

# Euclidean Algorithm

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The Euclidean Algorithm is a tool used to determine the greatest common divisor (GCD) of two numbers. It follows this three step procedure:

1. Let  $a$  be the larger of the two numbers in magnitude and  $b$  be the smaller.
2. Divide  $a$  by  $b$  and get the remainder  $R$ . If  $R = 0$  then the GCD of  $a$  and  $b$  is  $R$ .
3. If  $R \neq 0$ , replace  $a$  with  $b$  and replace  $b$  with  $R$  and repeat steps 2 and 3 until  $R = 0$ .

Examples:

1. Find the GCD of 126 and 78.

*Solution:*

$$126 = 1 \times 78 + 46$$

$$78 = 1 \times 46 + 30$$

$$46 = 1 \times 30 + 16$$

$$30 = 1 \times 16 + 14$$

$$16 = 1 \times 14 + 2$$

$$14 = 7 \times 2 + 0$$

Therefore  $\text{GCD}(126, 78) = 2$ .

2. Find the GCD of 456 and 112.

*Solution:*

$$456 = 4 \times 112 + 8$$

$$112 = 14 \times 8 + 0$$

Therefore  $\text{GCD}(456, 112) = 8$ .

3. Find the GCD of 97 and 18.

*Solution:*

$$97 = 5 \times 18 + 7$$

$$18 = 2 \times 7 + 4$$

$$7 = 1 \times 4 + 3$$

$$4 = 1 \times 3 + 1$$

$$3 = 3 \times 1 + 0$$

Therefore  $\text{GCD}(97, 18) = 1$ .

4. Find the GCD of 234 and 63.

*Solution:*

$$234 = 3 \times 63 + 45$$

$$63 = 1 \times 45 + 18$$

$$45 = 2 \times 18 + 9$$

$$18 = 2 \times 9 + 0$$

Therefore  $\text{GCD}(234, 63) = 9$ .

5. Find the GCD of 140 and 49.

*Solution:*

$$140 = 2 \times 49 + 42$$

$$49 = 1 \times 42 + 7$$

$$42 = 6 \times 7 + 0$$

Therefore  $\text{GCD}(140, 49) = 7$ .

Questions:

1. Find the GCD of 412 and 56.

*Solution:*

$$412 = 7 \times 56 + 20$$

$$56 = 2 \times 20 + 16$$

$$20 = 1 \times 16 + 4$$

$$16 = 4 \times 4 + 0$$

Therefore  $\text{GCD}(412, 56) = 4$ .

2. Find the GCD of 414 and 230.

*Solution:*

$$414 = 1 \times 230 + 184$$

$$230 = 1 \times 184 + 46$$

$$184 = 4 \times 46$$

Therefore  $\text{GCD}(414, 230) = 46$ .

3. Find the GCD of 321 and 144.

*Solution:*

$$321 = 2 \times 144 + 33$$

$$144 = 4 \times 33 + 12$$

$$33 = 2 \times 12 + 9$$

$$12 = 1 \times 9 + 3$$

$$9 = 3 \times 3 + 0$$

Therefore  $\text{GCD}(321, 144) = 3$ .

4. Find the GCD of 1026 and 453.

*Solution:*

$$1026 = 2 \times 453 + 120$$

$$453 = 3 \times 120 + 93$$

$$120 = 1 \times 93 + 27$$

$$93 = 3 \times 27 + 12$$

$$27 = 2 \times 12 + 3$$

$$12 = 4 \times 3 + 0$$

Therefore  $\text{GCD}(1026, 453) = 3$ .

5. Find the GCD of 554 and 214.

*Solution:*

$$554 = 2 \times 214 + 126$$

$$214 = 1 \times 126 + 88$$

$$126 = 1 \times 88 + 38$$

$$88 = 2 \times 38 + 12$$

$$38 = 3 \times 12 + 2$$

$$12 = 6 \times 2 + 0$$

Therefore  $\text{GCD}(554, 214) = 2$ .

6. Find the GCD of 351 and 189.

*Solution:*

$$351 = 1 \times 189 + 162$$

$$189 = 1 \times 162 + 27$$

$$162 = 6 \times 27 + 0$$

Therefore  $\text{GCD}(351, 189) = 27$ .

7. Find the GCD of 213 and 47.

*Solution:*

$$213 = 4 \times 47 + 25$$

$$47 = 1 \times 25 + 22$$

$$25 = 1 \times 22 + 3$$

$$22 = 7 \times 3 + 1$$

$$3 = 3 \times 1 + 0$$

Therefore  $\text{GCD}(213, 47) = 1$ .

8. Find the GCD of 1452 and 670.

*Solution:*

$$1452 = 2 \times 670 + 112$$

$$670 = 5 \times 112 + 110$$

$$112 = 1 \times 110 + 2$$

$$110 = 55 \times 2 + 0$$

Therefore  $\text{GCD}(1452, 670) = 2$ .