Discrete Math 1510 Assignment 1

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Question 1

a) **Claim Is (p^q)->p a tautology.**

For a proposition to be a tautology I must be true for all outcomes.

P	Q	p^q	(p^q)->p
T	T	T	T
T	F	F	T
F	T	T	T
F	T	F	T

For this proposition (p^q)->p, all outcomes are true therefore the proposition is a tautology.

b) **Claim is (p^ ¬p) a contradiction.**

P	¬p	(p^¬P)
T	F	F
F	T	F

This compound proposition is false in all outcomes therefore contradiction.

c) **Claim Is (p^q) a tautology.**

P	Q	(p^q)
T	T	T
T	F	F
F	T	F
F	F	F

the proposition (p^q) produces one or more false outcome. Therefore, the proposition is not a tautology.

Question 2 Consider the following argument:

Intuition comes from much study now, or in the past (or both).

∴If you don't have much study in the past, intuition requires much study now.

a) Yes, this a valid argument as we can rewrite it as intuition = now or past.

Therefore i=n∨p

b) Translate it into formal logic and prove your answer to (a)

If I = True, p = false, n =?

N	p	n v p
T	T	T
T	**F**	**T**
F	T	T
F	F	F

Therefore, if now is true and the past is false. Now or past is true

Therefore p = false, I = true therefore n = true.

Question 3"Find out whether the following compound proposition is satisfiable or not, and prove your answer"

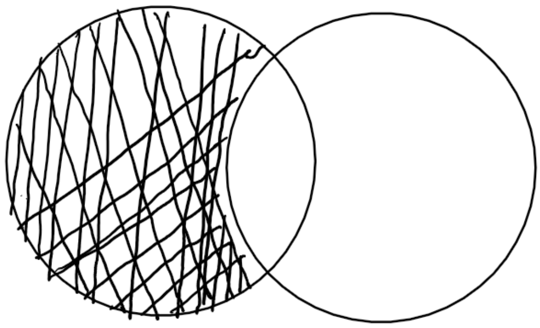
(p∨q∨¬q)∧(p∨¬p)∧(¬p∨¬q)∧(q∨¬q)

P	q	¬p	¬q	(p v q)	(p v ¬p)	(¬p v ¬q)	(q v ¬q)	((p v q) v ¬q)	((p ∨ q) ∨ ¬q) ∧ (p v ¬p))	(((p ∨ q) ∨ ¬q) ∧ (p v ¬p) ∧ (¬p v ¬q))	(((((p ∨ q) v ¬q) ∧ (p v ¬p)) ∧ (¬p v ¬q)) ∧ (q v ¬q))
T	T	F	F	T	T	F	T	T	T	F	F
T	F	F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T	T	T	T	T

Based on the proposition's final outcomes there at least one truth therefore making the proposition satisfiable.

Question 4 Suppose P(x) and Q(x) are propositional functions and D is their domain. Let A={x∈D:P(x) is true} and B={x∈D:Q(x) is true}.

(a) Draw a Venn Diagram of this situation and shade in A\B. (Alternatively, describe such a drawing in words.)



b) Given an x∈A\B, what can be said about the truth value of Q(x)?

Q(x) is false because B is true, and it is outside of B.

c) Given x∈A\B, what is the truth value of P(x)∩¬Q(x)?

The truth values of P(x)∩¬Q(x) is:

p(x)	Q(x)	¬Q(x)	P(x) ^ ¬Q(x)
T	F	T	T

P(x)∩¬Q(x) final outcome value is true.

Question 5 Prove the following by contradiction.

(a) There are no integers a and b for which 15a+ 20b= 1.

Proving the following by contradiction.

Given 15a + 20b = 1, Divide both sides by 5 making it

$\frac{15a+20b}{5} = \frac{1}{5}$ therefore becoming $3a + 4b = \frac{1}{5}$

Therefore, if a is an integer and b is an integer 3a+4b must be an integer because an integer plus an integer is an integer and the final is a fraction therefore contradiction.

(b) There does not exist a smallest positive rational number

Assume the smallest possible rational number exists and it is known an x. If you divide x by 2 making it $\frac{x}{2}$

$\frac{x}{2}$ is less than x therefore contradiction

Question 6 Prove by induction that for every positive integer n,

12+ 22+⋯+n2=n(n+ 1)(2n+ 1)6

Given $1^2 + 2^2 + \cdots + 2^2 = \frac{n(n+1)(2n+1)}{6}$

Base case:

1^2= 1 -> 1(1+1)(2x1 + 1) = $\frac{2x3}{6} = \frac{6}{6} = 1$ therefore this is true.

