ECE 4704: Principles of Robotic Systems (Spring 2023)
Homework 5
Due: April 21st, 11:59PM

April 14, 2023

Your submission must be your original work. Please follow the submission instructions posted on canvas exactly.

Problem 1 (50 points)**.** Derive the equations of motion for the manipulator shown in Figure 1 using the Lagrangian formulation. Assume that gravity acts vertically downwards. Note that $\tilde{I}_i$ in the diagram is the inertia matrix about the center of mass of link i. Also, note that the manipulator is composed of a revolute joint (θ_1) and a prismatic joint (d_2).

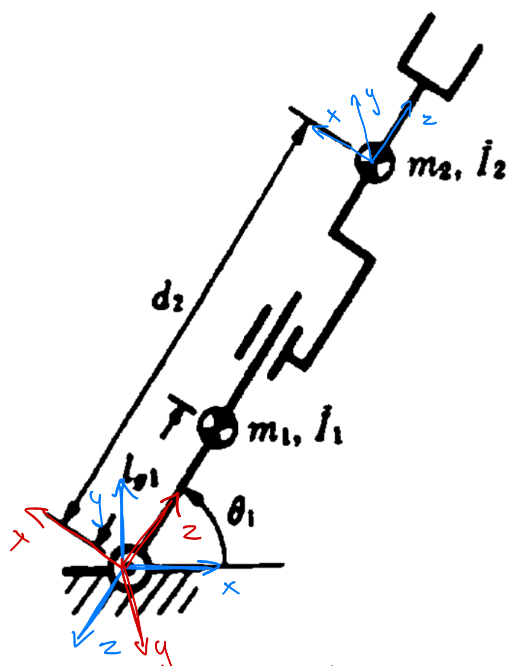

Figure 1: Manipulator for Problem 1.

Problem 2 (50 points)**.** Figure 2 depicts a manipulator with its end-effector in contact with a smooth surface. While applying a normal force f_N to the surface, the manipulator is moving at constant speed v_t along the tangential direction. Compute the required joint torques τ_1, τ_2 in the case shown in the figure. The length of each link is 1m and the mass of each link is 1kg. HINT: you need to consider both the statics and dynamics of the manipulator.

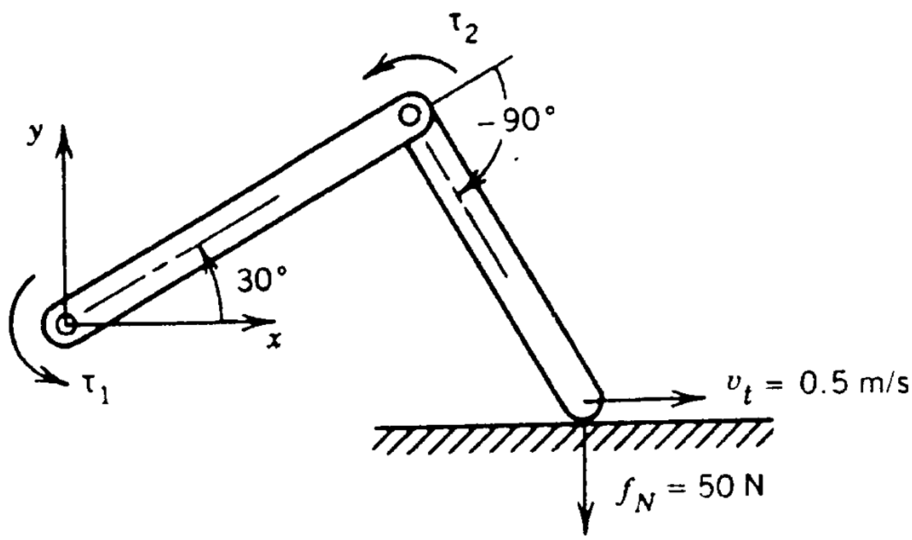

Figure 2: Manipulator for Problem 2.

