Rationalizing

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1. *Solve* $\lim_{x \to 8} \frac{\sqrt{x+1}-3}{x-8}$

$$\lim_{x\to 8} \frac{\sqrt{x+1}-3}{x-8} \cdot \frac{\sqrt{x+1}+3}{\sqrt{x+1}+3} = \lim_{x\to 8} \frac{x+1-9}{(x-8)(\sqrt{x+1}+3)} = \lim_{x\to 8} \frac{x-8}{(x-8)(\sqrt{x+1}+3)}$$

$$= \lim_{x\to 8} \frac{1}{\sqrt{x+1}+3} = \frac{1}{\sqrt{8+1}+3} \boxed{= \frac{1}{6}}$$

2. *Solve* $\lim_{x \to 5} \frac{\sqrt{x^2+11}-6}{x-5}$

$$\lim_{x\to 5} \frac{\sqrt{x^2+11}-6}{x-5} \cdot \frac{\sqrt{x^2+11}+6}{\sqrt{x^2+11}+6} = \lim_{x\to 5} \frac{x^2+11-36}{(x-5)(\sqrt{x^2+11}+6)} = \lim_{x\to 5} \frac{x^2-25}{(x-5)(\sqrt{x^2+11}+6)}$$

$$= \lim_{x\to 5} \frac{(x-5)(x+5)}{(x-5)(\sqrt{x^2+11}+6)} = \lim_{x\to 5} \frac{x+5}{\sqrt{x^2+11}+6} = \frac{5+5}{\sqrt{5^2+11}+6} = \frac{10}{12} \boxed{= \frac{5}{6}}$$

3. *Solve* $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

$$\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \lim_{x\to 0} \frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})} = \lim_{x\to} \frac{x}{x(\sqrt{x+2}+\sqrt{2})} = \lim_{x\to 0} \frac{1}{(\sqrt{x+2}+\sqrt{2})}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}} \boxed{= \frac{1}{2\sqrt{2}}}$$

4. *Solve* $\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3}$

$$\lim_{x\to 0} \frac{x}{\sqrt{x+9}-3} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x+9-9} = \lim_{x\to 0} \frac{x(\sqrt{x+9}+3)}{x} = \lim_{x\to 0} (\sqrt{x+9}+3)$$

$$= \sqrt{9}+3 \boxed{= 6}$$

5. *Solve* $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{(x-1)(\sqrt{x}+1)}{x-1} = \lim_{x\to 1} \sqrt{x}+1$$

$$= \sqrt{1}+1 \boxed{= 2}$$

6. *Solve* $\lim_{x\to 3} \frac{\sqrt{7-x}-2}{1-\sqrt{4-x}}$

$$\lim_{x\to 3} \frac{\sqrt{7-x}-2}{1-\sqrt{4-x}} \cdot \frac{1+\sqrt{4-x}}{1+\sqrt{4-x}} = \lim_{x\to 3} \frac{(\sqrt{7-x}-2)(1+\sqrt{4-x})}{1-4+x} = \lim_{x\to 3} \frac{(\sqrt{7-x}-2)(1+\sqrt{4-x})}{-(x-3)}$$

$$= \lim_{x\to 3} \frac{(\sqrt{7-x}-2)(1+\sqrt{4-x})}{-(x-3)} \cdot \frac{\sqrt{7-x}+2}{\sqrt{7-x}+2} = \lim_{x\to 3} \frac{(7-x-4)(1+\sqrt{4-x})}{-(x-3)(\sqrt{7-x}+2)} = \lim_{x\to 3} \frac{-(x-3)(1+\sqrt{4-x})}{-(x-3)(\sqrt{7-x}+2)}$$

$$= \lim_{x\to 3} \frac{1+\sqrt{4-x}}{\sqrt{7-x}+2} = \frac{1+\sqrt{4-3}}{\sqrt{7-x}+2} = \frac{1+\sqrt{1}}{\sqrt{4}+2} = \frac{2}{4} \boxed{= \frac{1}{2}}$$

7. *Solve* $\lim_{x\to 0} \frac{x}{3-\sqrt{x+9}}$

$$\lim_{x\to 0} \frac{x}{3-\sqrt{x+9}} \cdot \frac{(3+\sqrt{x+9})}{(3+\sqrt{x+9})} = \lim_{x\to 0} \frac{x(3+\sqrt{x+9})}{9-(x+9)} = \lim_{x\to 0} \frac{x(3+\sqrt{x+9})}{-x} = \lim_{x\to 0} -(3+\sqrt{x+9})$$

$$= -(3+\sqrt{0+9}) \boxed{= -6}$$

8. *Solve* $\lim_{x \to 0} \dfrac{\sqrt{x+9} - \sqrt{-x+9}}{x}$

$$\lim_{x \to 0} \frac{\sqrt{x+9} - \sqrt{-x+9}}{x} \cdot \frac{\sqrt{x+9} + \sqrt{-x+9}}{\sqrt{x+9} + \sqrt{-x+9}} = \lim_{x \to 0} \frac{x+9-(-x+9)}{x(\sqrt{x+9} + \sqrt{-x+9})} = \lim_{x \to 0} \frac{2x}{x(\sqrt{x+9} + \sqrt{-x+9})}$$

$$= \lim_{x \to 0} \frac{2}{\sqrt{x+9} + \sqrt{-x+9}} = \frac{2}{\sqrt{0+9} + \sqrt{-0+9}} = \frac{2}{6} \boxed{= \frac{1}{3}}$$

9. *Solve* $\lim_{x \to 4} \dfrac{\sqrt{x+5} - 3}{x-4}$

$$\lim_{x \to 4} \frac{\sqrt{x+5} - 3}{x-4} \cdot \frac{\sqrt{x+5} + 3}{\sqrt{x+5} + 3} = \lim_{x \to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5} + 3)} = \lim_{x \to 4} \frac{x-4}{(x-4)(\sqrt{x+5} + 3)}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{x+5} + 3} = \frac{1}{\sqrt{4+5} + 3} \boxed{= \frac{1}{6}}$$

10. *Solve* $\lim_{x \to 5} \dfrac{x-2}{\sqrt{x-1} - 1}$

$$\lim_{x \to 5} \frac{x-2}{\sqrt{x-1} - 1} \cdot \frac{\sqrt{x-1} + 1}{\sqrt{x-1} + 1} = \lim_{x \to 5} \frac{(x-2)(\sqrt{x-1} + 1)}{x-1-1} = \lim_{x \to 5} \frac{(x-2)(\sqrt{x-1} + 1)}{(x-2)}$$

$$= \lim_{x \to 5} \sqrt{x-1} + 1 = \sqrt{5-1} + 1 \boxed{= 3}$$