

Sample Midterm

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1. Find the limit if it exists:

(a) $\lim_{x \rightarrow -\infty} \frac{5x - 3}{\sqrt{9x^2 + 6}}$

(b) $\lim_{x \rightarrow 5} \frac{\sqrt{x^2 + 11} - 6}{x - 5}$

(c) $\lim_{x \rightarrow 0} 6x^2 \cot(3x) \csc(2x)$

2. Is the function $f(x) = \frac{1}{e^{2x} + \sin(x) - 100}$ continuous?

3. Find the domain of the function $f(x) = \frac{\ln(x-3)}{\sqrt{9-x}}$

4. Show that $\lim_{x \rightarrow 0^+} \sqrt{x} \left(1 - \sin^2 \left(\frac{2\pi}{x} \right) \right) = 0$

5. (a) State the formal definition of a derivative.

(b) Using the formal definition of a derivative, find the derivative of $f(x) = \sqrt{x - 5}$

6. Suppose c is a constant and the function f is given by:

$$f(x) = \begin{cases} c^x & x < 1 \\ 3cx - 2 & x \geq 1 \end{cases}$$

- (a) Calculate $\lim_{x \rightarrow 1^-} f(x)$ and $\lim_{x \rightarrow 1^+} f(x)$.
- (b) Find all values of c so that the function $f(x)$ is continuous everywhere.

7. Find the derivative of the following functions but **DO NOT** simplify your answers:

(a) $f(x) = \frac{e^x}{\sin(x^2)}$

(b) $f(x) = 2^{\csc(x)} \cdot \cot(-5x)$

(c) $f(x) = \sec^2\left(e^{\tan(\sqrt{x})}\right)$

8. A poster is to have an area of 600 in^2 with 1 inch margins at the bottom and sides and a 2 inch margin on top. What dimensions will give the largest printed area?

9. Use implicit differentiation to find the derivative of $x \cdot \tan(y) = xy + \cos(xy)$

10. Find the equation of the tangent line to the graph of $y = \tan(2x) + 3 \sec(x)$ at the point $(0, 3)$.

11. Make a detailed graph of $f(x) = \frac{4x}{x^2 + 1}$