Exponents

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Laws of Exponents:

Table 1: For $a \neq 0$, $b \neq 0$

Product Rule	$a^x \cdot a^y = a^{x+y}$
Quotient Rule	$\frac{a^x}{a^y} = a^{x-y}$
Power Rule	$(a^x)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{x/y} = \sqrt[y]{a^x}$

Examples:

1. Write $\frac{3^3 \cdot 3^2 \cdot 3^{-1}}{3^6}$ in the simplest form.

Solution:

$$\begin{aligned}\frac{3^3 \cdot 3^2 \cdot 3^{-1}}{3^6} &= 3^{3+2-1-6} \\ &= 3^{-2} \\ &= \frac{1}{3^2} \\ &= \frac{1}{9}\end{aligned}$$

2. Simplify $2x^3y^6 \times 8x^5y^2$

Solution:

$$\begin{aligned} 2x^3y^6 \times 8x^5y^2 &= 16x^{3+5}y^{6+2} \\ &= 16x^8y^8 \end{aligned}$$

3. Write $\dfrac{5^{2/3} \cdot 5^{1/4}}{5^{11/12}}$ in the simplest form.

Solution:

$$\begin{aligned} \frac{5^{2/3} \cdot 5^{1/4}}{5^{11/12}} &= 5^{2/3+1/4-11/12} \\ &= 5^{8/12+3/12-11/12} \\ &= 5^0 \\ &= 1 \end{aligned}$$

4. Simplify $\dfrac{15x^2y^3z^7}{5xy^{-1}z^4}$

Solution:

$$\begin{aligned} \frac{15x^2y^3z^7}{5xy^{-1}z^4} &= \frac{15}{5}x^{2-1}y^{3-(-1)}z^{7-4} \\ &= 3xy^4z^3 \end{aligned}$$

Questions:

1. Write $\dfrac{2^3(4 \cdot 5)^3}{2^5 \cdot 5^4}$ in the simplest form.

Solution:

$$\begin{aligned} \frac{2^3(4 \cdot 5)^3}{2^5 \cdot 5^4} &= \frac{2^3 \cdot 4^3 \cdot 5^3}{2^5 \cdot 5^4} \\ &= \frac{2^3 \cdot (2^2)^3 \cdot 5^3}{2^5 \cdot 5^4} \\ &= \frac{2^3 \cdot 2^6 \cdot 5^3}{2^5 \cdot 5^4} \\ &= 2^{3+6-5} \cdot 5^{3-4} \\ &= 2^4 \cdot 5^{-1} \\ &= \frac{2^4}{5} \\ &= \frac{16}{5} \end{aligned}$$

2. Simplify $5a^2b^3 - \dfrac{6b^4}{2a^{-2}b}$

Solution:

$$\begin{aligned} 5a^2b^3 - \frac{6b^4}{2a^{-2}b} &= 5a^2b^3 - 3a^2b^{4-1} \\ &= 5a^2b^3 - 3a^2b^3 \\ &= 2a^2b^3 \end{aligned}$$

3. Simplify $\left(\dfrac{2^2 \times 2^3}{2^2}\right)^{1/3} - 3^2 \times 3^{-2}$

Solution:

$$\begin{aligned} \left(\frac{2^2 \times 2^3}{2^2}\right)^{1/3} - 3^2 \times 3^{-2} &= \left(2^{2+3-2}\right)^{1/3} - 3^{2-2} \\ &= \left(2^3\right)^{1/3} - 3^0 \\ &= 2^{3\times 1/3} - 1 \\ &= 2^1 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

4. Simplify $\dfrac{8a^3b^{-2}}{(2a^{-3}b^2)(ab)}$

Solution:

$$\begin{aligned} \frac{8a^3b^{-2}}{(2a^{-3}b^2)(ab)} &= \frac{8}{2}a^{3-(-3)-1}b^{-2-2-1} \\ &= 4a^5b^{-5} \\ &= \frac{4a^5}{b^5} \end{aligned}$$

5. Simplify $(3x^{-3}y^2)(2x^2)(4y^4)^{-2}$

Solution:

$$\begin{aligned}(3x^{-3}y^2)(2x^2)(4y^4)^{-2} &= (3x^{-3}y^2)(2x^2)(4^{-2}y^{4\times -2}) \\ &= (3x^{-3}y^2)(2x^2)(4^{-2}y^{-8}) \\ &= \frac{3\times 2}{4^2}x^{-3+2}y^{2-8} \\ &= \frac{6}{16}x^{-1}y^{-6} \\ &= \frac{6}{16xy^6}\end{aligned}$$