Surface Area of Rotations

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Let $y = f(x) \geq 0$, if $f(x)$ is a smooth (continuously differentiable) function on the interval $[a, b]$, then the surface area generated by revolving the function about the line $y = r$ can be calculated by

$$S = \int_a^b 2\pi(f(x) - r)\sqrt{1 + \big(f'(x)\big)^2}\, dx.$$

Similarly, if $x = g(y) \geq 0$, where $g(y)$ is a smooth function on $[c, d]$, then the surface area generated by revolving the function around the line $x = r$ can be calculated by

$$S = \int_c^d 2\pi(g(y) - r)\sqrt{1 + \big(g'(y)\big)^2}\, dy.$$

Questions:

1. Find the surface area of the solid created by the rotating the function $f(x) = \sqrt{16 - x^2}$ around the x-axis for $-2 \leq x \leq 2$.

2. Find the surface area of the solid created by the rotating the function $f(x) = x^3$ around the x-axis for $0 \leq x \leq 1$

3. Find the surface area of the solid created by the rotating the function $9x = y^2 + 18$ around the x-axis for $2 \leq x \leq 6$.

4. Find the surface area of the solid created by the rotating the function $x = e^{-2y}$ around the y-axis for $0 \leq x \leq 1$.

5. Find the surface area of the solid created by the rotating the function $f(x) = \frac{1}{3}x$ around the x-axis for $1 \leq x \leq 3$.

6. Find the surface area of the solid created by the rotating the function $f(x) = \sqrt{x}$ around the x-axis for $2 \leq x \leq 6$.

7. Find the surface area generated by revolving the curve $y = \frac{x^3}{3} + \frac{1}{4x}$, $1 \leq x \leq 3$ about the line $y = -2$.