

Integral of $\sec^3(x)$

Stephen Styles

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$$\int \sec^3(x) dx = \int \sec(x) \sec^2(x) dx$$

Let $u = \sec(x)$, then $du = \sec(x) \tan(x)$

Let $dv = \sec^2(x) dx$, then $v = \tan(x)$

$$\begin{aligned} \int \sec^3(x) dx &= uv - \int v du \\ &= \sec(x) \tan(x) - \int \tan(x) \sec(x) \tan(x) dx \\ &= \sec(x) \tan(x) - \int \tan^2(x) \sec(x) dx \\ &= \sec(x) \tan(x) - \int (\sec^2(x) - 1) \sec(x) dx \\ &= \sec(x) \tan(x) - \int (\sec^3(x) - \sec(x)) dx \\ &= \sec(x) \tan(x) - \int \sec^3(x) dx + \int \sec(x) dx \end{aligned}$$

Adding $\int \sec^3(x) dx$ to both sides we get:

$$\begin{aligned} 2 \int \sec^3(x) dx &= \sec(x) \tan(x) + \int \sec(x) dx \\ &= \sec(x) \tan(x) + \ln |\sec(x) + \tan(x)| + C \end{aligned}$$

Therefore:

$$\int \sec^3(x) dx = \frac{\sec(x) \tan(x) + \ln |\sec(x) + \tan(x)|}{2} + C$$