# Exponents

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Laws of Exponents:

Table 1: For $a \neq 0$, $b \neq 0$

| | |
|---|---|
| Product Rule | $a^x \cdot a^y = a^{x+y}$ |
| Quotient Rule | $\dfrac{a^x}{a^y} = a^{x-y}$ |
| Power Rule | $(a^x)^y = a^{xy}$ |
| Power of a Product Rule | $(ab)^x = a^x b^x$ |
| Power of a Fraction Rule | $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$ |
| Zero Exponent | $a^0 = 1$ |
| Negative Exponent | $a^{-x} = \dfrac{1}{a^x}$ |
| Fractional Exponent | $a^{x/y} = \sqrt[y]{a^x}$ |

Examples:

1. Write $\dfrac{3^3 \cdot 3^2 \cdot 3^{-1}}{3^6}$ in the simplest form.

*Solution*:

$$\frac{3^3 \cdot 3^2 \cdot 3^{-1}}{3^6} = 3^{3+2-1-6}$$
$$= 3^{-2}$$
$$= \frac{1}{3^2}$$
$$= \frac{1}{9}$$

2. Simplify $2x^3y^6 \times 8x^5y^2$

*Solution:*

$$2x^3y^6 \times 8x^5y^2 = 16x^{3+5}y^{6+2}$$
$$= 16x^8y^8$$

3. Write $\dfrac{5^{2/3} \cdot 5^{1/4}}{5^{11/12}}$ in the simplest form.

*Solution*:

$$\frac{5^{2/3} \cdot 5^{1/4}}{5^{11/12}} = 5^{2/3+1/4-11/12}$$
$$= 5^{8/12+3/12-11/12}$$
$$= 5^0$$
$$= 1$$

4. Simplify $\dfrac{15x^2y^3z^7}{5xy^{-1}z^4}$

*Solution:*

$$\frac{15x^2y^3z^7}{5xy^{-1}z^4} = \frac{15}{5}x^{2-1}y^{3-(-1)}z^{7-4}$$
$$= 3xy^4z^3$$

Questions:

1. Simplify $3^5 \times 3^2 \times 3^{-4}$

2. Simplify $2^2 \times 2^{-1} \times 2^{-7}$

3. Simplify $6^7 \times 6^{12} \times 6^{-5} \times 6^9$

4. Simplify $7^3 \times 7 \times 7^{-12} \times 7^{14}$

5. Simplify $2^{10} \times 2^{-9} \times 2^{-6} \times 2^{-2}$

6. Simplify $13^4 \times 13^{15} \times 13^{-2} \times 13^6 \times 13^{-9}$

7. Simplify $x^3 \times x^4 \times x^{12}$

8. Simplify $x^{-3} \times x^5 \times x^{-4} \times x^{-6}$

9. Simplify $x^{14} \times x^{-21} \times x^{-61} \times x^{34}$

10. Simplify $2^5 \times 3^{-5} \times 3^{-4} \times 2^2 \times 3^7$

11. Simplify $4^3 \times 4^3 \times 5^6 \times 4^{-7} \times 5^{-3}$

12. Simplify $6^7 \times 7^6 \times 6^{12} \times 7^{-18} \times 6^{-15}$

13. Simplify $2^8 \times 3^{-12} \times 2^9 \times 5^4 \times 3^7$

14. Simplify $\dfrac{2^5 \times 2^7}{2^4}$

15. Simplify $\dfrac{x^{-4} \times x^{10}}{x^5 \times x^6}$

16. Simplify $\dfrac{5^3 \times 5^5 \times 5^2}{5^6 \times 5^9}$

17. Simplify $\dfrac{2^5 \times 2^7}{2^4}$

18. Simplify $\dfrac{3^3 \times 2^6}{2^3 \times 3^5}$

19. Simplify $\dfrac{x^6 \times y^{14}}{x^8 \times y^9}$

20. Simplify $\dfrac{7^3 \times 7^{-2} \times 7^8 \times 7^5}{7^9 \times 7^6}$

21. Simplify $\dfrac{2^{13}}{2^5 \times 2^4 \times 2^8 \times 2^3}$

22. Write $\dfrac{2^3(4 \times 5)^3}{2^5 \times 5^4}$ in the simplest form.

23. Simplify $5a^2b^3 - \dfrac{6b^4}{2a^{-2}b}$

24. Simplify $\left(\dfrac{2^2 \times 2^3}{2^2}\right)^{1/3} - 3^2 \times 3^{-2}$

25. Simplify $\dfrac{8a^3b^{-2}}{(2a^{-3}b^2)(ab)}$

26. Simplify $(3x^{-3}y^2)(2x^2)(4y^4)^{-2}$