Induction

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1. Prove by induction that $\displaystyle\sum_{k=1}^{n} k = \frac{n(n-1)}{2}$

2. Prove by induction that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n-1)(2n-1)}{6}$

3. Prove by induction that $n^3 - n$ is divisible by 3 for all positive integers.

4. Prove that the sequence $x_{n+1} = \dfrac{x_n + \sqrt{3x_n}}{2}$ is an increasing sequence where $x_1 = 1$.

5. Prove that $n! > 2^n$ for all positive integers greater than or equal to 4.

6. Prove that for any real number $x > -1$ and any positive integer n,
$(1 + x)^n \geq 1 + nx$.

7. Using induction, prove that the sequence $a_{n+1} = \dfrac{2a_n}{3 + a_n}$ is monotone with $a_1 = 1$ and bounded below by 0.

8. A sequence $\{a_n\}$ is given by $a_1 = 2$, $a_n = \sqrt{2 + a_{n-1}}$

   (a) Show by induction that $\{a_n\}$ is increasing and bounded above by 3.

   (b) Find $\lim\limits_{a \to \infty} a_n$.