Mathematical Laws

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The **Commutative Law** allows you to switch the order of the terms you are performing an operation on.

For addition:

$$a + b = b + a$$

For multiplication:

$$a \times b = b \times a$$

The **Associative Law** allows you tells us that how we group things doesn't matter.

For addition:

$$(a + b) + c = a + (b + c)$$

For multiplication:

$$(a \times b) \times c = a \times (b \times c)$$

The **Distributive Law** allows you to multiply a number into a larger term, or pull out a common factor form a few terms.

$$a \times (b + c) = a \times b + a \times c$$

Questions:

1. Simplify $x^2 - 7x + 2 - 3x - 2x^2 + 9$

Solution:

$$\begin{aligned} & x^2 - 7x + 2 - 3x - 2x^2 + 9 \\ &= x^2 - 2x^2 - 7x - 3x + 2 + 9 \\ &= (1 - 2)x^2 + (-7 - 3)x + (2 + 9) \\ &= -x^2 - 10x + 11 \end{aligned}$$

2. Simplify $-7x^4 + 5x^3 + 13x - 1 - 5x^5 + 7x^4 - 10x^3 - 7x - 3$

Solution:

$$\begin{aligned}
&-7x^4 + 5x^3 + 13x - 1 - 5x^5 + 7x^4 - 10x^3 - 7x - 3 \\
&= -5x^5 - 7x^4 + 7x^4 + 5x^3 - 10x^3 + 13x - 7x - 1 - 3 \\
&= -5x^5 + (-7 + 7)x^4 + (5 - 10)x^3 + (13 - 7)x + (-1 - 3) \\
&= -5x^5 + -5x^3 + 6x - 4
\end{aligned}$$

3. Simplify $x^3 + 2x^2 - 4x^3 + x - 13x^2 + 3 - 2x + 4$

Solution:

$$\begin{aligned}
&x^3 + 2x^2 - 4x^3 + x - 13x^2 + 3 - 2x + 4 \\
&= x^3 - 4x^3 + 2x^2 - 13x^2 + x - 2x + 3 + 4 \\
&= (1 - 4)x^3 + (2 - 13)x^2 + (1 - 2)x + (3 + 4) \\
&= -3x^3 - 11x^2 - x + 7
\end{aligned}$$

4. Simplify $4 \times (5x^2 + 6x - 9)$

Solution:

$$\begin{aligned}
&4 \times (5x^2 + 6x - 9) \\
&= (4 \times 5x^2) + (4 \times 6x) - (4 \times 9) \\
&= 20x^2 + 24x - 36
\end{aligned}$$

5. Simplify $-2 \times (x^2 - 7x - 17)$

Solution:

$$\begin{aligned}
&-2 \times (x^2 - 7x - 17) \\
&= (-2 \times x^2) + (-2 \times -7x) + (-2 \times -17) \\
&= -2x^2 + 14x + 34
\end{aligned}$$

6. Simplify $3 \times (4x^3 - 2x^2 - 5x + 12)$

Solution:

$$\begin{aligned}
&3 \times (4x^3 - 2x^2 - 5x + 12) \\
&= (3 \times 4x^3) + (3 \times -2x^2) + (3 \times -5x) + (3 \times 12) \\
&= 12x^3 - 6x^2 - 15x + 36
\end{aligned}$$

7. Simplify $2 \times (2x^2 - 5x + 6) - 3 \times (x^2 + 3x + 4)$

Solution:

$$
\begin{aligned}
&2 \times (2x^2 - 5x + 6) - 3 \times (x^2 + 3x + 4) \\
&= (2 \times 2x^2) + (2 \times -5x) + (2 \times 6) + (-3 \times x^2) + (-3 \times 3x) + (-3 \times 4) \\
&= 4x^2 - 10x + 12 - 3x^2 - 9x - 12 \\
&= 4x^2 - 3x^2 - 10x - 9x + 12 - 12 \\
&= (4 - 3)x^2 + (-10 - 9)x + (12 - 12) \\
&= x^2 - 19x
\end{aligned}
$$

8. Simplify $-1 \times (7x^2 - 6x - 6 + 4x^2 + 2x - 5)$

Solution:

$$
\begin{aligned}
&- 1 \times (7x^2 - 6x - 6 + 4x^2 + 2x - 5) \\
&= -1 \times (7x^2 + 4x^2 - 6x + 2x - 6 - 5) \\
&= -1 \times \big((7 + 4)x^2 + (-6 + 2)x + (-6 - 5)\big) \\
&= -1 \times (11x^2 - 4x - 11) \\
&= (-1 \times 11x^2) + (-1 \times -4x) + (-1 \times -11) \\
&= -11x^2 + 4x + 11
\end{aligned}
$$

9. Simplify $4a - 3b + 2 \times \left(\frac{3}{2}a - \frac{3}{4}b\right)$

Solution:

$$
\begin{aligned}
&4a - 3b + 2 \times \left(\frac{3}{2}a - \frac{3}{4}b\right) \\
&= 4a - 3b + 2 \times \left(\frac{3}{2}a\right) - 2 \times \left(\frac{3}{4}b\right) \\
&= 4a - 3b + 3a - \frac{3}{2}b \\
&= 4a + 3a - 3b - \frac{3}{2}b \\
&= (4 + 3)a + \left(- 3 - \frac{3}{2}\right)b \\
&= 7a + \left(- \frac{6}{2} - \frac{3}{2}\right)b \\
&= 7a - \frac{9}{2}b
\end{aligned}
$$

10. Simplify $\dfrac{5x^2 - 6x + 9 - x^2 - 10x + 5}{8}$

Solution:

$$\frac{5x^2 - 6x + 9 - x^2 - 10x + 5}{8}$$

$$= \frac{5x^2 - x^2 - 6x - 10x + 9 + 5}{8}$$

$$= \frac{(5-1)x^2 + (-6-10)x + (9+5)}{8}$$

$$= \frac{4x^2 - 16x + 14}{8}$$

$$= \frac{4}{8}x^2 - \frac{16}{8}x + \frac{14}{8}$$

$$= \frac{1}{2}x^2 - 2x + \frac{7}{4}$$