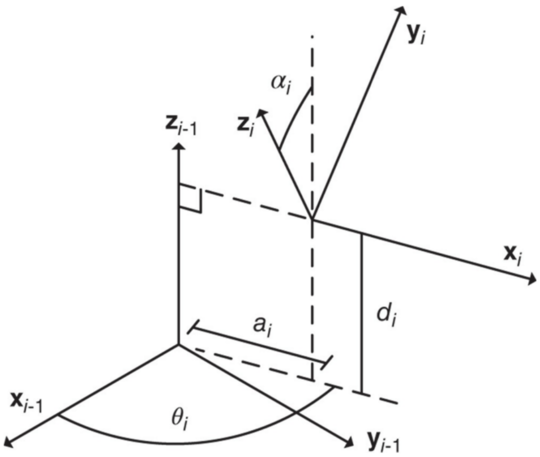
Project 1



Theorem 3.2 (DH convention homogeneous transformation)

Suppose that frames $i-1$ and i are two consecutive frames in a kinematic chain that satisfies the assumptions of the DH convention. The homogeneous transformation that relates the homogeneous coordinates in the frames $i-1$ and i is given by

$$\mathbf{H}_i^{i-1} = \begin{bmatrix} \mathbf{R}_i^{i-1} & \mathbf{d}_{i-1,i}^{i-1} \\ 0 & 1 \end{bmatrix},$$

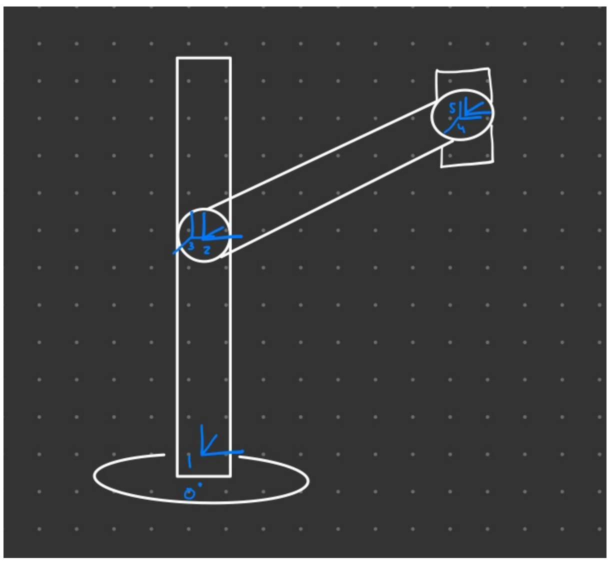
with the rotation matrix $\mathbf{R}_i^{i-1}$ defined as

$$\mathbf{R}_i^{i-1} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix},$$

and the relative offset $\mathbf{d}_{i-1,i}$ given by

$$\mathbf{d}_{i-1,i}^{i-1} = \begin{Bmatrix} a_i\cos\theta_i \\ a_i\sin\theta_i \\ d_i \end{Bmatrix}.$$

The parameters θ_i, α_i, d_i , and a_i are the rotation, twist, displacement, and offset of link i , respectively.



This is a robot that could be used for pick and place, e.g. in sorting tasks. The design is inspired by a robot I competed against in a robotics competition. It has an arm mounted on an elevator mounted on a turret. The arm has a 2-dof wrist.

Joint Variables:
θ_1: Turret rotation
$d_{1,2}$: Elevator displacement
θ_3: Elbow pitch
θ_4: Wrist pitch
θ_5: Wrist yaw

Design Parameters:
l_{arm}: 0.5

Joint	Rotation θ	Twist α	Displacement d	Offset a

Joint	Rotation θ	Twist α	Displacement d	Offset a
1	θ_1	0	0	0
2	0	$\frac{\pi}{2}$	$d_{1,2}$	0
3	θ_3	0	0	l_{arm}
4	θ_4	$-\frac{\pi}{2}$	0	0
5	θ_5	0	0	0

Turret

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elevator

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Elbow

$$\begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & 0.5\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0.5\sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wrist

