Implicit Differentiation

Stephen Styles

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1. Find the derivative of $\cos(x^2) = xe^y$

$$-\sin(x^2)\cdot 2x = e^y + xe^y \frac{dy}{dx}$$

$$-2x\sin(x^2) - e^y = xe^y \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-2x\sin(x^2) - e^y}{xe^y}$$

2. Find the derivative of $x^2 + y^2 = 3x$

$$2x + 2y \cdot \frac{dy}{dx} = 3$$

$$2y \cdot \frac{dy}{dx} = 3 - 2x$$

$$\frac{dy}{dx} = \frac{3-2x}{2y}$$

3. Find the derivative of $\sec(xy) + e^y = 3x - 4$

$$\sec(xy)\left(x\frac{dy}{dx} + y\right) + e^y \frac{dy}{dx} = 3$$

$$\sec(xy)\times\frac{dy}{dx} + \sec(xy)y + e^y\frac{dy}{dx} = 3$$

$$\sec(xy)\times\frac{dy}{dx} + e^y\frac{dy}{dx} = 3 - \sec(xy)y$$

$$\frac{dy}{dx}\left[\sec(xy)x + e^y\right] = 3 - \sec(xy)y$$

$$\frac{dy}{dx} = \frac{3 - \sec(xy)y}{\sec(xy)x + e^y}$$

4. Find the derivative of $5y^2 + \sin(y) = x^2$

$10y\frac{dy}{dx} + \cos(y)\frac{dy}{dx} = 2x$

$\frac{dy}{dx}\left[10y + \cos(y)\right] = 2x$

$\frac{dy}{dx} = \frac{2x}{10y + \cos(y)}$

5. Find the derivative of $\sin(x + y) = y^2\cos(x)$

$\cos(x+y)\left(1 + \frac{dy}{dx}\right) = 2y\frac{dy}{dx}\cos(x) + y^2(-\sin(x))$

$\cos(x+y) + \frac{dy}{dx}\cos(x+y) = 2y\frac{dy}{dx}\cos(x) - y^2\sin(x)$

$\frac{dy}{dx}\cos(x+y) - \frac{dy}{dx}2y\cos(x) = -\cos(x+y) - y^2\sin(x)$

$\frac{dy}{dx}\left[\cos(x+y) - 2y\cos(x)\right] = -\cos(x+y) - y^2\sin(x)$

$\frac{dy}{dx} = \frac{-\cos(x+y) - y^2\sin(x)}{\cos(x+y) - 2y\cos(x)}$

6. Find the derivative of $\csc(y) + \ln(x) = x^{-2}$

$-\csc(y)\cot(y)\frac{dy}{dx} + \frac{1}{x} = -2x^{-3}$

$-\csc(y)\cot(y)\frac{dy}{dx} = -2x^{-3} - \frac{1}{x}$

$\frac{dy}{dx} = \frac{2x^{-3} + \frac{1}{x}}{\csc(y)\cot(y)}$