Euclidean Algorithm

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The Euclidean Algorithm is a tool used to determine the greatest common divisor (GCD) of two numbers. It follows this three step procedure:

1. Let a be the larger of the two numbers in magnitude and b be the smaller.
2. Divide a by b and get the remainder R. If $R = 0$ then the GCD of a and b is R.
3. If $R \neq 0$, replace a with b and replace b with R and repeat steps 2 and 3 until $R = 0$.

Examples:

1. Find the GCD of 126 and 78.

   Solution:

   $$\begin{aligned} 126 &= 1 \times 78 + 46 \\ 78 &= 1 \times 48 + 30 \\ 48 &= 1 \times 30 + 18 \\ 30 &= 1 \times 18 + 12 \\ 18 &= 1 \times 12 + 6 \\ 12 &= 2 \times 6 + 0 \end{aligned}$$

   Therefore $\text{GCD}(126, 78) = 6$.

2. Find the GCD of 456 and 112.

   Solution:

   $$\begin{aligned} 456 &= 4 \times 112 + 8 \\ 112 &= 14 \times 8 + 0 \end{aligned}$$

   Therefore $\text{GCD}(456, 112) = 8$.

3. Find the GCD of 97 and 18.

Solution:

$$\begin{aligned} 97 &= 5 \times 18 + 7 \\ 18 &= 2 \times 7 + 4 \\ 7 &= 1 \times 4 + 3 \\ 4 &= 1 \times 3 + 1 \\ 3 &= 3 \times 1 + 0 \end{aligned}$$

Therefore $\text{GCD}(97, 18) = 1$.

4. Find the GCD of 234 and 63.

Solution:

$$\begin{aligned} 234 &= 3 \times 63 + 45 \\ 63 &= 1 \times 45 + 18 \\ 45 &= 2 \times 18 + 9 \\ 18 &= 2 \times 9 + 0 \end{aligned}$$

Therefore $\text{GCD}(234, 63) = 9$.

5. Find the GCD of 140 and 49.

Solution:

$$\begin{aligned} 140 &= 2 \times 49 + 42 \\ 49 &= 1 \times 42 + 7 \\ 42 &= 6 \times 7 + 0 \end{aligned}$$

Therefore $\text{GCD}(140, 49) = 7$.

Questions:

1. Find the GCD of 412 and 56.

2. Find the GCD of 414 and 230.

3. Find the GCD of 321 and 144.

4. Find the GCD of 1026 and 453.

5. Find the GCD of 554 and 214.

6. Find the GCD of 351 and 189.

7. Find the GCD of 213 and 47.

8. Find the GCD of 1452 and 670.