Limits to Infinity

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1. *Solve* $\lim_{x\to\infty} \dfrac{x^3-2}{2x^4-7x^2+x}$ $= \lim_{x\to\infty} \dfrac{x^4\left(\frac{1}{x}-\frac{2}{x^4}\right)}{x^4\left(2-\frac{7}{x^2}+\frac{1}{x^3}\right)} = \lim_{x\to\infty} \dfrac{\overset{\to 0}{\frac{1}{x}}-\overset{\to 0}{\frac{2}{x^4}}}{2-\underset{\to 0}{\frac{7}{x^2}}+\underset{\to 0}{\frac{1}{x^3}}} = \dfrac{0}{2} = \underline{2}$

2. *Solve* $\lim_{x\to\infty} \dfrac{9x^2}{x+1}$ $= \lim_{x\to\infty} \dfrac{x(9x)}{x\left(1+\frac{1}{x}\right)} = \lim_{x\to\infty} \dfrac{\overset{\to\infty}{9x}}{\left(1+\underset{\to 0}{\frac{1}{x}}\right)} = \dfrac{\infty}{1} = \underline{\infty}$

3. *Solve* $\lim_{x\to\infty} \dfrac{4x^3-2x+7}{3x^3+12x^2+x-4}$ $= \lim_{x\to\infty} \dfrac{x^3\left(4-\frac{2}{x^2}+\frac{7}{x^3}\right)}{x^3\left(3+\frac{12}{x}+\frac{1}{x^2}-\frac{4}{x^3}\right)} = \lim_{x\to\infty} \dfrac{4-\overset{\to 0}{\frac{2}{x^2}}+\overset{\to 0}{\frac{7}{x^3}}}{3+\underset{\to 0}{\frac{12}{x}}+\underset{\to 0}{\frac{1}{x^2}}-\underset{\to 0}{\frac{4}{x^3}}} = \underline{\dfrac{4}{3}}$

4. *Solve* $\lim_{x\to\infty} \dfrac{7x^2-5}{x^2-25}$ $= \lim_{x\to\infty} \dfrac{x^2\left(7-\frac{5}{x^2}\right)}{x^2\left(1-\frac{25}{x^2}\right)} = \lim_{x\to\infty} \dfrac{7-\overset{\to 0}{\frac{5}{x^2}}}{1-\underset{\to 0}{\frac{25}{x^2}}} = \underline{7}$

5. *Solve* $\lim_{x\to\infty} \frac{3x^2+4}{3x^2+4x-5}$ $= \lim_{x\to\infty} \frac{x^2(3+\frac{4}{x^2})}{x^2(3+\frac{4}{x}-\frac{5}{x^2})} = \lim_{x\to\infty} \frac{3+\overset{\to 0}{\frac{4}{x^2}}}{3+\underset{\to 0}{\frac{4}{x}}-\underset{\to 0}{\frac{5}{x^2}}} = \frac{3}{3} = \underline{1}$

6. *Solve* $\lim_{x\to\infty} \cos\left(\frac{x-2}{x^3+5x-2}\right)$ $= \cos\left(\lim_{x\to\infty} \frac{x-2}{x^3+5x-2}\right) = \cos\left(\lim_{x\to\infty} \frac{x^3(\frac{1}{x^2}-\frac{2}{x^3})}{x^3(1+\frac{5}{x^2}-\frac{2}{x^3})}\right) = \cos\left(\lim_{x\to\infty} \frac{\overset{\to 0}{\frac{1}{x^2}}-\overset{\to 0}{\frac{2}{x^3}}}{1+\underset{\to 0}{\frac{5}{x^2}}-\underset{\to 0}{\frac{2}{x^3}}}\right)$

$= \cos\left(\frac{0}{1}\right) = \cos(0) = \underline{1}$

7. *Solve* $\lim_{x\to\infty} e^{\frac{x}{x^2+1}}$ $= e^{\lim_{x\to\infty}\frac{x}{x^2+1}} = e^{\lim_{x\to\infty}\frac{x^2(\frac{1}{x})}{x^2(1+\frac{1}{x^2})}} = e^{\lim_{x\to\infty}\frac{(\frac{1}{x})^{\to 0}}{1+\underset{\to 0}{\frac{1}{x^2}}}} = e^0 = \underline{1}$

