

[03-60-231] Assignment 1

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1 Question 1

1.1 Part b

Construct a truth table for $((p \Rightarrow q) \Rightarrow (r \Rightarrow p)) \Rightarrow (r \Rightarrow p)$

p	q	r	$(p \Rightarrow q)$	$(r \Rightarrow p)$	$((p \Rightarrow q) \Rightarrow (r \Rightarrow p))$	$(r \Rightarrow p)$	$((p \Rightarrow q) \Rightarrow (r \Rightarrow p)) \Rightarrow (r \Rightarrow p)$
F	F	F	T	T	T	T	T
F	F	T	T	F	F	F	T
F	T	F	T	T	T	T	T
F	T	T	T	F	F	F	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	T	T	T	T	T
T	T	T	T	T	T	T	T

2 Question 3

2.1 Part f

Prove that $(\alpha \Rightarrow \beta) \vee (\beta \Rightarrow \alpha)$

Solution: (direct proof)

1. $(\alpha \vee \neg\alpha)$ Axiom
2. $(\alpha \vee \neg\alpha) \vee \beta$ 1, I1
3. $(\neg\alpha \vee \alpha) \vee \beta$ 2, E10
4. $\neg\alpha \vee (\alpha \vee \beta)$ 3, E12
5. $\neg\alpha \vee (\beta \vee \alpha)$ 4, E10
6. $((\neg\alpha \vee \beta) \vee \alpha)$ 5, E12
7. $((\neg\alpha \vee \beta) \vee \alpha) \vee \neg\beta$ 6, I1
8. $(\neg\alpha \vee \beta) \vee (\alpha \vee \neg\beta)$ 7, E12
9. $(\neg\alpha \vee \beta) \vee (\neg\beta \vee \alpha)$ 8, E10
10. $(\alpha \Rightarrow \beta) \vee (\neg\beta \vee \alpha)$ 9, E18
11. $(\alpha \Rightarrow \beta) \vee (\beta \Rightarrow \alpha)$ 10, E18

Hence, $\vdash((\alpha \Rightarrow \beta) \vee (\beta \Rightarrow \alpha))$

□

2.2 Part n

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Prove that $((\alpha \vee \beta) \wedge (\alpha \Rightarrow \gamma)) \Rightarrow (\gamma \vee \beta)$

Solution: (direct proof)

1. $((\alpha \vee \beta) \wedge (\alpha \Rightarrow \gamma))$	Hypothesis
2. $(\alpha \vee \beta)$	1, I2
3. $(\neg\neg\alpha \vee \beta)$	1, E15
4. $(\neg\alpha \Rightarrow \beta)$	3, E18
5. $(\neg\beta \Rightarrow \neg\neg\alpha)$	4, E19
6. $(\neg\beta \Rightarrow \alpha)$	5, E15
7. $((\alpha \Rightarrow \gamma) \wedge (\alpha \vee \beta))$	1, E9
8. $(\alpha \Rightarrow \gamma)$	7, I2
9. $(\neg\beta \Rightarrow \gamma)$	6, 8, I5
10. $(\neg\neg\beta \vee \gamma)$	9, E18
11. $(\beta \vee \gamma)$	10, E15
12. $(\gamma \vee \beta)$	11, E10

Hence, $\vdash((\alpha \vee \beta) \wedge (\alpha \Rightarrow \gamma)) \Rightarrow (\gamma \vee \beta)$

□

3 Question 4

3.1 Part IV

Prove that $(\alpha \Leftrightarrow \beta) \wedge (\beta \Leftrightarrow \gamma) \Rightarrow (\alpha \Leftrightarrow \gamma)$

Solution: (direct proof)

1. $(\alpha \Leftrightarrow \beta) \wedge (\beta \Leftrightarrow \gamma)$	Hypothesis
2. $(\alpha \Leftrightarrow \beta)$	1, I2
3. $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$	2, E20
4. $(\alpha \Rightarrow \beta)$	3, I2
5. $(\beta \Rightarrow \alpha) \wedge (\alpha \Rightarrow \beta)$	3, E9
6. $(\beta \Rightarrow \alpha)$	5, I2
7. $(\beta \Leftrightarrow \gamma) \wedge (\alpha \Leftrightarrow \beta)$	1, E9
8. $(\beta \Leftrightarrow \gamma)$	7, I2
9. $(\beta \Rightarrow \gamma) \wedge (\gamma \Rightarrow \beta)$	8, E20
10. $(\beta \Rightarrow \gamma)$	9, I2
11. $(\gamma \Rightarrow \beta) \wedge (\beta \Rightarrow \gamma)$	9, E9
12. $(\gamma \Rightarrow \beta)$	11, I2

- | | |
|--|------------|
| 13. $(\alpha \Rightarrow \gamma)$ | 4, 10, I5 |
| 14. $(\gamma \Rightarrow \alpha)$ | 12, 6, I5 |
| 15. $(\alpha \Rightarrow \gamma) \wedge (\gamma \Rightarrow \alpha)$ | 13, 14, I6 |
| 16. $(\alpha \Leftrightarrow \gamma)$ | 15, E20 |

Hence, $\vdash (\alpha \Leftrightarrow \beta) \wedge (\beta \Leftrightarrow \gamma) \Rightarrow (\alpha \Leftrightarrow \gamma)$ \square

4 Question 6

4.1 Part c

Prove that $((p \wedge q) \Rightarrow r) \wedge ((p \vee q) \Rightarrow u) \wedge (r \Rightarrow \neg u) \Rightarrow (p \Rightarrow \neg q)$
Solution: (direct proof)

