Euclidean Algorithm

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The Euclidean Algorithm is a tool used to determine the greatest common divisor (GCD) of two numbers. It follows this three step procedure:

1. Let a be the larger of the two numbers in magnitude and b be the smaller.
2. Divide a by b and get the remainder R. If $R = 0$ then the GCD of a and b is R.
3. If $R \neq 0$, replace a with b and replace b with R and repeat steps 2 and 3 until $R = 0$.

Examples:

1. Find the GCD of 126 and 78.

   Solution:

   $$\begin{aligned} 126 &= 1 \times 78 + 46 \\ 78 &= 1 \times 48 + 30 \\ 48 &= 1 \times 30 + 18 \\ 30 &= 1 \times 18 + 12 \\ 18 &= 1 \times 12 + 6 \\ 12 &= 2 \times 6 + 0 \end{aligned}$$

   Therefore $\text{GCD}(126, 78) = 6$.

2. Find the GCD of 456 and 112.

   Solution:

   $$\begin{aligned} 456 &= 4 \times 112 + 8 \\ 112 &= 14 \times 8 + 0 \end{aligned}$$

   Therefore $\text{GCD}(456, 112) = 8$.

3. Find the GCD of 97 and 18.

Solution:

$$\begin{aligned} 97 &= 5 \times 18 + 7 \\ 18 &= 2 \times 7 + 4 \\ 7 &= 1 \times 4 + 3 \\ 4 &= 1 \times 3 + 1 \\ 3 &= 3 \times 1 + 0 \end{aligned}$$

Therefore $\text{GCD}(97, 18) = 1$.

4. Find the GCD of 234 and 63.

Solution:

$$\begin{aligned} 234 &= 3 \times 63 + 45 \\ 63 &= 1 \times 45 + 18 \\ 45 &= 2 \times 18 + 9 \\ 18 &= 2 \times 9 + 0 \end{aligned}$$

Therefore $\text{GCD}(234, 63) = 9$.

5. Find the GCD of 140 and 49.

Solution:

$$\begin{aligned} 140 &= 2 \times 49 + 42 \\ 49 &= 1 \times 42 + 7 \\ 42 &= 6 \times 7 + 0 \end{aligned}$$

Therefore $\text{GCD}(140, 49) = 7$.

Questions:

1. Find the GCD of 412 and 56.

   Solution:

   $$\begin{aligned} 412 &= 7 \times 56 + 20 \\ 56 &= 2 \times 20 + 16 \\ 20 &= 1 \times 16 + 4 \\ 16 &= 4 \times 4 + 0 \end{aligned}$$

   Therefore $\text{GCD}(412, 56) = 4$.

2. Find the GCD of 414 and 230.

   Solution:

   $$\begin{aligned} 414 &= 1 \times 230 + 184 \\ 230 &= 1 \times 184 + 46 \\ 184 &= 4 \times 46 \end{aligned}$$

   Therefore $\text{GCD}(414, 230) = 46$.

3. Find the GCD of 321 and 144.

   Solution:

   $$\begin{aligned} 321 &= 2 \times 144 + 33 \\ 144 &= 4 \times 33 + 12 \\ 33 &= 2 \times 12 + 9 \\ 12 &= 1 \times 9 + 3 \\ 9 &= 3 \times 3 + 0 \end{aligned}$$

   Therefore $\text{GCD}(321, 144) = 3$.

4. Find the GCD of 1026 and 453.

Solution:

$$\begin{aligned} 1026 &= 2 \times 453 + 120 \\ 453 &= 3 \times 120 + 93 \\ 120 &= 1 \times 93 + 27 \\ 93 &= 3 \times 27 + 12 \\ 27 &= 2 \times 12 + 3 \\ 12 &= 4 \times 3 + 0 \end{aligned}$$

Therefore $\text{GCD}(1026, 453) = 3$.

5. Find the GCD of 554 and 214.

Solution:

$$\begin{aligned} 554 &= 2 \times 214 + 126 \\ 214 &= 1 \times 126 + 88 \\ 126 &= 1 \times 88 + 38 \\ 88 &= 2 \times 38 + 12 \\ 38 &= 3 \times 12 + 2 \\ 12 &= 6 \times 2 + 0 \end{aligned}$$

Therefore $\text{GCD}(554, 214) = 2$.

6. Find the GCD of 351 and 189.

Solution:

$$\begin{aligned} 351 &= 1 \times 189 + 162 \\ 189 &= 1 \times 162 + 27 \\ 162 &= 6 \times 27 + 0 \end{aligned}$$

Therefore $\text{GCD}(351, 189) = 27$.

7. Find the GCD of 213 and 47.

Solution:

$$\begin{aligned} 213 &= 4 \times 47 + 25 \\ 47 &= 1 \times 25 + 22 \\ 25 &= 1 \times 22 + 3 \\ 22 &= 7 \times 3 + 1 \\ 3 &= 3 \times 1 + 0 \end{aligned}$$

Therefore $\text{GCD}(213, 47) = 1$.

8. Find the GCD of 1452 and 670.

Solution:

$$\begin{aligned} 1452 &= 2 \times 670 + 112 \\ 670 &= 5 \times 112 + 110 \\ 112 &= 1 \times 110 + 2 \\ 110 &= 55 \times 2 + 0 \end{aligned}$$

Therefore $\text{GCD}(1452, 670) = 2$.