Definition of a Derivative

$$\frac{d}{dx}f(x) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Basic Rules

Scalar Multiplication Rule: $\big[cf(x)\big]' = cf'(x)$

Sum Rule: $\big[f(x)+g(x)\big]' = f'(x)+g'(x)$

Chain Rule: $\big[f(g(x))\big]' = f'\big(g(x)\big)\cdot g'(x)$

Product Rule: $\big[f(x)\cdot g(x)\big]' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Quotient Rule: $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g(x)^2}$

Inverse Rule: $\big[f^{-1}(x)\big]' = \dfrac{1}{f'\big(f^{-1}(x)\big)}$

Polynomial Derivatives

$$\frac{d}{dx}C = 0, \text{ where } C \text{ is a constant} \qquad \frac{d}{dx}x^n = nx^{n-1}$$

Exponential Derivatives

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}a^x = a^x\ln(a)$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \qquad \frac{d}{dx}\log_a(x) = \frac{1}{x\ln(a)}$$

Trig Derivatives

$$\frac{d}{dx}\sin\big(f(x)\big) = \cos\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\cos\big(f(x)\big) = -\sin\big(f(x)\big)f'(x)$$

$$\frac{d}{dx}\sec\big(f(x)\big) = \sec\big(f(x)\big)\tan\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\csc\big(f(x)\big) = -\csc\big(f(x)\big)\cot\big(f(x)\big)f'(x)$$

$$\frac{d}{dx}\tan\big(f(x)\big) = \sec^2\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\cot\big(f(x)\big) = -\csc^2\big(f(x)\big)f'(x)$$

Inverse Trig Derivatives

$$\frac{d}{dx}\sin^{-1}\big(f(x)\big) = \frac{f'(x)}{\sqrt{1-\big(f(x)\big)^2}} \qquad \frac{d}{dx}\cos^{-1}\big(f(x)\big) = \frac{-f'(x)}{\sqrt{1-\big(f(x)\big)^2}}$$

$$\frac{d}{dx}\sec^{-1}\big(f(x)\big) = \frac{f'(x)}{\big|f(x)\big|\sqrt{\big(f(x)\big)^2-1}} \qquad \frac{d}{dx}\csc^{-1}\big(f(x)\big) = \frac{-f'(x)}{\big|f(x)\big|\sqrt{\big(f(x)\big)^2-1}}$$

$$\frac{d}{dx}\tan^{-1}\big(f(x)\big) = \frac{f'(x)}{1+\big(f(x)\big)^2} \qquad \frac{d}{dx}\cot^{-1}\big(f(x)\big) = \frac{-f'(x)}{1+\big(f(x)\big)^2}$$

Hyperbolic Trig Derivatives

$$\frac{d}{dx}\sinh\big(f(x)\big) = \cosh\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\cosh\big(f(x)\big) = \sinh\big(f(x)\big)f'(x)$$

$$\frac{d}{dx}\operatorname{sech}\big(f(x)\big) = -\operatorname{sech}\big(f(x)\big)\tanh\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\operatorname{csch}\big(f(x)\big) = -\operatorname{csch}\big(f(x)\big)\coth\big(f(x)\big)f'(x)$$

$$\frac{d}{dx}\tanh\big(f(x)\big) = \operatorname{sech}^2\big(f(x)\big)f'(x) \qquad \frac{d}{dx}\coth\big(f(x)\big) = -\operatorname{csch}^2\big(f(x)\big)f'(x)$$

Inverse Hyperbolic Trig Derivatives

$$\frac{d}{dx}\sinh^{-1}\big(f(x)\big) = \frac{f'(x)}{\sqrt{1+\big(f(x)\big)^2}} \qquad \frac{d}{dx}\cosh^{-1}\big(f(x)\big) = \frac{f'(x)}{\sqrt{\big(f(x)\big)^2-1}}$$

$$\frac{d}{dx}\operatorname{sech}^{-1}\big(f(x)\big) = \frac{-f'(x)}{f(x)\sqrt{1-\big(f(x)\big)^2}} \qquad \frac{d}{dx}\operatorname{csch}^{-1}\big(f(x)\big) = \frac{-f'(x)}{\big|f(x)\big|\sqrt{\big(f(x)\big)^2+1}}$$

$$\frac{d}{dx}\tanh^{-1}\big(f(x)\big) = \frac{f'(x)}{1-\big(f(x)\big)^2} \qquad \frac{d}{dx}\coth^{-1}\big(f(x)\big) = \frac{f'(x)}{1-\big(f(x)\big)^2}$$