the output as a function of frequency. Plot your results.
   c. Determine the resonance frequency (the frequency of maximum output of the filter)
   d. Compare with the theoretical value of resonant frequency determined by $f_0 = \frac{1}{2\pi\sqrt{LC}}$

9. Determine the transfer function ($H(s)=V_O/V_{in}$) for the circuit in Fig. 4 and then use Python to generate a Bode Plot for the circuit in Fig. 4 and compare to your measurements.

10. Use Laplace Transformation to solve analytically expected response for V_o in Fig. 4 with a constant forcing function V_{pp} (similar to the "on" of your square wave **AND** for a forcing function that is sinusoidal as in step 8.

REPORT:

1. Present your data from each step in the Experimental Section and discuss. Pay particular attention to comparisons between theoretical and measured results. (Make sure all your figures have a caption that tell me what I am looking at.)
2. Discuss the value of Laplace transformations in circuit analysis.
3. Discuss the usefulness of a Bode plot and what information one can obtain from it.
4. If the values of R and C in Fig. 3 were changed, hypothesize the effects on the Bode plot and cutoff frequency.
5. Discuss your theoretical calculations for the RLC circuit and compare to your measurements.