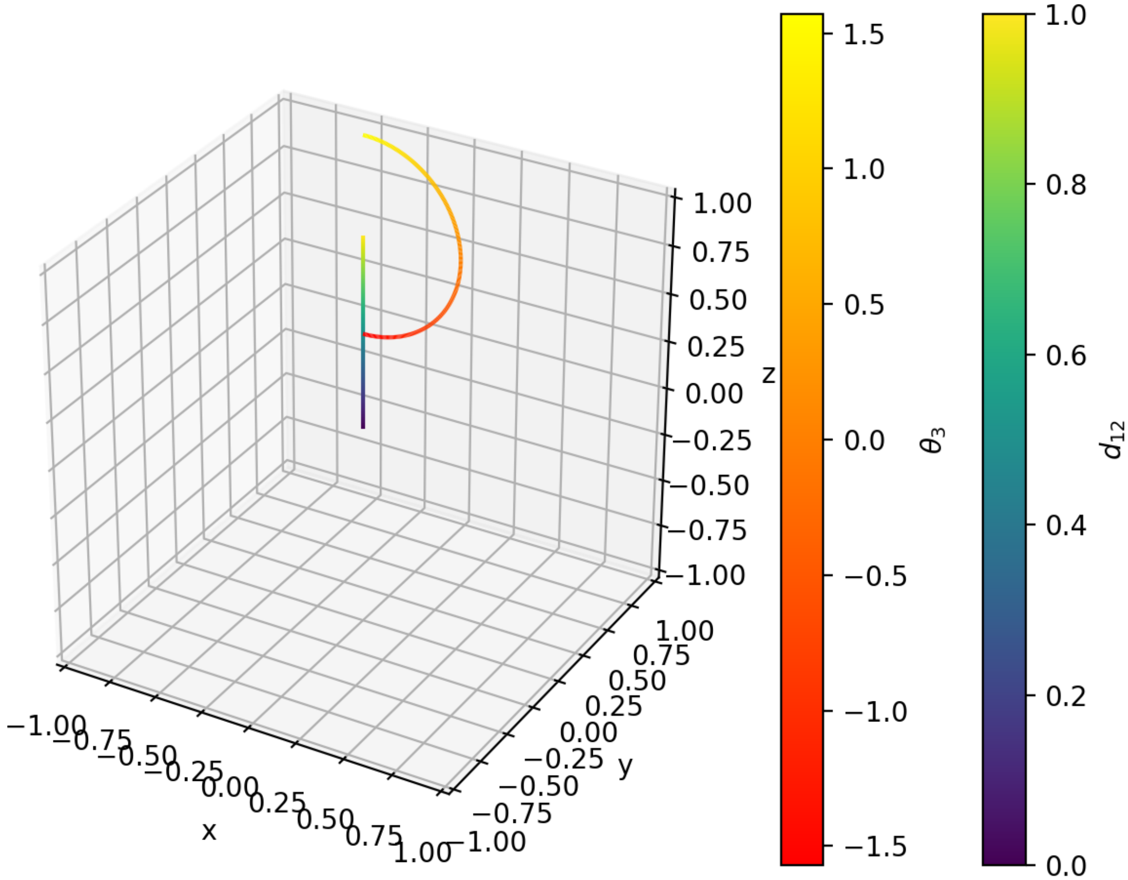
$$\begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & 0 \\ \sin(\theta_4) & \cos(\theta_4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Base-to-End-Effector

$$\begin{bmatrix} (-\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_3)\cos(\theta_4))\cos(\theta_5) - \sin(\theta_1)\sin(\theta_5) & -(-\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) + \cos \\ (-\sin(\theta_1)\sin(\theta_3)\sin(\theta_4) + \sin(\theta_1)\cos(\theta_3)\cos(\theta_4))\cos(\theta_5) + \sin(\theta_5)\cos(\theta_1) & -(-\sin(\theta_1)\sin(\theta_3)\sin(\theta_4) + \sin \\ (\sin(\theta_3)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_3))\cos(\theta_5) & -(\sin(\theta_3)\cos(\theta \\ 0 & \end{bmatrix}$$

