

# Power Rule

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Chain Rule:

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

1. Find the derivative of  $f(x) = (5x^2 + 3x - 1)^{1/2}$

$$\begin{array}{ll} h(x) = x^{1/2} & h'(x) = \frac{1}{2} x^{-1/2} \\ g(x) = 5x^2 + 3x - 1 & g'(x) = 10x + 3 \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = x^{1/2} \\ g(x) = 5x^2 + 3x - 1 \end{array}} \right\} \Rightarrow f'(x) = h'(g(x)) \cdot g'(x) \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad = \frac{1}{2} (5x^2 + 3x - 1)^{-1/2} \cdot (10x + 3)$$

2. Find the derivative of  $f(x) = (3x^2 + 1)^5$

$$\begin{array}{ll} h(x) = x^5 & h'(x) = 5x^4 \\ g(x) = 3x^2 + 1 & g'(x) = 6x \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = x^5 \\ g(x) = 3x^2 + 1 \end{array}} \right\} \Rightarrow f'(x) = 5(3x^2 + 1)^4 \cdot 6x$$

3. Find the derivative of  $f(x) = e^{x^2 - x + 2}$

$$\begin{array}{ll} h(x) = e^x & h'(x) = e^x \\ g(x) = x^2 - x + 2 & g'(x) = 2x - 1 \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = e^x \\ g(x) = x^2 - x + 2 \end{array}} \right\} \Rightarrow f'(x) = e^{x^2 - x + 2} \cdot (2x - 1)$$

4. Find the derivative of  $f(x) = \sin(\cos(x))$

$$\begin{array}{l} h(x) = \sin(x) \\ g(x) = \cos(x) \end{array} \quad \begin{array}{l} h'(x) = \cos(x) \\ g'(x) = -\sin(x) \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = \sin(x) \\ g(x) = \cos(x) \end{array}} \right\} \Rightarrow \underline{f'(x) = \cos(\cos(x)) \cdot (-\sin(x))}$$

5. Find the derivative of  $f(x) = \ln(x^2 + 1)$

$$\begin{array}{l} h(x) = \ln x \\ g(x) = x^2 + 1 \end{array} \quad \begin{array}{l} h'(x) = \frac{1}{x} \\ g'(x) = 2x \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = \ln x \\ g(x) = x^2 + 1 \end{array}} \right\} \Rightarrow \underline{f'(x) = \frac{1}{x^2 + 1} \cdot 2x}$$

6. Find the derivative of  $f(x) = e^{\sin(x^2)}$

$$\begin{array}{l} h(x) = e^x \\ g(x) = \sin(x) \\ k(x) = x^2 \end{array} \quad \begin{array}{l} h'(x) = e^x \\ g'(x) = \cos(x) \\ k'(x) = 2x \end{array} \quad \left. \vphantom{\begin{array}{l} h(x) = e^x \\ g(x) = \sin(x) \\ k(x) = x^2 \end{array}} \right\} \Rightarrow \underline{f'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x}$$

$$f(x) = h(g(k(x)))$$

$$f'(x) = h'(g(k(x))) \cdot g'(k(x)) \cdot k'(x)$$

7. Find the derivative of  $f(x) = \tan(\csc(e^{3x+1}))$

$$\begin{array}{l} h(x) = \tan(x) \\ g(x) = \csc(x) \\ k(x) = e^x \\ s(x) = 3x + 1 \end{array} \quad \begin{array}{l} h'(x) = \sec^2(x) \\ g'(x) = -\csc(x) \cot(x) \\ k'(x) = e^x \\ s'(x) = 3 \end{array}$$

$$\Rightarrow \underline{f'(x) = \sec^2(\csc(e^{3x+1})) \cdot -\csc(e^{3x+1}) \cot(e^{3x+1}) \cdot e^{3x+1} \cdot 3}$$