Logarithmic Differentiation

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1. Find the derivative of $y = x^x$

$\ln y = x \ln x$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x \ln x)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln x + 1$

$\Rightarrow \frac{dy}{dx} = y(\ln x + 1)$

$= x^x(\ln x + 1)$

2. Find the derivative of $y = \frac{(x-2)^2}{\sqrt{x^2+1}}$

$\ln y = \ln\left(\frac{(x-2)^2}{\sqrt{x^2+1}}\right)$

$= 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$

Take derivative of both sides

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x-2} - \frac{1}{2}\cdot\frac{1}{x^2+1}\cdot 2x$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x-2} - \frac{x}{x^2+1}$

$\frac{dy}{dx} = y\left(\frac{2}{x-2} - \frac{x}{x^2+1}\right)$

$= \frac{(x-2)^2}{\sqrt{x^2+1}}\left(\frac{2}{x-2} - \frac{x}{x^2+1}\right)$

3. Find the derivative of $y = sin(x)^{2x}$

$\ln y = \ln(\sin(x)^{2x})$

$= 2x \ln \sin(x)$

Take derivative of both sides

$\frac{1}{y}\frac{dy}{dx} = 2\ln(\sin(x)) + \frac{2x}{\sin(x)}\cdot\cos(x)$

$= 2\ln(\sin(x)) + 2x\cot(x)$

$\frac{dy}{dx} = y(2\ln(\sin(x)) + 2x\cot(x))$

$= \sin(x)^{2x}(2\ln(\sin(x)) + 2x\cot(x))$

4. Find the derivative of $y = \dfrac{(1-x)(2+x)^2(3-x)^3}{(4+x)^4}$

$\ln(y) = \ln\left(\dfrac{(1-x)(2+x)^2(3-x)^3}{(4+x)^4}\right)$

$= \ln(1-x) + 2\ln(2+x) + 3\ln(3-x) - 4\ln(4+x)$

Take derivative of both sides

$\frac{1}{y}\frac{dy}{dx} = \frac{-1}{1-x} + \frac{2}{2+x} - \frac{3}{3-x} - \frac{4}{4+x}$

$\frac{dy}{dx} = y\left(\frac{-1}{1-x} + \frac{2}{2+x} - \frac{3}{3-x} - \frac{4}{4+x}\right)$

$\frac{dy}{dx} = \frac{(1-x)(2+x)^2(3-x)^3}{(4+x)^4}\left(\frac{-1}{1-x} + \frac{2}{2+x} - \frac{3}{3-x} - \frac{4}{4-x}\right)$

5. Find the derivative of $y = (\ln(\sin(x)))^x$

$\ln y = \ln\left(\ln(\sin(x))^x\right)$

$= x\ln(\ln(\sin(x)))$

Take derivative of both sides

$\frac{1}{y}\frac{dy}{dx} = \ln(\ln(\sin(x))) + x\frac{1}{\ln(\sin(x))}\cdot\frac{1}{\sin(x)}\cdot\cos(x)$

$= \ln(\ln(\sin(x))) + \frac{x\cot(x)}{\ln(\sin(x))}$

$\frac{dy}{dx} = y\left(\ln(\ln(\sin(x))) + \frac{x\cot(x)}{\ln(\sin(x))}\right)$

$= \ln(\sin(x))^x\left(\ln(\ln(\sin(x))) + \frac{x\cot(x)}{\ln(\sin(x))}\right)$

6. Find the derivative of $y = \dfrac{(6x^2+9)^2}{\sqrt{3x^3-2}}$

$\ln y = \ln\left(\dfrac{(6x^2+9)^2}{\sqrt{3x^3-2}}\right)$

$= 2\ln(6x^2+9) - \frac{1}{2}\ln(3x^3-2)$

Take derivative of both sides

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{6x^2+9}\cdot 12x - \frac{1}{2}\cdot\frac{1}{3x^3-2}\cdot 9x^2$

$= \frac{24x}{6x^2+9} - \frac{9}{2}\frac{x^2}{3x^3-2}$

$\frac{dy}{dx} = y\left(\frac{24x}{6x^2+9} - \frac{9}{2}\frac{x^2}{3x^3-2}\right)$

$= \frac{(6x^2+9)^2}{\sqrt{3x^3-2}}\left(\frac{24x}{6x^2+9} - \frac{9}{2}\frac{x^2}{3x^3-2}\right)$