Forward Kinematics Plots



Elbow position varied with elevator travel & wrist position varied with elbow angle plotted in the base coordinate frame

Inverse Kinematics

Desired state: (x, y, z, θ)

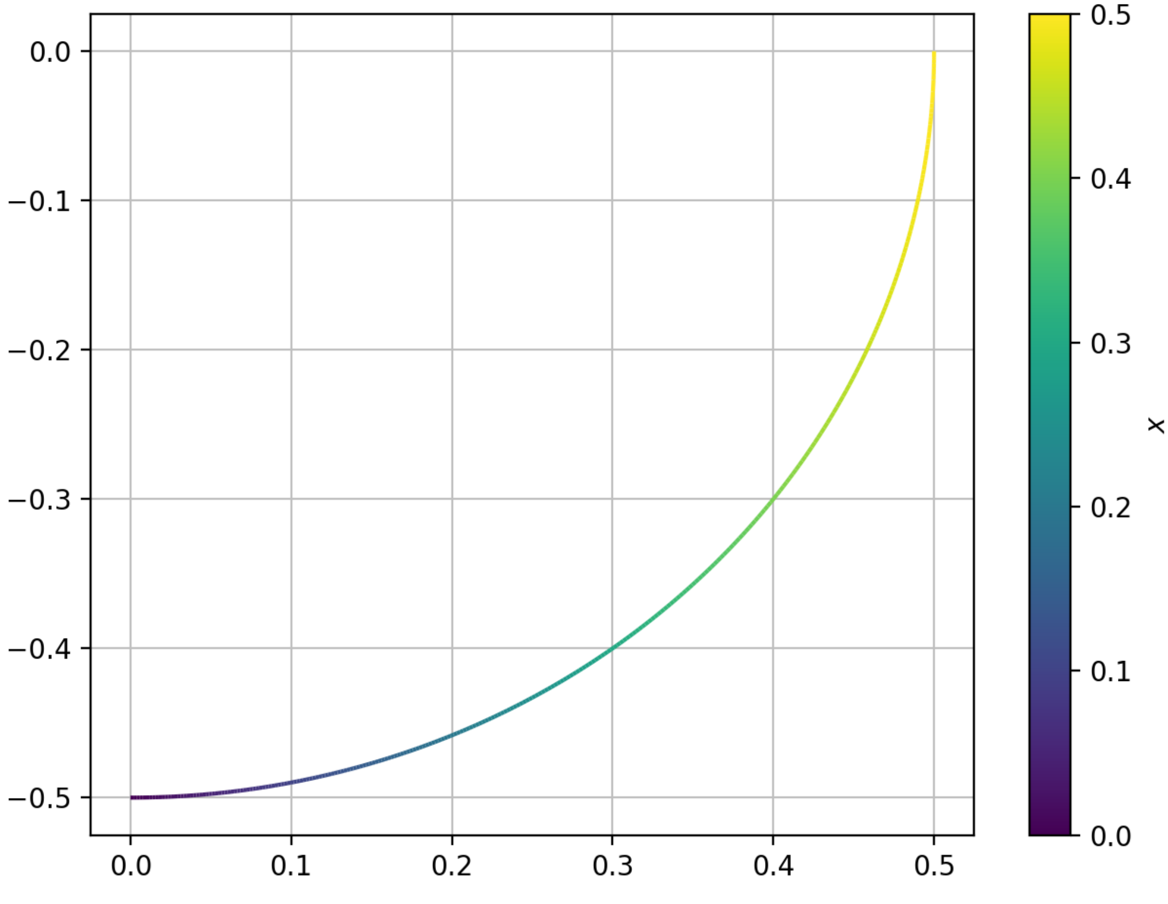
For θ_1, $d_{1,2}$, θ_3:

$$\left(6.28318530717959 - \operatorname{acos}\left(-\frac{x}{\sqrt{x^2+y^2}}\right),\ z - 1.0\sqrt{-x^2-y^2+0.25},\ \operatorname{acos}\left(-2.0\sqrt{x^2+y^2}\right)\right)$$