	θ	α	d	a
1	$\pi/2 + \theta_1$	$\pi/2$	0	0
2	0	0	d_1	0

$$H_0^2 = \begin{bmatrix} -\sin(\theta_1) & 0 & \cos(\theta_1) & d_2\cos(\theta_1) \\ \cos(\theta_1) & 0 & \sin(\theta_1) & d_2\sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1\omega_1 = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$${}^2\omega_2 = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -d_2\sin(\theta_1) & \cos(\theta_1) \\ d_2\cos(\theta_1) & \sin(\theta_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$${}^1v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$${}^2v_2 = \begin{bmatrix} d_2\dot{\theta}_1 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$${}^1v_{c_1} = \begin{bmatrix} l_{c1}\dot{\theta}_1 \\ 0 \\ 0 \end{bmatrix}$$

$${}^2v_{c_2} = \begin{bmatrix} d_2\dot{\theta}_1 \\ 0 \\ \dot{d}_2 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 0.5I_1\dot{\theta}_1^2 + 0.5l_{c1}^2 m_1\dot{\theta}_1^2 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0.5I_2\dot{\theta}_1^2 + 0.5m_2 d_2^2\dot{\theta}_1^2 + 0.5m_2\dot{d}_2^2 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} g l_{c1} m_1 \sin(\theta_1) \end{bmatrix}$$

$$V_2 = \begin{bmatrix} g m_2 d_2 \sin(\theta_1) \end{bmatrix}$$

$$\tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q_i} - \left[{}^0J^T\, {}^0_eF_{ext}\right] =$$

$$\begin{bmatrix} 1.0I_1\ddot{\theta}_1 + 1.0I_2\ddot{\theta}_1 + f_x d_2\sin(\theta_1) - f_y d_2\cos(\theta_1) + g l_{c1} m_1\cos(\theta_1) + g m_2 d_2\cos(\theta_1) - g_z + 1.0 l_{c1}^2 m_1\ddot{\theta}_1 + 1.0m_2 d_2^2\ddot{\theta}_1 + 2.0m_2 d_2\dot{\theta}_1\dot{d}_2 \\ -f_x\cos(\theta_1) - f_y\sin(\theta_1) + g m_2\sin(\theta_1) - 1.0m_2 d_2\dot{\theta}_1^2 + 1.0m_2\ddot{d}_2 \end{bmatrix}$$

Source code attached

$$J = \begin{bmatrix} -L_1 \sin(\theta_1) - L_2 \sin(\theta_1)\cos(\theta_2) - L_2 \sin(\theta_2)\cos(\theta_1) & -L_2 \sin(\theta_1)\cos(\theta_2) - L_2 \sin(\theta_2)\cos(\theta_1) \\ L_1 \cos(\theta_1) - L_2 \sin(\theta_1)\sin(\theta_2) + L_2 \cos(\theta_1)\cos(\theta_2) & -L_2 \sin(\theta_1)\sin(\theta_2) + L_2 \cos(\theta_1)\cos(\theta_2) \end{bmatrix}$$

$$J\begin{pmatrix} 30^\circ \\ -90^\circ \end{pmatrix}^{-1} \begin{bmatrix} 0 \\ -50 \end{bmatrix} = \begin{bmatrix} -0.25 \\ 0.683012701892219 \end{bmatrix}$$

$L =$

$$\left[0.5I_{c1}\dot{\theta}_1^2 + 0.5I_{c2}\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 + 0.5L_1^2 m_1 \sin^2(\theta_1)\dot{\theta}_1^2 - g\left(m_1\left(L_1 + L_{c1}\right)\sin(\theta_1) + m_2\left(L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_{c2}\sin(\theta_1 + \theta_2)\right)\right) + 0.5m_1\left(L_1\cos(\theta_1) + L_{c1}\right)^2 \dot{\theta}_1^2 + 0.5m_2\left(L_1 \sin(\theta_1 - \theta_2)\dot{\theta}_1 + L_2 \sin(\theta_2)\dot{\theta}_1 + L_2 \sin(\theta_2)\dot{\theta}_2\right)^2 + 0.5m_2\left(L_1 \cos(\theta_1 - \theta_2)\dot{\theta}_1 + L_2\cos(\theta_2)\dot{\theta}_1 + L_2 \cos(\theta_2)\dot{\theta}_2 + L_{c2}\dot{\theta}_1 + L_{c2}\dot{\theta}_2\right)^2\right]$$

$$T = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q_i} - \left[{}^{0}J^{T}\,{}^{0}_{e}F_{ext}\right] = \begin{bmatrix} 97.1114846863766 + 0 \\ 32.5991265877365 + 0 \end{bmatrix}$$