Induction hypothesis:

Assume that integer n = k is true.

Induction Step:

$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

Prove true for n = k+1

therefore the left side becomes And right side becomes

$$(1^2 + 2^2 + \cdots + k^2) + (k+1) = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{k(k+1)(2k+1) + (k+1)^2}{6} = \frac{(k^2 + 2k + k + 2)(2k+3)}{6}$$

$$\frac{(k^2 + k)(2k+1) + 6(k^2 + 2k + 1)}{6} = \frac{(2k^3 + 3k^2 + 4k^2 + 6k + 2k^2 + 3k + 4k + 6)}{6}$$

$$\frac{2k^3 + k^2 + 2k^2 + k + 6k^2 + 12k + 6}{6} = \frac{2k^3 + 9k^2 + 13k + 6}{6}$$

$$\frac{2k^3 + 9k^2 + 13k + 6}{6} = \frac{2k^3 + 9k^2 + 13k + 6}{6}$$

Therefore, left-hand side is equal to the right-hand side. the inductive step proves the statement true for every positive integer n.

Question 7 Let S={1,2,3,4}

a) **Write out the Cartesian Product S×S.**
Given set S = {1,2,3,4} The Cartesian product of set S is:
{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)}

b) **Let R be the binary relation on S defined by**
aRb if and only if a divides b.
i) How many elements are there in the relation R?

aRb => $\frac{b}{a}$ = integer the following are those elements

$$\frac{1}{1} = 1,\ \frac{2}{1} = 2,\ \frac{3}{1} = 3,\ \frac{4}{1} = 4,\ \frac{2}{2} = 1\,,\ \frac{3}{3} = 1,\ \frac{4}{4} = 1,\ \frac{4}{2} = 2$$

R=(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)

Therefore, there is 8 elements in the relation.

ii) Represent this relation by its adjacency matrix.

$$\begin{array}{c c} & \begin{array}{cccc} A\ 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} B \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{array}{c} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \end{array}$$

iii) Which of the following properties applies to this relation? Is it reflexive, symmetric, anti-symmetric, transitive?

the following properties that apply are

Anti-symmetric, Transitive and reflexive.

iv). Is R a POSET on S? If yes, draw or describe its Hasse diagram.

No this relation is not POSET.

Question 8 Recall that Z={...,−2,−1,0,1,2, ...} is the set of integers. Define relation aRb to hold if and only if a-b= 2k for some k ∈ Z

a) Show that R is an equivalence relation on Z.
To prove R is an equivalence relation we need to prove R is reflexive, symmetric and transitive.

Reflexive) if for example of set z = {…-2,-1,0,1…} and the cartesian product is {(-2,-2),(-2,-1),(-2,0)(-2,1),(-1,-2),(-1,-1),(-1,0),(-1,1),(0,-2),(0,-1),(0,0),(0,1),(1-2),(1-1),(1,0),(1,1)}
It contains (-2,-2), (-1,-1) (-0,-0) (1,1) (2,2) there for it is reflexive.

Symmetric) To prove symmetry, let a and b be integers. Suppose that a ≡ b (mod n), suppose a-b = kn where k is an integer.

B-a = -(a-b) which = - (kn) = (-k)n therefore, n divides b-a and b ≡ a (mod n). for this example it has
(-2,-1) and (-1,-2) therefore it is symmetric.

Transitive) a,b ε z, if aRb and bRc then a-b and b-c are integers, therefore the sum (a-b)+(b-c) = a-c is also an integer and therefore, aRc transitive by default.
Therefore, R is a equivalence relation of z.

b) Explicitly Describe the equivalence class
The equivalence class of z2 will be the all of the numbers in the set that have a remainder of zero or 1 after completing the modular arithmetic fitting into the class of [0] and [1]

c) Recall [a] :={x:x Ra}. What is [0]? What is [1]? What is [17]? Are any of these the same as each other?
[0] = {…-4, -2, 0, 2, 4…}
[1] = {…-3, -1, 1, 3, 5, …}
[17] = {…-17, -15, -13, -11, -9, -7, -5, -3, -1, 1, 3, 5, 7, 9…}
[1] and [17] are the same.

d) Choose convenient names for the equivalence classes of R and show that they form a partition of z
[0] = {… -6, -4, -2, 0, 2, 4, 6…}
[1] = {… -5, -3, -1, 1, 3, 5, 7…}
[2] = {… -4, -4, 0, 2, 5, 6, 8…}
[3] = {… -3, -2, 1, 3, 6, 7, 9…}
[4] = {… -2, -1, 2, 4, 7, 8, 10…}
[5] = {… -1, 0, 3, 5, 8, 9, 11…}
Class [0], [2], and [4] are equal and classes [1], [3] and [5] are equal therefore set [0] plus [1] = z

e) Define [a] + [b] ={x+y:xRa and yRb}

i) What is [0] + [0]?

