Environmental Instruments Canada

Thoron Detection

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PIMS

Team

Edward Timko — Team Academic Mentor. Currently a PIMS PDF at the University of Manitoba. PhD in Mathematics and MSc in Physics from Indiana University, BSc in Mathematics and Physics from the Univeristy of Dayton.

Stephen Styles — MSc in Statistics in progress, BSc in Applied Mathematics from the University of Alberta.

Liam Wrubleski — BSc Electrical and Computer Engineering, BSc Mathematics from the University of Calgary.

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Industry

Kai Kaletsch — Industry Mentor, President of Environmental Instruments Canada (EIC)

EIC — Experienced specialists in radiation safety, occupational hygiene and ventilation in the uranium industry and public service. They have worked with all the operating uranium mines in Saskatchewan, Canada as well as on uranium projects in Mongolia, Kazakhstan and the United States. These projects range from exploration to decommissioning.

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Introduction

- Residential radon progeny exposure is the second leading cause of lung cancer in smokers, and the leading cause of lung cancer in non-smokers.
- Uranium and thorium in the soil decay into radon, which can then seep into basements and low-lying areas of the house.
- The two main radon isotopes are Rn-222, which is part of the U-238 decay chain, and Rn-220, also called thoron, which is part of the Th-232 decay chain.
- Thoron has been ignored in the past as it has a relatively short half life and usually decays before reaching a house, however its decay products can reach living areas.

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- Radon is chemically inert, and is most often detected by its decays.
- Environmental Instruments Canada (EIC) produces a Radon Sniffer which is used by radon mitigators and building scientists to find radon entry points.
- The sniffer pumps in air through a filter, only passing in radon to the chamber.
- The sniffer counts alpha particles, but cannot distinguish between decays of Rn-222, Rn-220, or their progeny.
- These sniffers currently assume all radon is Rn-222.

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Problem Statement

Find a sampling schedule and algorithm to determine the concentrations of Rn-222 and Rn-220 that minimizes the variance in the estimated values of their concentrations.

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Linear Regression

Using the expected number of decays and the observed counts, we use a linear regression model to estimate the initial amount of each isotope. Thus we can fit the model:

$$y_i = N_{222}x_{1i} + N_{220}x_{2i} + \epsilon_i$$

This allows us to estimate N_{222} and N_{220} when given a sample vector y and inputs X where $X = [x_1, x_2]$ as

$$\begin{bmatrix} \hat{N}_{222} \\ \hat{N}_{220} \end{bmatrix} = (X^T X)^{-1} X^T y$$

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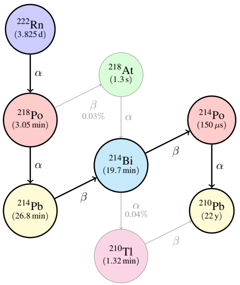
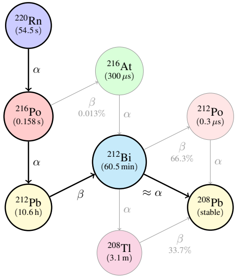
Decay Chains

We model the decay chain as an ordered sequence of nuclides. These nuclides decay either by emitting an alpha particle or a beta particle, with a half-life of $t_{1/2}$.





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Decay Rate

For a collection of $N(0)$ unstable atoms, all of a single nuclide, the number $N(t)$ of atoms *expected* to be remaining at time t is given by

$$N(t) = N(0)e^{-\lambda t}$$

for a value $\lambda > 0$ called the *decay constant* for that nuclide.

The expected rate of decay at time t is given by

$$-\frac{dN}{dt} = \lambda N(t).$$

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System of ODEs

Let $N_0(t), N_1(t), \ldots, N_\ell(t)$ denote the expected number of atoms of each type in a given decay chain remaining at time t. Let λ_j denote the decay constant of the j-th member of the chain. The evolution of the system is described by the following ODEs:

$$\frac{dN_0}{dt} = -\lambda_0 N_0, \quad \frac{dN_j}{dt} = -\lambda_j N_j + \lambda_{j-1} N_{j-1} \quad (0 < j \leq \ell)$$

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Bateman Equation

Note that our detector does not measure $N_j(t)$ but instead a sum of expressions of the form

$$\int_{s_1}^{s_2} \lambda_j N_j(s) ds,$$

where $(s_1, s_2]$ is an interval of time during which the detector is counting. Nevertheless, it will be in our interest to solve for N_j first.

