Calculating Volumes Using Cylindrical Shells

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The formula to calculate the volume using cylindrical shells is:

$$V = \int_a^b 2\pi\, r(x) f(x)\, dx$$

Where $r(x)$ is the radius to the line you are rotating about. Using we just consider rotations around the y-axis, so in this cause $r(x) = x$. However, if we consider rotations around the line $x = a$, then our radius would be $r(x) = x - a$.

Examples:

1. Find the volume of the solid generated by rotating the region bounded by the curves $y = x$ and $y = x^2$, about the y-axis.

   Solution:

   $$x = x^2 \text{ if and only if } x = 0, 1$$

   $$\begin{aligned} V &= \int_0^1 2\pi x(x - x^2)\, dx \\ &= 2\pi \int_0^1 x^2 - x^3\, dx \\ &= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Bigg|_0^1 \\ &= 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) \\ &= \frac{\pi}{6} \end{aligned}$$

2. Find the volume of the solid generated by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = x$, about the line $x = -4$.

   Solution:

3. Find the volume of the solid generated by rotating the region bounded by the curves $y = \frac{1}{x\sqrt{x^2+1}}$, and $x = 1$, and $x = 2$ about the line y-axis.

Solution:

$$
\begin{aligned}
V &= \int_1^2 2\pi x \frac{1}{x\sqrt{x^2+1}}\,dx \\
&= 2\pi \int_0^1 \frac{1}{x^2+1}\,dx \\
&= 2\pi \sinh^{-1}(x)\Big|_1^2 \\
&= 2\pi\big(\sinh^{-1}(2) - \sinh^{-1}(1)\big)
\end{aligned}
$$