Power Rule - Integration

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Abstract

If there are any mistakes in this worksheet/solution please email me at sjstyles@ualberta.ca. As well, if this worksheet does not cover enough details for the specific topic, notify me and I can expand it to include more questions.

Let us define

$$F(x) = \int f(x)dx$$

then,

$$\frac{d}{dx}F(x) = f(x).$$

Some properties of integration:

1. $\int cf(x)dx = c\int f(x)dx$ for all constants c.
2. $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$.
3. $\int c\ dx = cx$

The power rule for integration works in the opposite direction as it does for differentiation. If you recall:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Then the power rule for integration is defined as following:

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \text{ for all } n \neq -1$$

If $n = -1$ then the integral becomes:

$$\int x^{-1}dx = \int \frac{1}{x}dx = \ln(|x|) + c$$

Remember, we must include the "+c" at the end because unless we have more information about the function (i.e. initial values), if we differentiate $F(x)$, the constant c will just disappear and we will regain $f(x)$ for any value of c.

In our previous definition for the power rule, we see that

$$\begin{aligned}\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}+c\right) &= \frac{1}{n+1}\left(\frac{d}{dx}x^{n+1}\right)+\frac{d}{dx}c\\ &= \frac{1}{n+1}\big((n+1)x^n\big)\\ &= x^n\end{aligned}$$

Which is what we want since $\dfrac{d}{dx}F(x)=f(x)$.

Example:

Find $\displaystyle\int 3x^5-2x^{\frac{\pi}{2}}+x-7+\frac{2}{x}-\frac{6}{x^3}dx$

Solution:

$$\begin{aligned}\int 3x^5-2x^{\frac{\pi}{2}}+x-7+\frac{2}{x}-\frac{6}{x^3}dx &= 3\left(\frac{1}{5+1}x^{5+1}\right)-2\left(\frac{1}{\frac{\pi}{2}+1}x^{\frac{\pi}{2}+1}\right)+\frac{1}{1+1}x^{1+1}\\ &\quad -7x+2\ln(|x|)-6\left(\frac{1}{-3+1}x^{-3+1}\right)+c\\ &= \frac{1}{2}x^6-\frac{2}{\frac{\pi}{2}+1}x^{\frac{\pi}{2}+1}+\frac{1}{2}x^2-7x+2\ln(|x|)+3x^{-2}+c\end{aligned}$$

1. Find $\int 7x^3 \, dx$

2. Find $\int 5x^2 - 3 \, dx$

3. Find $\int x^2 - 2x + 1 \, dx$

4. Find $\int -3x^4 + 2x^3 - \frac{1}{x} \, dx$

5. Find $\int x^{-2} - 7x^{-3}\ dx$

6. Find $\int 7x^6 + x^{\frac{3}{2}} - 2 + \frac{2}{x^3}\ dx$

7. Find $\int -x^{99} - \frac{50}{x} + 7x^{\pi} - \frac{1}{x^{101}}\ dx$

8. Find $\int 7\sqrt{x} + \sqrt[3]{x} - \frac{1}{x^{\frac{4}{5}}}\ dx$