8. *Solve* $\lim_{x\to-\infty} \frac{4x^2+x-1}{x+1}$ $= \lim_{x\to-\infty} \frac{x(4x+1-\frac{1}{x})}{x(1+\frac{1}{x})} = \lim_{x\to-\infty} \frac{\overset{\to -\infty}{4x}+1-\overset{\to 0}{\frac{1}{x}}}{1+\underset{\to 0}{\frac{1}{x}}} = \frac{-\infty}{1} = \underline{-\infty}$

9. *Solve* $\lim_{x\to-\infty} \frac{3x^3}{\sqrt{9x^6+x}}$ $= \lim_{x\to-\infty} \frac{3x^3}{\sqrt{x^6(9+\frac{1}{x^5})}} = \lim_{x\to-\infty} \frac{3x^3}{|x^3|\sqrt{9+\frac{1}{x^5}}} = \lim_{x\to-\infty} \frac{3x^3}{-x^3\sqrt{9+\frac{1}{x^5}}}$

$= \lim_{x\to-\infty} \frac{-3}{\sqrt{9+\underset{\to 0}{\frac{1}{x^5}}}} = \frac{-3}{\sqrt{9}} = \frac{-3}{3} = \underline{-1}$

10. *Solve* $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+2}}{3x-6}$ $= \lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2(1+\frac{2}{x^2})}}{x(3-\frac{6}{x})} = \lim\limits_{x\to-\infty} \dfrac{|x|\sqrt{1+\frac{2}{x^2}}}{x(3-\frac{6}{x})} = \lim\limits_{x\to-\infty} \dfrac{-x\sqrt{1+\frac{2}{x^2}}}{x(3-\frac{6}{x})} = \lim\limits_{x\to-\infty} \dfrac{-\sqrt{1+\underbrace{\frac{2}{x^2}}_{\to 0}}}{3-\underbrace{\frac{6}{x}}_{\to 0}}$

$= \dfrac{-\sqrt{1}}{3} = \underline{-\dfrac{1}{3}}$

11. *Solve* $\lim\limits_{x\to-\infty} \dfrac{-2x}{x-\sqrt{x^2+2x}}$ $= \lim\limits_{x\to-\infty} \dfrac{-2x}{x-\sqrt{x^2(1+\frac{2}{x})}} = \lim\limits_{x\to-\infty} \dfrac{-2x}{x-|x|\sqrt{1+\frac{2}{x}}} = \lim\limits_{x\to-\infty} \dfrac{-2x}{x-(-x)\sqrt{1+\frac{2}{x}}} = \lim\limits_{x\to-\infty} \dfrac{-2x}{x+x\sqrt{1+\frac{2}{x}}}$

$= \lim\limits_{x\to-\infty} \dfrac{x(-2)}{x(1+\sqrt{1+\frac{2}{x}})} = \lim\limits_{x\to-\infty} \dfrac{-2}{1+\sqrt{1+\underbrace{\frac{2}{x}}_{\to 0}}} = \dfrac{-2}{1+\sqrt{1}} = \dfrac{-2}{2} = \underline{-1}$

12. *Solve* $\lim\limits_{x\to-\infty} 3x+\sqrt{9x^2+x}$ $\cdot \dfrac{3x-\sqrt{9x^2+x}}{3x-\sqrt{9x^2+x}} = \lim\limits_{x\to-\infty} \dfrac{9x^2-(\sqrt{9x^2+x})^2}{3x-\sqrt{9x^2+x}} = \lim\limits_{x\to-\infty} \dfrac{9x^2-9x^2-x}{3x-\sqrt{9x^2+x}}$

$= \lim\limits_{x\to-\infty} \dfrac{-x}{3x-\sqrt{9x^2+x}} = \lim\limits_{x\to-\infty} \dfrac{-x}{3x-\sqrt{x^2(9+\frac{1}{x})}} = \lim\limits_{x\to-\infty} \dfrac{-x}{3x-|x|\sqrt{9+\frac{1}{x}}} = \lim\limits_{x\to-\infty} \dfrac{-x}{3x-(-x)\sqrt{9+\frac{1}{x}}}$

$= \lim\limits_{x\to-\infty} \dfrac{-x}{3x+x\sqrt{9+\frac{1}{x}}} = \lim\limits_{x\to-\infty} \dfrac{-x}{x(3+\sqrt{9+\frac{1}{x}})} = \lim\limits_{x\to-\infty} \dfrac{-1}{3+\sqrt{9+\underbrace{\frac{1}{x}}_{\to 0}}} = \dfrac{-1}{3+\sqrt{9}} = \dfrac{-1}{3+3} = \underline{\dfrac{-1}{6}}$