[0] +[0] = [0]

	……	**-6**	**-4**	**-2**	**0**	**2**	**4**	**6**	**……**
…..	……	……	……	……	……	……	……	……	……
-6	……	-12	-10	-8	-6	-4	-2	0	……
-4	……	-10	-8	-6	-4	-2	0	2	……
-2	……	-8	-6	-4	-2	0	2	4	……
0	……	-6	-4	-2	0	2	4	6	……
2	……	-4	-2	0	2	4	6	8	……
4	……	-2	0	2	4	6	8	10	……
6	……	0	2	4	6	8	10	12	……
……	……	……	……	……	……	……	……	……	……

ii) What is [0] + [1]?

[0] +[1] = [1]

	……	**-5**	**-3**	**-1**	**1**	**3**	**5**	**7**	**……**
…..	……	……	……	……	……	……	……	……	……
-6	……	-11	-9	-7	-5	-3	-1	1	……
-4	……	-9	-7	-5	-3	-1	1	3	……
-2	……	-7	-5	-3	-1	1	3	5	……
0	……	-5	-3	-1	1	3	5	7	……
2	……	-3	-1	1	3	5	7	9	……
4	……	-1	1	3	5	7	9	11	……
6	……	1	3	5	7	9	11	13	……
……	……	……	……	……	……	……	……	……	……

iii. What is [1] + [1]?

[1] +[1] = [2]

	……	**-5**	**-3**	**-1**	**1**	**3**	**5**	**7**	**……**
…..	……	……	……	……	……	……	……	……	……
-5	……	-10	-8	-6	-4	-2	0	2	……
-3	……	-8	-6	-4	-2	0	2	4	……
-1	……	-6	-4	-2	0	2	4	6	……
1	……	-4	-2	0	2	4	6	8	……
3	……	-2	0	2	4	6	8	10	……
5	……	0	2	4	6	8	10	12	……
7	……	2	4	6	8	10	12	14	……
……	……	……	……	……	……	……	……	……	……

iv) Write out the addition table for the equivalence classes.

Addition	[0]	[1]
[0]	[0]	[1]
[1]	[1]	[0]

f) Think back to logic. The addition table you have just written may remind you of the table for one of the binary logical operations. Which one?

The binary logical operator used in XOR

Question 9 (a) Define relation F to hold if and only if a−b= 5k for some k∈Z

The equivalence classes of this relation can be labelled [0],[1],[2],[3] and [4]. The set of the equivalence classes, together with operations of addition and multiplication defined by the rules:

[a] + [b] :={x+y:xRa and yRb}, and [a][b] :={x y:xRa and yRb}

form a new mathematical structure called Z5. Figure out the addition and multiplication tables for Z5.

Equivalent classes are for z5 are as follows.

[0] = {…, -10, -5, 0, 5, 10, …} = {5k: k ε z}

[1] = {…., -9, -4, 1, 6, 11…,} = {5k+1: k ε z}

[2] = {…., -8, -3, 2, 7 12…,} = {5k+2: k ε z}

[3] = {…., -7, -2, 3, 8, 13…,} = {5k+3: k ε z}

[4] = {…., -6, -1, 4, 9, 14…,} = {5k+4: k ε z}

Below is the Addition table for z5

Addition	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

Below is the Multiplication table for z5

Multiplication	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]
[2]	[0]	[2]	[4]	[1]	[3]
[3]	[0]	[3]	[1]	[4]	[2]
[4]	[0]	[4]	[3]	[2]	[1]

b) this equivalence relation produces a mathematical structure called Z6. Figure out the addition and multiplication tables for Z6.

Equivalent classes for z6

[0] = {…, -10, -5, 0, 5, 10, …}

[1] = {…., -9, -4, 1, 6, 11, …}

[2] = {…., -8, -3, 2, 7 12, …}

[3] = {…., -7, -2, 3, 8, 13, …}

[4] = {…., -6, -1, 4, 9, 14, …}

[5] = {…., -5, -0, 5, 10, 15, …}

Below is the Addition table for z6

Addition	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

Below is the Multiplication table for z6

Multiplication	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]
[2]	[0]	[2]	[4]	[0]	[2]	[4]
[3]	[0]	[3]	[0]	[3]	[0]	[3]
[4]	[0]	[4]	[2]	[0]	[4]	[2]
[5]	[0]	[5]	[4]	[3]	[2]	[1]