• 玻色气体的热力学性质

重新考虑 $k=0$ 项；

$$\ln Q = -\frac{V}{(2\pi)^3}\int d^3k\cdot \ln(1-ze^{-\beta\varepsilon_k}) - \underbrace{\ln(1-z)}_{k=0\text{项. }O(\ln N)\text{. 可去!}}.$$

$$= \frac{V}{\lambda^3} g_{5/2}(z).$$

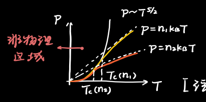
$$pV = k_B T \ln Q \Rightarrow p = \frac{k_B T}{\lambda^3} g_{5/2}(z).$$

当 $T<T_c$：$z=1$.

$$p = \frac{k_B T}{\lambda^3}\zeta\left(\frac{5}{2}\right) \simeq 1.341\frac{k_B T}{\lambda^3}.$$ 与n无关！（注：T_c依赖于粒子数密度）

$$p \sim T\cdot T^{3/2} = T^{5/2}.$$

解释：基态粒子对压强没有贡献，只有激发态粒子（只与T相关）有贡献！



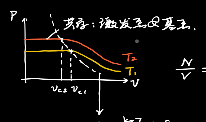
温度升高时，从左下趋近（化学势为负）

[注：是平滑过渡！T_c 前 $p\propto T$（非简并）还远着呢！]

$p\sim v$ 关系.

$$n\lambda_T^3 = \zeta\left(\frac{3}{2}\right).$$

固定T，减小v ($n\uparrow$) $\Rightarrow$ 临界密度 $n_c(T)$, $v_c(T)$.



$$\frac{N}{V} = \frac{N_{ext}}{V} + \frac{N_0}{V}.$$ 随着$V\downarrow$，$N_{ext}\downarrow$，被"挤"到了基态！

$\downarrow$ $n_{ext}(T)$，只与T有关

$$\begin{cases} p = \frac{k_B T}{\lambda^3}\zeta\left(\frac{5}{2}\right) \\ v_c = \lambda^3/\zeta\left(\frac{3}{2}\right) \end{cases} \Rightarrow p_c v_c^{5/3} = \text{Const}.$$ （临界线）

相变潜热

$$\frac{dp}{dT} = \frac{L}{T(v_2-v_1)}.$$ 1：激发态 2：基态 $v_2=0$.

$$= -\frac{L}{T\cdot v_c}$$

$$p = \frac{k_B T}{\lambda^3}\zeta\left(\frac{5}{2}\right) \Rightarrow \frac{dp}{dT} = \frac{5}{2}\frac{p}{T}$$

$$L = -\frac{5}{2}pv_c = -\frac{5}{2}k_B T\cdot\frac{\zeta\left(\frac{5}{2}\right)}{\zeta\left(\frac{3}{2}\right)} < 0.$$

相变潜热为负：激→凝相变放热，合理！

内能与比热

$$E = \frac{3}{2}k_B T\frac{V}{\lambda^3}g_{5/2}(z).$$

$T\le T_c$时：$E = \frac{3}{2}k_B T\cdot\frac{V}{\lambda^3}\zeta\left(\frac{5}{2}\right)$

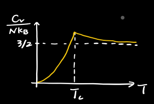
$$C_v = \frac{dE}{dT} = \frac{5}{2}\frac{E}{T} = \frac{15}{4}k_B\cdot\frac{V}{\lambda^3}\zeta\left(\frac{5}{2}\right).$$

$T>T_c$时：

$$C_v = \frac{15}{4}k_B\cdot\frac{V}{\lambda^3}g_{5/2}(z) + \frac{3}{2}k_B T\cdot\frac{V}{\lambda^3}g_{5/2}'(z)\cdot\frac{dz}{dT}.$$

$$g_m(z) = \sum_{l=1}^{\infty}\frac{z^l}{l^m}.\quad g_m'(z) = \sum_{l=0}^{\infty}\frac{z^{l-1}}{l^{m-1}} = \frac{1}{z}g_{m-1}(z).$$

$$\frac{dz}{dT} = ?\quad \frac{dN}{dT} = \frac{d}{dT}\left(\frac{V}{\lambda^3}g_{3/2}(z)\right) = 0 \Rightarrow \frac{dz}{dT}.$$



$T\to 0$时，$C_v\sim T^{3/2}\to 0$.