Surface Area of Rotations

Stephen Styles

September 26, 2020

Let $y = f(x) \ge 0$, if f(x) is a smooth (continuously differentiable) function on the interval [a, b], then the surface area generated by revolving the function about the line y = r can be calculated by

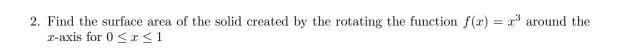
$$S = \int_{a}^{b} 2\pi (f(x) - r) \sqrt{1 + (f'(x))^{2}} dx.$$

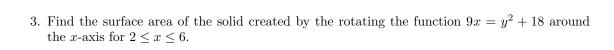
Similarly, if $x = g(y) \ge 0$, where g(y) is a smooth function on [c, d], then the surface area generated by revolving the function around the line x = r can be calculated by

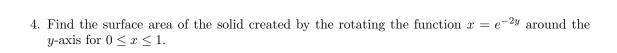
$$S = \int_{c}^{d} 2\pi (g(y) - r) \sqrt{1 + (g'(y))^{2}} \, dy.$$

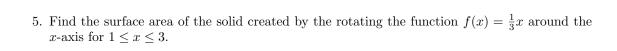
Questions:

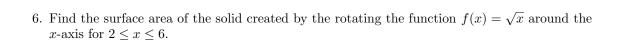
1. Find the surface area of the solid created by the rotating the function $f(x) = \sqrt{16 - x^2}$ around the x-axis for $-2 \le x \le 2$.











7. Find the surface area generated by revolving the curve $y = \frac{x^3}{3} + \frac{1}{4x}$, $1 \le x \le 3$ about the line y = -2.