The solutions of the system are provided by the *Bateman Equation* when $N_1(0) = \cdots = N_\ell(0) = 0$:

$$N_j(t) = \frac{N_0(0)}{\lambda_j} \sum_{r=0}^{j} \left(\prod_{q=0, q \neq r}^{j} \frac{\lambda_q}{\lambda_q - \lambda_r} \right) \lambda_r e^{-\lambda_r t}.$$

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Bateman Equation

Thus the number of decays by atoms of the j-th type in the time interval $(s_1, s_2]$ is

$$\int_{s_1}^{s_2} \lambda_j N_j(s) ds = N_0(0) \sum_{r=0}^{j} \left(\prod_{q=0, q \neq r}^{j} \frac{\lambda_q}{\lambda_q - \lambda_r} \right) \left(e^{-\lambda_r s_1} - e^{-\lambda_r s_2} \right)$$

Since our detector only detects alpha decays, the number of counts we expect to see in the time interval $(s_1, s_2]$ is given by by

$$\sum_{j\ \alpha\text{-decays}} \int_{s_1}^{s_2} \lambda_j N_j(s) ds.$$

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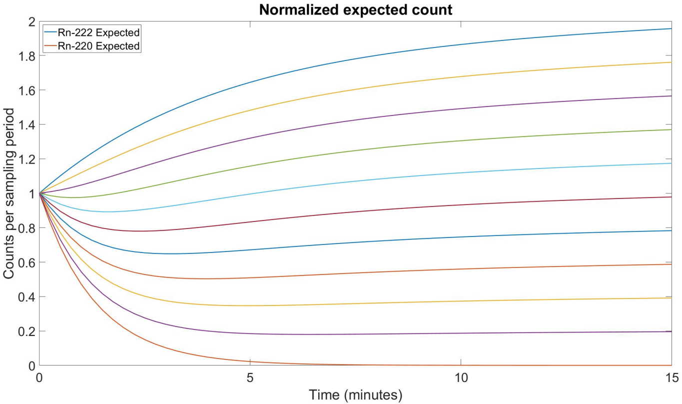
Normalized Count Rates

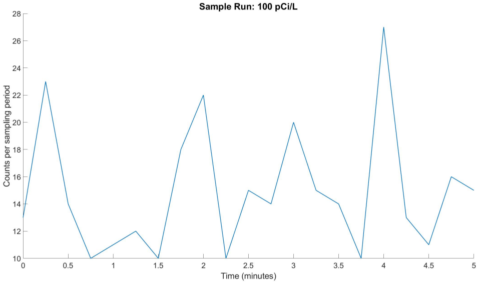
Normalizing to the expected counts in the first sampling period, here equal to 1 second, we can see the difference in behaviour of different mixtures of radon and thoron.



Generating Data

Since we are testing new sampling times and periods, and we have limited real world datasets to test our model with, we need to generate data that will simulate radon and thoron decay.

The decay times of an atom follow an exponential distribution with parameter $\lambda = \frac{\ln(2)}{t_{1/2}}$. From here we can follow the evolution of an atom along the decay chain.



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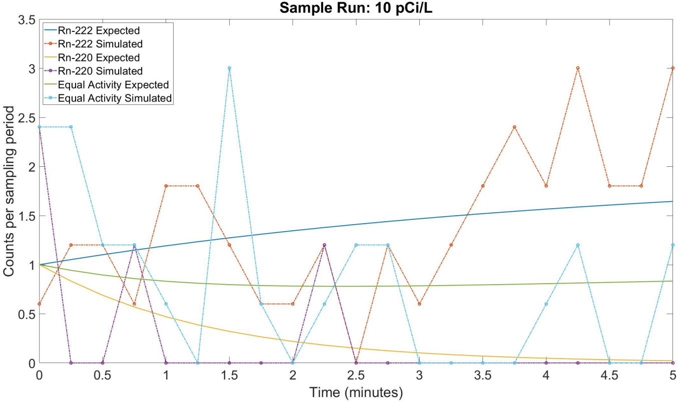
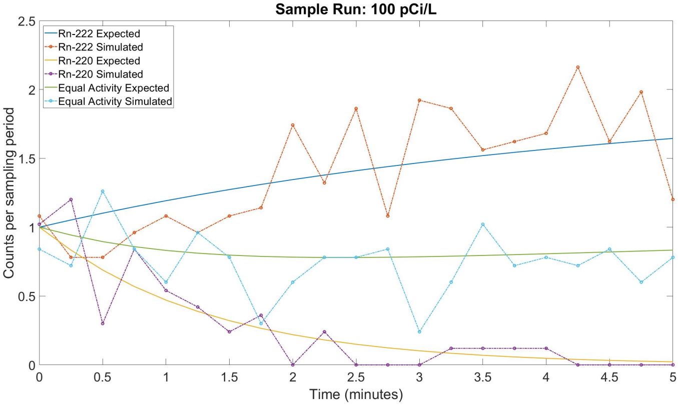
10 pCi/L

