Power Rule

Stephen Styles

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Chain Rule:

$$\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$$

1. Find the derivative of $f(x) = (5x^2 + 3x - 1)^{1/2}$

$h(x) = x^{\frac{1}{2}}$ $\quad h'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$g(x) = 5x^2 + 3x - 1$ $\quad g'(x) = 10x + 3$

$\Rightarrow f'(x) = h'(g(x)) \cdot g'(x)$

$= \frac{1}{2}(5x^2 + 3x - 1)^{-\frac{1}{2}} \cdot (10x + 3)$

2. Find the derivative of $f(x) = (3x^2 + 1)^5$

$h(x) = x^5$ $\quad h'(x) = 5x^4$

$g(x) = 3x^2 + 1$ $\quad g'(x) = 6x$

$\Rightarrow f'(x) = 5(3x^2 + 1)^4 \cdot 6x$

3. Find the derivative of $f(x) = e^{x^2 - x + 2}$

$h(x) = e^x$ $\quad h'(x) = e^x$

$g(x) = x^2 - x + 2$ $\quad g'(x) = 2x - 1$

$\Rightarrow f'(x) = e^{x^2 - x + 2} \cdot (2x - 1)$

4. Find the derivative of $f(x) = \sin(\cos(x))$

$h(x) = \sin(x)$ $\quad h'(x) = \cos(x)$

$g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

$\Rightarrow f'(x) = \cos(\cos(x)) \cdot (-\sin(x))$

5. Find the derivative of $f(x) = \ln(x^2 + 1)$

$h(x) = \ln x$ $\quad h'(x) = \frac{1}{x}$

$g(x) = x^2 + 1$ $\quad g'(x) = 2x$

$\Rightarrow f'(x) = \frac{1}{x^2+1} \cdot 2x$

6. Find the derivative of $f(x) = e^{\sin(x^2)}$

$h(x) = e^x$ $\quad h'(x) = e^x$

$g(x) = \sin(x)$ $\quad g'(x) = \cos(x)$

$k(x) = x^2$ $\quad k'(x) = 2x$

$\Rightarrow f'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$

$f(x) = h(g(k(x)))$

$f'(x) = h'(g(k(x))) \cdot g'(k(x)) \cdot k'(x)$

7. Find the derivative of $f(x) = \tan(\csc(e^{3x+1}))$

$h(x) = \tan(x)$ $\quad h'(x) = \sec^2(x)$

$g(x) = \csc(x)$ $\quad g'(x) = -\csc(x)\cot(x)$

$k(x) = e^x$ $\quad k'(x) = e^x$

$s(x) = 3x + 1$ $\quad s'(x) = 3$

$\Rightarrow f'(x) = \sec^2(\csc(e^{3x+1})) \cdot -\csc(e^{3x+1})\cot(e^{3x+1}) \cdot e^{3x+1} \cdot 3$