- | | |
|--|----------|
| 1. $p \wedge q \Rightarrow r$ | P1 |
| 2. $p \vee q \Rightarrow u$ | P2 |
| 3. $r \Rightarrow \neg u$ | P3 |
| 4. $p \wedge q \Rightarrow \neg u$ | 1, 3, I5 |
| 5. $\neg u \Rightarrow \neg(p \vee q)$ | 2, E19 |
| 6. $p \wedge q \Rightarrow \neg(p \vee q)$ | 4, 5, I5 |
| 7. $\neg(p \wedge q) \vee \neg(p \vee q)$ | 6, E18 |
| 8. $(\neg p \vee \neg q) \vee \neg(p \vee q)$ | 7, E16 |
| 9. $(\neg p \vee \neg q) \vee (\neg p \wedge \neg q)$ | 8, E17 |
| 10. $((\neg p \vee \neg q) \vee \neg p) \wedge ((\neg p \vee \neg q) \vee \neg q)$ | 9, E14 |
| 11. $((\neg p \vee \neg q) \vee \neg p)$ | 10, I2 |
| 12. $((\neg q \vee \neg p) \vee \neg p)$ | 11, E10 |
| 13. $(\neg q \vee (\neg p \vee \neg p))$ | 12, E12 |
| 14. $(\neg q \vee \neg p)$ | 13, E4 |
| 15. $(\neg p \vee \neg q)$ | 14, E10 |
| 16. $p \Rightarrow \neg q$ | 15, E18 |

Hence, $\vdash ((p \wedge q) \Rightarrow r) \wedge ((p \vee q) \Rightarrow u) \wedge (r \Rightarrow \neg u) \Rightarrow (p \Rightarrow \neg q)$ \square

4.2 Part i

4.2 Part i

Prove that $((p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow s) \wedge (\neg r)) \Rightarrow (r \vee s)$

Solution: (direct proof)

- | | |
|--------------------------------|----------|
| 1. $p \vee q$ | P1 |
| 2. $p \Rightarrow r$ | P2 |
| 3. $q \Rightarrow s$ | P3 |
| 4. $\neg r$ | P4 |
| 5. $\neg \neg p \vee q$ | 1, E15 |
| 6. $\neg p \Rightarrow q$ | 5, E18 |
| 7. $\neg r \Rightarrow \neg p$ | 2, E19 |
| 8. $\neg r \Rightarrow q$ | 7, 6, I5 |
| 9. $\neg r \Rightarrow s$ | 8, 3, I5 |
| 10. $\neg \neg r \vee s$ | 9, E18 |
| 11. $r \vee s$ | 10, E15 |

Hence, $\vdash((p \vee q) \wedge (p \Rightarrow r) \wedge (q \Rightarrow s) \wedge (\neg r)) \Rightarrow (r \vee s)$

□

5 Question 7

5.1 Part c

Let c be the contract is satisfied.

Let b be the building is completed by November 30th.

Let e be the electrical subcontractor be done by November 10th.

Let m be the bank loses money.

Prove that $((c \Leftrightarrow b) \wedge (e \Leftrightarrow b) \wedge (m \Leftrightarrow \neg c)) \Rightarrow (e \Leftrightarrow \neg m)$

Solution: (direct proof)

- | | |
|--|--------|
| 1. $(c \Leftrightarrow b)$ | P1 |
| 2. $(e \Leftrightarrow b)$ | P2 |
| 3. $(m \Leftrightarrow \neg c)$ | P3 |
| 4. $(c \Rightarrow b) \wedge (b \Rightarrow c)$ | 1, E20 |
| 5. $(c \Rightarrow b)$ | 4, I2 |
| 6. $(b \Rightarrow c) \wedge (c \Rightarrow b)$ | 4, E9 |
| 7. $(b \Rightarrow c)$ | 6, I2 |
| 8. $(e \Rightarrow b) \wedge (b \Rightarrow e)$ | 2, E20 |
| 9. $(e \Rightarrow b)$ | 9, I2 |
| 10. $(b \Rightarrow e) \wedge (e \Rightarrow b)$ | 9, E9 |

5.1 Part c

11. $(b \Rightarrow e)$	12, I2
12. $(m \Rightarrow \neg c) \wedge (\neg c \Rightarrow m)$	3, E20
13. $(m \Rightarrow \neg c)$	15, I2
14. $(\neg c \Rightarrow m) \wedge (m \Rightarrow \neg c)$	15, E9
15. $(\neg c \Rightarrow m)$	18, I2
16. $(c \Rightarrow \neg m)$	16, E19
17. $(\neg m \Rightarrow c)$	19, E19
18. $(e \Rightarrow c)$	9, 7, I5
19. $(c \Rightarrow e)$	5, 11, I5
20. $(e \Rightarrow \neg m)$	18, 16, I5
21. $(\neg m \Rightarrow e)$	17, 19, I5
22. $(e \Rightarrow \neg m) \wedge (\neg m \Rightarrow e)$	20, 21, I6
23. $(e \Leftrightarrow \neg m)$	22, E20

Hence, $\vdash((c \Leftrightarrow b) \wedge (e \Leftrightarrow b) \wedge (m \Leftrightarrow \neg c)) \Rightarrow (e \Leftrightarrow \neg m)$ □

Hence, the electrical subcontractor completes his work by November 10 if and only if the bank does not lose money.