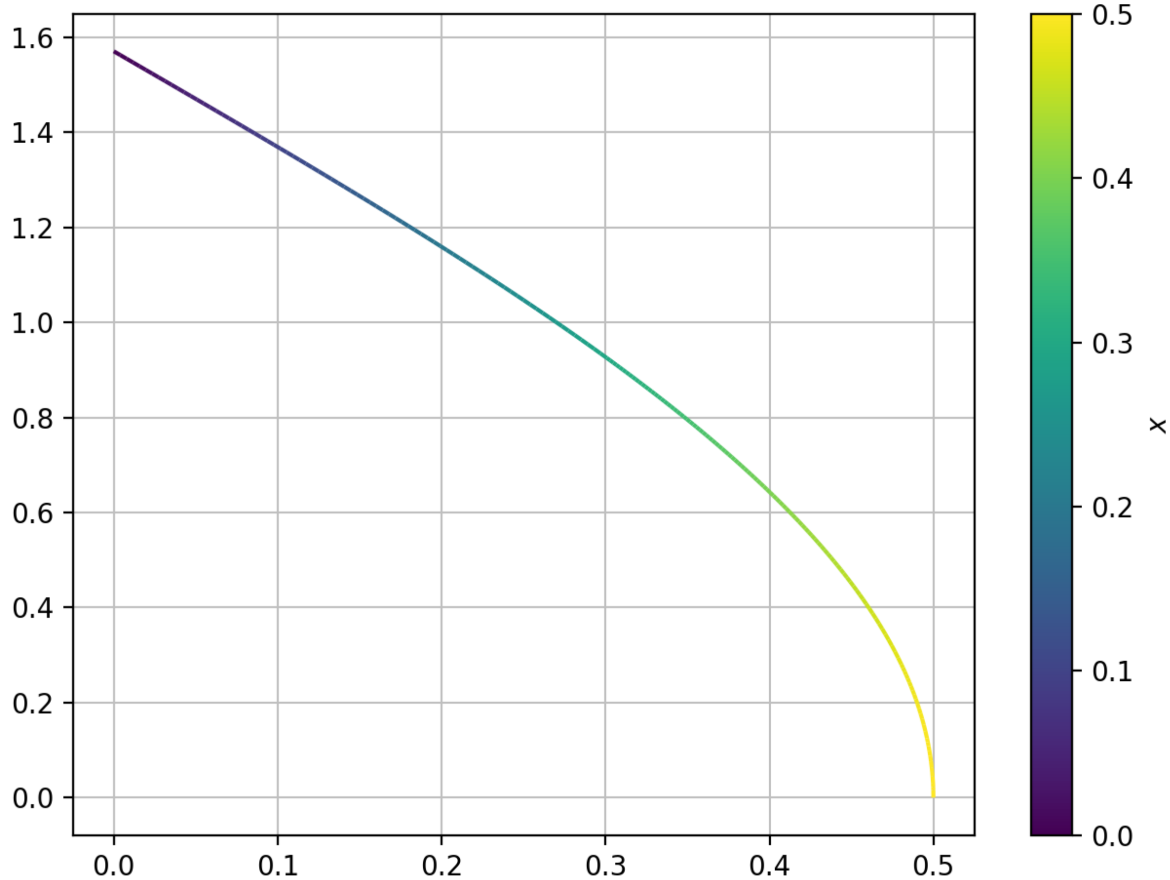
$$\left(\operatorname{acos}\left(\frac{x}{\sqrt{x^2+y^2}}\right),\ z - 1.0\sqrt{-x^2-y^2+0.25},\ \operatorname{acos}\left(2.0\sqrt{x^2+y^2}\right)\right)$$

For θ_4: $\theta_4 = -\theta_3$. This keeps the wrist level

For θ_5: $\theta_5 = \theta - \theta_1$



Elevator position (Y-axis) as end-effector X-position changes (X-axis).



Elbow position (Y-axis) as end effector X-position changes (X-axis)