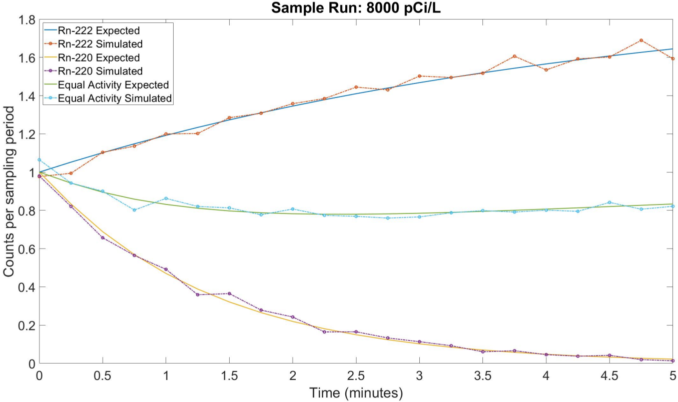


We can add a sample run of generated data onto the theoretical curves and compare the observed counts to the expected counts.

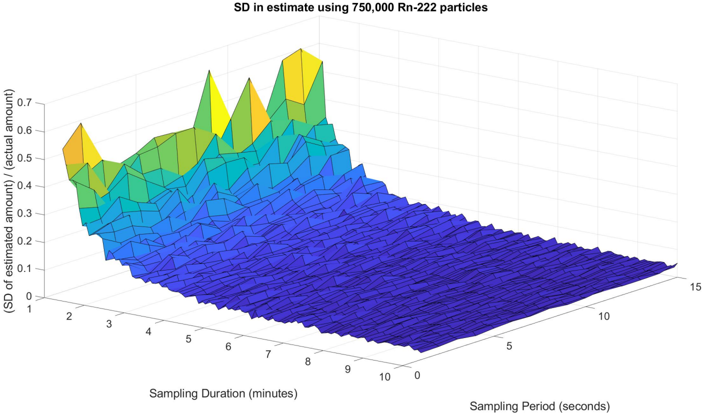
100 pCi/L


Sample Run: 100 pCi/L
Rn-222 Expected
Rn-222 Simulated
Rn-220 Expected
Rn-220 Simulated
Equal Activity Expected
Equal Activity Simulated
Counts per sampling period
Time (minutes)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

8000 pCi/L


Sample Run: 8000 pCi/L
Counts per sampling period
Time (minutes)
Rn-222 Expected
Rn-222 Simulated
Rn-220 Expected
Rn-220 Simulated
Equal Activity Expected
Equal Activity Simulated
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0.5
1.5
2
2.5
3
3.5
4
4.5
5

Grid Search



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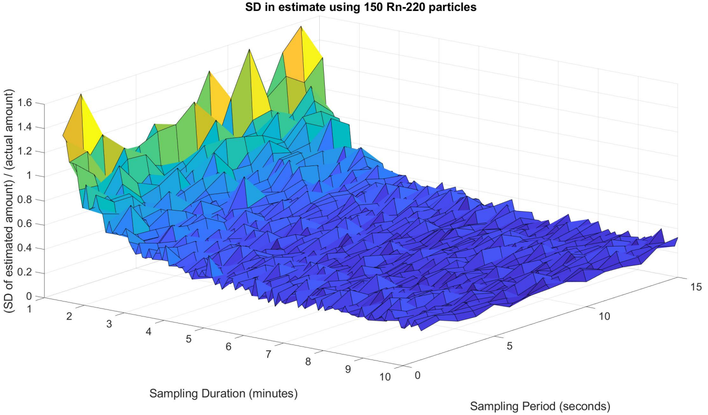
Grid Search



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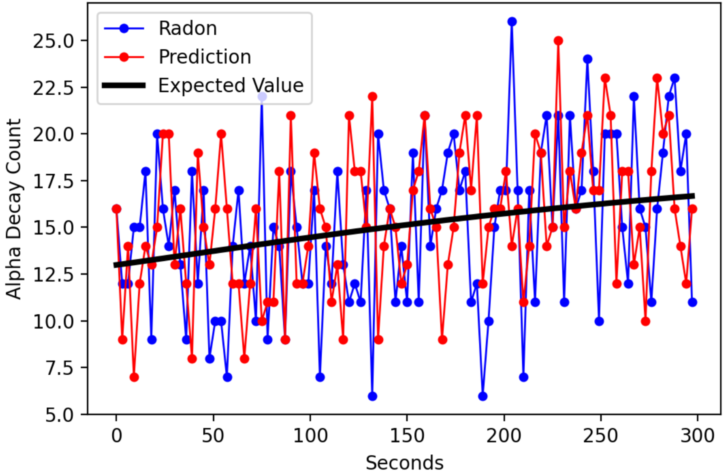
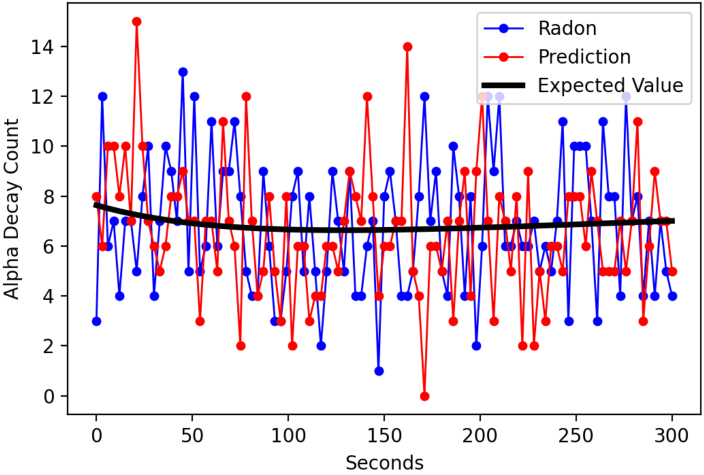
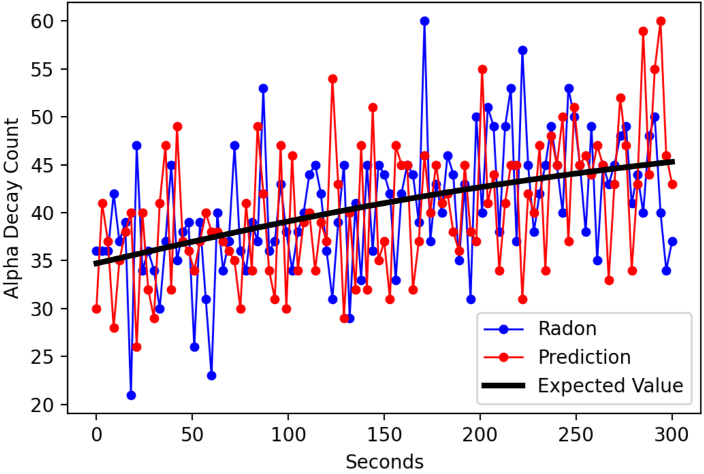
Data Fitting - Radon Trial 1



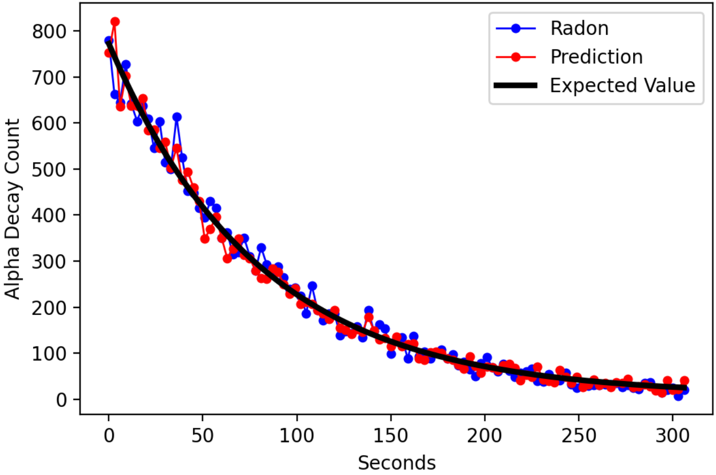
Data Fitting - Radon Trial 2



Data Fitting - Radon Trial 3



Data Fitting - Thoron Trial



Conclusion

We would like to thank:

- PIMS for running this workshop and providing us with this opportunity
- Kai & EIC, for letting us work on this problem and helping us through it
- The *Math*^{Industry} organizers for all their work in setting up this workshop
- You, for listening to our presentation

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